គណិតវិទ្យា និង វិទ្យាសាស្ត្រ
MATHEMATICS & SCIENCE

លីមីតនៃអនុគមន៍

សម្រាប់សិស្សានុសិស្សថ្នាក់ទី១២

សិក្សាបន្ថែមនៅផ្ទះក្នុងអំឡុងពេលពិភពលោក

រងគ្រោះដោយជំងឺ COVID-19

រៀបរៀងដោយ ៖ ពុធ ឫទ្ធី

ខែឧសភា ឆ្នាំ ២០២០

លីមីតនៃអនុគមន៍

១. គេឱ្យអនុគមន៍ $f(x)=\dfrac{4x^2+kx+7k-6}{2x^2-5x-3}$ ។ កំណត់តម្លៃ k ដើម្បីឱ្យ $\lim\limits_{x\to3} f(x)$ ជាចំនួនពិត ។

ចម្លើយ

កំណត់តម្លៃ k ដើម្បីឱ្យ $\lim\limits_{x\to3} f(x)$ ជាចំនួនពិត

របៀបទី១

តាង $p(x)=4x^2+kx+7k-6$

$q(x)=2x^2-5x-3=(2x^2-6x)+(x-3)=(x-3)(2x+1)$

ដោយ $\lim\limits_{x\to3} f(x)=\lim\limits_{x\to3}\dfrac{4x^2+kx+7k-6}{2x^2-5x-3}=\dfrac{30+10k}{0}$

គេបានលីមីត $\lim\limits_{x\to3} f(x)$ ជាចំនួនពិតកាលណាមានរាង $\dfrac{0}{0}$

នាំឱ្យ $p(x)=0 \Leftrightarrow 30+10k=0 \Rightarrow k=-\dfrac{30}{10}=-3$

ដូចនេះ លីមីត $\lim\limits_{x\to3} f(x)$ ជាចំនួនពិតកាលណា $\boxed{k=-3}$ ។

របៀបទី២

តាង $p(x)=4x^2+kx+7k-6$

$q(x)=2x^2-5x-3=(2x^2-6x)+(x-3)=(x-3)(2x+1)$

ធ្វើប្រមាណវិធីចែកនៃពហុធា $p(x)$ នឹង $x-3$

$$\begin{array}{r|l} & 4x+(k+12) \\ x-3 & x^2 \quad +kx \qquad +(7k-6) \\ & -\ \underline{(4x^2-12x)} \\ & \qquad (k+12)x \quad +(7k-6) \\ & -\ \underline{[(k+12)x\ -3(k+12)]} \\ & \qquad 10k+30 \end{array}$$

គេបានលីមីត $\lim_{x\to 3} f(x)$ ជាចំនួនពិតកាលណាពហុធា $p(x)$

ចែកដាច់នឹង $x-3$

ឬពហុធា $p(x)$ ចែកនឹង $x-3$ បានសំណល់សូន្យ

គេបាន $10k+30=0 \Rightarrow k=-3$

ដូចនេះ លីមីត $\lim_{x\to 3} f(x)$ ជាចំនួនពិតកាលណា $\boxed{k=-3}$ ។

២. រកគ្រប់អាស៊ីមតូតនៃអនុគមន៍ $f(x)=\dfrac{2x+1}{x^2-2x-8}$ ។

<u>ចម្លើយ</u>

រកគ្រប់អាស៊ីមតូតនៃអនុគមន៍ $f(x)$

អនុគមន៍ $x^2-2x-8\neq 0 \Leftrightarrow (x+2)(x-4)\neq 0 \Rightarrow \begin{cases} x\neq -2 \\ x\neq 4 \end{cases}$

គេបានអនុគមន៍ $f(x)$ មានដែនកំណត់ $D=\mathbb{R}\setminus\{-2,4\}$

ឬ $D=(-\infty,-2)\cup(-2,4)\cup(4,+\infty)$

គណនាលីមីតចុងដែននៃ $f(x)$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{2x+1}{x^2-2x-8} = \lim_{x\to+\infty} \frac{x\left(2+\frac{1}{x}\right)}{x^2\left(1-\frac{2}{x}-\frac{8}{x^2}\right)}$$

$$= \lim_{x\to+\infty} \frac{2+\frac{1}{x}}{x\left(1-\frac{2}{x}-\frac{8}{x^2}\right)} = 0$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{2x+1}{x^2-2x-8} = \lim_{x\to-\infty} \frac{x\left(2+\frac{1}{x}\right)}{x^2\left(1-\frac{2}{x}-\frac{8}{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{2+\frac{1}{x}}{x\left(1-\frac{2}{x}-\frac{8}{x^2}\right)} = 0$$

