

លីមីត

និង

ជេរីវេ

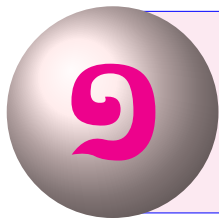
ថ្នាក់ទី១១

សន ពៅ

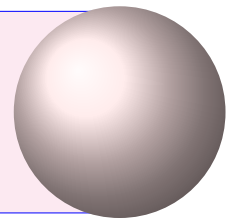
២០២១

មាតិកា

មេរៀនទី ១	លីមីត និងដេរីវេ	១
១	លីមីត	១
១.១	សញ្ញាណលីមីត	១
១.២	ប្រមាណវិធីលីមីត	៣
១.៣	លីមីតរាងមិនកំណត់ $\frac{0}{0}$	៤
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១.៥	លីមីតអនន្តនៃអនុគមន៍	១៣



លីមីត និងដេរីវេ



១. លីមីត

ក្នុងជំពូកនេះ យើងចាប់ផ្តើមសិក្សាគណិតវិភាគ សម្រាប់ប្រើក្នុងការដោះស្រាយចំណោទបន្ទាត់ប៉ះ ចំណោទតម្លៃបរមា ចំណោទផ្ទៃក្រឡាជាដើម។

១.១. សញ្ញាណលីមីត

ឧទាហរណ៍ ១. គេឱ្យអនុគមន៍ $y = f(x) = x^2 - 1$ និងតារាងតម្លៃលេខ

x	0.99	0.999	0.9999	1.0001	1.001	1.01
y	-0.0199	-0.001999	-0.00019999	0.00020001	0.002001	0.0201

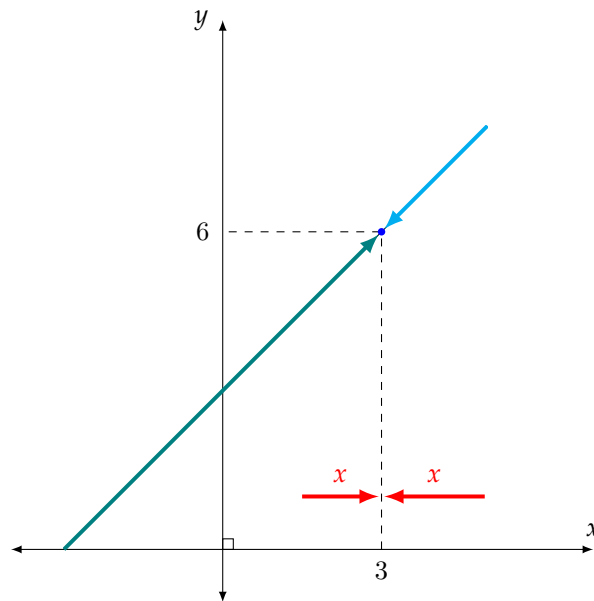
តាមតារាងតម្លៃលេខ បើ x ខិតជិត 1 ខាងឆ្វេង និងខាងស្តាំ នោះគេបាន y ខិតជិត 0 ។

គេថាអនុគមន៍ $y = f(x)$ មានលីមីត 0 កាលណា x ខិតជិត 1 ហើយកំណត់សរសេរ

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (x^2 - 1) = 0$$

ឧទាហរណ៍ ២. គេឱ្យអនុគមន៍ $f(x) = \frac{x^2 - 9}{x - 3}$ ចំពោះ $x \neq 3$ ។

ចំពោះ $x \neq 3$ គេបាន $f(x) = \frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{x - 3} = x + 3$



តាមក្រាប បើ x ខិតជិត ៣ តែ $x \neq 3$ នោះអនុគមន៍ $y = f(x)$ ខិតជិត ៦ ហើយកំណត់សរសេរ

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$$

ជាទូទៅ

បើ x ខិតជិត a ហើយអនុគមន៍ $y = f(x)$ ខិតជិត b ណាមួយ នោះគេកំណត់សរសេរ

$$\lim_{x \rightarrow a} f(x) = b$$

លំហាត់គំរូទី ១. បំពេញតារាងតម្លៃលេខខាងក្រោម រួចទាញរក $\lim_{x \rightarrow 0} \frac{1}{x}$ ដែល $x > 0$ ។

x	0.1	0.01	0.001	0.0001
y				

លំហាត់គំរូទី ២. បំពេញតារាងតម្លៃលេខខាងក្រោម រួចទាញរក $\lim_{x \rightarrow 0} \frac{1}{x}$ ដែល $x < 0$ ។

x	-0.1	-0.01	-0.001	-0.0001
y				

លំហាត់ប្រតិបត្តិ ១.

១. បំពេញតារាងតម្លៃលេខខាងក្រោម រួចទាញរក $\lim_{x \rightarrow 2} f(x)$ ។

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x) = \frac{x^2 - 4}{x - 2}$							

២. សង់ក្រាបនៃអនុគមន៍ $f(x) = x^2 + 2$ រួចទាញរកលីមីត $\lim_{x \rightarrow 2} f(x)$ ។

១.២. ប្រមាណវិធីលីមីត

អនុគមន៍ $y = f(x)$ និង $y = g(x)$ មានលីមីតកាលណា $x \rightarrow a$ ។ គេបាន

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} c = c \quad \text{ដែល } f(x) = c \text{ ជាអនុគមន៍ថេរ។}$$

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} x = a \quad \text{ដែល } f(x) = x$$

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad \text{ដែល } g(a) \neq 0$$

ជាទូទៅ

- បើ $f(x) = kx^n$ នោះ $\lim_{x \rightarrow a} f(x) = f(a) = ka^n$
- បើ $f(x)$ ជាពហុធា នោះ $\lim_{x \rightarrow a} f(x) = f(a)$
- បើ $f(x) = \frac{g(x)}{h(x)}$ ជាប្រភាគសនិទាន នោះ $\lim_{x \rightarrow a} f(x) = \frac{g(a)}{h(a)}$ ដែល $h(a) \neq 0$

លំហាត់គំរូទី ៣. គណនាលីមីតខាងក្រោម

$$ក. \lim_{x \rightarrow 0} (x^2 - 3x + 1)$$

$$ខ. \lim_{x \rightarrow -1} (x^3 - 3x^2 + 6x - 1)$$

$$គ. \lim_{x \rightarrow 1} (x^{2021} - x^2)$$

$$ឃ. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x + 2}$$

$$ង. \lim_{x \rightarrow 5} \frac{x^2 + 2x + 1}{x + 5}$$

$$ច. \lim_{x \rightarrow -1} \frac{x^3 - 7x + 1}{x^3 - 1}$$

ដំណោះស្រាយ. គណនាលីមីតខាងក្រោម

$$ក. \lim_{x \rightarrow 0} (x^2 - 3x + 1) = \underline{\hspace{2cm}}$$

$$ខ. \lim_{x \rightarrow -1} (x^3 - 3x^2 + 6x - 1) = \underline{\hspace{2cm}}$$

$$គ. \lim_{x \rightarrow 1} (x^{2021} - x^2) = \underline{\hspace{2cm}}$$

$$ឃ. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x + 2} = \underline{\hspace{2cm}}$$

$$ង. \lim_{x \rightarrow 5} \frac{x^2 + 2x + 1}{x + 5} = \underline{\hspace{2cm}}$$

$$ច. \lim_{x \rightarrow -1} \frac{x^3 - 7x + 1}{x^3 - 1} = \underline{\hspace{2cm}}$$

១.៣. លីមីតរាងមិនកំណត់ $\frac{0}{0}$

ជាទូទៅ

បើភាគយក និងភាគបែងមានលីមីតសូន្យ នោះគេត្រូវគណនាតាមជំហានខាងក្រោម

- ជំហានទី១ : បំបែកតួភាគយក និងភាគបែងជាផលគុណកត្តា
- ជំហានទី២ : សម្រួលកត្តារួមចោល
- ជំហានទី៣ : រកលីមីតនៃកន្សោមថ្មី។

លំហាត់គំរូទី ៤. គណនាលីមីតខាងក្រោម

$$ក. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$ខ. \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - x - 2}$$

$$គ. \lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x^2 - 3x - 4}$$

$$ឃ. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x^3 - 27}$$

$$ង. \lim_{x \rightarrow 1} \frac{x^3 - x^2 - x + 1}{3x^2 - 6x + 3}$$

$$ច. \lim_{x \rightarrow 1} \frac{x^{10} - 1}{x^5 - x^2 + x - 1}$$

[illegible]

ក. រកចំនួនពិត a និង b ដើម្បីឱ្យ $\lim_{x \rightarrow 1} \frac{x^2 + ax + b}{x - 1} = 5$ ។

២. រកចំនួនពិត p និង q ដើម្បីឱ្យ $\lim_{x \rightarrow -2} \frac{x^2 + px - 6}{2x^2 + 3x - 2} = q$ ។

គ. រកចំនួនពិត m និង n ដើម្បីឱ្យ $\lim_{x \rightarrow 2} \frac{x^2 - mx + 8}{x^2 - (n+2)x + 2n} = \frac{1}{5}$ ។

ដំណោះស្រាយ.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

លំហាត់ប្រតិបត្តិ ២.

