គណិតវិទ្យាថ្ងៃនេះ

Mathtoday

សៀវភៅគណិតវិទ្យាកម្រិតវិទ្យាល័យ

លីមីតនៃអនុគមន៍

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សម្រាប់ថ្នាក់ទី១២ ថ្នាក់វិទ្យាសាស្ត្រពិតនិងថ្នាក់វិទ្យាសាស្ត្រសង្គម

អ្នករៀបរៀង លឹម ផល្គុន

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ស្របតាមកម្មវិធីសិក្សាគោលរបស់ក្រសួងអប់រំយុវជននិងកីឡា

ជំពូកទី០១

សង្ខេបមេរៀនលីមីតនៃអនុគមន៍

១-លីមីតនៃអនុគមន៍ត្រង់ចំនួនកំណត់

✧ **និយមន័យ**

អនុគមន៍ f មានលីមីត L កាលណា x ខិតជិត a បើគ្រប់ចំនួន $\varepsilon > 0$ មានចំនួន $\delta > 0$ ដែល $0 < |x-a| < \delta$ នាំឲ្យ $|f(x)-L| < \varepsilon$ ។

គេកំណត់សរសេរ $\lim\limits_{x \to a} f(x) = L$ ។

ឧទាហរណ៍ ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x \to 2}(3x+1) = 7$?

យើងត្រូវបង្ហាញថា ចំពោះគ្រប់ចំនួន $\varepsilon > 0$ មាន $\delta > 0$ ដែល $|(3x+1)-7| < \varepsilon$ កាលណា $0 < |x-2| < \delta$ ។

គេបាន $|(3x+1)-7| < \varepsilon \Leftrightarrow |3(x-2)| < \varepsilon$ គ្រប់ $\varepsilon > 0$

សមមូល $|x-2| < \dfrac{\varepsilon}{3}$ យក $\delta = \dfrac{\varepsilon}{3}$ នោះ $|x-2| < \delta$ ។

សម្រាយនេះបញ្ជាក់ថាគ្រប់ $\varepsilon > 0$ មាន $\delta = \dfrac{\varepsilon}{2} > 0$ ដែល $|x-2| < \delta$ នាំឲ្យ $|(3x+1)-7| < \varepsilon$ ។ ដូចនេះ $\lim\limits_{x \to 2}(3x+1) = 7$ ។

✧ **និយមន័យ**

អនុគមន៍ f ខិតទៅរក $+\infty$ ឬ $-\infty$ កាលណា x ខិតជិត a បើគ្រប់ចំនួន $M > 0$ មានចំនួន $\delta > 0$ ដែល $0 < |x-a| < \delta$ នាំឲ្យ $f(x) > M$ ឬ $f(x) < -M$ ។

គេកំណត់សរសេរ $\lim\limits_{x \to a} f(x) = +\infty$ ឬ $\lim\limits_{x \to a} f(x) = -\infty$ ។

ឧទាហរណ៍ គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\dfrac{2x+3}{x-2}$

ចូរស្រាយតាមនិយមន័យថា $\lim\limits_{x\to 2^+} f(x)=+\infty$ ។

គេមាន $f(x)=\dfrac{2x+3}{x-2}=\dfrac{2(x-2)+7}{x-2}=2+\dfrac{7}{x-2}$ ។ យើងនឹងរកចំនួន $M>0$

ដែល $f(x)>M$ ។ ដើម្បីឲ្យ $f(x)>M$ យើងគ្រាន់តែឲ្យ $\dfrac{7}{x-2}>M$ និង $x>2$

គេទាញបាន $0<x-2<\dfrac{7}{M}$ យក $\delta=\dfrac{7}{M}>0$ នោះ $0<x-2<\delta$

សម្រាយនេះបញ្ជាក់ថាគ្រប់ចំនួន $M>0$ មាន $\delta=\dfrac{7}{M}>0$

ដែល $0<|x-2|<\delta$ នាំឲ្យ $f(x)>M$ ។ ដូចនេះ $\lim\limits_{x\to 2^+} f(x)=+\infty$ ។

២-លីមីតនៃអនុគមន៍ត្រង់អនន្ត

✧ **និយមន័យ** អនុគមន៍ f មានលីមីត L កាលណា x ខិតទៅ $+\infty$ ឬ $-\infty$

បើគ្រប់ចំនួន $\varepsilon>0$ មានចំនួន $N>0$ ដែល $x>N$ ឬ $x<-N$ នាំឲ្យ

$|f(x)-L|<\varepsilon$ ។ គេកំណត់សរសេរ $\lim\limits_{x\to+\infty} f(x)=L$ ឬ $\lim\limits_{x\to-\infty} f(x)=L$ ។

ឧទាហរណ៍ គេមានអនុគមន៍ f កំណត់ដោយ $f(x)=\dfrac{4x+1}{2x+3}$

ចូរស្រាយថា $\lim\limits_{x\to-\infty} f(x)=2$ និង $\lim\limits_{x\to+\infty} f(x)=2$ ។

គេមាន $f(x)=\dfrac{4x+1}{2x+3}=\dfrac{2(2x+3)-5}{2x+3}=2-\dfrac{5}{2x+3}$ នាំឲ្យ $|f(x)-2|=\dfrac{5}{|2x+3|}$

ចំពោះ $|f(x)-2|<\varepsilon$, $\varepsilon>0$ គេបាន $\dfrac{5}{|2x+3|}<\varepsilon$ នាំឲ្យ $|2x+3|>\dfrac{5}{\varepsilon}$

គេទាញ $\left[\begin{array}{l}2x+3>\dfrac{5}{\varepsilon}\\[2ex] 2x+3<-\dfrac{5}{\varepsilon}\end{array}\right.$ សមមូល $\left[\begin{array}{l}x>\dfrac{5}{2\varepsilon}-\dfrac{3}{2}\\[2ex] x<-\dfrac{5}{2\varepsilon}-\dfrac{3}{2}\end{array}\right.$ ហេតុនេះចំពោះគ្រប់ $\varepsilon>0$

មាន $A=\dfrac{5}{2\varepsilon}$ ឬ $A=\dfrac{5}{2\varepsilon}+\dfrac{3}{2}$ ដែល $x>A$ ឬ $x<-A$ នាំឲ្យ $|f(x)-2|<\varepsilon$

ដូចនេះ $\lim_{x\to -\infty} f(x)=2$ និង $\lim_{x\to +\infty} f(x)=2$ ។

✧ **និយមន័យ** អនុគមន៍ f មានលីមីត $+\infty$ កាលណា x ខិតទៅ $+\infty$ បើគ្រប់ចំនួន $M>0$ មានចំនួន $N>0$ ដែល $x>N$ នាំឲ្យ $f(x)>M$ ។

គេកំណត់សរសេរ $\lim_{x\to +\infty} f(x)=+\infty$ ។

✧ **និយមន័យ** អនុគមន៍ f មានលីមីត $+\infty$ កាលណា x ខិតទៅ $-\infty$ បើគ្រប់ចំនួន $M>0$ មានចំនួន $N>0$ ដែល $x<-N$ នាំឲ្យ $f(x)>M$ ។

គេកំណត់សរសេរ $\lim_{x\to -\infty} f(x)=+\infty$ ។

៣-ប្រមាណវិធីលើលីមីត

បើ $\lim_{x\to a} f(x)=L$, $\lim_{x\to a} g(x)=M$ និង $\lim_{x\to a} h(x)=N$ ដែល L , M និង N ជាចំនួនពិតនោះគេបាន ៖

ក. $\lim_{x\to a}[f(x)+g(x)-h(x)]=L+M-N$

ខ. $\lim_{x\to a}[\alpha f(x)+\beta g(x)-\gamma h(x)]=\alpha L+\beta M-\gamma N$ ($\alpha,\beta,\gamma\in\mathbb{R}$)។

គ. $\lim_{x\to a}[f(x)g(x)h(x)]=L.M.N$

ឃ. $\lim_{x\to a}[\frac{f(x)}{g(x)}]=\frac{L}{M}$ ដែល $M\neq 0$

៤-លីមីតនៃអនុគមន៍អសនិទាន

រូបមន្តសំខាន់ៗគួរកត់សម្គាល់ ៖

ក. $\lim_{x\to a}\sqrt[n]{x}=\sqrt[n]{a}$

ខ. $\lim_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to a} f(x)}$

៥-លីមីតនៃអនុគមន៍បណ្តាក់

បើ f និង g ជាអនុគមន៍ពីរដែលមានលីមីត $\lim_{x\to a} g(x) = L$ និង $\lim_{x\to L} f(x) = f(L)$

នោះ $\lim_{x\to a} f[g(x)] = f(L)$ ។

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to 2} \ln(\frac{4x+7}{x+3})$

តាង $g(x) = \frac{4x+7}{x+3}$ និង $f(x) = \ln x$ នោះ $f[g(x)] = \ln(\frac{4x+7}{x+3})$

គេមាន $\lim_{x\to 2} g(x) = \lim_{x\to 2} \frac{4x+7}{x+3} = \frac{8+7}{2+3} = 3$

ដូចនេះ $\lim_{x\to 2} f[g(x)] = \lim_{x\to 3} f(x) = f(3) = \ln 3$ ។

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to +\infty} \ln(\frac{4x+7}{x+3})$

តាង $g(x) = \frac{4x+7}{x+3}$ និង $f(x) = \ln x$ នោះ $f[g(x)] = \ln(\frac{4x+7}{x+3})$

គេមាន $\lim_{x\to +\infty} g(x) = \lim_{x\to +\infty} \frac{4x+7}{x+3} = \lim_{x\to +\infty} \frac{4+\frac{7}{x}}{1+\frac{3}{x}} = \frac{4}{1} = 4$

ដូចនេះ $\lim_{x\to +\infty} f[g(x)] = \lim_{x\to 4} f(x) = \lim_{x\to 4} \ln x = \ln 4 = 2\ln 2$ ។

៦-លីមីតតាមការប្រៀបធៀប

ក.បើ f និង g ជាអនុគមន៍ពីរហើយ A ជាចំនួនពិតមួយដែលចំពោះ $\forall x \ge A$ គេមាន

$f(x) \ge g(x)$ និង $\lim_{x\to +\infty} g(x) = +\infty$ នោះ $\lim_{x\to +\infty} f(x) = +\infty$ ។

ខ.បើ f និង g ជាអនុគមន៍ពីរហើយ A ជាចំនួនពិតមួយដែលចំពោះ $\forall x \ge A$ គេមាន

$f(x) \le g(x)$ និង $\lim_{x\to +\infty} g(x) = -\infty$ នោះ $\lim_{x\to +\infty} f(x) = -\infty$ ។

គ.បើ f, g និង h ជាអនុគមន៍បី ហើយ A ជាចំនួនពិតមួយដែលចំពោះ $\forall x \ge A$

គេមាន $g(x) \le f(x) \le h(x)$ និង $\lim_{x\to +\infty} g(x) = \lim_{x\to +\infty} h(x) = \lambda$ នោះ $\lim_{x\to +\infty} f(x) = \lambda$

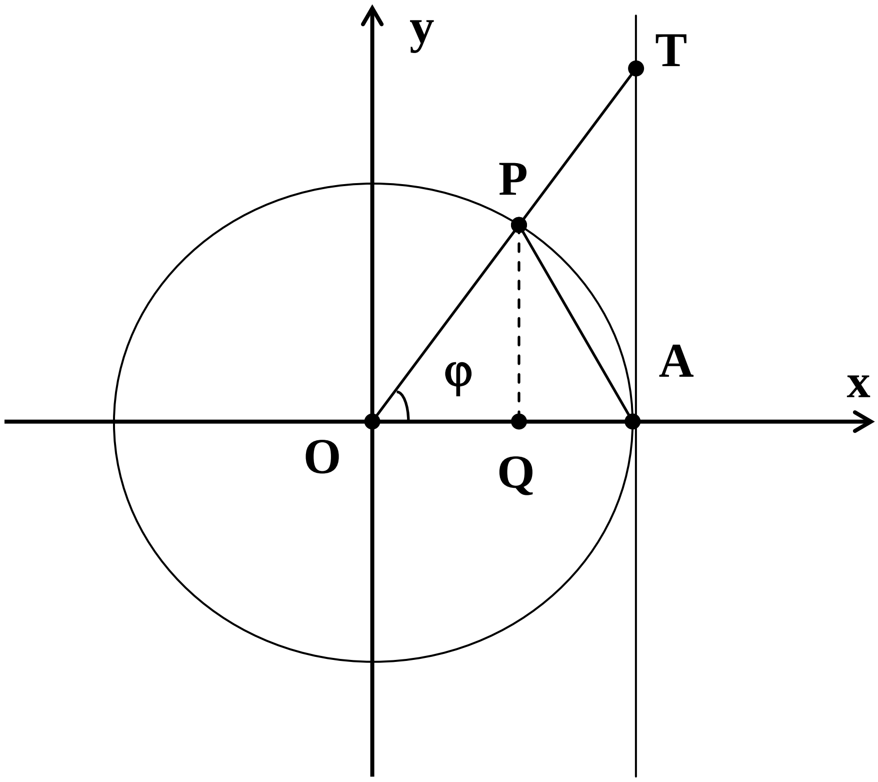
ឃ.បើ f និង g ជាអនុគមន៍ពីរ ហើយ A ជាចំនួនពិតមួយដែលចំពោះ $\forall x \geq A$ គេមាន $f(x) \leq g(x)$ និង $\lim\limits_{x\to+\infty} f(x) = \lambda$, $\lim\limits_{x\to+\infty} g(x) = \lambda'$ នោះ $\lambda \leq \lambda'$ ។ (λ និង λ' ជាចំនួនពិត) ។

៨-លីមីតនៃអនុគមន៍ត្រីកោណមាត្រ

ទ្រឹស្តីបទ បើ x ជារង្វាស់មុំ ឬ ធ្នូគិតជារ៉ាដ្យង់នោះគេបាន ៖

ក. $\lim\limits_{x\to0}\dfrac{\sin x}{x} = 1$ ខ. $\lim\limits_{x\to0}\dfrac{1-\cos x}{x} = 0$ គ. $\lim\limits_{x\to0}\dfrac{\tan x}{x} = 1$