គេបាន បន្ទាត់ $d_1 : y = 0$ ជាអាស៊ីមតូតដេកនៃអនុគមន៍

$$\lim_{x\to-2} f(x) = \lim_{x\to-2} \frac{2x+1}{x^2-2x-8} = \frac{-3}{0} = \pm\infty$$

នាំឱ្យ បន្ទាត់ $d_2 : x = -2$ ជាអាស៊ីមតូតឈរនៃអនុគមន៍

$$\lim_{x\to4} f(x) = \lim_{x\to4} \frac{2x+1}{x^2-2x-8} = \frac{9}{0} = \pm\infty$$

នាំឱ្យ បន្ទាត់ $d_3 : x = 4$ ជាអាស៊ីមតូតឈរនៃអនុគមន៍

ដូចនេះ អនុគមន៍ $f(x)$ មានអាស៊ីមតូតបីគឺ

$$\boxed{d_1 : y = 0, d_2 : x = -2, d_3 : x = 4}$$ ។

៣. រកដែនកំណត់នៃ x ដែលធ្វើឱ្យអនុគមន៍

$$f(x)=\frac{1}{x}-\sqrt{\frac{x+6}{x^2+1}}+(3x^2+5)+\sin x$$ ជាប់ ។

ចម្លើយ

រកដែនកំណត់នៃ x ដែលធ្វើឱ្យអនុគមន៍ជាប់

តាង $p(x)=\frac{1}{x},q(x)=\sqrt{\frac{x+6}{x^2+1}},r(x)=3x^2+5,t(x)=\sin x$

ដែនកំណត់ដែលឱ្យអនុគមន៍ $f(x)$ ជាប់ជាប្រសព្វនៃដែនកំណត់ដែលធ្វើឱ្យអនុគមន៍ p,q,r,t ជាប់

អនុគមន៍ $p(x)$ ជាប់ជានិច្ចចំពោះ $x\in D_1=\mathbb{R}\setminus\{0\}$

អនុគមន៍ $p(x)$ ជាប់ជានិច្ចចំពោះ $x\in D_2\left\{x\mid\frac{x+6}{x^2+1}\geq 0,\ x^2+1\neq 0\right\}$

តែ $\forall x\in\mathbb{R},x^2+1>0$ គេបាន $\frac{x+6}{x^2+1}\geq 0\Leftrightarrow x+6\geq 0$

$\Leftrightarrow x\geq -6$

គេបាន $D_2=[-6,+\infty)$

អនុគមន៍ $p(x)$ និង $t(x)$ ជាប់ជានិច្ចចំពោះ $x\in D_3=\mathbb{R}$

គេបានអនុគមន៍ $f(x)$ ជាប់លើចន្លោះ $D=D_1\cap D_2\cap D_3$

$=[-6,+\infty)-\{0\}$

ដូចនេះ អនុគមន៍ $f(x)$ ជាប់លើចន្លោះ $\boxed{D=[-6,+\infty)-\{0\}}$ ។

៤. គណនាលីមីត

ក. $\lim_{x\to 0} x^2\cos\frac{1}{x}$ ខ. $\lim_{x\to 0}\sqrt{\sqrt{x}+x^3}\sin\frac{\pi}{x}$

ចម្លើយ

គណនាលីមីត

ក. $\lim_{x\to 0} x^2\cos\frac{1}{x}$

ចំពោះ $\forall x\neq 0$ គេមាន $-1\le\cos\frac{1}{x}\le 1$

$$-x^2\le x^2\cos\frac{1}{x}\le x^2 \text{ (ព្រោះ } x^2\ge 0\text{)}$$

$$\lim_{x\to 0}\left(-x^2\right)\le\lim_{x\to 0} x^2\cos\frac{1}{x}\le\lim_{x\to 0} x^2$$

$$0\le\lim_{x\to 0} x^2\cos\frac{1}{x}\le 0$$

តាមលីមីតតាមការប្រៀបធៀបគេបាន $\lim_{x\to 0} x^2\cos\frac{1}{x}=0$

ដូចនេះ $\boxed{\lim_{x\to 0} x^2\cos\frac{1}{x}=0}$ ។

ខ. $\lim_{x\to 0}\sqrt{\sqrt{x}+x^3}\sin\frac{\pi}{x}$

ចំពោះ $\forall x\neq 0$ គេមាន $-1\le\sin\frac{\pi}{x}\le 1$

ដោយ $\sqrt{\sqrt{x}+x^3}\ge 0$ គេបាន

$$-\sqrt{\sqrt{x}+x^3}\le\sqrt{\sqrt{x}+x^3}\sin\frac{\pi}{x}\le\sqrt{\sqrt{x}+x^3}$$