១. គណនាលីមីតខាងក្រោម

- ក. $\lim_{x \rightarrow 0} \frac{(x+1)^2 - 1}{x}$ ខ. $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x - 1}$
- គ. $\lim_{x \rightarrow 1} \frac{x^2 + 4x - 5}{x^2 + 2x - 3}$ ឃ. $\lim_{x \rightarrow -1} \frac{x^2 - 5x - 6}{2x^2 + 3x + 1}$
- ង. $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x^2 + x - 2}$ ច. $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^3 - 8}$
- ឆ. $\lim_{x \rightarrow 3} \frac{3x^2 - 11x + 6}{2x^2 - 5x - 3}$ ជ. $\lim_{x \rightarrow -1} \frac{x^3 + 5x^2 + 7x + 3}{x^3 + 4x^2 + 5x + 2}$
- ឈ. $\lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^3 - x^2 - x + 1}$ ញ. $\lim_{x \rightarrow 1} \frac{x^4 - 1}{2x^4 - x^2 - 1}$
- ដ. $\lim_{x \rightarrow -1} \frac{x^3 + 4x^2 + 5x + 2}{x^3 - 3x - 2}$ ថ. $\lim_{x \rightarrow 2} \frac{x^3 - 5x^2 + 8x - 4}{x^3 - 3x^2 + 4}$
- ឌ. $\lim_{x \rightarrow 2} \frac{x^3 - 6x^2 + 12x - 8}{x^3 - 3x^2 + 4}$ ឍ. $\lim_{x \rightarrow 0} \frac{(1+x)^3 - (1+3x)}{x^2 + x^5}$
- ណ. $\lim_{x \rightarrow -1} \frac{x^3 - 2x - 1}{x^4 + 2x + 1}$ ត. $\lim_{x \rightarrow -5} \frac{x^2 + 2x - 15}{x + 5}$
- ថ. $\lim_{x \rightarrow 2} \frac{3x^2 - 7x + 2}{4x^2 - 5x - 6}$ ទ. $\lim_{x \rightarrow -2} \frac{x^3 + 5x^2 + 8x + 4}{x^3 + 7x^2 + 16x + 12}$

២. រកតម្លៃនៃចំនួនពិត a ដើម្បីឱ្យលីមីតខាងក្រោមជាលីមីតនៃចំនួនកំណត់ រួចគណនាលីមីតនោះ

ក. $\lim_{x \rightarrow 1} \frac{x^2 - a}{x - 1}$ ខ. $\lim_{x \rightarrow 2} \frac{x^2 - ax + 3}{x - 2}$ គ. $\lim_{x \rightarrow -1} \frac{x^2 + 3x - a}{x^3 + 1}$

៣. គណនាលីមីតខាងក្រោម

- ក. $\lim_{x \rightarrow -1} \frac{(x^3 - 2x - 1)(x + 1)}{x^4 + 4x^2 - 5}$ ខ. $\lim_{x \rightarrow 2} \frac{x^3 - 6x^2 + 12x - 8}{x^3 - 3x^2 + 4}$
- គ. $\lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x + x^2}$ ឃ. $\lim_{x \rightarrow -1} \frac{(x^2 + 3x + 2)^2}{x^3 + 2x^2 - x - 2}$
- ង. $\lim_{x \rightarrow -3} \frac{(x^2 + 2x - 3)^2}{x^3 + 4x^2 + 3x}$ ច. $\lim_{x \rightarrow -1} \frac{(x^3 - 2x - 1)^2}{x^4 + 2x + 1}$
- ឆ. $\lim_{x \rightarrow 0} \frac{(1+x)^3 - (1+3x)}{x + x^5}$ ជ. $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{2x^2 - x - 1}$

$$\text{ឈ. } \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x^2 - x - 2}$$

$$\text{ដ. } \lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^3 - x^2 - x + 1}$$

$$\text{ញ. } \lim_{x \rightarrow -1} \frac{x^3 + 5x^2 + 7x + 3}{x^3 + 4x^2 + 5x + 2}$$

$$\text{ថ. } \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{(x^2 - x - 2)^2}$$

៤. គណនាលីមីតខាងក្រោម:

$$\text{ក. } \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$$

$$\text{គ. } \lim_{x \rightarrow 1} \frac{x^9 - 3x + 2}{x^6 + 5x - 6}$$

$$\text{ង. } \lim_{x \rightarrow 1} \frac{x^{2m} - 1}{x^{2n} - 1}, \quad m, n \in \mathbb{N}$$

$$\text{ឆ. } \lim_{x \rightarrow 1} \frac{x + x^2 + x^3 + \dots + x^{2021} - 2021}{x + x^2 + x^3 + \dots + x^{2020} - 2020}$$

$$\text{ឈ. } \lim_{x \rightarrow 1} \frac{x^{50} - 7x + 6}{x^{20} + 3x - 4}$$

$$\text{ដ. } \lim_{x \rightarrow 1} \frac{(x-1)(x^3 + x - 2)}{x^3 - x^2 - x + 1}$$

$$\text{ជ. } \lim_{x \rightarrow -3} \frac{x^4 - 6x^2 - 27}{x^3 + 3x^2 + x + 3}$$

$$\text{ណ. } \lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{n}{1-x^n} \right), \quad n \in \mathbb{N}$$

$$\text{ច. } \lim_{x \rightarrow 0} \frac{(x+1)^n - (nx+1)}{x^2}, \quad n \in \mathbb{N}$$

$$\text{ខ. } \lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^3 - x^2 + 2x - 2}$$

$$\text{ឃ. } \lim_{x \rightarrow 2} \frac{x-2}{x^3 - x^2 - x - 2}$$

$$\text{ច. } \lim_{x \rightarrow -1} \frac{x^7 + 1}{x^5 + 1}$$

$$\text{ជ. } \lim_{x \rightarrow 1} \frac{x^n - nx + n - 1}{(x-1)^2}, \quad n \in \mathbb{N}$$

$$\text{ញ. } \lim_{x \rightarrow \frac{1}{3}} \frac{3x^2 - \frac{1}{3}}{x + \frac{1}{3}}$$

$$\text{ថ. } \lim_{x \rightarrow 1} \frac{4x^6 - 5x^5 + x}{(1-x)^2}$$

$$\text{ឈ. } \lim_{x \rightarrow 1} \left[\left(\frac{4}{x^2 - x^{-1}} - \frac{1 - 3x + x^2}{1 - x^3} \right)^{-1} + 3 \frac{x^4 - 1}{x^3 - x^{-1}} \right]$$

$$\text{គ. } \lim_{x \rightarrow 1} \left(\frac{m}{1-x^m} - \frac{n}{1-x^n} \right), \quad m, n \in \mathbb{N}$$

$$\text{ទ. } \lim_{x \rightarrow 1} \frac{nx^{n+1} - (n+1)x^n + 1}{x^{p+1} - x^p - x + 1}, \quad n, p \in \mathbb{N}$$