សម្រាយបញ្ជាក់ ៖



តាង φ ជាមុំគិតជារ៉ាដ្យង់ ដែល $0 < \varphi < \dfrac{\pi}{2}$ ។ តាង S_{OAT} , $S_{O\overset{\frown}{AP}}$ និង S_{OAP} រៀងគ្នាជាផ្ទៃក្រឡានៃត្រីកោណOAT ផ្ទៃក្រឡាចំរៀកថាស $O\overset{\frown}{AP}$ និងផ្ទៃក្រឡានៃត្រីកោណ OAP។ តាមរូបខាងលើគេមាន $S_{OAT} \geq S_{O\overset{\frown}{AP}} \geq S_{OAP}$

ដោយ $S_{OAT} = \dfrac{1}{2}\times1\times\tan\varphi$, $S_{O\overset{\frown}{AP}} = \dfrac{1}{2}\times1^2\times\varphi$ និង $S_{OAP} = \dfrac{1}{2}\sin\theta$

នោះគេបាន $\frac{1}{2}\tan\varphi \geq \frac{1}{2}\varphi \geq \frac{1}{2}\sin\varphi$ ឬ $\tan\varphi \geq \varphi \geq \sin\varphi$

ដោយ $\tan\varphi = \frac{\sin\varphi}{\cos\varphi}$ នោះ $\frac{\sin\varphi}{\cos\varphi} \geq \varphi \geq \sin\varphi$ នោះគេទាញ $\cos\varphi \leq \frac{\sin\varphi}{\varphi} \leq 1$ ។

បើ $-\frac{\pi}{2} < \varphi < 0$ នោះ $0 < -\varphi < \frac{\pi}{2}$ នោះវិសមភាពខាងលើអាចសរសេរទៅជា

$\cos(-\varphi) \leq \frac{\sin(-\varphi)}{-\varphi} \leq 1$ ឬ $\cos\varphi \leq \frac{\sin\varphi}{\varphi} \leq 1$

ហេតុនេះគេបាន $\cos\varphi \leq \frac{\sin\varphi}{\varphi} \leq 1$ ចំពោះគ្រប់ $\varphi \in (-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2})$

ដោយ $\lim_{\varphi\to 0}\cos\varphi = 1$ នោះ $\lim_{x\to 0}\frac{\sin\varphi}{\varphi} = 1$ ។

ដោយជំនួស φ ដោ x នោះគេបាន $\lim_{x\to 0}\frac{\sin x}{x} = 1$ ពិត ។

ម្យ៉ាងទៀត $\lim_{x\to 0}\frac{1-\cos x}{x} = \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)}$

$$= \lim_{x\to 0}\frac{\sin x}{x}.\frac{\sin x}{1+\cos x} = 1\times 0 = 0$$

ដូចនេះ $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$ ពិត ។ តាមរូបមន្ត $\tan x = \frac{\sin x}{x}$

គេបាន $\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\sin x}{x}.\frac{1}{\cos x} = 1\times 1 = 1$ ។ ដូចនេះ $\lim_{x\to 0}\frac{\tan x}{x} = 1$ ។

សម្គាល់ ៖

ក. $\lim_{x\to 0}\frac{x}{\sin x} = 1$ ខ. $\lim_{x\to 0}\frac{x}{\tan x} = 1$

៩-លីមីតនៃអនុគមន៍អិចស្ប៉ូណង់ស្យែល

រូបមន្តសំខាន់ៗ ៖

ក. $\lim_{x\to+\infty} e^x = +\infty$ ខ. $\lim_{x\to-\infty} e^x = 0$ គ. $\lim_{x\to+\infty}\frac{e^x}{x} = +\infty$

ឃ. $\lim_{x\to+\infty}\frac{x}{e^x} = 0$ ង. $\lim_{x\to+\infty}\frac{e^x}{x^n} = +\infty$ ចៈ. $\lim_{x\to+\infty}\frac{x^n}{e^x} = +\infty$

១០-លីមីតនៃអនុគមន៍លោការីតនេពែ

រូបមន្តសំខាន់ៗ ៖

ក. $\lim_{x \to +\infty} \ln x = +\infty$

ខ. $\lim_{x \to 0^+} \ln x = -\infty$

គ. $\lim_{x \to +\infty} \frac{\ln x}{x} = 0$

ឃ. $\lim_{x \to 0^+} x \ln x = 0$

ង. $\lim_{x \to +\infty} \frac{\ln x}{x^n} = 0$

ច. $\lim_{x \to 0^+} x^n \ln x = 0$

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ជំពូកទី០២

វិធីសាស្ត្រគណនាលីមីតនៃអនុគមន៍មួយចំនួន

១.វិធីសាស្ត្រគណនាលីមីតតាមការដាក់ជាផលគុណកត្តា

ឧបមាថាគេមានលីមីត $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ (1) ។

បើតម្លៃលេខ $g(a)=0$ និង $f(a)=0$ នោះលីមីត (1) មានរាងមិនកំនត់ $\frac{0}{0}$ ។
ក្នុងករណីនេះដើម្បីគណនាលីមីតគេត្រូវដាក់ភាគយកនិងភាគបែងជាផលគុណនៃកត្តាដោយមាន $(x-a)$ ជាកត្តារួមរួចសម្រួលកត្តារួមនេះចោលបន្ទាប់មករកលីមីតនៃប្រភាគថ្មី ។

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to -2}\frac{x^3+8+4(x+2)}{x^2-4}$

គេបាន $\lim\limits_{x\to -2}\frac{x^3+8+4(x+2)}{x^2-4}=\lim\limits_{x\to -2}\frac{(x+2)(x^2-2x+4)+4(x+2)}{(x-2)(x+2)}$

$$=\lim_{x\to -2}\frac{(x+2)(x^2-2x+8)}{(x-2)(x+2)}$$

$$=\lim_{x\to -2}\frac{x^2-2x+8}{x-2}=-\frac{16}{4}=-4$$

ដូចនេះ $\lim\limits_{x\to -2}\frac{x^3+8+4(x+2)}{x^2-4}=-4$ ។

២.របៀបគណនាលីមីតអនុគមន៍អសនិទាន

ក.ករណីកន្សោមរាង $\sqrt{A}-B$

គេមាន $(\sqrt{A}-B)(\sqrt{A}+B)=A-B^2$ នោះគេបាន $\sqrt{A}-B=\frac{A-B^2}{\sqrt{A}+B}$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to1}\dfrac{\sqrt{2x^2-2x+1}-x}{(x-1)^2}$?

ដំណោះស្រាយ

យើងបាន $\lim\limits_{x\to1}\dfrac{\sqrt{2x^2-2x+1}-x}{(x-1)^2}=\lim\limits_{x\to1}\dfrac{2x^2-2x+1-x^2}{(x-1)^2(\sqrt{2x^2-2x+1}+x)}$

$$=\lim_{x\to1}\frac{(x-1)^2}{(x-1)^2(\sqrt{2x^2-2x+1}+x)}$$

$$=\lim_{x\to1}\frac{1}{\sqrt{2x^2-2x+1}+x}=\frac{1}{1+1}=\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to1}\dfrac{\sqrt{2x^2-2x+1}-x}{(x-1)^2}=\dfrac{1}{2}$ ។

ខ.ករណីកន្សោមរាង $A-\sqrt{B}$

គេមាន $(A-\sqrt{B})(A+\sqrt{B})=A^2-B$ នោះគេបាន $A-\sqrt{B}=\dfrac{A^2-B}{A+\sqrt{B}}$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to2}\dfrac{2x+1-\sqrt{3x^2+8x-3}}{(x-2)^2}$ ។

ដំណោះស្រាយ

យើងបាន $\lim\limits_{x\to2}\dfrac{2x+1-\sqrt{3x^2+8x-3}}{(x-2)^2}=\lim\limits_{x\to2}\dfrac{(2x+1)^2-(3x^2+8x-3)}{(x-2)^2(2x+1+\sqrt{3x^2+8x-3})}$

$$=\lim_{x\to2}\frac{4x^2+4x+1-3x^2-8x+3}{(x-2)^2(2x+1+\sqrt{3x^2+8x+3})}$$

$$=\lim_{x\to2}\frac{(x-2)^2}{(x-2)^2(2x+1+\sqrt{3x^2+8x+3})}$$

$$=\lim_{x\to2}\frac{1}{2x+1+\sqrt{3x^2+8x-3}}=\frac{1}{10}$$

គ.ករណីកន្សោមរាង $\sqrt{A}-\sqrt{B}$

គេមាន $(\sqrt{A}-\sqrt{B})(\sqrt{A}+\sqrt{B})=A-B$ នោះគេបាន $\sqrt{A}-\sqrt{B}=\dfrac{A-B}{\sqrt{A}+\sqrt{B}}$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to 2}\dfrac{\sqrt{x^3+1}-\sqrt{6x^2-12x+9}}{(x-2)^3}$ ។

ដំណោះស្រាយ

យើងបាន $\lim\limits_{x\to 2}\dfrac{\sqrt{x^3+1}-\sqrt{6x^2-12x+9}}{(x-2)^3}$

$$=\lim_{x\to 2}\frac{(x^3+1)-(6x^2-2x+9)}{(x-2)^3(\sqrt{x^3+1}+\sqrt{6x^2-12x+9})}$$

$$=\lim_{x\to 2}\frac{(x-2)^3}{(x-2)^3(\sqrt{x^3+1}+\sqrt{6x^2-12x+9})}$$

$$=\lim_{x\to 2}\frac{1}{(\sqrt{x^3+1}+\sqrt{6x^2-12x+9})}=\frac{1}{6}$$

ដូចនេះ $\lim\limits_{x\to 2}\dfrac{\sqrt{x^3+1}-\sqrt{6x^2-12x+9}}{(x-2)^3}=\dfrac{1}{6}$ ។

ឃ.ករណីកន្សោមរាង $\sqrt[3]{A}-B$

គេមាន $(\sqrt[3]{A}-B)(\sqrt[3]{A^2}+B\sqrt[3]{A}+B^2)=A-B^3$

នោះគេបាន $\sqrt[3]{A}-B=\dfrac{A-B^3}{\sqrt[3]{A^2}+B\sqrt[3]{A}+B^2}$ ។

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x^3+6x}-x-1}{(x-1)^3}$?

ដំណោះស្រាយ

យើងបាន $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x^3+6x}-x-1}{(x-1)^3}$

$$=\lim_{x\to1}\frac{(2x^3+6x)-(x+1)^3}{(x-1)^3\left[\sqrt[3]{(2x^3+6x)^2}+(x+1)\sqrt[3]{2x^3+6x}+(x+1)^2\right]}$$

$$=\lim_{x\to1}\frac{2x^3+6x-x^3-3x^2-3x-1}{(x-1)^3\left[\sqrt[3]{(2x^3+6x)^2}+(x+1)\sqrt[3]{2x^3+6x}+(x+1)^2\right]}$$

$$=\lim_{x\to1}\frac{(x-1)^3}{(x-1)^3\left[\sqrt[3]{(2x^3+6x)^2}+(x+1)\sqrt[3]{2x^3+6x}+(x+1)^2\right]}$$

$$=\lim_{x\to1}\frac{1}{\sqrt[3]{(2x^3+6x)^2}+(x+1)\sqrt[3]{2x^3+6x}+(x+1)^2}=\frac{1}{12}$$

ង.ករណីកន្សោមរាង $A-\sqrt[3]{B}$

គេមាន $(A-\sqrt[3]{B})(A^2+A\sqrt[3]{B}+\sqrt[3]{B^2})=A^3-B$

នោះគេបាន $A-\sqrt[3]{B}=\dfrac{A^3-B}{A^2+A\sqrt[3]{B}+\sqrt[3]{B^2}}$ ។

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to2}\dfrac{x-1-\sqrt[3]{4x^2-12x+7}}{(x-2)^3}$?

ដំណោះស្រាយ

តាង $L=\lim_{x\to2}\dfrac{x-1-\sqrt[3]{4x^2-12x+7}}{(x-2)^3}$

$$=\lim_{x\to2}\frac{(x-1)^3-(4x^2-12x+7)}{(x-2)^3\left[(x-1)^2+(x-1)\sqrt[3]{4x^2-12x+7}+\sqrt[3]{(4x^2-12x+7)^2}\right]}$$

$$=\lim_{x\to2}\frac{(x-2)^3}{(x-2)^3\left[(x-1)^2+(x-1)\sqrt[3]{4x^2-12x+7}+\sqrt[3]{(4x^2-12x+7)^2}\right]}$$

$$=\lim_{x\to2}\frac{1}{\left[(x-1)^2+(x-1)\sqrt[3]{4x^2-12x+7}+\sqrt[3]{(4x^2-12x+7)^2}\right]}=\frac{1}{3}$$

ដូចនេះ $L=\dfrac{1}{3}$ ។

ច.ករណីកន្សោមរាង $\sqrt[3]{A}-\sqrt[3]{B}$

គេមាន $(\sqrt[3]{A}-\sqrt[3]{B})(\sqrt[3]{A^2}+\sqrt[3]{A}.\sqrt[3]{B}+\sqrt[3]{B^2})=A-B$

គេបាន $\sqrt[3]{A}-\sqrt[3]{B}=\dfrac{A-B}{\sqrt[3]{A^2}+\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3-2x^2}-\sqrt[3]{x^2-3x+1}}{(x-1)^3}$?