$$\lim_{x\to 0}\left(-\sqrt{\sqrt{x}+x^3}\right)\le \lim_{x\to 0}\sqrt{\sqrt{x}+x^3}\sin\frac{\pi}{x}\le \lim_{x\to 0}\sqrt{\sqrt{x}+x^3}$$

$$0\le \lim_{x\to 0}\sqrt{\sqrt{x}+x^3}\sin\frac{\pi}{x}\le 0$$

តាមលីមីតតាមការប្រៀបធៀបគេបាន $\lim_{x\to 0}\sqrt{\sqrt{x}+x^3}\sin\frac{\pi}{x}=0$

ដូចនេះ $\boxed{\lim_{x\to 0}\sqrt{\sqrt{x}+x^3}\sin\frac{\pi}{x}=0}$ ។

៥. គណនាលីមីត

ក. $\lim_{x\to 1}\frac{x^2-1}{|x-1|}$ ខ. $\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x^2}-1}$

គ. $\lim_{x\to \pm\infty}\frac{x^2+5x+7}{|x+2|}$ ឃ. $\lim_{p\to 3}\frac{|p^2-7p|}{p^2+1}$

<u>ចម្លើយ</u>

គណនាលីមីត

ក. $\lim_{x\to 1}\frac{x^2-1}{|x-1|}$

គេមាន $|x-1|=\begin{cases} x-1 & \text{បើ } x\ge 1 \\ -(x-1) & \text{បើ } x<1 \end{cases}$

នាំឱ្យ $\lim_{x\to 1}\frac{x^2-1}{|x-1|}=\begin{cases} \lim_{x\to 1^-}\frac{x^2-1}{-(x-1)} & \text{បើ } x\ge 1 \\ \lim_{x\to 1^+}\frac{x^2-1}{x-1} & \text{បើ } x<1 \end{cases}$

$$= \begin{cases} \lim\limits_{x\to1^-} \dfrac{(x-1)(x+1)}{-(x-1)} & \text{បើ } x\ge 1 \\ \lim\limits_{x\to1^+} \dfrac{(x-1)(x+1)}{(x-1)} & \text{បើ } x<1 \end{cases}$$

$$= \begin{cases} -\lim\limits_{x\to1^-}(x+1) & \text{បើ } x\ge 1 \\ \lim\limits_{x\to1^+}(x+1) & \text{បើ } x<1 \end{cases}$$

$$= \begin{cases} -2 & \text{បើ } x\ge 1 \\ 2 & \text{បើ } x<1 \end{cases}$$

ដូចនេះ $\boxed{\lim\limits_{x\to1^-}\dfrac{x^2-1}{|x-1|}=-2, \lim\limits_{x\to1^+}\dfrac{x^2-1}{|x-1|}=2}$ ។

ខ. $$\lim_{x\to1}\frac{\sqrt[3]{x}-1}{\sqrt{x^2}-1}=\lim_{x\to1}\frac{\sqrt[3]{x}-1}{|x|-1}$$

$$=\lim_{x\to1}\frac{\left(\sqrt[3]{x}-1\right)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}{\left(|x|-1\right)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}$$

$$=\lim_{x\to1}\frac{x-1}{(x-1)\left(\sqrt[3]{x^2}+\sqrt[2]{x}+1\right)}$$

$$=\lim_{x\to1}\frac{1}{\sqrt[3]{x^2}+\sqrt[2]{x}+1}$$

$$=\frac{1}{3}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x^2}-1}=\frac{1}{3}}$ ។

គ. $\lim_{x\to\pm\infty}\frac{x^2+5x+7}{|x+2|}=\left[\begin{array}{l}\lim_{x\to+\infty}\frac{x^2+5x+7}{x+2}\\ \lim_{x\to-\infty}\frac{x^2+5x+7}{-(x+2)}\end{array}\right.$

$$=\left[\begin{array}{l}\lim_{x\to+\infty}\frac{x^2\left(1+\frac{5}{x}+\frac{7}{x^2}\right)}{x\left(1+\frac{2}{x}\right)}\\ \lim_{x\to-\infty}\frac{x^2\left(1+\frac{5}{x}+\frac{7}{x^2}\right)}{-x\left(1+\frac{2}{x}\right)}\end{array}\right.$$

$$=\left[\begin{array}{l}\lim_{x\to+\infty}\frac{x\left(1+\frac{5}{x}+\frac{7}{x^2}\right)}{1+\frac{2}{x}}\\ \lim_{x\to-\infty}\frac{x\left(1+\frac{5}{x}+\frac{7}{x^2}\right)}{-\left(1+\frac{2}{x}\right)}\end{array}\right.$$

$$=\left[\begin{array}{l}+\infty\\ +\infty\end{array}\right.$$

ដូចនេះ $\boxed{\lim_{x\to\pm\infty}\frac{x^2+5x+7}{|x+2|}=+\infty}$ ។

ឃ. $\lim_{p\to 3}\frac{|p^2-7p|}{p^2+1}=\frac{|9-21|}{9+1}=\frac{12}{10}=\frac{6}{5}$