លំហាត់គំរូទី ៦. គណនាលីមីតខាងក្រោម

$$\text{ក. } \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{2x - 2}$$

$$\text{គ. } \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{9 - x^2}$$

$$\text{ង. } \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 3} - 2}{x - 1}$$

$$\text{ឆ. } \lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 2} - \sqrt{2}}{x - 2}$$

$$\text{ឈ. } \lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{\sqrt{x+1} - \sqrt{1-x}}$$

$$\text{ខ. } \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{2 - x}$$

$$\text{ឃ. } \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{x^2}$$

$$\text{ច. } \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

$$\text{ជ. } \lim_{x \rightarrow 2} \frac{1+x - \sqrt{2x+5}}{x-2}$$

$$\text{ញ. } \lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{16 - x^2}$$

$$\text{ដ.} \lim_{x \rightarrow 1} \frac{\sqrt{23 + 2x} - 5}{\sqrt{2x + 1} - 3}$$

$$\text{ខ.} \lim_{x \rightarrow 0} \frac{\sqrt{x + 4} - 2}{\sqrt{x^2 + 9} - 3}$$

$$\text{គ.} \lim_{x \rightarrow 0} \frac{\sqrt{2x + 4} - 2}{\sqrt{2x + 1} - 1}$$

$$\text{ថ.} \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{3 + x^2} - 2}$$

$$\text{ធី.} \lim_{x \rightarrow 2} \frac{2 - \sqrt{5x - 8}}{x^2 - 4}$$

$$\text{ប.} \lim_{x \rightarrow 0} \frac{\sqrt{(1 + x)(1 + 2x)} - 1}{x}$$

$$\text{ព.} \lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{\sqrt{2x + 3} - \sqrt{x + 6}}$$

$$\text{ម.} \lim_{x \rightarrow 1} \frac{x\sqrt{x} - \sqrt{3x - 2}}{\sqrt{2x - 1} - x}$$

$$\text{រ.} \lim_{x \rightarrow 2} \frac{x\sqrt{x} - 2\sqrt{2}}{x - 2}$$

$$\text{ស.} \lim_{x \rightarrow 2} \frac{\sqrt{3x + 3} - 3}{x^2 - \sqrt{7x + 4}}$$

$$\text{វ.} \lim_{x \rightarrow 4} \frac{\sqrt{1 + 2x} - \sqrt{13 - x}}{\sqrt{5 + x} - 3}$$

$$\text{ឈ.} \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 3} - 2}{\sqrt{x^2 + 8} - 3}$$

$$\text{ត.} \lim_{x \rightarrow 1} \frac{\sqrt{1 + x} - \sqrt{3 - x}}{\sqrt{1 + x} - \sqrt{2}}$$

$$\text{ទ.} \lim_{x \rightarrow 8} \frac{\sqrt{1 + x} - 3}{\sqrt{3x + 1} - 5}$$

$$\text{ន.} \lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 5} - \sqrt{x + 7}}{x - 2}$$

$$\text{ដ.} \lim_{x \rightarrow 2} \frac{\sqrt{x^3 + 1} - 3}{x - 2}$$

$$\text{ឆ.} \lim_{x \rightarrow 2} \frac{\sqrt{2 + \sqrt{2 + x}} - 2}{x - 2}$$

$$\text{ឃ.} \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 3} - 2}{x^3 - 1}$$

$$\text{ប.} \lim_{x \rightarrow 1} \frac{\sqrt{2x + 1} - \sqrt{3}}{1 - \sqrt{x}}$$

$$\text{ស.} \lim_{x \rightarrow 1} \frac{\sqrt{2x^2 + 3x + 4} - \sqrt{x^2 + 3x + 5}}{x - 1}$$

លំហាត់គំរូទី ៧. គណនាលីមីតខាងក្រោម

$$\text{ក.} \lim_{x \rightarrow -8} \frac{\sqrt{1 - x} - 3}{2 + \sqrt[3]{x}}$$

$$\text{គ.} \lim_{x \rightarrow -2} \frac{\sqrt[3]{x - 6} + 2}{x^3 + 8}$$

$$\text{ង.} \lim_{x \rightarrow 8} \frac{\sqrt{9 + 2x} - 5}{\sqrt[3]{x} - 2}$$

$$\text{ឆ.} \lim_{x \rightarrow 0} \frac{\sqrt[3]{8 + 3x + x^2} - 2}{x + x^2}$$

$$\text{ខ.} \lim_{x \rightarrow 3} \frac{\sqrt{x + 13} - 2\sqrt{x + 1}}{x^2 - 9}$$

$$\text{ឃ.} \lim_{x \rightarrow 16} \frac{\sqrt[4]{x} - 2}{\sqrt{x} - 4}$$

$$\text{ច.} \lim_{x \rightarrow 0} \frac{\sqrt{1 - 2x + x^2} - (1 + x)}{x}$$

$$\text{ជ.} \lim_{x \rightarrow 2} \frac{x\sqrt{x} - 2\sqrt{2}}{x - 2}$$

$$\text{ឈ.} \lim_{x \rightarrow 8} \frac{\sqrt[3]{x^2} - 4}{x - 8}$$

$$\text{ដ.} \lim_{x \rightarrow 0} \frac{1 + x - \sqrt[2021]{1 + 2021x}}{x^2}$$

$$\text{ខ.} \lim_{x \rightarrow 3} \frac{x^3 - 27}{\sqrt{2x + 3} - x + 6}$$

$$\text{គា.} \lim_{x \rightarrow 1} \frac{\sqrt{x-1} - \sqrt{x^2+1} + \sqrt{x^4+1}}{\sqrt{x+1} + \sqrt{x^2-1} - \sqrt{x^3+1}}$$

$$\text{ច.} \lim_{x \rightarrow 1} \frac{1 - \sqrt[8]{x}}{1 - \sqrt[6]{x}}$$

$$\text{ឆ.} \lim_{x \rightarrow 1} \frac{\sqrt[3]{x^2} - 2\sqrt[3]{x} + 1}{(x-1)^2}$$

$$\text{ប.} \lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 2\sqrt[3]{x} + 1}{\sqrt[3]{x^2} + \sqrt{x} - 2}$$

$$\text{ព.} \lim_{x \rightarrow 0} \frac{\sqrt[n]{1+ax} - 1}{x}$$

$$\text{ម.} \lim_{x \rightarrow 0} \frac{1 - \sqrt[2005]{1+x}}{x}$$

$$\text{រ.} \lim_{x \rightarrow 1} \frac{\sqrt{1+3x+5x^2+7x^3} - 4}{x-1}$$

$$\text{ស.} \lim_{x \rightarrow 0} \frac{\sqrt[n]{1+a_1x+a_2x^2+\dots+a_kx^k} - 1}{x}$$

$$\text{ញ.} \lim_{x \rightarrow a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a}$$

$$\text{ថ.} \lim_{x \rightarrow 1} \frac{\sqrt{2x^2+2x} - \sqrt{2x+2}}{x^3 - 1}$$

$$\text{ឈ.} \lim_{x \rightarrow 2} \frac{\sqrt{x+2} + \sqrt[3]{x-1} - 3}{x-2}$$

$$\text{ត.} \lim_{x \rightarrow 27} \frac{3 - \sqrt[3]{x}}{27 - x}$$

$$\text{ទ.} \lim_{x \rightarrow 1} \frac{1 - \sqrt[n]{x}}{1 - \sqrt[p]{x}}$$

$$\text{ដ.} \lim_{x \rightarrow 1} \frac{\sqrt[4]{x} + 2\sqrt[6]{x} - 3}{\sqrt[3]{x} - 1}$$

$$\text{ជ.} \lim_{x \rightarrow 1} \frac{1 - \sqrt[6]{x}}{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x} - 3}$$

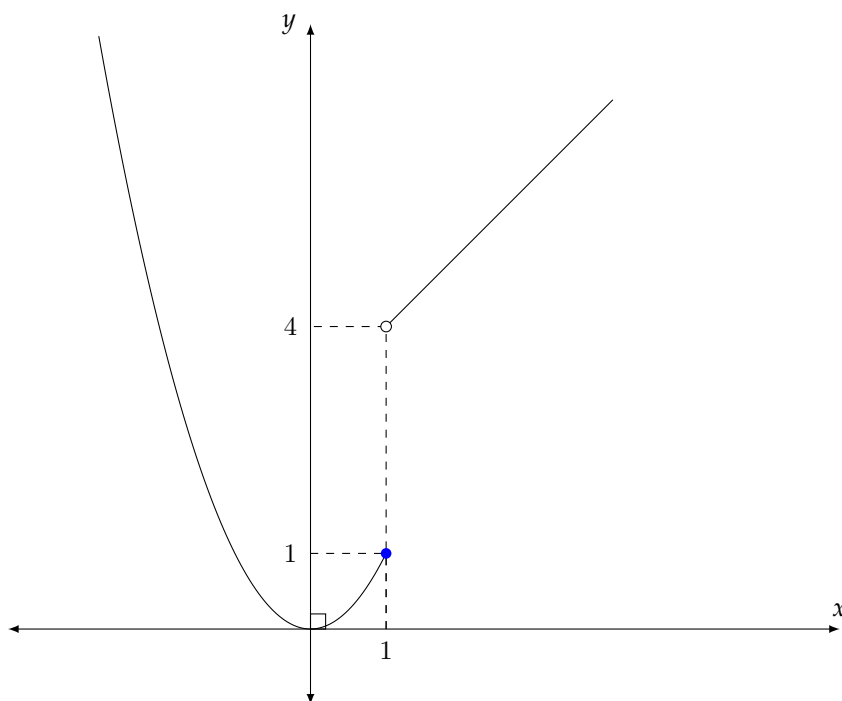
$$\text{ន.} \lim_{x \rightarrow 0} \frac{\sqrt[n]{1+ax} - \sqrt[p]{1+bx}}{x}$$

$$\text{ឃ.} \lim_{x \rightarrow 0} \frac{\sqrt[n]{1+ax} \sqrt[p]{1+bx} - 1}{x}$$

$$\text{ឈ.} \lim_{x \rightarrow 0} \frac{\sqrt[n]{1+ax} \sqrt[p]{1+bx} \sqrt[q]{1+cx} - 1}{x}$$