ដំណោះស្រាយ

យក $L=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3-2x^2}-\sqrt[3]{x^2-3x+1}}{(x-1)^3}$

$$=\lim_{x\to 1}\frac{(x^3-2x^2)-(x^2-3x+1)}{(x-1)^3\left[\sqrt[3]{(x^3-2x^2)^2}+\sqrt[3]{(x^3-2x^2)(x^2-3x+1)}+\sqrt[3]{(x^2-3x+1)^2}\right]}$$

$$=\lim_{x\to 1}\frac{(x-1)^3}{(x-1)^3\left[\sqrt[3]{(x^3-2x^2)^2}+\sqrt[3]{(x^3-2x^2)(x^2-3x+1)}+\sqrt[3]{(x^2-3x+1)^2}\right]}$$

$$=\lim_{x\to 1}\frac{1}{\left[\sqrt[3]{(x^3-2x^2)^2}+\sqrt[3]{(x^3-2x^2)(x^2-3x+1)}+\sqrt[3]{(x^2-3x+1)^2}\right]}=\frac{1}{3}$$

ដូចនេះ $L=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3-2x^2}-\sqrt[3]{x^2-3x+1}}{(x-1)^3}=\dfrac{1}{3}$ ។

ឆ.ករណីកន្សោមរាង $\sqrt[3]{A}+\sqrt[3]{B}$

គេមាន $(\sqrt[3]{A}+\sqrt[3]{B})(\sqrt[3]{A^2}-\sqrt[3]{A}.\sqrt[3]{B}+\sqrt[3]{B^2})=A+B$

នោះគេបាន $\sqrt[3]{A}+\sqrt[3]{B}=\dfrac{A+B}{\sqrt[3]{A^2}-\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x^2+4}+\sqrt[3]{4x}}{(x+2)^2}$ ។

ដំណោះស្រាយ

តាង $L=\lim\limits_{x\to -2}\frac{\sqrt[3]{x^2+4}+\sqrt[3]{4x}}{(x+2)^2}$

$$=\lim_{x\to -2}\frac{(x^2+4)+(4x)}{(x+2)^2\left[\sqrt[3]{(x^2+4)^2}-\sqrt[3]{4x(x^2+4)}+\sqrt[3]{(4x)^2}\right]}$$

$$=\lim_{x\to -2}\frac{(x+2)^2}{(x+2)^2\left[\sqrt[3]{(x^2+4)^2}-\sqrt[3]{4x(x^2+4)}+\sqrt[3]{(4x)^2}\right]}$$

$$\lim_{x\to -2}\frac{1}{\left[\sqrt[3]{(x^2+4)^2}-\sqrt[3]{4x(x^2+4)}+\sqrt[3]{(4x)^2}\right]}=\frac{1}{12}$$

ដូចនេះ $\lim\limits_{x\to -2}\frac{\sqrt[3]{x^2+4}+\sqrt[3]{4x}}{(x+2)^2}=\frac{1}{12}$ ។

៣.របៀបគណនាលីមីតត្រង់អនន្ត

☐ **រូបមន្តសំខាន់ៗ**

ក. $\lim\limits_{x\to\pm\infty} ax^2 = \begin{cases} +\infty & \text{បើ } a>0 \\ -\infty & \text{បើ } a<0 \end{cases}$

ខ. $\lim\limits_{x\to\pm\infty}\frac{a}{x}=0 \quad , \quad a\in\mathbb{R}$

☐ **របៀបគណនាលីមីតត្រង់អនន្តចំពោះកន្សោមរាង ៖**

$$L=\lim_{x\to\infty}\frac{a_m x^m+a_{m-1}x^{m-1}+\ldots.+a_1x+a_0}{b_n x^n+b_{n-1}x^{n-1}+\ldots..+b_1x+b_0}$$

ដើម្បីគណនាលីមីតនេះគេត្រូវ

✓ ដាក់តួ x ដែលមានស្វ័យគុណខ្ពស់ជាងគេជាកត្តារួម

✓ សម្រួលកត្តា x នោះចោល

✓ គណនាលីមីតដោយប្រើរូបមន្ត $\lim\limits_{x\to\infty}\frac{1}{x^\alpha}=0,\ \alpha>0$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to\infty}\frac{8x^7-3x^4+x+2}{2x^7+7x-3}$?

ដំណោះស្រាយ

យើងតាង $L=\lim_{x\to\infty}\frac{8x^7-3x^4+x+2}{2x^7+7x-3}$

$$=\lim_{x\to\infty}\frac{x^7(8-\frac{3}{x^3}+\frac{1}{x^6}+\frac{2}{x^7})}{x^7(2+\frac{7}{x^6}-\frac{3}{x^7})}$$

$$=\lim_{x\to\infty}\frac{8-\frac{3}{x^3}+\frac{1}{x^6}+\frac{2}{x^7}}{2+\frac{7}{x^6}-\frac{3}{x^7}}=\frac{8}{2}=4$$

ដូចនេះ $L=\lim_{x\to\infty}\frac{8x^7-3x^4+x+2}{2x^7+7x-3}=4$ ។

សម្គាល់ គេអាចគណនាតាមរបៀបខាងក្រោម ៖

$$L=\lim_{x\to\infty}\frac{8x^7-3x^4+x+2}{2x^7+7x-3}=\lim_{x\to\infty}\frac{8x^7}{2x^7}=\frac{8}{2}=4 \text{ ។}$$

✧ របៀបគណនាលីមីតត្រង់អនន្តចំពោះកន្សោមអសនិទាន

គេត្រូវប្រើរូបមន្តបំលែងដូចខាងក្រោម ៖

ក. $\sqrt{A}-\sqrt{B}=\frac{A-B}{\sqrt{A}+\sqrt{B}}$

ខ. $\sqrt[3]{A}-\sqrt[3]{B}=\frac{A-B}{\sqrt[3]{A^2}+\sqrt[3]{A}.\sqrt[3]{B}+\sqrt[3]{B^2}}$

គ. $\sqrt[3]{A}+\sqrt[3]{B}=\frac{A+B}{\sqrt[3]{A^2}-\sqrt[3]{A}.\sqrt[3]{B}+\sqrt[3]{B^2}}$

☐ ឃ. $\sqrt[n]{A}-\sqrt[n]{B}=\frac{A-B}{\sqrt[n]{A^{n-1}}+\sqrt[n]{A^{n-2}B}+\ldots+\sqrt[n]{B^{n-1}}}$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x \to +\infty} (\sqrt{x^2+7x+1} - \sqrt{x^2+4})$ ។

ដំណោះស្រាយ

គេបាន $\lim_{x \to +\infty} (\sqrt{x^2+7x+1} - \sqrt{x^2+4}) = \lim_{x \to +\infty} \dfrac{x^2+7x+1-x^2-4}{\sqrt{x^2+7x+1}+\sqrt{x^2+4}}$

$$= \lim_{x \to +\infty} \frac{7x-3}{x\sqrt{1+\frac{7}{x}+\frac{1}{x^2}}+x\sqrt{1+\frac{4}{x^2}}}$$

$$= \lim_{x \to +\infty} \frac{7-\frac{3}{x}}{\sqrt{1+\frac{7}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{4}{x^2}}} = \frac{7}{2}$$

✧ របៀបគណនាលីមីតត្រង់អនន្តចំពោះកន្សោមជាផលបូកនៃស្វ៊ីតចំនួនពិត

រូបមន្តផលបូកស្វ៊ីតសំខាន់ៗ ៖

ក. $1+2+3+......+n = \dfrac{n(n+1)}{2}$

ខ. $1^2+2^2+3^2+.....+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

គ. $1^3+2^3+3^3+....+n^3 = \dfrac{n^2(n+1)^2}{4}$

ឃ. ផលបូកស្វ៊ីតនព្វន្ត

$$S_n = a_1+a_2+a_3+....+a_n = \frac{n(a_1+a_n)}{2}$$

ង. ផលបូកស្វ៊ីតធរណីមាត្រ

$$S_n = a_1+a_2+a_3+....+a_n = a_1 \times \frac{1-q^n}{1-q}$$ ។

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{n\to+\infty}\dfrac{1+2+3+....+n}{n^2}$ ។

ដំណោះស្រាយ

យើងបាន $\lim\limits_{n\to+\infty}\dfrac{1+2+3+...+n}{n^2}=\lim\limits_{n\to+\infty}\dfrac{n(n+1)}{2n^2}$

$=\lim\limits_{n\to+\infty}\dfrac{n+1}{2n}=\lim\limits_{n\to+\infty}\dfrac{n}{2n}=\dfrac{1}{2}$ ។

ដូចនេះ $\lim\limits_{n\to+\infty}\dfrac{1+2+3+....+n}{n^2}=\dfrac{1}{2}$ ។

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{n\to+\infty}\dfrac{1^2+2^2+3^2+....+n^2}{n^3}$ ។

ដំណោះស្រាយ

យើងបាន $\lim\limits_{n\to+\infty}\dfrac{1^2+2^2+3^2+...+n^2}{n^3}=\lim\limits_{n\to+\infty}\dfrac{n(n+1)(2n+1)}{6n^3}$

$=\lim\limits_{n\to+\infty}\dfrac{(n+1)(2n+1)}{6n^2}$

$=\lim\limits_{n\to+\infty}\dfrac{(1+\frac{1}{n})(2+\frac{1}{n})}{6}=\dfrac{2}{6}=\dfrac{1}{3}$

ដូចនេះ $\lim\limits_{n\to+\infty}\dfrac{1^2+2^2+3^2+....+n^2}{n^3}=\dfrac{1}{3}$ ។

៤.របៀបគណនាលីមីតអនុគមន៍ត្រីកោណមាត្រ

រូបមន្តអនុគមន៍ត្រីកោណមាត្រដែលគួរចងចាំ

$A)$ ទំនាក់ទំនងគ្រឹះ

ក. $\sin^2\theta+\cos^2\theta=1$ ខ. $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ គ. $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$

ឃ. $1+\tan^2\theta=\dfrac{1}{\cos^2\theta}$ ង. $1+\cot^2\theta=\dfrac{1}{\sin^2\theta}$ ឋ. $\tan\theta=\dfrac{1}{\cot\theta}$

$B)$ រូបមន្តបំលែងមុំផ្សេងៗ

ក.មុំផ្ទុយគ្នា θ និង $-\theta$
$$\begin{cases} \sin(-\theta) = -\sin\theta \\ \cos(-\theta) = \cos\theta \\ \tan(-\theta) = -\tan\theta \\ \cot(-\theta) = -\cot\theta \end{cases}$$

ខ.មុំបំពេញគ្នា $(\frac{\pi}{2}-\theta)$ និង θ
$$\begin{cases} \sin(\frac{\pi}{2}-\theta) = \cos\theta \\ \cos(\frac{\pi}{2}-\theta) = \sin\theta \\ \tan(\frac{\pi}{2}-\theta) = \cot\theta \\ \cot(\frac{\pi}{2}-\theta) = \tan\theta \end{cases}$$

គ.មុំបន្ថែមគ្នា $\pi-\theta$ និង θ
$$\begin{cases} \sin(\pi-\theta) = \sin\theta \\ \cos(\pi-\theta) = -\cos\theta \\ \tan(\pi-\theta) = -\tan\theta \\ \cot(\pi-\theta) = -\cot\theta \end{cases}$$

ឃ.មុំមានផលសងស្មើនឹង π
$$\begin{cases} \sin(\pi+\theta) = -\sin\theta \\ \cos(\pi+\theta) = -\cos\theta \\ \tan(\pi+\theta) = \tan\theta \\ \cot(\pi+\theta) = \cot\theta \end{cases}$$

ង.មុំមានផលសងស្មើនឹង $\frac{\pi}{2}$
$$\begin{cases} \sin(\frac{\pi}{2}+\theta) = \cos\theta \\ \cos(\frac{\pi}{2}+\theta) = -\sin\theta \\ \tan(\frac{\pi}{2}+\theta) = -\cot\theta \\ \cot(\frac{\pi}{2}+\theta) = -\tan\theta \end{cases}$$

$C)$ **រូបមន្តផលបូកនិងផលដករវាងមុំពីរ**

ក. $\cos(a-b)=\cos a\cos b+\sin a\sin b$

ខ. $\cos(a+b)=\cos a\cos b-\sin a\sin b$

គ. $\sin(a+b)=\sin a\cos b+\sin b\cos a$

ឃ. $\sin(a-b)=\sin a\cos b-\sin b\cos a$

ង. $\tan(a\pm b)=\dfrac{\tan a\pm\tan b}{1\mp\tan a\tan b}$

$D)$ **រូបមន្តមុំឌុប**

ក. $\sin 2a=\sin a\cos a$

ខ. $\cos 2a=\cos^2 a-\sin^2 a=2\cos^2 a-1=1-2\sin^2 a$

គ. $\tan 2a=\dfrac{2\tan a}{1-\tan^2 a}$

$E)$ **រូបមន្តបំលែងពីផលគុណទៅផលបូក**

ក. $\sin a\cos b=\dfrac{1}{2}\left[\sin(a+b)+\sin(a-b)\right]$

ខ. $\sin b\cos a=\dfrac{1}{2}\left[\sin(a+b)-\sin(a-b)\right]$

គ. $\cos a\cos b=\dfrac{1}{2}\left[\cos(a+b)+\cos(a-b)\right]$

ឃ. $\sin a\sin b=-\dfrac{1}{2}\left[\cos(a+b)-\cos(a-b)\right]$

$F)$ **រូបមន្តបំលែងពីផលគុណទៅផលបូក**

ក. $\sin p+\sin q=2\sin\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$

ខ. $\sin p-\sin q=2\sin\dfrac{p-q}{2}\cos\dfrac{p+q}{2}$

គ. $\cos p+\cos q=2\cos\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$

ឃ. $\cos p - \cos q = -2\sin\frac{p-q}{2}\sin\frac{p+q}{2}$

ង. $\tan p + \tan q = \frac{\sin(p+q)}{\cos p\cos q}$

ប. $\tan p - \tan q = \frac{\sin(p-q)}{\cos p\cos q}$

ឆ. $\cot p + \cot q = \frac{\sin(p+q)}{\sin p\sin q}$

ជ. $\cot p - \cot q = -\frac{\sin(p-q)}{\sin p\sin q}$

$G)$ កន្សោម $\sin 3a$, $\cos 3a$ និង $\tan 3a$

ក. $\sin 3a = 3\sin a - 4\sin^3 a$

ខ. $\cos 3a = 4\cos^3 a - 3\cos a$

គ. $\tan 3a = \frac{3\tan a - \tan^3 a}{1-3\tan^2 a}$

$H)$ អនុគមន៍ត្រីកោណមាត្រនៃមុំ $\theta + 2k\pi$ និង θ

ក. $\sin(\theta + 2k\pi) = \sin\theta$

ខ. $\cos(\theta + 2k\pi) = \cos\theta$

គ. $\tan(\theta + 2k\pi) = \tan\theta$

ឃ. $\cot(\theta + 2k\pi) = \cot\theta$ (ដែល k ជាចំនួនគត់រ៉ឺឡាទីហ្វ)។

✧ របៀបគណនាដោយប្រើរូបមន្តគ្រឹះ

ក. $\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{x}{\sin x} = 1$

ខ. $\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{x}{\tan x} = 1$

គ. $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$

☐ **ឧទាហរណ៍** គណនាលីមីត

ក. $\lim\limits_{x\to0}\dfrac{1-\cos x+\sin x}{x}=\lim\limits_{x\to0}\dfrac{1-\cos x}{x}+\lim\limits_{x\to0}\dfrac{\sin x}{x}=0+1=1$