ដូចនេះ $\boxed{\lim_{p\to 3}\frac{|p^2-7p|}{p^2+1}=\frac{6}{5}}$ ។

៦. គណនាលីមីត

ក. $\lim_{x\to 5}\frac{5x-x^2}{x-5}$

ខ. $\lim_{x\to 1}\frac{x-1}{x^2+x-2}$

គ. $\lim_{x\to 1}\frac{x^4-1}{x^6-1}$

ឃ. $\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}$

<u>ចម្លើយ</u>

គណនាលីមីត

ក. $\lim_{x\to 5}\frac{5x-x^2}{x-5}=\lim_{x\to 5}\frac{-x(x-5)}{(x-5)}=\lim_{x\to 5}-x=-5$

ដូចនេះ $\boxed{\lim_{x\to 5}\frac{5x-x^2}{x-5}=-5}$ ។

ខ. $\lim_{x\to 1}\frac{x-1}{x^2+x-2}=\lim_{x\to 1}\frac{x-1}{(x^2-x)+(2x-2)}$

$=\lim_{x\to 1}\frac{x-1}{x(x-1)+2(x-1)}$

$=\lim_{x\to 1}\frac{x-1}{(x-1)(x+2)}$

$=\lim_{x\to 1}\frac{1}{x+2}$

$$=\frac{1}{3}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{x-1}{x^2+x-2}=\frac{1}{3}}$ ។

គ. $\lim_{x\to 1}\frac{x^4-1}{x^6-1}=\lim_{x\to 1}\frac{(x-1)(x^3+x^2+x+1)}{(x-1)(x^5+x^4+x^3+x^2+x+1)}$

$$=\lim_{x\to 1}\frac{x^3+x^2+x+1}{x^5+x^4+x^3+x^2+x+1}$$

$$=\frac{1^3+1^2+1+1}{1^5+1^4+1^3+1^2+1+1}$$

$$=\frac{4}{6}=\frac{2}{3}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{x^4-1}{x^6-1}=\frac{2}{3}}$ ។

ឃ. $\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}=\lim_{x\to 1}\frac{(\sqrt[3]{x}-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(x-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}$

$$=\lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}$$

$$=\lim_{x\to 1}\frac{1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

$$=\frac{1}{3}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}=\frac{1}{3}}$ ។

៧. គណនាលីមីត

ក. $\lim_{x\to 4}\frac{\sqrt{5+x}-3}{x-4}$

ខ. $\lim_{x\to 0}\frac{\sqrt{5-x}-\sqrt{5}}{x}$

គ. $\lim_{x\to -3}\frac{\sqrt{1+\sqrt{4+x}}-\sqrt{2}}{x+3}$

ឃ. $\lim_{x\to 6}\frac{\sqrt[3]{2+x}-2}{x-6}$

ង. $\lim_{x\to 1}\frac{\sqrt{x-1}+\sqrt{x}-1}{\sqrt{x^2-1}}$

ច. $\lim_{x\to 4}\frac{\sqrt{x-2\sqrt{x}+1}-1}{\sqrt{x}-2}$

<u>ចម្លើយ</u>

គណនាលីមីត

ក. $$\lim_{x\to 4}\frac{\sqrt{5+x}-3}{x-4}=\lim_{x\to 4}\frac{\left(\sqrt{5+x}-3\right)\left(\sqrt{5+x}+3\right)}{(x-4)\left(\sqrt{5+x}+3\right)}$$

$$=\lim_{x\to 4}\frac{5+x-9}{(x-4)\left(\sqrt{5+x}+3\right)}$$

$$=\lim_{x\to 4}\frac{x-4}{(x-4)\left(\sqrt{5+x}+3\right)}$$

$$=\lim_{x\to 4}\frac{1}{\sqrt{5+x}+3}$$

$$=\frac{1}{6}$$

ដូចនេះ $\boxed{\lim_{x\to 4}\frac{\sqrt{5+x}-3}{x-4}=\frac{1}{6}}$ ។

ខ. $\lim\limits_{x\to 0}\dfrac{\sqrt{5-x}-\sqrt{5}}{x}=\lim\limits_{x\to 0}\dfrac{\left(\sqrt{5-x}-\sqrt{5}\right)\left(\sqrt{5-x}+\sqrt{5}\right)}{x\left(\sqrt{5-x}+\sqrt{5}\right)}$