១.៤. លីមីតខាងឆ្វេង និងខាងស្តាំ

$$\text{ឧទាហរណ៍ ៣. គេឱ្យអនុគមន៍ } f(x) = \begin{cases} x^2 & \text{បើ } x \leq 1 \\ x+3 & \text{បើ } x > 1 \end{cases}$$



តាមក្រាបគេសង្កេតឃើញថា $f(x)$ ខិតជិត 1 កាលណា x ខិតជិត 1 ពីខាងឆ្វេង។ គេកំណត់សរសេរ

$$\lim_{x \rightarrow 1^-} f(x) = 1$$

ម្យ៉ាងទៀត $f(x)$ ខិតជិត 4 កាលណា x ខិតជិត 1 ពីខាងស្តាំ។ គេកំណត់សរសេរ

$$\lim_{x \rightarrow 1^+} f(x) = 4$$

ជាទូទៅ

- បើ $f(x)$ ខិតជិត L កាលណា x ខិតជិត x_0 ពីខាងឆ្វេងនោះ L ជាលីមីតឆ្វេងនៃ $f(x)$ ហើយកំណត់សរសេរ $\lim_{x \rightarrow x_0^-} f(x) = L$ ។
- បើ $f(x)$ ខិតជិត R កាលណា x ខិតជិត x_0 ពីខាងស្តាំនោះ R ជាលីមីតស្តាំនៃ $f(x)$ ហើយកំណត់សរសេរ $\lim_{x \rightarrow x_0^+} f(x) = R$ ។

លំហាត់គំរូទី ៨. គេឱ្យ $f(x) = \begin{cases} 3x^2 - 5x + 1 & \text{បើ } x \geq 0 \\ 3^x - 4^x + 1 & \text{បើ } x < 0 \end{cases}$ ។

ក. គណនាលីមីត $\lim_{x \rightarrow 0^-} f(x)$ និង $\lim_{x \rightarrow 0^+} f(x)$ ។

ខ. ប្រៀបធៀបតម្លៃលីមីតទាំងពីរ។

ដំណោះស្រាយ.

លំហាត់គំរូទី ៩. គេឱ្យ $f(x) = \sqrt{(x-1)^2}$ ។ គណនា $\lim_{x \rightarrow 1} f(x)$ ។

ដំណោះស្រាយ.

លំហាត់ប្រតិបត្តិ ៣.

១. សង់ក្រាបនៃអនុគមន៍ $f(x) = \begin{cases} 2x + 4 & \text{បើ } x > 1 \\ -x & \text{បើ } x \leq 1 \end{cases}$ ។

២. សង់ក្រាបនៃអនុគមន៍ $f(x) = \begin{cases} x^2 + 1 & \text{បើ } x > 0 \\ X^2 - 1 & \text{បើ } x \leq 0 \end{cases}$ ។

៣. គេឱ្យ $f(x) = \begin{cases} 3x - 1 & \text{បើ } x < 1 \\ 2x & \text{បើ } x \geq 1 \end{cases}$ ។

ក. គណនាលីមីត $\lim_{x \rightarrow 1^-} f(x)$ និង $\lim_{x \rightarrow 1^+} f(x)$ ។

ខ. តើអនុគមន៍ f មានលីមីតត្រង់ $x = 1$ ឬទេ?

៤. គេឱ្យ $f(x) = \begin{cases} \frac{x^2 - 4}{2 - x} & \text{បើ } x \geq 2 \\ \frac{|x - 2|}{x - 2} - 5 & \text{បើ } x < 2 \end{cases}$ ។

ក. គណនាលីមីត $\lim_{x \rightarrow 2^-} f(x)$ និង $\lim_{x \rightarrow 2^+} f(x)$ ។

ខ. តើអនុគមន៍ f មានលីមីតត្រង់ $x = 1$ ឬទេ?

១.៥. លីមីតអនន្តនៃអនុគមន៍

ក. លីមីតអនន្តនៃអនុគមន៍ស្វ័យគុណខាង

ឧទាហរណ៍ ៤. គេឱ្យអនុគមន៍ $y = x^2$ និងតារាងតម្លៃលេខ :

x		-10^4	-10^3	-10^2	-10	0	10	10^2	10^3	10^4	
$y = x^2$		10^8	10^6	10^4	10^2	0	10^2	10^4	10^6	10^8	

☞ តាមតារាងតម្លៃលេខខាងលើ បើ x យកតម្លៃវិជ្ជមានកាន់តែធំទៅៗគ្មានទីបញ្ចប់ នោះអនុគមន៍ $y = x^2$ មានតម្លៃវិជ្ជមានកាន់តែធំទៅៗគ្នាទីបញ្ចប់ដែរ។

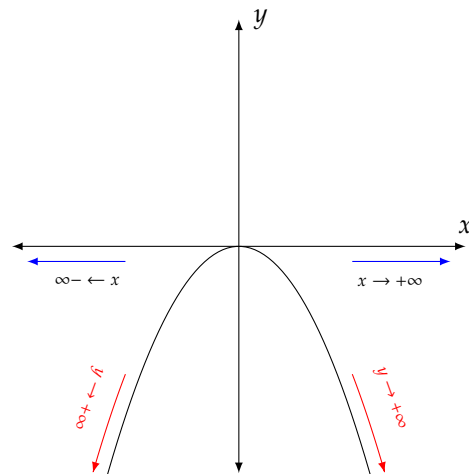
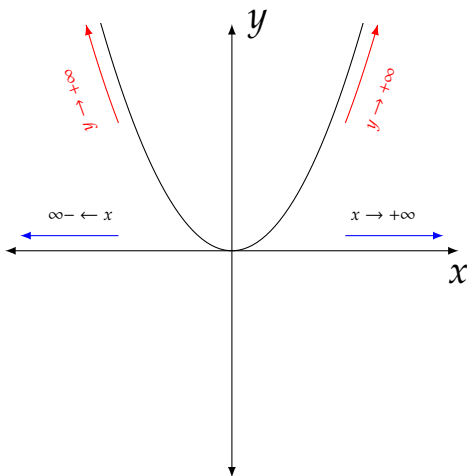
គេថាអនុគមន៍ $y = x^2$ មានលីមីត $+\infty$ កាលណា x ខិតជិត $+\infty$ ហើយកំណត់សរសេរ

$$\lim_{x \rightarrow +\infty} x^2 = +\infty$$

☞ តាមតារាងតម្លៃលេខខាងលើ បើ x យកតម្លៃអវិជ្ជមានកាន់តែតូចទៅៗគ្មានទីបញ្ចប់ នោះអនុគមន៍ $y = x^2$ មានតម្លៃវិជ្ជមានកាន់តែធំទៅៗគ្នាទីបញ្ចប់។

គេថាអនុគមន៍ $y = x^2$ មានលីមីត $+\infty$ កាលណា x ខិតជិត $-\infty$ ហើយកំណត់សរសេរ

$$\lim_{x \rightarrow -\infty} x^2 = +\infty$$



ជាទូទៅ

ចំពោះចំនួនពិត $a \neq 0$ គេបាន $\lim_{x \rightarrow \pm\infty} ax^2 = \begin{cases} +\infty & \text{បើ } a > 0 \\ -\infty & \text{បើ } a < 0 \end{cases}$