ខ. $\lim\limits_{x\to0}\dfrac{1-\cos x}{x^2}=\lim\limits_{x\to0}\dfrac{2\sin^2\frac{x}{2}}{x^2}=\dfrac{1}{2}\lim\limits_{x\to0}\left(\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2=\dfrac{1}{2}\times1^2=\dfrac{1}{2}$

គ. $\lim\limits_{x\to0}\dfrac{x+\tan x}{x+\tan 3x}=\lim\limits_{x\to0}\dfrac{1+\dfrac{\tan x}{x}}{1+\dfrac{\tan 3x}{3x}.3}=\dfrac{1+1}{1+3}=\dfrac{1}{2}$ ។

ឃ. $\lim\limits_{x\to0}\dfrac{\sin[\sin(\sin x)]}{x}=\lim\limits_{x\to0}\dfrac{\sin[\sin(\sin x)]}{\sin(\sin x)}.\dfrac{\sin(\sin x)}{\sin x}.\dfrac{\sin x}{x}=1$

ង. $\lim\limits_{x\to0}\dfrac{\sin x\sin 2x\sin 3x....\sin(nx)}{x^n}$

$=\lim\limits_{x\to0}\dfrac{\sin x}{x}.2\dfrac{\sin 2x}{2x}.3\dfrac{\sin 3x}{3x}....n.\dfrac{\sin(nx)}{nx}=1\times2\times3\times....\times n=n!$

✧ **របៀបគណនាត្រីកោណមាត្រដោយប្រើរូបមន្តប្តូរអថេរ**

ឧបមាថាគេមានលីមីត $L=\lim\limits_{x\to a}[f(x)]$ ដែល $a\neq0$

គេតាង $t=a-x\Rightarrow x=a-t$ កាលណា $x\to a$ នោះ $t\to0$

គេបានរូបមន្ត $L=\lim\limits_{x\to a}[f(x)]=\lim\limits_{t\to0}[f(a-t)]$ (ហៅថារូបមន្តប្តូរអថេរ)។

ឧទាហរណ៍ គណនាលីមីត $L=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1-\sin x}{(\frac{\pi}{2}-x)^2}$

ដំណោះស្រាយ

យើងតាង $\dfrac{\pi}{2}-x=t$ នោះ $x=\dfrac{\pi}{2}-t$

បើ $x\to\dfrac{\pi}{2}$ នោះ $t\to0$

គេបាន $L=\lim\limits_{t\to 0}\dfrac{1-\sin(\frac{\pi}{2}-t)}{t^2}=\lim\limits_{t\to 0}\dfrac{1-\cos t}{t^2}=\lim\limits_{t\to 0}\dfrac{\sin^2 t}{t^2}.\dfrac{1}{1+\cos t}=\dfrac{1}{2}$ ។

ដូចនេះ $L=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1-\sin x}{(\frac{\pi}{2}-x)^2}=\dfrac{1}{2}$ ។

ឧទាហរណ៍ គណនាលីមីត $A=\lim\limits_{x\to 1}\dfrac{1-\cos(2\pi x)}{(1-x)^2}$ ។

ដំណោះស្រាយ

គណនា $A=\lim\limits_{x\to 1}\dfrac{1-\cos(2\pi x)}{(1-x)^2}$ ដោយ $1-\cos(2\pi x)=2\sin^2(\pi x)$

គេបាន $A=\lim\limits_{x\to 1}\dfrac{2\sin^2(\pi x)}{(1-x)^2}$

តាង $u=1-x$ នោះ $x=1-u$ ហើយកាលណា $x\to 1$ នោះ $u\to 0$

គេបាន $A=2\lim\limits_{u\to 0}\dfrac{\sin^2(\pi-\pi u)}{u^2}=2\lim\limits_{u\to 0}\dfrac{\sin^2(\pi u)}{u^2}$

$$=2\lim\limits_{u\to 0}\left[\frac{\sin(\pi u)}{(\pi u)}\times\pi\right]^2=2\pi^2$$

ដូចនេះ $A=2\pi^2$ ។

ឧទាហរណ៍ ចូរគណនាលីមីត $A=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\sin(x-\frac{\pi}{3})}{3x-\pi}$ ។

ដំណោះស្រាយ

គណនា $A=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\sin(x-\frac{\pi}{3})}{3x-\pi}$

តាង $u = x - \frac{\pi}{3}$ នោះ $x = \frac{\pi}{3} + u$ ។ កាលណា $x \to \frac{\pi}{3}$ នោះ $u \to 0$

គេបាន $A = \lim_{u \to 0} \frac{2\sin u}{3(\frac{\pi}{3} + u) - \pi} = \lim_{u \to 0} \frac{2\sin u}{3u} = \frac{2}{3} \lim_{u \to 0} \frac{\sin u}{u} = \frac{2}{3}$

ដូចនេះ $A = \lim_{x \to \frac{\pi}{3}} \frac{2\sin(x - \frac{\pi}{3})}{3x - \pi} = \frac{2}{3}$ ។

ឧទាហរណ៍ ចូរគណនាលីមីត $A = \lim_{x \to \pi} (\pi^2 - x^2)\tan\frac{x}{2}$ ។

ដំណោះស្រាយ

គណនា $A = \lim_{x \to \pi} (\pi^2 - x^2)\tan\frac{x}{2}$

តាង $u = \pi - x$ នោះ $x = \pi - u$ ។ កាលណា $x \to \pi$ នោះ $u \to 0$

គេបាន $A = \lim_{x \to \pi} \left[\pi^2 - (\pi - u)^2\right] \tan\left(\frac{\pi}{2} - \frac{u}{2}\right)$

$$= \lim_{x \to \pi} \left(2\pi u - u^2\right) \cot\frac{u}{2}$$

$$= \lim_{x \to \pi} [u(2\pi - u)\frac{1}{\tan\frac{u}{2}}]$$

$$= \lim_{x \to \pi} [\ 2(2\pi - \mathrm{u})\frac{\frac{u}{2}}{\tan\frac{u}{2}}\ ] = 4\pi$$

ដូចនេះ $A = \lim_{x \to \pi} (\pi^2 - x^2)\tan\frac{x}{2} = 4\pi$

ឧទាហរណ៍ ចូរគណនាលីមីត $A=\lim\limits_{x\to\infty}(2x+1)\sin\left(\dfrac{\pi x}{x+2}\right)$ ។

ដំណោះស្រាយ

គណនា $A=\lim\limits_{x\to\infty}(2x+1)\sin\left(\dfrac{\pi x}{x+2}\right)$

គេមាន $\dfrac{\pi x}{x+2}=\dfrac{\pi(x+2)-2\pi}{x+2}=\pi-\dfrac{2\pi}{x+2}$

នោះ $\sin\left(\dfrac{\pi x}{x+2}\right)=\sin\left(\pi-\dfrac{2\pi}{x+2}\right)=\sin\left(\dfrac{2\pi}{x+2}\right)$

គេបាន $A=\lim\limits_{x\to\infty}(2x+1)\sin\left(\dfrac{2\pi}{x+2}\right)$

តាង $u=\dfrac{2\pi}{x+2}$ នោះ $x=\dfrac{2\pi-2u}{u}$

កាលណា $x\to\infty$ នោះ $u\to 0$

គេបាន $A=\lim\limits_{u\to 0}\left[2\left(\dfrac{2\pi-2u}{u}\right)+1\right]\sin u$

$$=\lim_{u\to 0}\frac{4\pi-4u+u}{u}\sin u$$

$$=\lim_{u\to 0}(4\pi-3u)\frac{\sin u}{u}=4\pi$$

ដូចនេះ $A=\lim\limits_{x\to\infty}(2x+1)\sin\left(\dfrac{\pi x}{x+2}\right)=4\pi$ ។

៥.លីមីតនៃអនុគមន៍អិចស្ប៉ូណង់ស្យែល

រូបមន្តសំខាន់ៗគួរកត់សម្គាល់៖

ក. $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1$

ខ. $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\ln a \quad ,(a>0)$

គ. $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e$

ឃ. $\lim_{x\to+\infty} a^{x} = \begin{cases} +\infty & \text{បើ } a>1 \\ 0 & \text{បើ } 0<a<1 \end{cases}$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\frac{e^{\sin x}-1}{x} = \lim_{x\to 0}\frac{e^{\sin x}-1}{\sin x}\times\frac{\sin x}{x} = 1\times 1 = 1$

ខ. $\lim_{x\to 0}\frac{e^{x^2}-\cos 2x}{x^2} = \lim_{x\to 0}\frac{e^{x^2}-(1-2\sin^2 x)}{x^2}$

$= \lim_{x\to 0}\frac{e^{x^2}-1}{x^2} + 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2$

$= 1 + 2\times 1^2 = 3$

គ. $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x} = \lim_{x\to 0}\frac{e^{ax}-1}{ax}\times a - \lim_{x\to 0}\frac{e^{bx}-1}{bx}\times b$

$= a-b \text{ , } a,b\in\mathbb{R}^*$

ដូចនេះ $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x} = a-b$ ។

៦.លីមីតនៃអនុគមន៍លោការីត

រូបមន្តសំខាន់ៗ៖

ក. $\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$

ខ. $\lim_{x\to 0^+}\ln x = -\infty$

គ. $\lim_{x\to+\infty}\ln x = +\infty$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\frac{\ln(1+\tan 2x)}{x} = \lim_{x\to 0}\frac{\ln(1+\tan 2x)}{\tan 2x}.\frac{\tan 2x}{2x}.2 = 2$

ខ. $\lim\limits_{x\to0}\dfrac{\ln(\cos 2x)}{x^2}=\lim\limits_{x\to0}\dfrac{\ln(1-2\sin^2 x)}{x^2}$

$=\lim\limits_{x\to0}\dfrac{\ln(1-2\sin^2 x)}{(-2\sin^2 x)}\times\dfrac{\sin^2 x}{x^2}\times(-2)=-2$

គ. $\lim\limits_{x\to0}\dfrac{\ln(1+4x-x^2)}{x}=\lim\limits_{x\to0}\dfrac{\ln(1+4x-x^2)}{4x-x^2}\times\dfrac{4x-x^2}{x}$

$=\lim\limits_{x\to0}\dfrac{\ln(1+4x-x^2)}{4x-x^2}.(4-x)=1\times4=4$

ឃ. $\lim\limits_{x\to0}\dfrac{\ln(3-2e^x)}{x}=\lim\limits_{x\to0}\dfrac{\ln(1+2(1-e^x)]}{2(1-e^x)}\times\dfrac{-2(e^x-1)}{x}=-2$

៧.របៀបគណនាលីមីតរាងមិនកំណត់ 1^∞

កាលណា $x\to x_0$ គេមាន $f(x)\to0$ និង $g(x)\to\infty$ នោះលីមីត $\lim\limits_{x\to x_0}\left[f(x)\right]^{g(x)}$ មានរាងមិនកំនត់ 1^∞ ។

ដើម្បីគណនាលីមីតនេះគេត្រូវប្រើរូបមន្តគ្រឹះ

$a)\ \lim\limits_{x\to0}(1+x)^{\frac{1}{x}}=e \qquad b)\ \lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$ ដែល $e=2.7182818...$ ។

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to0}\left(1+\dfrac{x}{2}\right)^{\frac{1}{x}}=\lim\limits_{x\to0}\left[\left(1+\dfrac{x}{2}\right)^{\frac{1}{x/2}}\right]^{\frac{1}{2}}=e^{\frac{1}{2}}=\sqrt{e}$

ខ. $\lim\limits_{x\to0}\left(1+\sin\dfrac{x}{2}\right)^{\frac{1}{x}}=\lim\limits_{x\to0}\left[\left(1+\sin\dfrac{x}{2}\right)^{\frac{1}{\sin\frac{x}{2}}}\right]^{\frac{\sin\frac{x}{2}}{\frac{x}{2}}.\frac{1}{2}}=e^{\frac{1}{2}}=\sqrt{e}$

គ. $\lim_{x\to+\infty}\left(1+\frac{2}{x}\right)^{x}=\left[\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^{2}=e^{2}$

ឃ. $\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x}=\lim_{x\to+\infty}\left[1+\left(\frac{-2}{x+1}\right)^{\frac{x+1}{-2}}\right]^{\frac{-2x}{x+1}}=e^{-2}=\frac{1}{e^{2}}$ ។