$$=\lim_{x\to 0}\frac{5-x-5}{x\left(\sqrt{5-x}+\sqrt{5}\right)}$$

$$=\lim_{x\to 0}\frac{-x}{x\left(\sqrt{5-x}+\sqrt{5}\right)}$$

$$=\lim_{x\to 0}\frac{-1}{\sqrt{5-x}+\sqrt{5}}$$

$$=-\frac{1}{2\sqrt{5}}$$

$$=-\frac{\sqrt{5}}{10}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\dfrac{\sqrt{5-x}-\sqrt{5}}{x}=-\dfrac{\sqrt{5}}{10}}$ ។

គ. $\lim\limits_{x\to -3}\dfrac{\sqrt{1+\sqrt{4+x}}-\sqrt{2}}{x+3}$

$$=\lim_{x\to -3}\frac{\left(\sqrt{1+\sqrt{4+x}}-\sqrt{2}\right)\left(\sqrt{1+\sqrt{4+x}}+\sqrt{2}\right)}{(x+3)\left(\sqrt{1+\sqrt{4+x}}+\sqrt{2}\right)}$$

$$=\lim_{x\to -3}\frac{1+\sqrt{4+x}-2}{(x+3)\left(\sqrt{1+\sqrt{4+x}}+\sqrt{2}\right)}$$

$$=\lim_{x\to -3}\frac{\sqrt{4+x}-1}{(x+3)\left(\sqrt{1+\sqrt{4+x}}+\sqrt{2}\right)}$$

$$=\lim_{x\to -3}\frac{\left(\sqrt{4+x}-1\right)\left(\sqrt{4+x}+1\right)}{(x+3)\left(\sqrt{1+\sqrt{4+x}}+\sqrt{2}\right)\left(\sqrt{4+x}+1\right)}$$

$$=\lim_{x\to -3}\frac{4+x-1}{(x+3)\left(\sqrt{1+\sqrt{4+x}}+\sqrt{2}\right)\left(\sqrt{4+x}+1\right)}$$

$$=\lim_{x\to -3}\frac{x+3}{(x+3)\left(\sqrt{1+\sqrt{4+x}}+\sqrt{2}\right)\left(\sqrt{4+x}+1\right)}$$

$$=\lim_{x\to -3}\frac{1}{\left(\sqrt{1+\sqrt{4+x}}+\sqrt{2}\right)\left(\sqrt{4+x}+1\right)}$$

$$=\frac{1}{\left(\sqrt{1+\sqrt{4-3}}+\sqrt{2}\right)\left(\sqrt{4-3}+1\right)}$$

$$=\frac{1}{4\sqrt{2}}$$

$$=\frac{\sqrt{2}}{8}$$

ដូចនេះ $\boxed{\lim_{x\to -3}\frac{\sqrt{1+\sqrt{4+x}}-\sqrt{2}}{x+3}=\frac{\sqrt{2}}{8}}$ ។

ឃ. $\lim_{x\to 6}\frac{\sqrt[3]{2+x}-2}{x-6}$

$$=\lim_{x\to 6}\frac{\left(\sqrt[3]{2+x}-2\right)\left(\sqrt[3]{(2+x)^2}+2\sqrt[3]{2+x}+4\right)}{(x-6)\left(\sqrt[3]{(2+x)^2}+2\sqrt[3]{2+x}+4\right)}$$

$$=\lim_{x\to 6}\frac{2+x-8}{(x-6)\left(\sqrt[3]{(2+x)^2}+2\sqrt[3]{2+x}+4\right)}$$

$$=\lim_{x\to 6}\frac{x-6}{(x-6)\left(\sqrt[3]{(2+x)^2}+2\sqrt[3]{2+x}+4\right)}$$

$$=\lim_{x\to 6}\frac{1}{\sqrt[3]{(2+x)^2}+2\sqrt[3]{2+x}+4}$$

$$=\frac{1}{4+4+4}$$

$$=\frac{1}{12}$$

ដូចនេះ $\boxed{\lim_{x\to 6}\frac{\sqrt[3]{2+x}-2}{x-6}=\frac{1}{12}}$ ។

ង. $\lim_{x\to 1}\frac{\sqrt{x-1}+\sqrt{x}-1}{\sqrt{x^2-1}}$

អនុគមន៍ $f(x)=\frac{\sqrt{x-1}+\sqrt{x}-1}{\sqrt{x^2-1}}$ កំណត់លើ $(1,+\infty)$

នោះ $\lim_{x\to 1}\frac{\sqrt{x-1}+\sqrt{x}-1}{\sqrt{x^2-1}}$ កំណត់បានតែលីមីតខាងស្តាំប៉ុណ្ណោះ

$$\lim_{x\to 1^+}\frac{\sqrt{x-1}+\sqrt{x}-1}{\sqrt{x^2-1}}=\lim_{x\to 1^+}\frac{\sqrt{x-1}+\sqrt{x}-1}{\sqrt{(x-1)(x+1)}}$$