លំហាត់គំនូរទី ១០. គណនាលីមីតខាងក្រោម

ក. $\lim_{x \rightarrow +\infty} 3x^2$

ខ. $\lim_{x \rightarrow -\infty} -5x^3$

គ. $\lim_{x \rightarrow -\infty} 9x^{12}$

ឃ. $\lim_{x \rightarrow \pm\infty} -8x^{2021}$

ដំណោះស្រាយ. គណនាលីមីតខាងក្រោម

ក. $\lim_{x \rightarrow +\infty} 3x^2 =$

ខ. $\lim_{x \rightarrow -\infty} -5x^3 =$

គ. $\lim_{x \rightarrow -\infty} 9x^{12} =$

ឃ. $\lim_{x \rightarrow \pm\infty} -8x^{2021} =$

ឧទាហរណ៍ ៥. គេឱ្យអនុគមន៍ $y = \frac{1}{x}$ និងតារាងតម្លៃលេខ :

x	10	10^2	10^3	10^4	10^5	
$y = \frac{1}{x}$	0.1	0.01	0.001	0.0001	0.00001	

x	-10	-10^2	-10^3	-10^4	-10^5	
$y = \frac{1}{x}$	-0.1	-0.01	-0.001	-0.0001	-0.00001	

- ☞ តាមតារាងតម្លៃលេខខាងលើ បើ x យកយម្លៃវិជ្ជមានកាន់តែធំទៅៗគ្មានទីបញ្ចប់ នោះអនុគមន៍ $y = \frac{1}{x}$ មានតម្លៃវិជ្ជមានកាន់តែតូចទៅៗជិតស្មើសូន្យ។
 គេថាអនុគមន៍ $y = \frac{1}{x}$ មានលីមីត 0 កាលណា x ខិតជិត $+\infty$ ហើយកំណត់សរសេរ

$$\lim_{x \rightarrow +\infty} \frac{1}{x} = 0$$

- ☞ តាមតារាងតម្លៃលេខខាងលើ បើ x យកយម្លៃអវិជ្ជមានកាន់តែតូចទៅៗគ្មានទីបញ្ចប់ នោះអនុគមន៍ $y = \frac{1}{x}$ មានតម្លៃអវិជ្ជមានកាន់តែតូចទៅៗជិតស្មើសូន្យ។
 គេថាអនុគមន៍ $y = \frac{1}{x}$ មានលីមីត 0 កាលណា x ខិតជិត $-\infty$ ហើយកំណត់សរសេរ

$$\lim_{x \rightarrow -\infty} \frac{1}{x} = 0$$

ជាទូទៅ

ចំពោះគ្រប់ចំនួនពិត a គេបាន $\lim_{x \rightarrow \pm\infty} \frac{a}{x} = 0$ ។

លំហាត់គំរូទី ១១. គណនាលីមីតខាងក្រោម

ក. $\lim_{x \rightarrow +\infty} \frac{7}{x^2}$

ខ. $\lim_{x \rightarrow -\infty} \frac{-9}{x+1}$

គ. $\lim_{x \rightarrow -\infty} \frac{2021}{-x}$

ឃ. $\lim_{x \rightarrow \pm\infty} \frac{-1}{x^{12}}$

ដំណោះស្រាយ. គណនាលីមីតខាងក្រោម

ក. $\lim_{x \rightarrow +\infty} \frac{7}{x^2} =$ _____

ខ. $\lim_{x \rightarrow -\infty} \frac{-9}{x+1} =$ _____

គ. $\lim_{x \rightarrow -\infty} \frac{2021}{-x} =$ _____

ឃ. $\lim_{x \rightarrow \pm\infty} \frac{-1}{x^{12}} =$ _____

លំហាត់ប្រតិបត្តិ ៤.**១. គណនាលីមីតខាងក្រោម**

ក. $\lim_{x \rightarrow +\infty} (-4x^2)$

ខ. $\lim_{x \rightarrow +\infty} (2020x^3)$

គ. $\lim_{x \rightarrow -\infty} (-x^3)$

ឃ. $\lim_{x \rightarrow -\infty} (x+1)^3$

២. គណនាលីមីតខាងក្រោម

ក. $\lim_{x \rightarrow +\infty} \frac{3}{x+1}$

ខ. $\lim_{x \rightarrow +\infty} \frac{-5}{x^5}$

គ. $\lim_{x \rightarrow -\infty} \frac{10}{-x^7}$

ឃ. $\lim_{x \rightarrow -\infty} \frac{3}{x^{-3}}$

ខ. លីមីតរាងមិនកំណត់ $\infty - \infty$ **ឧទាហរណ៍ ៦. គណនាលីមីត $\lim_{x \rightarrow +\infty} (x^2 - 3x + 1)$**

បើ $x \rightarrow +\infty$ នោះ $\lim_{x \rightarrow +\infty} (x^2 - 3x + 1)$ មានរាងមិនកំណត់ $\infty - \infty$ ព្រោះ $\lim_{x \rightarrow +\infty} x^2 = +\infty$

និង $\lim_{x \rightarrow +\infty} (-3x + 1) = -\infty$ ។

- ជំហានទី១ : ដាក់តួដែលមានដឺក្រេធំជាងគេជាកត្តារួម

$$x^2 - 3x + 1 = x^2 \left(1 - \frac{3}{x} + \frac{1}{x^2}\right)$$

- ជំហានទី២ : រកលីមីតនៃកន្សោមថ្មី

$$\lim_{x \rightarrow +\infty} (x^2 - 3x + 1) = \lim_{x \rightarrow +\infty} x^2 \left(1 - \frac{3}{x} + \frac{1}{x^2}\right) = (+\infty)^2(1 - 0 + 0) = +\infty$$

ជំនួយ

- បើលីមីតនៃពហុធាមានរាងមិនកំណត់ $\infty - \infty$ នៅត្រង់អនន្ត នោះគេត្រូវ :
 - ដាក់តួដែលមានដឺក្រេធំជាងគេជាកត្តារួម
 - គណនាលីមីតនៃកន្សោមថ្មី។
- លីមីតនៃពហុធានៅត្រង់អនន្ត គឺជាលីមីតនៃតួដែលមានដឺក្រេធំជាងគេ។

លំហាត់គំរូទី ១២. គណនាលីមីតខាងក្រោម

ក. $\lim_{x \rightarrow +\infty} (-3x^2 + 4x - 19)$

ខ. $\lim_{x \rightarrow +\infty} (4x - 7x^2 - 9x^3)$

គ. $\lim_{x \rightarrow -\infty} [(1 - 2x)^7 - (2 - x)^7]$

ឃ. $\lim_{x \rightarrow +\infty} (\sqrt{2x^2 - 3x + 1} - 4x + 4)$

ង. $\lim_{x \rightarrow -\infty} (\sqrt{x^2 - 3x + 1} - 2x)$

ច. $\lim_{x \rightarrow -\infty} (\sqrt[3]{x^3 - 5x + 1} - \sqrt[5]{x^5 + 1})$

ដំណោះស្រាយ. គណនាលីមីតខាងក្រោម

ក. $\lim_{x \rightarrow +\infty} (-3x^2 + 4x - 19) = \lim_{x \rightarrow +\infty} x^2 \left(-3 + \frac{4}{x} - \frac{19}{x^2}\right) = +\infty(-3 + 0 - 0) = -\infty$

ខ. $\lim_{x \rightarrow +\infty} (4x - 7x^2 - 9x^3) = \lim_{x \rightarrow +\infty} x^3 \left(\frac{4}{x^2} - \frac{7}{x} - 9\right) = +\infty(0 - 0 - 9) = -\infty$

គ. $\lim_{x \rightarrow -\infty} [(1 - 2x)^7 - (2 - x)^7] = \lim_{x \rightarrow -\infty} \left\{ \left[x \left(\frac{1}{x} - 2 \right)^7 \right] - \left[x \left(\frac{2}{x} - 1 \right)^7 \right] \right\}$

$$\begin{aligned}
&= \lim_{x \rightarrow -\infty} \left[x^7 \left(\frac{1}{x} - 2 \right)^7 - x^7 \left(\frac{2}{x} - 1 \right)^7 \right] \\
&= \lim_{x \rightarrow -\infty} \left\{ x^7 \left[\left(\frac{1}{x} - 2 \right)^7 - \left(\frac{2}{x} - 1 \right)^7 \right] \right\} \\
&= (-\infty)^7 [(0 - 2)^7 - (0 - 1)^7] = -\infty(-2^7 + 1) = +\infty
\end{aligned}$$

$$\begin{aligned}
\text{ឃ. } \lim_{x \rightarrow +\infty} \left(\sqrt{2x^2 - 3x + 1} - 4x + 4 \right) &= \lim_{x \rightarrow +\infty} \left[\sqrt{x^2 \left(2 - \frac{3}{x} + \frac{1}{x^2} \right)} - 4x + 4 \right] \\
&= \lim_{x \rightarrow +\infty} \left(x \sqrt{2 - \frac{3}{x} + \frac{1}{x^2}} - 4x + 4 \right) \\
&= \lim_{x \rightarrow +\infty} x \left(\sqrt{2 - \frac{3}{x} + \frac{1}{x^2}} - 4 + \frac{4}{x} \right) \\
&= +\infty(\sqrt{2 - 0 + 0} - 4 + 0) = +\infty(\sqrt{2} - 4) = -\infty
\end{aligned}$$

$$\begin{aligned}
\text{ង. } \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 - 3x + 1} - 2x \right) &= \lim_{x \rightarrow -\infty} \left[\sqrt{x^2 \left(1 - \frac{3}{x} + \frac{1}{x^2} \right)} - 2x \right] \\
&= \lim_{x \rightarrow -\infty} \left(|x| \sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} - 2x \right) \\
&= \lim_{x \rightarrow -\infty} \left(-x \sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} - 2x \right)
\end{aligned}$$