ង. $\lim_{x\to0}\left(\frac{x^{2}-x+1}{x^{2}+x+1}\right)^{\frac{1}{\sin x}}$

គេមាន $\frac{x^{2}-x+1}{x^{2}+x+1}=1+\left(\frac{x^{2}-x+1}{x^{2}+x+1}-1\right)=1+\frac{-2x}{x^{2}+x+1}$

$$\lim_{x\to0}\left(\frac{x^{2}-x+1}{x^{2}+x+1}\right)^{\frac{1}{\sin x}}=\lim_{x\to0}\left(1+\frac{-2x}{x^{2}+x+1}\right)^{\frac{1}{\sin x}}$$

$$=\lim_{x\to0}\left[\left(1+\frac{-2x}{x^{2}+x+1}\right)^{\frac{x^{2}+x+1}{-2x}}\right]^{-\frac{x}{\sin x}\cdot\frac{2}{x^{2}+x+1}}$$

$$=e^{-2}=\frac{1}{e^{2}}$$

ដូចនេះ $\lim_{x\to0}\left(\frac{x^{2}-x+1}{x^{2}+x+1}\right)^{\frac{1}{\sin x}}=\frac{1}{e^{2}}$ ។

៨.របៀបគណនាលីមីតរាងមិនកំនត់ដោយប្រើទ្រឹស្តីបទឡូពីតាល់

ក.ទ្រឹស្តីបទ

សន្មតថាគេមានអនុគមន៍ពីរ f និង g មានដេរីវេត្រង់ $x=x_0$ ហើយ $g'(x_0)\neq0$ ។

.បើ $\lim_{x\to x_0}\frac{f(x)}{g(x)}$ មានរាង $\frac{0}{0}$ ឬ $\frac{\infty}{\infty}$ នោះ $\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}=\frac{f'(x_0)}{g'(x_0)}$ ។

.បើ $\lim\limits_{x\to x_0}\frac{f'(x)}{g'(x)}$ មានរាងមិនកំនត់ $\frac{0}{0}$ ឬ $\frac{\infty}{\infty}$ ដដែលហើយផលធៀប $\frac{f''(x)}{g''(x)}$ ផ្ទៀងផ្ទាត់លក្ខខណ្ឌឡូពីតាល់នោះគេអាចអនុវត្តន៍តាមវិធានទូទៅដូចខាងក្រោម ៖

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}=\lim_{x\to x_0}\frac{f''(x)}{g''(x)}=....=\lim_{x\to x_0}\frac{f^{(n)}(x)}{g^{(n)}(x)}$$ ។

ខ.រូបមន្តដេរីវេសំខាន់ៗ

អនុគមន៍	ដេរីវេ
ក. $y=k$	$y'=0$
ខ. $y=x^n$	$y'=nx^{n-1}$
គ. $y=\frac{1}{x}$	$y'=-\frac{1}{x^2}$
ឃ. $y=\sqrt{x}$	$y'=\frac{1}{2\sqrt{x}}$
ង. $y=e^x$	$y'=e^x$
ច. $y=a^x$	$y'=a^x\ln a$
ឆ. $y=\ln x$	$y'=\frac{1}{x}$
ជ. $y=\sin x$	$y=\cos x$
ឈ. $y=\cos x$	$y'=-\sin x$
ញ. $y=\tan x$	$y'=\frac{1}{\cos^2 x}=1+\tan^2 x$
ដ. $y=\cot x$	$y'=-\frac{1}{\sin^2 x}$
ឋ. $y=\arcsin x$	$y'=\frac{1}{\sqrt{1-x^2}}$
ទ. $y=\arccos x$	$y'=-\frac{1}{\sqrt{1-x^2}}$
ធ. $y=\arctan x$	$y'=\frac{1}{1+x^2}$

គ.រូបមន្តដេរីវេផ្សេងៗទៀត

អនុគមន៍	ដេរីវេ
ក. $y = u^n$	$y' = n.u'.u^{n-1}$
ខ. $y = \sqrt{u}$	$y' = \frac{u'}{2\sqrt{u}}$
គ. $y = u.v$	$y' = u'v + v'u$
ឃ. $y = \frac{u}{v}$	$y' = \frac{u'v - v'u}{v^2}$
ង. $y = \ln u$	$y' = \frac{u'}{u}$
ច. $y = \sin u$	$y' = u'.\cos u$
ឆ. $y = \cos u$	$y' = -u'\sin u$
ជ. $y = e^u$	$y' = u'.e^u$
ឈ. $y = \tan u$	$y' = u'(1 + \tan^2 u)$
ញ. $y = \arcsin u$	$y' = \frac{u'}{\sqrt{1-u^2}}$
ដ. $y = \arccos u$	$y' = -\frac{u'}{\sqrt{1-u^2}}$
ឋ. $y = \arctan u$	$y' = \frac{u'}{1+u^2}$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម

ក. $\lim\limits_{x \to 1} \frac{2x^5 + 3x^4 - 5}{x^2 - 1}$ មានរាង $\frac{0}{0}$

$= \lim\limits_{x \to 1} \frac{(2x^5 + 3x^4 - 5)'}{(x^2 - 1)'}$

$= \lim\limits_{x \to 1} \frac{10x^4 + 12x^3}{2x} = \frac{10 + 12}{2} = 11$

ដូចនេះ $\lim\limits_{x \to 1} \frac{2x^5 + 3x^4 - 5}{x^2 - 1} = 11$ ។

ខ. $\lim_{x\to 2}\frac{x^4+x^3-x-22}{x^2-4}$ មានរាង $\frac{0}{0}$

$$=\lim_{x\to 2}\frac{(x^4+x^3-x-22)'}{(x^2-4)'}=\lim_{x\to 2}\frac{4x^3+3x^2}{2x}$$

$$=\frac{4(2)^3+3(2)^2}{2(2)}=\frac{32+12}{4}=11$$

ដូចនេះ $\lim_{x\to 2}\frac{x^4+x^3-x-22}{x^2-4}=11$

គ. $\lim_{x\to 1}\frac{x^3-1}{\sqrt{x}-1}$ រាង $\frac{0}{0}$

$$=\lim_{x\to 1}\frac{(x^3-1)'}{(\sqrt{x}-1)'}=\lim_{x\to 1}\frac{3x^2}{\frac{1}{2\sqrt{x}}}=\frac{3}{\frac{1}{2}}=6$$

ដូចនេះ $\lim_{x\to 1}\frac{x^3-1}{\sqrt{x}-1}=6$ ។

ឃ. $\lim_{x\to 1}\frac{\sin \pi x}{x^2-1}$ រាង $\frac{0}{0}$

$$=\lim_{x\to 1}\frac{(\sin \pi x)'}{(x^2-1)'}=\lim_{x\to 1}\frac{\pi\cos \pi x}{2x}=\frac{\pi\cos\pi}{2}=-\frac{\pi}{2}$$

ដូចនេះ $\lim_{x\to 1}\frac{\sin \pi x}{x^2-1}=-\frac{\pi}{2}$ ។

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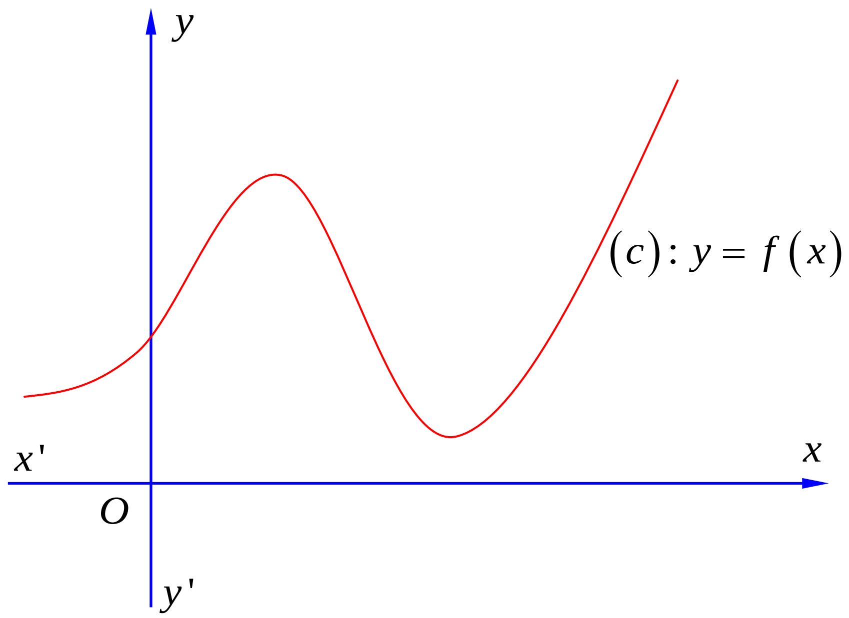
ជំពូកទី០៣

សង្ខេបមេរៀនសិក្សាភាពជាប់នៃអនុគមន៍

១.សញ្ញាណនៃអនុគមន៍ជាប់

កាលណាគេគូសក្រាបនៃអនុគមន៍ $y=f(x)$ លើចន្លោះ I មួយនៃដែនកំណត់ ដោយមិនលើកខ្មៅដៃនោះគេបានគំនូសជាខ្សែកោងជាប់គេថាអនុគមន៍ f ជាអនុគមន៍ជាប់ត្រង់គ្រប់ចំណុចនៃចន្លោះ I ។



២.ភាពជាប់ត្រង់មួយចំណុច

និយមន័យ អនុគមន៍ $y=f(x)$ ជាអនុគមន៍ជាប់ត្រង់ $x=a$ កាលណា f បំពេញលក្ខខណ្ឌទាំងបីខាងក្រោម ៖

ក. f កំណត់ចំពោះ $x = a$

ខ. f មានលីមីតកាលណា x ខិតជិត a

គ. $\lim\limits_{x\to a} f(x) = f(a)$ ។

ឧទាហរណ៍ គេឲ្យអនុគមន៍ f កំណត់ដោយ

$$f(x) = \begin{cases} \dfrac{3\sin x - \sin 3x}{x^3} & \text{បើ } x \neq 0 \\ 4 & \text{បើ } x = 0 \end{cases}$$

ចូរសិក្សាភាពជាប់នៃអនុគមន៍ f ត្រង់ $x = 0$ ។

ដំណោះស្រាយ

សិក្សាភាពជាប់នៃអនុគមន៍ f ត្រង់ $x = 0$

តាមសម្មតិកម្ម $f(x) = \begin{cases} \dfrac{3\sin x - \sin 3x}{x^3} & \text{បើ } x \neq 0 \\ 4 & \text{បើ } x = 0 \end{cases}$

គេមាន $f(0) = 4$ កំណត់

គណនា $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{3\sin x - \sin 3x}{x^3} = \lim\limits_{x\to 0} \dfrac{3\sin x - (3\sin x - 4\sin^3 x)}{x^3}$

$$= \lim_{x\to 0} \frac{3\sin x - 3\sin x + 4\sin^3 x}{x^3} = \lim_{x\to 0} \frac{4\sin^3 x}{x^3}$$

$$= 4\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^3 = 4$$

ដោយ $\lim\limits_{x\to 0} f(x) = f(0) = 4$ នោះ f ជាអនុគមន៍ជាប់ត្រង់ $x = 0$ ។

ដូចនេះ f ជាអនុគមន៍ជាប់ត្រង់ $x = 0$ ។

៣.លក្ខណៈភាពជាប់នៃអនុគមន៍

បើ f និង g ជាអនុគមន៍ជាប់ត្រង់ចំណុច $x=a$ នោះគេបាន ៖

ក. $f(x)+g(x)$ ជាអនុគមន៍ជាប់ត្រង់ ចំណុច $x=a$ ។

ខ. $f(x)-g(x)$ ជាអនុគមន៍ជាប់ត្រង់ ចំណុច $x=a$ ។

គ. $f(x).g(x)$ ជាអនុគមន៍ជាប់ត្រង់ ចំណុច $x=a$ ។

ឃ. $\lambda f(x)$ ជាអនុគមន៍ជាប់ត្រង់ ចំណុច $x=a$ ។ (λ ជាចំនួនពិត)

ង. $\dfrac{f(x)}{g(x)}$ ជាអនុគមន៍ជាប់ត្រង់ ចំណុច $x=a$ ដែល $g(a)\neq 0$ ។

៤.ភាពជាប់លើចន្លោះ

និយមន័យ ៖

❖ អនុគមន៍ f ជាប់លើចន្លោះបើក (a,b) លុះត្រាតែ f ជាប់ចំពោះគ្រប់តម្លៃ x នៃចន្លោះបើកនោះ ។

❖ អនុគមន៍ f ជាប់លើចន្លោះបិទ $[a,b]$ លុះត្រាតែ f ជាប់លើ (a,b) និងមាន លីមីត $\lim\limits_{x\to a^+} f(x)=f(a)$ និង $\lim\limits_{x\to b^-} f(x)=f(b)$ ។

ឧទាហរណ៍ គេឲ្យអនុគមន៍ f កំណត់ដោយ $f\left(x\right)=\begin{cases} -\dfrac{\pi}{2} & \text{បើ } x=-1 \\ \dfrac{\sin \pi x}{1-x^2} & \text{បើ } -1<x<1 \\ \dfrac{\pi}{2} & \text{បើ } x=1 \end{cases}$

ចូរសិក្សាភាពជាប់នៃអនុគមន៍ f ជាប់លើចន្លោះ $\left[-1,1\right]$ ។

ដំណោះស្រាយ

សិក្សាភាពជាប់នៃអនុគមន៍ f ជាប់លើចន្លោះ $[-1,1]$

គេមាន $f(-1)=-\frac{\pi}{2}$ និង $f(1)=\frac{\pi}{2}$

អនុគមន៍ f ដែល $f(x)=\frac{\sin \pi x}{1-x^2}$ ជាប់លើចន្លោះបើក $(-1,1)$។

យើងមាន $\lim\limits_{x\to -1^+} f(x)=\lim\limits_{x\to -1^+}\frac{\sin \pi x}{1-x^2}$ យក $x=-1-u$

កាលណា $x\to -1^+$ នោះ $u\to 0^-$

គេបាន $\lim\limits_{x\to -1^+} f(x)=\lim\limits_{u\to 0^-}\frac{\sin(-\pi-\pi u)}{1-(-1-u)^2}=\lim\limits_{u\to 0^-}\frac{\sin \pi u}{u(-2-u)}=-\frac{\pi}{2}=f(-1)$

ហើយ $\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-}\frac{\sin \pi x}{1-x^2}$ យក $x=1-u$