តាង $t=x-1\Rightarrow x=t+1$

បើ $x\to1^{+}$ គេបាន $t\to0^{+}$

គេបាន $\lim\limits_{x\to1^{+}}\dfrac{\sqrt{x-1}+\sqrt{x}-1}{\sqrt{x^{2}-1}}=\lim\limits_{t\to0^{+}}\dfrac{\sqrt{t}+\sqrt{t+1}-1}{\sqrt{t\left(t+1+1\right)}}$

$$=\lim_{t\to0^{+}}\frac{\left(\sqrt{t}-1\right)+\sqrt{t+1}}{\sqrt{t\left(t+2\right)}}$$

$$=\lim_{t\to0^{+}}\frac{\left[\left(\sqrt{t}-1\right)+\sqrt{t+1}\right]\left[\left(\sqrt{t}-1\right)-\sqrt{t+1}\right]\sqrt{t\left(t+2\right)}}{\sqrt{t\left(t+2\right)}\times\sqrt{t\left(t+2\right)}\left[\left(\sqrt{t}-1\right)-\sqrt{t+1}\right]}$$

$$=\lim_{t\to0^{+}}\frac{\left[\left(\sqrt{t}-1\right)^{2}-\left(t+1\right)\right]\sqrt{t\left(t+2\right)}}{t\left(t+2\right)\left[\left(\sqrt{t}-1\right)-\sqrt{t+1}\right]}$$

$$=\lim_{t\to0^{+}}\frac{\left(t-2\sqrt{t}+1-t-1\right)\sqrt{t\left(t+2\right)}}{t\left(t+2\right)\left[\left(\sqrt{t}-1\right)-\sqrt{t+1}\right]}$$

$$=\lim_{t\to0^{+}}\frac{-2\sqrt{t}\sqrt{t\left(t+2\right)}}{t\left(t+2\right)\left[\left(\sqrt{t}-1\right)-\sqrt{t+1}\right]}$$

$$=\lim_{t\to0^{+}}\frac{-2\sqrt{t^{2}\left(t+2\right)}}{t\left(t+2\right)\left[\left(\sqrt{t}-1\right)-\sqrt{t+1}\right]}$$

$$=\lim_{t\to0^{+}}\frac{-2t\sqrt{t+2}}{t\left(t+2\right)\left[\left(\sqrt{t}-1\right)-\sqrt{t+1}\right]}$$

$$=\lim_{t\to 0^+}\frac{-2\sqrt{t+2}}{(t+2)\left[\left(\sqrt{t}-1\right)-\sqrt{t+1}\right]}$$

$$=\frac{-2\sqrt{2}}{2(-1-1)}$$

$$=\frac{\sqrt{2}}{2}$$

ដូចនេះ $\boxed{\lim_{x\to 1^+}\frac{\sqrt{x-1}+\sqrt{x}-1}{\sqrt{x^2-1}}=\frac{\sqrt{2}}{2}}$ ។

ច. $\lim_{x\to 4}\frac{\sqrt{x-2\sqrt{x}+1}-1}{\sqrt{x}-2}$

$$=\lim_{x\to 4}\frac{\left(\sqrt{x-2\sqrt{x}+1}-1\right)\left(\sqrt{x-2\sqrt{x}+1}+1\right)\left(\sqrt{x}+2\right)}{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)\left(\sqrt{x-2\sqrt{x}+1}+1\right)}$$

$$=\lim_{x\to 4}\frac{\left(x-2\sqrt{x}+1-1\right)\left(\sqrt{x}+2\right)}{(x-4)\left(\sqrt{x-2\sqrt{x}+1}+1\right)}$$

$$=\lim_{x\to 4}\frac{\left(x-2\sqrt{x}\right)\left(\sqrt{x}+2\right)}{(x-4)\left(\sqrt{x-2\sqrt{x}+1}+1\right)}$$

$$=\lim_{x\to 4}\frac{\sqrt{x}\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)}{(x-4)\left(\sqrt{x-2\sqrt{x}+1}+1\right)}$$