ព្រោះ: $x \rightarrow -\infty \implies |x| = -x$

$$\begin{aligned}
&= \lim_{x \rightarrow -\infty} x \left(-\sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} - 2 \right) \\
&= -\infty(-1 - 2) = +\infty
\end{aligned}$$

$$\begin{aligned}
\text{ច. } \lim_{x \rightarrow -\infty} \left(\sqrt[3]{2x^3 - 5x + 1} - \sqrt[5]{x^5 + 1} \right) &= \lim_{x \rightarrow -\infty} \left[\sqrt[3]{2x^3 \left(1 - \frac{5}{x^2} + \frac{1}{x^3} \right)} - \sqrt[5]{x^5 \left(1 + \frac{1}{x^5} \right)} \right] \\
&= \lim_{x \rightarrow -\infty} \left(x \sqrt[3]{2 - \frac{5}{x^2} + \frac{1}{x^3}} - x \sqrt[5]{1 + \frac{1}{x^5}} \right) \\
&= \lim_{x \rightarrow -\infty} x \left(\sqrt[3]{2 - \frac{5}{x^2} + \frac{1}{x^3}} - \sqrt[5]{1 + \frac{1}{x^5}} \right) \\
&= -\infty(\sqrt[3]{2} - 1) = -\infty
\end{aligned}$$

លំហាត់ប្រតិបត្តិ ៥. គណនាលីមីតខាងក្រោម

ក. $\lim_{x \rightarrow -\infty} (x^7 - 6x^5 + 10x)$

ខ. $\lim_{x \rightarrow +\infty} \left(\frac{4}{3}x^2 - \frac{5}{3}x + 2 \right)$

គ. $\lim_{x \rightarrow +\infty} [(x+1)(x-2) - (3x-2)(1-x)]$

ឃ. $\lim_{x \rightarrow -\infty} [(x+1)^7 - 100(x+100)^7]$

ង. $\lim_{x \rightarrow +\infty} (\sqrt{3-2x+x^2} - 3x)$

ច. $\lim_{x \rightarrow -\infty} (\sqrt{4x^2 - 5x + 1} - 7x + 9)$

ឆ. $\lim_{n \rightarrow +\infty} [(2n+1)^3 - (n-3)^3]$

ជ. $\lim_{n \rightarrow +\infty} [n(n+1)^2 - (n+2)^2(n+3)]$

ឈ. $\lim_{n \rightarrow +\infty} [(3n+1)^4 - (4-5n)^4]$

ញ. $\lim_{n \rightarrow +\infty} [(n+1)^5 - (2n-1)^5]$

គ. លីមីតរាងមិនកំណត់ $\frac{\infty}{\infty}$

ឧទាហរណ៍ ៧. គណនា $\lim_{x \rightarrow +\infty} \frac{3x-5}{2x+4}$ និង $\lim_{x \rightarrow -\infty} \frac{3x-5}{2x+4}$

បើ $x \rightarrow +\infty$ ឬ $x \rightarrow -\infty$ នោះ $\lim_{x \rightarrow +\infty} \frac{3x-5}{2x+4}$ និង $\lim_{x \rightarrow -\infty} \frac{3x-5}{2x+4}$ មានរាងមិនកំណត់ $\frac{\infty}{\infty}$

- ជំហានទី១ : ដាក់តួដែលមានដឺក្រេខ្ពស់ជាងគេនៃភាគយក និងភាគបែងជាកត្តារួម ហើយសម្រួលកត្តារួមចោល

$$\frac{3x-5}{2x+4} = \frac{x \left(3 - \frac{5}{x} \right)}{x \left(2 + \frac{4}{x} \right)} = \frac{3 - \frac{5}{x}}{2 + \frac{4}{x}}$$

- ជំហានទី២ : គណនាលីមីតនៃកន្សោមថ្មី

$$\begin{aligned} \circ \lim_{x \rightarrow +\infty} \frac{3x-5}{2x+4} &= \lim_{x \rightarrow +\infty} \frac{3 - \frac{5}{x}}{2 + \frac{4}{x}} = \frac{3-0}{2+0} = \frac{3}{2} \\ \circ \lim_{x \rightarrow -\infty} \frac{3x-5}{2x+4} &= \lim_{x \rightarrow -\infty} \frac{3 - \frac{5}{x}}{2 + \frac{4}{x}} = \frac{3-0}{2+0} = \frac{3}{2} \end{aligned}$$

ជាទូទៅ

- បើលីមីតនៃប្រភាគសនិទានមានរាងមិនកំណត់ $\frac{\infty}{\infty}$ នៅត្រង់អនន្ត នោះគេត្រូវ :
 - ដាក់តួដែលមានដឺក្រេធំជាងគេនៅភាគយក នៅភាគបែងជាកត្តារួម ហើយសម្រួលចោល
 - គណនាលីមីតនៃប្រភាគថ្មី។
- លីមីតនៃប្រភាគសនិទាននៅត្រង់អនន្ត គឺជាផលធៀបរវាងលីមីតនៃតួដែលមានដឺក្រេធំជាងគេនៅភាគយក និងលីមីតនៃតួដែលមានដឺក្រេធំជាងគេនៅភាគបែង។

លំហាត់គំរូទី ១៣. គណនាលីមីតខាងក្រោម

$$ក. \lim_{x \rightarrow +\infty} \frac{x^2 - 3x + 1}{2x^2 + 4x - 1}$$

$$គ. \lim_{x \rightarrow +\infty} \frac{(x+1)^2 + (2-x)^2}{(x+1)^3 - x^3}$$

$$ង. \lim_{x \rightarrow +\infty} \frac{x^3 + (2x-1)^2}{x(x+1) - (3x-1)^2}$$

$$ឆ. \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 4x + 1} + 7x}{\sqrt{9x^2 - 8x + 1} - 2x}$$

$$ខ. \lim_{x \rightarrow -\infty} \frac{4 - 5x + 6x^2}{(3x - 1)^2}$$

$$ឃ. \lim_{x \rightarrow -\infty} \frac{5x^3 - 3x^2 + 8}{3x(2-x)^3}$$

$$ច. \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 6x - 9} + 5x}{3x + 8}$$

$$ជ. \lim_{n \rightarrow +\infty} \frac{(n+1)!}{n! + (n+2)!}$$

លំហាត់ប្រតិបត្តិ ៦.**១. គណនាលីមីតខាងក្រោម**

$$ក. \lim_{x \rightarrow +\infty} \frac{1 - x^2 + 7x^3}{5 + x - x^3}$$

$$គ. \lim_{x \rightarrow +\infty} \frac{x^2 + 2x - 12}{1 - x^3}$$

$$ង. \lim_{x \rightarrow +\infty} \frac{x - \sqrt{9x^2 - 1}}{\sqrt{x^2 + 1} + 3x}$$

$$ឆ. \lim_{x \rightarrow +\infty} \frac{(3-x)^4 - (2-x)^4}{(1-x)^4 + (1+x)^4}$$

$$ខ. \lim_{x \rightarrow -\infty} \frac{x^5 + 2x + 7}{x^2 + 12x - 1}$$

$$ឃ. \lim_{x \rightarrow -\infty} \frac{3x^2 + 1}{\sqrt{4x^2 + 4}}$$

$$ច. \lim_{x \rightarrow +\infty} \frac{(3-x)^2 - (3+x)^2}{(3-x)^2 + (3+x)^2}$$

$$ជ. \lim_{x \rightarrow \pm\infty} \frac{x + 10(x-1)^2}{2x + 3(3x-2)(x-3)}$$

$$\text{ឈ.} \quad \lim_{x \rightarrow +\infty} \frac{(x+2)(2x+3)}{x^2+1}$$

$$\text{ដ.} \quad \lim_{n \rightarrow \pm\infty} \frac{(3-n)^4 - (2-n)^4}{(1-n)^3 + (1+n)^3}$$