កាលណា $x\to 1^-$ នោះ $u\to 0^+$

គេបាន $\lim\limits_{x\to 1^-} f(x)=\lim\limits_{u\to 0^+}\frac{\sin(\pi-\pi u)}{1-(1-u)^2}=\lim\limits_{u\to 0^+}\frac{\sin \pi u}{u(2-u)}=\frac{\pi}{2}=f(1)$

ដោយ $\lim\limits_{x\to -1^+} f(x)=f(-1)$ និង $\lim\limits_{x\to 1^-} f(x)=f(1)$

ដូចនេះ f ជាអនុគមន៍ជាប់លើចន្លោះ $[-1,1]$។

ឧទាហរណ៍ គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases} -\frac{\pi}{2} & \text{បើ } x=0 \\ \frac{\sin \pi x}{x^2-2x} & \text{បើ } 0<x<2 \\ \frac{\pi}{2} & \text{បើ } x=2 \end{cases}$

ចូរសិក្សាភាពជាប់នៃអនុគមន៍ f ជាប់លើចន្លោះ $[0,2]$។

ដំណោះស្រាយ

សិក្សាភាពជាប់នៃអនុគមន៍ f ជាប់លើចន្លោះ $\left[0,2\right]$

គេមាន $f\left(0\right)=-\frac{\pi}{2}$ និង $f\left(2\right)=\frac{\pi}{2}$

អនុគមន៍ f ដែល $f\left(x\right)=\frac{\sin\pi x}{x^2-2x}$ ជាប់លើចន្លោះបើក $\left(0,2\right)$ ។

យើងមាន $\lim\limits_{x\to0^+}f\left(x\right)=\lim\limits_{x\to0^+}\frac{\sin\pi x}{x^2-2x}=\lim\limits_{x\to0^+}\frac{\sin\pi x}{\pi x}\times\frac{\pi}{x-2}=-\frac{\pi}{2}=f\left(0\right)$

ហើយ $\lim\limits_{x\to2^-}f\left(x\right)=\lim\limits_{x\to2^-}\frac{\sin\pi x}{x^2-2x}$ យក $x=2-u$

កាលណា $x\to2^-$ នោះ $u\to0^+$

គេបាន $\lim\limits_{x\to2^-}f\left(x\right)=\lim\limits_{u\to0^+}\frac{\sin(2\pi-\pi u)}{(2-u)^2-2\left(2-u\right)}=\lim\limits_{u\to0^+}\frac{\sin\pi u}{u\left(2-u\right)}=\frac{\pi}{2}=f\left(2\right)$

ដោយ $\lim\limits_{x\to0^+}f\left(x\right)=f\left(0\right)$ និង $\lim\limits_{x\to2^-}f\left(x\right)=f\left(2\right)$

ដូចនេះ f ជាអនុគមន៏ជាប់លើចន្លោះ $\left[0,2\right]$ ។

៥.ភាពជាប់នៃអនុគមន៍បណ្តាក់

អនុគមន៍ g ជាប់ត្រង់ $x=a$ និងអនុគមន៍ f ជាប់ត្រង់ $x=g(a)$

នោះអនុគមន៍បណ្តាក់ $\left(f\,o\,g\right)(x)=f[g(x)]$ ជាប់ត្រង់ $x=a$ ។

ឧទាហរណ៏ គេឲ្យអនុគមន៍ g និង f កំណត់ដោយ $g(x)=\frac{4x}{x^2+1}$ និង $f(x)=\ln x$

តើអនុគមន៍ $\left(f\,o\,g\right)(x)=f[g(x)]$ ជាប់ត្រង់ $x=1$ ដែរឬទេ ?

ដំណោះស្រាយ

គេមាន $g(1)=\frac{4}{1^2+1}=2$

ហើយ $\lim_{x\to1} g(x)=\lim_{x\to1}\frac{4x}{x^2+1}=\frac{4}{1+1}=2=g(1)$ នោះ g ជាអនុគមន៍ជាប់ត្រង់ $x=1$ ។

ហើយបើ $x=g(1)=2$ នោះ $f(2)=\ln 2$ និង $\lim_{x\to2} f(x)=\lim_{x\to2}\ln x=\ln = f(2)$

នោះ f ជាអនុគមន៍ជាប់ត្រង់ $x=2$ ។

សម្រាយខាងលើបញ្ជាក់ថា $(f\circ g)(x)=f[g(x)]$ ជាប់ត្រង់ $x=1$ ។

៦.អនុគមន៍បន្លាយតាមភាពជាប់

បើ f ជាអនុគមន៍មិនកំណត់ត្រង់ $x=a$ និងមានលីមីត $\lim_{x\to a} f(x)=\lambda$

នោះអនុគមន៍បន្លាយនៃ f តាមភាពជាប់ត្រង់ $x=a$ កំណត់ដោយ ៖

$$g(x)=\begin{cases} f(x) & \text{បើ } x\neq a \\ \lambda & \text{បើ } x=a \end{cases}$$

ឧទាហរណ៍ គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\frac{\ln\left(\cos x+\sqrt{1+x^2}\right)-\ln 2}{x^2}$

ដែល $x\neq 0$ ។ រកអនុគមន៍បន្លាយតាមភាពជាប់នៃ f ត្រង់ $x=0$ ។

ដំណោះស្រាយ

រកអនុគមន៍បន្លាយតាមភាពជាប់នៃ f ត្រង់ $x=0$

យើងមាន $\lim_{x\to0} f(x)=\lim_{x\to0}\frac{\ln\left(\cos x+\sqrt{1+x^2}\right)-\ln 2}{x^2}$

$$=\lim_{x\to0}\frac{\ln\left[\frac{\cos x+\sqrt{1+x^2}}{2}\right]}{x^2}$$

$$=\lim_{x\to0}\frac{\ln\left[1+\left(\frac{\cos x+\sqrt{1+x^2}-2}{2}\right)\right]}{\frac{\cos x+\sqrt{1+x^2}-2}{2}}\times\lim_{x\to0}\frac{\cos x+\sqrt{1+x^2}-2}{2x^2}$$

$$=\lim_{x\to0}\frac{\cos x-1}{2x^2}+\lim_{x\to0}\frac{\sqrt{1+x^2}-1}{2x^2}$$

$$=-\frac{1}{2}\lim_{x\to0}\frac{\sin^2 x}{x^2}\times\frac{1}{1+\cos x}+\frac{1}{2}\lim_{x\to0}\frac{1}{\sqrt{1+x^2}+1}=-\frac{1}{4}+\frac{1}{4}=0$$

បើ g ជាអនុគមន៍បន្លាយតាមភាពជាប់នៃ f ត្រង់ $x=0$ នោះគេបាន៖

$$g(x)=\begin{cases}\dfrac{\ln\left(\cos x+\sqrt{1+x^2}\right)-\ln 2}{x^2} & \text{បើ } x\neq 0\\ 0 & \text{បើ } x=0\end{cases}$$

ឧទាហរណ៍ គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\dfrac{1}{x^2}\ln\left(\dfrac{1}{2}+\sqrt{x^2+\dfrac{1}{4}}\right)$

ដែល $x\neq 0$ ។ ចូររកអនុគមន៍បន្លាយតាមភាពជាប់នៃ f ត្រង់ $x=0$ ។

ដំណោះស្រាយ

រកអនុគមន៍បន្លាយតាមភាពជាប់នៃ f ត្រង់ $x=0$

យើងមាន $\lim\limits_{x\to0}f(x)=\lim\limits_{x\to0}\dfrac{1}{x^2}\ln\left(\dfrac{1}{2}+\sqrt{x^2+\dfrac{1}{4}}\right)$

$$=\lim_{x\to0}\frac{\ln\left(1+\left(\sqrt{x^2+\frac{1}{4}}-\frac{1}{2}\right)\right)}{\left(\sqrt{x^2+\frac{1}{4}}-\frac{1}{2}\right)}\times\lim_{x\to0}\frac{\sqrt{x^2+\frac{1}{4}}-\frac{1}{2}}{x^2}$$

$$=\lim_{x\to0}\frac{x^2+\frac{1}{4}-\frac{1}{4}}{x^2\left(\sqrt{x^2+\frac{1}{4}}+\frac{1}{2}\right)}=\lim_{x\to0}\frac{1}{\sqrt{x^2+\frac{1}{4}}+\frac{1}{2}}=\frac{1}{\frac{1}{2}+\frac{1}{2}}=1$$

តាង g ជាអនុគមន៍បន្លាយតាមភាពជាប់នៃ f ត្រង់ $x=0$ នោះគេបាន៖

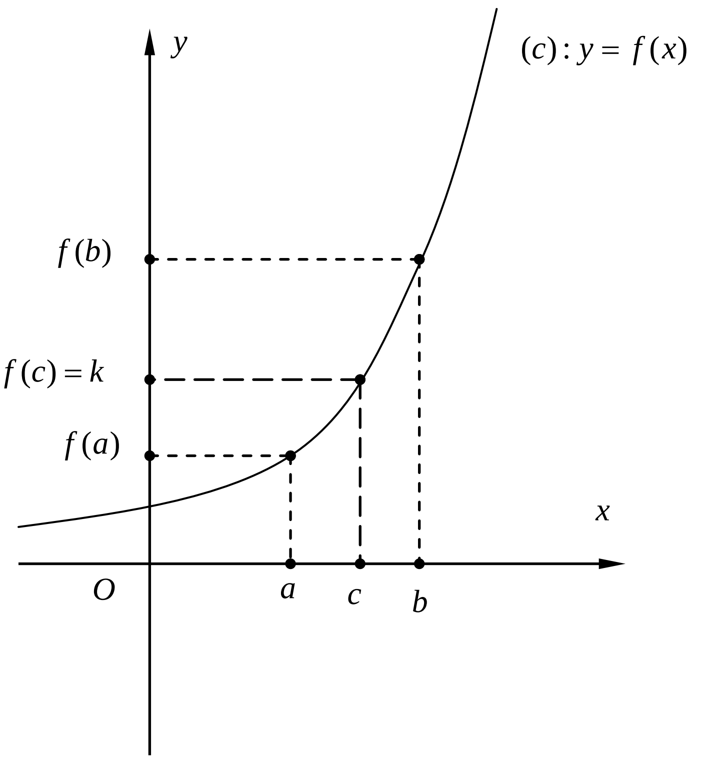
$$g(x)=\begin{cases}\dfrac{1}{x^2}\ln\left(\dfrac{1}{2}+\sqrt{x^2+\dfrac{1}{4}}\right) & \text{បើ } x\neq 0\\ 1 & \text{បើ } x=0\end{cases}$$

៧.ទ្រឹស្តីបទតម្លៃកណ្តាល

ទ្រឹស្តីបទ បើអនុគមន៍ f ជាប់លើចន្លោះបិទ $[a,b]$ និង k ជាចំនួនមួយនៅចន្លោះ $f(a)$ និង $f(b)$ នោះមានចំនួនពិត c មួយយ៉ាងតិចក្នុងចន្លោះបិទ $[a,b]$ ដែល $f(c)=k$ ។

វិបាក បើអនុគមន៍ f ជាប់ហើយកើនដាច់ខាត ឬ ចុះដាច់ខាតលើចន្លោះបិទ $[a,b]$ នោះចំពោះគ្រប់ចំនួន kនៅចន្លោះ $f(a)$ និង $f(b)$សមីការ $f(x)=k$ មានចម្លើយតែមួយគត់នៅចន្លោះ$[a,b]$ ។

សម្រាយបញ្ជាក់



អនុគមន៍ f ជាប់ និង កើនដាច់ខាត លើចន្លោះបិទ$[a,b]$ ។

f ជាអនុគមន៍កើនដាច់ខាតមានន័យថាមានពីរចំនួន α , β នៃ$[a,b]$ដែល $\alpha<\beta$ នាំឲ្យ $f(\alpha)<f(\beta)$ ។ ឲ្យចំនួន k នៅចន្លោះ $f(a)$ និង $f(b)$ ($f(a)<k<f(b)$) និង f ជាអនុគមន៍ជាប់ ។

តាមទ្រឹស្តីបទបញ្ជាក់ថាមានចំនួន c នៅចន្លោះ a និង b ដែល $f(c)=k$ ។

ឧបមាថាមានចំនួន c' មួយទៀតផ្សេងពី c ដែល $f(c')=k$ នោះគេបាន $f(c)=f(c')$ ដែលផ្ទុយពីសម្មតិកម្មដែលថា f ជាអនុគមន៍កើនដាច់ខាត។

ដូចនេះមានចំនួន cតែមួយគត់ដែលផ្ទៀងផ្ទាត់ $f(c)=k$ មានន័យថា សមីការ $f(x)=k$ មានចម្លើយតែមួយគត់ ។

សម្គាល់ ទ្រឹស្តីបទតម្លៃកណ្តាលអាចប្រើបានចំពោះ $k=0$ជាពិសេសបើ f ជាអនុគមន៍ជាប់លើចន្លោះបិទ$[a,b]$ និង $f(a).f(b)<0$ នោះមានចំនួន c មួយយ៉ាងតិចនៃ $[a,b]$ ដែល $f(c)=0$ ។

ឧទាហរណ៍

គេឲ្យសមីការ$(E): ax^2+bx+c=0$ ដែល $ac\neq 0$ និង$a,b,c\in\mathbb{R}$

បើ $7a+4b+c=0$ និង $|a|>|c|$ ចូរស្រាយថាសមីការ(E)មានឫសយ៉ាងតិចមួយស្ថិតនៅក្នុងចន្លោះចំនួន 1 និង 2។