$$=\lim_{x\to 4}\frac{\sqrt{x}(x-4)}{(x-4)\left(\sqrt{x-2\sqrt{x}+1}+1\right)}$$

$$=\lim_{x\to 4}\frac{\sqrt{x}}{\sqrt{x-2\sqrt{x}+1}+1}$$

$$=\frac{2}{1+1}$$

$$=1$$

ដូចនេះ $\boxed{\lim_{x\to 4}\frac{\sqrt{x-2\sqrt{x}+1}-1}{\sqrt{x}-2}=1}$ ។

៨. គណនាលីមីត

ក. $\lim_{x\to\infty}\left(\frac{x-1}{x+3}\right)^{x+2}$

ខ. $\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-2}\right)^{x^2}$

គ. $\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}}$

ឃ. $\lim_{x\to\infty}\left(x\left[\ln(x+1)-\ln x\right]\right)$

រំឭក

$a).\ \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e$

$b).\ \lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$

$c).$ បើ $\lim_{x\to a}u(x)=\infty$ គេបាន $\lim_{x\to a}\left[1+\frac{1}{u(x)}\right]^{u(x)}=e$

$d).$ បើ $\lim_{x\to a}u(x)=0$ គេបាន $\lim_{x\to a}\left[1+u(x)\right]^{\frac{1}{u(x)}}=e$

$$f).\ \lim_{x\to a}\left[f(x)\right]^{g(x)}=\left[\lim_{x\to a}f(x)\right]^{\lim\limits_{x\to a}g(x)}$$

<u>ចម្លើយ</u>

គណនាលីមីត

$$ក.\ \lim_{x\to\infty}\left(\frac{x-1}{x+3}\right)^{x+2}=\lim_{x\to\infty}\left(\frac{x+3-4}{x+3}\right)^{x+2}$$

$$=\lim_{x\to\infty}\left(1+\frac{-4}{x+3}\right)^{x+2}$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x+3}{-4}}\right)^{\frac{x+3}{-4}\times\frac{-4(x+2)}{x+3}}$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x+3}{-4}}\right)^{\frac{x+3}{-4}}\right]^{\frac{-4(x+2)}{x+3}}$$

$$=\left[\lim_{x\to\infty}\left(1+\frac{1}{\frac{x+3}{-4}}\right)^{\frac{x+3}{-4}}\right]^{\lim\limits_{x\to\infty}\frac{-4(x+2)}{x+3}}$$

$$=e^{\lim\limits_{x\to\infty}\frac{-4x}{x+3}}$$ (ប្រើរូបមន្ត c)

$$=e^{-4}$$ (ព្រោះ $\lim\limits_{x\to\infty}\frac{-4x}{x+3}=-4$)

ដូចនេះ $\boxed{\lim_{x\to\infty}\left(\frac{x-1}{x+3}\right)^{x+2}=e^{-4}=\frac{1}{e^4}}$ ។

ខ. $\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-2}\right)^{x^2}=\lim_{x\to\infty}\left(\frac{x^2-2+3}{x^2-2}\right)^{x^2}$

$$=\lim_{x\to\infty}\left(1+\frac{3}{x^2-2}\right)^{x^2}$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x^2-2}{3}}\right)^{\frac{x^2-2}{3}\times\frac{3x^2}{x^2-2}}$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x^2-2}{3}}\right)^{\frac{x^2-2}{3}}\right]^{\frac{3x^2}{x^2-2}}$$

$$=\left[\lim_{x\to\infty}\left(1+\frac{1}{\frac{x^2-2}{3}}\right)^{\frac{x^2-2}{3}}\right]^{\lim_{x\to\infty}\frac{3x^2}{x^2-2}}$$

$$=e^{\lim_{x\to\infty}\frac{3x^2}{x^2-2}}$$ (ប្រើរូបមន្ត c)

$$=e^3$$ (ព្រោះ $\lim_{x\to\infty}\frac{3x^2}{x^2-2}=3$)

ដូចនេះ $\boxed{\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-2}\right)^{x^2}=e^3}$ ។

គ. $\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}}=\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}\times\frac{\sin x}{x}}$

$$=\lim_{x\to 0}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\frac{\sin x}{x}}$$

$$=\left[\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}}\right]^{\lim\limits_{x\to 0}\frac{\sin x}{x}}$$

$$=e^{\lim\limits_{x\to 0}\frac{\sin x}{x}}$$ (ប្រើរូបមន្ត d)

$$=e$$ (ព្រោះ $\lim_{x\to 0}\frac{\sin x}{x}=1$)