$$\text{ខ.} \quad \lim_{n \rightarrow \pm\infty} \frac{(3-4n)^2}{(n-3)^3 - (n+3)^3}$$

$$\text{គ.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{x}}$$

$$\text{ថ.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{x}}$$

$$\text{ឆ.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+2x-3} + 2x}{\sqrt{x^2+4} + x}$$

$$\text{ប.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt[4]{2+x^4}}{\sqrt[3]{5+27x^3}}$$

$$\text{ព.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + \sqrt{x} + \sqrt{x}}{\sqrt{x} + 1}$$

$$\text{ម.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + \sqrt{x+1}}{x+1}$$

$$\text{រ.} \quad \lim_{n \rightarrow +\infty} \frac{n\sqrt[3]{5n^2} + \sqrt[4]{9n^8+1}}{(n+\sqrt{n})\sqrt{7-n} + n^2}$$

$$\text{ស.} \quad \lim_{n \rightarrow +\infty} \frac{\sqrt{n^3+1} - \sqrt{n-1}}{\sqrt[3]{n^3+1} - \sqrt{n-1}}$$

$$\text{ហ.} \quad \lim_{n \rightarrow +\infty} \frac{\sqrt{3n-1} - \sqrt[3]{125n^3+n}}{\sqrt[3]{n} - n}$$

$$\text{ញ.} \quad \lim_{x \rightarrow +\infty} \frac{(4x^2+1)^3(3x^3+2)^2}{6(2x^3+1)^4}$$

$$\text{ដ.} \quad \lim_{n \rightarrow \pm\infty} \frac{(1+2n)^3 - 8n^3}{(1+2n)^2 + 4n^2}$$

$$\text{ឆ.} \quad \lim_{n \rightarrow \pm\infty} \frac{(3-n)^3}{(n+1)^2 - (n+1)^3}$$

$$\text{គ.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + 3x}{2\sqrt{x} - 4x}$$

$$\text{ថ.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+3}}$$

$$\text{ឆ.} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt[4]{x} + 1}{\sqrt[3]{x^2}}$$

$$\text{ប.} \quad \lim_{x \rightarrow \pm\infty} \frac{\sqrt{x^2+1} - x + 2}{x+3}$$

$$\text{ព.} \quad \lim_{x \rightarrow +\infty} \frac{x + \sqrt{x^2-1}}{\sqrt{x^2-4}}$$

$$\text{ម.} \quad \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+3} + x}{x+2}$$

$$\text{ឈ.} \quad \lim_{n \rightarrow +\infty} \frac{\sqrt{n-1} - \sqrt{n^2-1}}{\sqrt[3]{3n^3+3} + \sqrt[5]{n^6+1}}$$

$$\text{ស.} \quad \lim_{n \rightarrow +\infty} \frac{\sqrt[3]{n^2-1} + 7n^3}{\sqrt[4]{n^{12}+n+1} - n}$$

$$\text{ហ.} \quad \lim_{n \rightarrow +\infty} \frac{n\sqrt[5]{n} - \sqrt[3]{27n^6+n^2}}{(n+\sqrt[4]{n})\sqrt{9+n^2}}$$

២. គណនាលីមីតខាងក្រោម

$$\text{ក.} \quad \lim_{x \rightarrow +\infty} (\sqrt{x^2-1} - x)$$

$$\text{គ.} \quad \lim_{x \rightarrow +\infty} (\sqrt{x^2+2} - x + 1)$$

$$\text{ង.} \quad \lim_{x \rightarrow -\infty} (\sqrt{x^2+8x-1} - \sqrt{x^2-3})$$

$$\text{ខ.} \quad \lim_{x \rightarrow +\infty} (\sqrt{x^2+3x-1} - x)$$

$$\text{ឃ.} \quad \lim_{x \rightarrow +\infty} (\sqrt{x} + \sqrt{x} - \sqrt{x})$$

$$\text{ច.} \quad \lim_{x \rightarrow -\infty} (\sqrt{x^2+2} - \sqrt{x^2-3})$$

$$\text{ឆ. } \lim_{x \rightarrow +\infty} (\sqrt{x+1} - \sqrt{x^2+3})$$

$$\text{ជ. } \lim_{x \rightarrow +\infty} (\sqrt{x^2+x} - x^2)$$

$$\text{ឈ. } \lim_{x \rightarrow \pm\infty} (x\sqrt{x^2+1} - x)$$

$$\text{ញ. } \lim_{x \rightarrow +\infty} (\sqrt{2x^2+1} - \sqrt{x+x^2})$$

$$\text{ដ. } \lim_{x \rightarrow +\infty} (\sqrt{2x^2+x} + 3x - 1)$$

$$\text{ថ. } \lim_{x \rightarrow +\infty} (\sqrt{x^2+x} - \sqrt[3]{x^3-x})$$

$$\text{ខ. } \lim_{x \rightarrow +\infty} (\sqrt{x^2+x} - x)$$

$$\text{ឈ. } \lim_{x \rightarrow +\infty} (\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x})$$

$$\text{ណ. } \lim_{x \rightarrow +\infty} (\sqrt{x^2-1} - \sqrt{x^2+3x})$$

$$\text{ត. } \lim_{x \rightarrow +\infty} (\sqrt{x^2+2x} - \sqrt{x^2-4x})$$

$$\text{ថ. } \lim_{x \rightarrow +\infty} (\sqrt[4]{x^4+4} - x)$$

$$\text{ទ. } \lim_{x \rightarrow +\infty} (\sqrt[3]{x^3+3x^2} - \sqrt{x^2-2x})$$

៣. គណនាលីមីតខាងក្រោម

$$\text{ក. } \lim_{n \rightarrow +\infty} n (\sqrt{n^2+1} - \sqrt{n^2-1})$$

$$\text{ខ. } \lim_{n \rightarrow +\infty} n (\sqrt{n(n-2)} - \sqrt{n^2-3})$$

$$\text{គ. } \lim_{n \rightarrow +\infty} (n - \sqrt[3]{n^3-5}) n\sqrt{n}$$

$$\text{ឃ. } \lim_{n \rightarrow +\infty} (\sqrt{(n^2+1)(n^2-4)} - \sqrt{n^4-9})$$

$$\text{ង. } \lim_{n \rightarrow +\infty} \frac{1}{\sqrt{n}} (\sqrt{n^5-8} - n\sqrt{n(n^2+5)})$$

$$\text{ច. } \lim_{n \rightarrow +\infty} (\sqrt{n^2-3n+2} - n)$$

$$\text{ឆ. } \lim_{n \rightarrow +\infty} (n + \sqrt[3]{4-n^3})$$

$$\text{ជ. } \lim_{n \rightarrow +\infty} (\sqrt{n(n+2)} - \sqrt{n^2-2n+3})$$

$$\text{ឈ. } \lim_{n \rightarrow +\infty} (\sqrt{n^2+3n-2} - \sqrt{n^2-3})$$

$$\text{ញ. } \lim_{n \rightarrow +\infty} n^2 (\sqrt{n(n^4-1)} - \sqrt{n^5-1})$$

$$\text{ដ. } \lim_{n \rightarrow +\infty} n (\sqrt[3]{5+8n^3} - \sqrt[3]{3+n^3})$$

$$\text{ថ. } \lim_{n \rightarrow +\infty} (\sqrt[3]{(n+2)^2} - \sqrt[3]{(n-3)^2})$$

$$\text{ខ. } \lim_{n \rightarrow +\infty} \frac{\sqrt{(n+1)^3} - \sqrt{n(n-1)(n-3)}}{\sqrt{n}}$$

$$\text{ឈ. } \lim_{n \rightarrow +\infty} (\sqrt{(n+2)(n+1)} - \sqrt{(n-1)(n+3)})$$

$$\text{ណ. } \lim_{n \rightarrow +\infty} \sqrt{n} (\sqrt{n+2} - \sqrt{n-3})$$

$$\text{ត. } \lim_{n \rightarrow +\infty} \frac{\sqrt{n(n^5+9)} - \sqrt{(n^4-1)(n^2+5)}}{n}$$

$$\text{ថ. } \lim_{n \rightarrow +\infty} (\sqrt{b(n+5)} - n)$$

$$\text{ទ. } \lim_{n \rightarrow +\infty} \sqrt{n^3+8} (\sqrt{n^3+2} - \sqrt{n^3-1})$$