ដំណោះស្រាយ

ស្រាយថាសមីការ(E)មានឫសយ៉ាងតិចមួយស្ថិតនៅក្នុងចន្លោះចំនួន 1 និង 2

គេមាន$(E): ax^2+bx+c=0$ ដែល $ac\neq 0$ និង$a,b,c\in\mathbb{R}$

យក $f(x)=ax^2+bx+c$ ជាអនុគមន៍ជាប់លើ$[1,2]$

យើងមាន $f(1)=a+b+c=-\frac{3}{4}(a-c)+\frac{1}{4}(7a+4b+c)=-\frac{3}{4}(a-c)$

ហើយ $f(2)=4a+2b+c=\frac{1}{2}(a+c)+\frac{1}{2}(7a+4b+c)=\frac{1}{2}(a+c)$

ព្រោះ$7a+4b+c=0$ (តាមបម្រាប់)

យើងបាន $f(1)f(2)=-\frac{3}{8}(a^2-c^2)=-\frac{3}{8}(|a|^2-|c|^2)<0$ព្រោះ$|a|>|c|$

តាមទ្រឹស្តីបទតម្លៃកណ្តាលបញ្ជាក់ថាយ៉ាងហើយណាស់មានចំនួនពិត $x_0\in(1,2)$

ដែល $f(x_0)=0$ ។

ដូចនេះសមីការ(E)មានឫសយ៉ាងតិចមួយស្ថិតនៅក្នុងចន្លោះចំនួន 1 និង 2។

ឧទាហរណ៍

ក.ស្រាយបញ្ជាក់ថាសមីការ $x\tan x=\cos x$ យ៉ាងហោចណាស់មានឫសពិតមួយ

ចន្លោះ $[0,\frac{\pi}{4}]$ ។

ខ.ស្រាយបញ្ជាក់ថាសមីការ $(x^n-1)\cos x+\sqrt{2}\sin x-1=0$

យ៉ាងហោចណាស់មានឫសពិតមួយចន្លោះ $(0,1)$ ។

ដំណោះស្រាយ

ក.ស្រាយបញ្ជាក់ថាសមីការយ៉ាងហោចណាស់មានឫសពិតមួយចន្លោះ $[0,\frac{\pi}{4}]$

គេមានសមីការ $x\tan x=\cos x$

តាងអនុគមន៍ $f(x)=x\tan x-\cos x$

មានដែនកំណត់ $D_f=\mathbb{R}-\{\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}\}$ ។

អនុគមន៍ f ជាប់លើចន្លោះ $[\,0\,,\frac{\pi}{4}\,]$ ។

គេមាន $f(0)=-1$ និង $f(\frac{\pi}{4})=\frac{\pi}{4}-\frac{\sqrt{2}}{2}>0$

ដោយ $f(0)\times f(\frac{\pi}{4})<-(\frac{\pi}{4}-\frac{\sqrt{2}}{2})<0$ នោះតាមទ្រឹស្តីបទតម្លៃ

កណ្តាលយ៉ាងហោចណាស់មានចំនួនពិត c មួយនៃ $[\,0\,,\frac{\pi}{4}\,]$

ដែល $f(c)=0$ ។ ដូចនេះសមីការ $x\tan x=\cos x$ យ៉ាងហោចណាស់

មានឫសពិតមួយចន្លោះ $[\,0\,,\frac{\pi}{4}\,]$ ។

ខ.ស្រាយបញ្ជាក់ថាសមីការ $(x^n-1)\cos x+\sqrt{2}\sin x-1=0$

យ៉ាងហោចណាស់មានឫសពិតមួយចន្លោះ $(\,0\,,1\,)$ ។

តាងអនុគមន៍ $f(x)=(x^n-1)\cos x+\sqrt{2}\sin x-1$

គេមាន $f(0)=-2$ និង $f(1)=\sqrt{2}\sin 1-1>\sqrt{2}\sin\frac{\pi}{4}-1=0$

ដោយ $f(0)\times f(1)<0$ នោះតាមទ្រឹស្តីបទតម្លៃកណ្តាលយ៉ាងហោចណាស់

មានចំនួនពិត c មួយនៃ $[\,0\,,1\,]$ ដែល $f(c)=0$ ។

ដូចនេះសមីការ $(x^n-1)\cos x+\sqrt{2}\sin x-1=0$ យ៉ាងហោចណាស់មានឫស

ពិតមួយចន្លោះ$(0,1)$។

ជំពូកទី០៤

វិធីសាស្ត្រគណនាលីមីតនៃអនុគមន៍មួយចំនួន

លំហាត់ទី០១

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 2}\left(x^3+x-5\right)$ ខ. $\lim_{x\to 3}\left(2x^2-5x+1\right)$ គ. $\lim_{x\to -1}\left(x^4+6x^2+1\right)$

ឃ. $\lim_{x\to 2}\left(x+\sqrt{x^3+1}\right)$ ង. $\lim_{x\to 3}\left(\sqrt{x+3+\sqrt{x+6}}\right)$ ច. $\lim_{x\to 3}\left(x^2-\sqrt[3]{x^2-1}\right)$

ឆ. $\lim_{x\to 2}\frac{x^4+3x^2-8}{x^2+1}$ ជ. $\lim_{x\to 3}\frac{x^2+5x-4}{2x-1}$ ឈ. $\lim_{x\to 1}\frac{\sqrt{x^2+x+2}}{3x+1}$

លំហាត់ទី០២

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\left(\sin x+\cos x\right)$ ខ. $\lim_{x\to \frac{\pi}{3}}\left(\cos x+\sqrt{3}\sin x\right)$ គ. $\lim_{x\to 0}\frac{3+5\cos x}{3-\cos x}$

ឃ. $\lim_{x\to \frac{\pi}{6}}\left(\sin x+2\cos^2 x\right)$ ង. $\lim_{x\to \frac{\pi}{2}}\frac{2\sin x-\cos x}{1+\sin x}$ ច. $\lim_{x\to \frac{\pi}{3}}\frac{1+4\sin^2 x}{1+2\cos x}$

ឆ. $\lim_{x\to \frac{\pi}{3}}\frac{2\cos^2 x+3\cos x+1}{\cos x+\sqrt{3}\sin x}$ ជ. $\lim_{x\to \frac{\pi}{4}}\frac{3+\sqrt{2}\sin x}{3-\sqrt{2}\cos x}$ ឈ. $\lim_{x\to \frac{\pi}{3}}\frac{1+\sqrt{3}\tan x}{1+4\sin^2 x}$

លំហាត់ទី០៣

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to1}\dfrac{x-1}{x^2-1}$ ខ. $\lim\limits_{x\to3}\dfrac{x^2-4x+3}{x-3}$ គ. $\lim\limits_{x\to1}\dfrac{x^2+x-2}{x^2-3x+2}$

ឃ. $\lim\limits_{x\to2}\dfrac{x^3-8}{x^3-2x^2+2x-4}$ ង. $\lim\limits_{x\to1}\dfrac{(x-1)^2}{x^3-x^2-x+1}$ ច. $\lim\limits_{x\to3}\dfrac{x^3-27}{x^3-3x^2-3x+9}$

ឆ. $\lim\limits_{x\to-1}\dfrac{x^4+x^3-x^2+1}{x^3+1}$ ជ. $\lim\limits_{x\to1}\dfrac{x^3+x-2}{x^3-x^2+x-1}$ ញ. $\lim\limits_{x\to2}\dfrac{x^4-16}{x^3-2x^2+4x-8}$

លំហាត់ទី០៤

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to1}\dfrac{x^3-1}{x^2+x-2}$ ខ. $\lim\limits_{x\to3}\dfrac{x^3-3x^2+45x-135}{x^3-27}$

គ. $\lim\limits_{x\to2}\dfrac{x^3-8}{x^2-4}$ ឃ. $\lim\limits_{x\to-1}\dfrac{x^4+x^3+4x+4}{x^3+1}$

ង. $\lim\limits_{x\to2}\dfrac{x^4-2x^3+x^2-4}{x^3-2x^2-x+2}$ ច. $\lim\limits_{x\to3}\dfrac{x^2-9}{x^2-3x}$

ឆ. $\lim\limits_{x\to1}\dfrac{x^3+x-2}{x^2-1}$ ជ. $\lim\limits_{x\to2}\dfrac{x^2-6x+8}{x^2-3x+2}$

ឈ. $\lim\limits_{x\to0}\dfrac{(1+x)^4-4x-1}{x^4+x^2}$ ញ. $\lim\limits_{x\to0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}$

លំហាត់ទី០៥

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to4}\dfrac{x^3-64}{x^2-16}$ ខ. $\lim\limits_{x\to1}\dfrac{x^3-6x^2+9x-4}{x^3-x^2-x+1}$

គ. $\lim\limits_{x\to2}\dfrac{5x^2-8x-4}{x^3-8}$ ឃ. $\lim\limits_{x\to3}\dfrac{x^2-9}{x^2-3x}$

ង. $\lim\limits_{x\to 1}\dfrac{x^3+x-2}{x^2-1}$

ច. $\lim\limits_{x\to 2}\dfrac{x^2-6x+8}{x^2-3x+2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{(1+x)^4-4x-1}{x^4+x^2}$

ជ. $\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}$

ឈ. $\lim\limits_{x\to 4}\dfrac{x^3-64}{x^2-16}$

ញ. $\lim\limits_{x\to 2}\dfrac{x^3-8+4(x-2)}{x^2-4}$

លំហាត់ទី០៦

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 3}\dfrac{x^3-3x^2+x-3}{2x^2-7x+3}$

ខ. $\lim\limits_{x\to 1}\dfrac{3x^4-4x^3+1}{(x-1)^2}$

គ. $\lim\limits_{x\to 1}\dfrac{x^4-4x+3}{x^3-x^2-x+1}$

ឃ. $\lim\limits_{x\to 0}\dfrac{(1+ax)^n-1}{x}$

ង. $\lim\limits_{x\to 0}\dfrac{(1+x)^3-(3x+1)}{x^2}$

ច. $\lim\limits_{x\to 0}\dfrac{1-(1+x)(1+2x)(1+3x)}{x}$

ឆ. $\lim\limits_{x\to 1}\dfrac{x^n-1}{x-1}$ ដែល $n\in\mathbb{N}$

ជ. $\lim\limits_{x\to 1}\left(\dfrac{1}{1-x}-\dfrac{3}{1-x^3}\right)$

ឈ. $\lim\limits_{x\to 2}\dfrac{x^3-3x^2+4}{x^3-4x^2+4x}$

ញ. $\lim\limits_{x\to 1}\dfrac{x^3-x^2+x-1}{x^3-1}$

លំហាត់ទី០៧

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to -1}\dfrac{x^2-1}{x+1}$

ខ. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}$

គ. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x-3}$

ឃ. $\lim\limits_{x\to -4}\dfrac{x^3+64}{x+4}$

ង. $\lim\limits_{x\to 2}\left(\dfrac{1}{2-x}-\dfrac{12}{8-x^3}\right)$

ច. $\lim\limits_{x\to 2}\left[\dfrac{x(x+1)}{x-2}-\dfrac{6x^2}{x^2-4}\right]$

ឆ. $\lim_{x\to1}\dfrac{x^3+x-2}{x^2-1}$

ជ. $\lim_{x\to2}\dfrac{x^5-32}{x^4-16}$

ឈ. $\lim_{x\to1}\dfrac{x^5+x^3-1}{x^4-1}$

ញ. $\lim_{x\to2}\dfrac{x^3-8}{x^4-2x^3-x^2+4}$

លំហាត់ទី០៨

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to1}\dfrac{x^4-2x^2+1}{x^3-3x+2}$

ខ. $\lim_{x\to3}\dfrac{x^3-3x^2+7x-21}{x^3-3x^2-x+3}$

គ. $\lim_{x\to2}\dfrac{x^6-64}{x^4-2x^3+8x-16}$

ឃ. $\lim_{x\to1}\dfrac{2x^2-5x+3}{x^2-3x+2}$

ង. $\lim_{x\to2}\dfrac{x^3-3x-2}{x^2-5x+6}$

ច. $\lim_{x\to1}\dfrac{x^3-3x+2}{2x^3-3x+1}$

ឆ. $\lim_{x\to1}\dfrac{x^4-4x+3}{(x-1)^2}$

ជ. $\lim_{x\to2}\dfrac{x^3-2x^2+2x-4}{x^2-6x+8}$

ឈ. $\lim_{x\to\frac{\pi}{6}}\dfrac{2\sin^2x-3\sin x+1}{4\sin^2x-1}$

ញ. $\lim_{x\to\frac{\pi}{6}}\dfrac{8\sin^3x-1}{2\sin^2x-3\sin x+1}$

លំហាត់ទី០៩

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to1}\left(\dfrac{2}{1-x^2}-\dfrac{3}{1-x^3}\right)$

ខ. $\lim_{x\to0}\dfrac{(1+x)^4-4x-1}{x^3+x^2}$

គ. $\lim_{x\to2}\dfrac{x+x^2+x^3-12}{x-2}$

ឃ. $\lim_{x\to1}\dfrac{x+2x^2+3x^3-6}{x-1}$

ង. $\lim_{x\to1}\dfrac{x^3-3x^2+3x-1}{(x^3-x)^3}$

ច. $\lim_{x\to-1}\dfrac{x^2+3x+2}{x^2-2x-3}$

ឆ. $\lim\limits_{x\to\frac{1}{2}}\dfrac{2x^2-x}{2x^2-3x+1}$

ជ. $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}$ $(\ n\in\mathbb{N}\ )$ ។

ឈ. $\lim\limits_{x\to1}\dfrac{x+x^2+x^3-3}{x-1}$

ញ. $\lim\limits_{x\to1}\dfrac{x+x^2+x^3+\ldots+x^n-n}{x-1}$ ។

លំហាត់ទី១០

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\tan^2x-(1+\sqrt{3})\tan x+\sqrt{3}}{\sin x-\sqrt{3}\cos x}$

ខ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\tan^2x-3\tan x+2}{\sin x-\cos x}$

គ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos 2x}{\cos^3x-\sin^3x}$

ឃ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{4\cos^2x-1}{\sin x-\sin 2x}$