ដូចនេះ $\boxed{\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}}=e}$ ។

ឃ. $\lim_{x\to\infty}\left(x\left[\ln(x+1)-\ln x\right]\right)=\lim_{x\to\infty}\left(x\ln\frac{x+1}{x}\right)$

$$=\lim_{x\to\infty}\ln\left(\frac{x+1}{x}\right)^x$$

$$=\ln\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right]$$

$$=\ln e$$

$$=1$$

ដូចនេះ $\boxed{\lim_{x\to\infty}\left(x\left[\ln(x+1)-\ln x\right]\right)=1}$ ។

៩. គណនាលីមីត

ក. $\lim_{x\to0}\frac{\sin 5x}{3x}$ ខ. $\lim_{x\to\infty} x\sin\frac{1}{x}$

គ. $\lim_{x\to0}\frac{1-\cos x}{\sin 4x^2}$ ឃ. $\lim_{x\to1}\frac{\sin(x-1)}{1-x+x^2-x^3}$

ចម្លើយ

គណនាលីមីត

ក. $\lim_{x\to0}\frac{\sin 5x}{3x}=\lim_{x\to0}\left(\frac{\sin 5x}{5x}\times\frac{5}{3}\right)$

$=1\times\frac{5}{3}$

$=\frac{5}{3}$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sin 5x}{3x}=\frac{5}{3}}$ ។

ខ. $\lim_{x\to\infty} x\sin\frac{1}{x}$

តាង $t=\frac{1}{x}\Rightarrow x=\frac{1}{t}$

បើ $x\to\infty$ គេបាន $t\to0$

គេបាន $\lim_{x\to\infty} x\sin\frac{1}{x}=\lim_{t\to0}\frac{1}{t}\sin t=\lim_{t\to0}\frac{\sin t}{t}=1$

ដូចនេះ $\boxed{\lim_{x\to\infty} x\sin\frac{1}{x}=1}$ ។

គ. $\lim_{x\to0}\frac{1-\cos x}{\sin 4x^2}=\lim_{x\to0}\left(\frac{1-\cos x}{1}\times\frac{1}{\sin 4x^2}\right)$

$$=\lim_{x\to0}\left(\frac{2\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{4x^2}{\sin 4x^2}\times\frac{\left(\frac{x}{2}\right)^2}{4x^2}\right)$$

$$=2\lim_{x\to0}\left(\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{4x^2}{\sin 4x^2}\times\frac{1}{16}\right)$$

$$=2\left(1\times1\times\frac{1}{16}\right)$$

$$=\frac{1}{8}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{1-\cos x}{\sin 4x^2}=\frac{1}{8}}$ ។

ឃ. $\lim_{x\to1}\frac{\sin(x-1)}{1-x+x^2-x^3}=\lim_{x\to1}\frac{\sin(x-1)}{-(x-1)-x^2(x-1)}$

$$=\lim_{x\to1}\frac{\sin(x-1)}{-(x-1)(x^2+1)}$$

តាង $t=x-1\Rightarrow x=t+1$

បើ $x\to1$ គេបាន $t\to0$

គេបាន $\lim_{x\to1}\frac{\sin(x-1)}{1-x+x^2-x^3}=\lim_{t\to0}\frac{\sin t}{-t\left[(t+1)^2+1\right]}$

$$=-\lim_{t\to0}\left(\frac{\sin t}{t}\times\frac{1}{(t+1)^2+1}\right)$$

$$=-1\times\frac{1}{2}$$

$$=-\frac{1}{2}$$

ដូចនេះ $\boxed{\lim_{x\to1}\frac{\sin(x-1)}{1-x+x^2-x^3}=-\frac{1}{2}}$ ។

១០. គេឱ្យអនុគមន៍ $g(y)=\begin{cases}y^2+5 & \text{បើ } y<-2\\ 1-3y & \text{បើ } y\geq-2\end{cases}$ ។

គណនា $\lim_{y\to6}g(y), \lim_{y\to-2}g(y)$ ។

<u>ចម្លើយ</u>

គណនា $\lim_{y\to6}g(y), \lim_{y\to-2}g(y)$

គេមាន $g(y)=\begin{cases}y^2+5 & \text{បើ } y<-2\\ 1-3y & \text{បើ } y\geq-2\end{cases}$

គេបាន $\lim_{y\to6}g(y)=\lim_{y\to6}(1-3y)=1-18=-17$

និង $\lim_{x\to-2}g(y)=\left[\begin{array}{l}\lim_{y\to-2^-}(y^2+5)=9\\ \lim_{y\to-2^+}(1-3y)=7\end{array}\right.$

ដូចនេះ $\boxed{\lim_{y\to6}g(y)=-17,\ \lim_{y\to-2^-}g(y)=9,\ \lim_{y\to-2^+}g(y)=7}$ ។

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