$$\text{ឆ. } \lim_{n \rightarrow +\infty} \frac{\sqrt{(n^3+1)(n^2+3)} - \sqrt{n(n^4+2)}}{2\sqrt{n}}$$

$$\text{ន. } \lim_{n \rightarrow +\infty} \frac{\sqrt{(n^5-1)(n^2-1)} - n\sqrt{n(n^4+1)}}{n}$$

$$\text{ប. } \lim_{n \rightarrow +\infty} \frac{\sqrt{(n^5+1)(n^2-1)} - \sqrt{n^6-1}}{n}$$

$$\text{ដ. } \lim_{n \rightarrow +\infty} (n - \sqrt{n(n-1)})$$

ឃ. លីមីតនៃស្វ៊ីត

ឧទាហរណ៍ ៨. គេឲ្យស្វ៊ីតចំនួនពិត (u_n) កំណត់ដោយ $u_n = \frac{2n+1}{n^2(n+1)^2}$ ។

ក. កំណត់ចំនួនពិត A និង B ដើម្បីឲ្យ $u_n = \frac{A}{n^2} + \frac{B}{(n+1)^2}$

ខ. គណនាលីមីតនៃ $S_n = u_1 + u_2 + \cdots + u_n$ កាលណា $n \rightarrow +\infty$ ។

ដំណោះស្រាយ. គេមាន $u_n = \frac{2n+1}{n^2(n+1)^2}$

ក. កំណត់ចំនួនពិត A និង B

$$\text{គេមាន } u_n = \frac{A}{n^2} + \frac{B}{(n+1)^2} = \frac{A(n+1)^2 + Bn^2}{n^2(n+1)^2} = \frac{(A+B)n^2 + (2A)n + A}{n^2(n+1)^2}$$

$$\text{គេបាន } \frac{(A+B)n^2 + (2A)n + A}{n^2(n+1)^2} = \frac{2n+1}{n^2(n+1)^2}$$

$$\implies (A+B)n^2 + (2A)n + A = 2n+1$$

$$\text{ផ្អែមមេគុណត្រូវគ្នា គេបាន } \begin{cases} A+B=0 \\ 2A=2 \\ A=1 \end{cases} \iff \begin{cases} A=1 \\ B=-1 \end{cases}$$

$$\text{ដូចនេះ: } \boxed{A=1, B=-1}$$

ខ. គណនាលីមីតនៃ $S_n = u_1 + u_2 + \cdots + u_n$ កាលណា $n \rightarrow +\infty$

$$\text{គេមាន } u_n = \frac{2n+1}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$\text{គេបាន } S_n = u_1 + u_2 + u_3 + \cdots + u_n$$

$$= \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \cdots + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$= 1 - \frac{1}{(n+1)^2}$$

$$\text{ដូចនេះ: } \lim_{n \rightarrow +\infty} S_n = \lim_{n \rightarrow +\infty} \left[1 - \frac{1}{(n+1)^2}\right] = 1$$

លំហាត់ប្រតិបត្តិ ៧.

១. គណនាលីមីតខាងក្រោម

$$\text{ក. } \lim_{n \rightarrow +\infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \cdots + \frac{n-1}{n^2} \right)$$

$$\text{ខ. } \lim_{n \rightarrow +\infty} \left[\frac{1 + 3 + 5 + \cdots + (2n-1)}{n+1} - \frac{2n+1}{2} \right]$$

$$\text{គ. } \lim_{n \rightarrow +\infty} \frac{1 + 2 + 3 + \cdots + n}{\sqrt{9n^4 + 1}}$$

$$\text{ឃ. } \lim_{n \rightarrow +\infty} \frac{1 + 3 + 5 + \cdots + (2n-1)}{1 + 2 + 3 + \cdots + n}$$

$$\text{ង. } \lim_{n \rightarrow +\infty} \left[\frac{1 + 3 + 5 + \cdots + (2n-1)}{n+3} - n \right]$$

$$\text{ច. } \lim_{n \rightarrow +\infty} \frac{1 + 4 + 7 + \cdots + (3n-2)}{\sqrt{5n^4 + n + 1}}$$

$$\text{ឆ. } \lim_{n \rightarrow +\infty} \frac{1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^n}}{1 + \frac{1}{5} + \frac{1}{5^2} + \cdots + \frac{1}{5^n}}$$

$$\text{ជ. } \lim_{n \rightarrow +\infty} \frac{1 - 3 + 5 - 7 + 9 - 11 + \cdots + (4n-3) - (4n-1)}{\sqrt{n^2 + 1} + \sqrt{n^2 + n + 1}}$$

$$\text{ឈ. } \lim_{n \rightarrow +\infty} \left(\frac{5}{6} + \frac{13}{36} + \cdots + \frac{3^n + 2^n}{6^n} \right)$$

$$\text{ញ. } \lim_{n \rightarrow +\infty} \left(\frac{3}{4} + \frac{5}{16} + \frac{9}{64} + \cdots + \frac{1 + 2^n}{4^n} \right)$$

$$\text{ដ. } \lim_{n \rightarrow +\infty} \frac{2 + 4 + 6 + \cdots + (2n)}{1 + 3 + 5 + \cdots + (2n-1)}$$

$$\text{ប. } \lim_{n \rightarrow +\infty} \left(\frac{1 + 5 + 9 + \cdots + (4n-3)}{n+1} - \frac{4n+1}{2} \right)$$

$$\text{ខ្ល. } \lim_{n \rightarrow +\infty} \left(\frac{7}{10} + \frac{29}{100} + \cdots + \frac{2^n + 5^n}{10^n} \right)$$

$$\text{ឈ. } \lim_{n \rightarrow +\infty} \left[\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} \right]$$

$$\text{ឈា. } \lim_{n \rightarrow +\infty} \left[\frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{n}{(n+1)!} \right]$$

៣. គណនាលីមីត $\lim_{n \rightarrow +\infty} \left[\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)} \right]$

៤. គណនាលីមីត $\lim_{x \rightarrow 1} \sum_{r=1}^n \frac{x^r - 1}{x - 1}$ ។

៥. គណនាលីមីត $\lim_{n \rightarrow +\infty} \frac{1}{n^4} \left[\left(\sum_{k=1}^n k \right) + 2 \left(\sum_{k=1}^{n-1} k \right) + 3 \left(\sum_{k=1}^{n-2} k \right) + \cdots + n \right]$ ។

៦. គណនាលីមីត $\lim_{n \rightarrow +\infty} \prod_{r=3}^n \left(\frac{r^3 - 1}{r^3 + 1} \right)$ ។

៧. គណនាលីមីត $\lim_{n \rightarrow +\infty} \frac{1}{n^3} \left(\sum_{k=1}^n \lfloor k^2 x \rfloor \right)$ ដែល $\lfloor x \rfloor$ តាងឱ្យផ្នែកគត់ធំបំផុត $\leq x$ ។

៨. គេឱ្យ $f(x) = x - \lfloor x \rfloor$ ។ គណនាលីមីត $\lim_{n \rightarrow +\infty} \frac{[f(x)]^{2n} - 1}{[f(x)]^{2n} + 1}$ ។

៩. គណនាលីមីត $\lim_{x \rightarrow 5} \frac{x^2 - 9x + 20}{x - \lfloor x \rfloor}$ ដែល $\lfloor x \rfloor$ តាងឱ្យផ្នែកគត់ធំបំផុត $\leq x$ ។

១០. គណនាលីមីត $\lim_{n \rightarrow +\infty} \frac{\{x\} + \{2x\} + \{3x\} + \cdots + \{nx\}}{n^2}$ ដែល $\{k\} = k - \lfloor k \rfloor$ ផ្នែកទសភាគនៃ k ។

១១. តាង $f(n) = (n^2 + n + 1)^2 + 1$ ។ គេកំណត់ស្វ៊ីត

$$U_n = \frac{f(1) \times f(3) \times f(5) \times \cdots \times f(2n-1)}{f(2) \times f(4) \times f(6) \times \cdots \times f(2n)} \quad \text{ដែល } n \in \mathbb{N}$$

គណនា $\lim_{n \rightarrow \infty} \left(n \sqrt{U_n} \right)$ ។

១២. គេឱ្យស្វ៊ីតពីរ (a_n) និង (b_n) ជាស្វ៊ីតនៃចំនួនពិតធម្មជាតិដែលផ្ទៀងផ្ទាត់

$$a_n + \sqrt{2}b_n = (2 + \sqrt{2})^n$$

គណនា $\lim_{n \rightarrow \infty} \frac{a_n}{b_n}$ ។