ង. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\tan^3x}{\cos 2x}$

ច. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\cos 3x}{\cos x-\sin 2x}$

ឆ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\tan^3x}$

ជ. $\lim\limits_{x\to0}\dfrac{1-\cos 2x}{\sin^2x+\sin^3x}$

ឈ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{3-4\sin^2x}{2\cos x-1}$

ញ. $\lim\limits_{x\to0}\dfrac{2x-x^2-\sin^2x}{\cos x+x-1}$

លំហាត់ទី១១

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\cos x-\sin 2x}{4\cos^2x-3}$

ខ. $\lim\limits_{x\to0}\dfrac{\cos x-\sqrt{x^2+1}}{x^2+\sin^2x}$

គ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\tan^4x+\cot^4x-2}{1-\sin 2x}$

ឃ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\sqrt[3]{\tan x}}$

ង. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{1-8\sin^3x}{4\cos^2x-3}$

ច. $\lim\limits_{x\to0}\dfrac{x^4-\sin^4x}{\sqrt{x^2+1}-\cos x}$

ឆ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos^3 x-\sin^3 x}{1-\tan^2 x}$

ជ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{3-4\sin^2 x}{2\cos x-1}$

ឈ. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{4\cos^2 x-3}{1-2\sin x}$

ញ. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{2-\sin x-\sin^2 x}{1-\sin x}$

លំហាត់ទី១២

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{\sin x(1-\cos^3 x)}$

ខ. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{2\sin x-1}$

គ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\tan^3 x}{\cos x-\sin x}$

ឃ. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{\cos x-\sin 2x}$

ង. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\cos x-1}{3-4\sin^2 x}$

ង. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos^4 x-\sin^4 x}{1-\tan x}$

ឋ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\tan^2 x-(2+\sqrt{3})\tan x+2\sqrt{3}}{\sin x-\sqrt{3}\cos x}$

ឆ. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x-5\sin x+2}{3-4\cos^2 x}$

លំហាត់ទី១៣

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-1}{x^2}$

ខ. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3}-2}{x-1}$

គ. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

ឃ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2-2}-\sqrt{2}}{x-2}$

ង. $\lim\limits_{x\to 0}\dfrac{1+x-\sqrt{2x+1}}{x^2}$

ច. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-\sqrt{x+7}}{x-2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{\sqrt{(1+x)(1+2x)}-1}{x}$

ជ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^3+1}-3}{x-2}$

ឈ. $\lim\limits_{x\to1}\dfrac{\sqrt{2x^2+3x+4}-\sqrt{x^2+3x+5}}{x-1}$ ញ. $\lim\limits_{x\to3}\dfrac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}$

លំហាត់ទី១៤

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to2}\dfrac{\sqrt{2+\sqrt{2+x}}-2}{x-2}$ ខ. $\lim\limits_{x\to1}\dfrac{x\sqrt{x}-\sqrt{3x-2}}{\sqrt{2x-1}-x}$

គ. $\lim\limits_{x\to2}\dfrac{\sqrt{x^3+1}-3}{x^2-2x}$ ឃ. $\lim\limits_{x\to1}\dfrac{\sqrt{2x^2-2x+1}-x}{(x-1)^2}$

ង. $\lim\limits_{x\to2}\dfrac{2x+1-\sqrt{3x^2+8x-3}}{(x-2)^2}$ ច. $\lim\limits_{x\to2}\dfrac{\sqrt{x^3+1}-\sqrt{6x^2-12x+9}}{(x-2)^3}$

ឆ. $\lim\limits_{x\to3}\dfrac{x^3-27}{\sqrt{x^2-5}-2}$ ជ. $\lim\limits_{x\to2}\dfrac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}$

ញ. $\lim\limits_{x\to3}\dfrac{x-\sqrt{2x+3}}{x-3}$ ញ. $\lim\limits_{x\to1}\dfrac{(x-1)^2}{x-\sqrt{2x-1}}$

លំហាត់ទី១៥

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to4}\dfrac{\sqrt{x^2+9}-5}{\sqrt{x}-2}$ ខ. $\lim\limits_{x\to0}\dfrac{\sqrt{x^2-x+1}-\sqrt{x^2+x+1}}{x}$

គ. $\lim\limits_{x\to2}\dfrac{x-2}{\sqrt{x^2+12}-4}$ ឃ. $\lim\limits_{x\to1}\dfrac{(x-2)^2}{x-2\sqrt{x-1}}$

លំហាត់ទី១៦

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x}-2}{x-2}$ ខ. $\lim\limits_{x\to0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}$

គ. $\lim\limits_{x\to2}\dfrac{\sqrt[3]{3x+2}-\sqrt[3]{x+6}}{x-2}$ ឃ. $\lim\limits_{x\to0}\dfrac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$

ង. $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x-1}-x+1}{x-2}$

ច. $\lim\limits_{x\to 2}\dfrac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

ឆ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2-2}-\sqrt{2}}{x-2}$

ជ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-\sqrt{4x+1}}{(x-2)^2}$

ឈ. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+x+2}-\sqrt{5x-1}}{x^2-1}$

ញ. $\lim\limits_{x\to 3}\dfrac{\sqrt{x^2+x+4}-4}{x-3}$

លំហាត់ទី១៧

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 3}\dfrac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}$

ខ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2-2}-\sqrt{4x-6}}{(\sqrt{x}-\sqrt{2})^2}$

គ. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}$

ឃ. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x^3+6x}-x-1}{(x-1)^3}$

ង. $\lim\limits_{x\to 8}\dfrac{\sqrt[3]{x}-2}{x-8}$

ច. $\lim\limits_{x\to 2}\dfrac{x-1-\sqrt[3]{3x^2-9x+7}}{(x-2)^3}$

ឆ. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}$

ជ. $\lim\limits_{x\to 2}\dfrac{x-\sqrt[3]{x^2+4}}{x-2}$

ឈ. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3-2x^2}-\sqrt[3]{x^2-3x+1}}{(x-1)^3}$

ញ. $\lim\limits_{x\to 2}\dfrac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

លំហាត់ទី១៨

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3}-2}{x^3-1}$

ខ. $\lim\limits_{x\to 2}\dfrac{x\sqrt{x}-2\sqrt{2}}{x-2}$

គ. $\lim_{x\to 8}\frac{\sqrt[3]{x^2}-4}{x-8}$

ឃ. $\lim_{x\to a}\frac{\sqrt[n]{x}-\sqrt[n]{a}}{x-a}$

ង. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

ច. $\lim_{x\to 0}\frac{1+x-\sqrt[2006]{1+2006x}}{x^2}$

ឆ. $\lim_{x\to 0}\frac{1-\sqrt{(1+x)(1+2x)(1+3x)\ldots(1+nx)}}{x}$

ជ. $\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$

ឈ. $\lim_{x\to 1}\frac{\sqrt{2x^2+2x}-\sqrt{2x+2}}{x^3-1}$

ញ. $\lim_{x\to 2}\frac{\sqrt{x+2}+\sqrt[3]{x-1}-3}{x-2}$

លំហាត់ទី១៩

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\frac{\sin 2x}{\sin 6x}$

ខ. $\lim_{x\to 0}\frac{\sin^2 3x}{6x^2}$

គ. $\lim_{x\to 0}\frac{\sin^3 2x}{x^2\sin 6x}$

ឃ. $\lim_{x\to 0}\frac{2x+\sin 4x}{3x}$

ង. $\lim_{x\to 0}\frac{x-\sin 3x}{x+\sin 3x}$

ច. $\lim_{x\to 0}\frac{5x^2+\sin^2 2x}{x\sin 3x}$

ឆ. $\lim_{x\to 0}\frac{x^2\sin 4x}{x^3+\sin^3 x}$

ជ. $\lim_{x\to 0}\frac{3x\sin 4x}{\sin^2 6x}$

ឈ. $\lim_{x\to 0}\frac{\sin 2x\sin 6x}{\sin^2 3x}$

ញ. $\lim_{x\to 0}\frac{x^2\sin 2x}{\sin^3 x}$

លំហាត់ទី២០

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\frac{1-\cos 2x}{4x^2}$

ខ) $\lim_{x\to 0}\frac{\sin^2 6x}{3x\sin 4x}$

គ) $\lim_{x\to 0}\frac{\sin 3x\sin 4x}{6x^2}$

ឃ) $\lim_{x\to 0}\frac{2\sin 3x-\sin 4x}{x}$

ង) $\lim_{x\to 0}\frac{1-\cos^4 x}{4x^2}$

ច) $\lim_{x\to 0}\frac{2x}{x+\sin 3x}$

ឆ) $\lim_{x\to 0}\frac{x\sin 2x+\sin^2 4x}{9x^2}$

ជ) $\lim_{x\to 0}\frac{1+2x\sin 4x-\cos^2 2x}{6x^2}$

ឈ) $\lim_{x\to 0}\frac{x\sin^3 2x}{(1-\cos 2x)^2}$

ញ) $\lim_{x\to 0}\frac{1+x\sin 3x-\cos 6x}{3x^2+\sin^2 2x}$

លំហាត់ទី២១

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\frac{\sin^2 x-\sin^2 3x}{1-\cos 4x}$

ខ) $\lim_{x\to 0}\frac{\cos x-\cos 3x}{1-\cos 2x}$

គ) $\lim_{x\to 0}\frac{x^2-\sin^2 x}{x^2-x\sin x}$

ឃ) $\lim_{x\to 0}\frac{1-2\cos 2x+\cos^2 2x}{x^3\sin 2x}$

ង) $\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x(1-\cos 4x)}$

ច) $\lim_{x\to 0}\frac{3\sin 2x-\sin 6x}{x^3+\sin^3 x}$

ឆ) $\lim_{x\to 0}\frac{2-\cos 2x-\cos 4x}{1-\cos^4 5x}$

ជ) $\lim_{x\to 0}\frac{\sin^3(2x^2)}{(\tan x-\sin x)^2}$

ឈ) $\lim_{x\to 0}\frac{1-\cos^4(2x^3)}{(3\sin x-\sin 3x)^2}$

ញ) $\lim_{x\to 0}\frac{1-\cos^3 2x}{1-\cos^4 3x}$

លំហាត់ទី២២

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\frac{1-\cos 2x\cos 6x}{\sin^2 5x}$

ខ) $\lim_{x\to 0}\frac{1-\cos 4x}{1-\cos 6x}$

គ) $\lim_{x\to 0}\frac{1-\cos^2 6x}{1-\cos^3 2x}$

ឃ) $\lim_{x\to 0}\frac{1-\cos^2 2x\ \cos 4x}{x^2}$

ង) $\lim_{x\to 0}\frac{\cos^2 2x-\cos 6x}{3\sin^2 x+\sin^2 2x}$

ច) $\lim_{x\to 0}\frac{1-\sqrt{\cos^3 4x}}{3x^2}$

ឆ) $\lim_{x\to 0}\dfrac{\cos 4x-\sqrt{\cos 2x}}{x^2}$

ជ) $\lim_{x\to 0}\dfrac{1-\sqrt[3]{\cos 3x}}{x^2}$

ឈ) $\lim_{x\to 0}\dfrac{\cos 2x-\sqrt[3]{\cos 4x}}{x^2}$

ញ) $\lim_{x\to 0}\dfrac{1-\cos 2x\sqrt{\cos 4x}}{x^2}$

លំហាត់ទី២៣

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\dfrac{x\sin 3x}{\sin^2 5x}$

ខ) $\lim_{x\to 0}\dfrac{x(1-\cos 2x)^2}{\tan^3 2x-\sin^3 2x}$

គ) $\lim_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

ឃ) $\lim_{x\to 0}\dfrac{1+\sin x-\cos x}{1-\sin x-\cos x}$

ង) $\lim_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

ច) $\lim_{x\to 0}\dfrac{1-\cos x\cos 2x}{x^2}$

ឆ) $\lim_{x\to 0}\dfrac{1+x\sin x-\cos 2x}{\sin^2 x}$

ជ) $\lim_{x\to 0}\dfrac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$

ឈ) $\lim_{x\to 0}\dfrac{x(a-b)}{\sin ax-\sin bx}$

ញ) $\lim_{x\to \frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{4\sin^2 x-1}$

លំហាត់ទី២៤

ចូរគណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0}\dfrac{2\sin x-\sin 2x}{3\sin x-\sin 3x}$

ឃ. $\lim_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}$

ខ. $\lim_{x\to 0}\dfrac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}$

ង. $\lim_{x\to 0}\dfrac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x-\sqrt{\cos 3x}}}$

គ. $\lim_{x\to 0}\dfrac{1-\cos(1-\cos x)}{x^4}$

ច. $\lim_{x\to 0}\dfrac{1-\sqrt[3]{\cos 3x}}{x(1-\cos\sqrt{x})}$ ។

លំហាត់ទី២៥

ចូរគណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to0}\dfrac{\sin x\sin 2x\sin 3x}{x^3}$

ខ. $\lim\limits_{x\to0}\dfrac{\sin x+\sin 2x+\sin 3x}{x}$

គ. $\lim\limits_{x\to0}\dfrac{x+\sin 3x}{x}$

ឃ. $\lim\limits_{x\to0}\dfrac{\sin 2x}{\sin 6x}$

ង. $\lim\limits_{x\to0}\dfrac{x+\sin 3x}{x+\tan x}$

ច. $\lim\limits_{x\to0}\dfrac{1-\cos x}{x^2}$

ឆ. $\lim\limits_{x\to0}\dfrac{1-\cos 2x}{1-\cos 4x}$

ជ. $\lim\limits_{x\to0}\dfrac{1-\cos^3 2x}{x\sin x}$

ឈ. $\lim\limits_{x\to0}\dfrac{\tan x-\sin x}{x^3}$

ញ. $\lim\limits_{x\to0}\dfrac{2\sin x-\sin 2x}{3\sin x-\sin 3x}$

ដ. $\lim\limits_{x\to0}\dfrac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}$

ឋ. $\lim\limits_{x\to0}\dfrac{1-\cos(1-\cos x)}{x^4}$

ឌ. $\lim\limits_{x\to0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

ឍ. $\lim\limits_{x\to0}\dfrac{1-\sqrt{\cos x}}{1-\sqrt{\cos 2x}}$

ណ. $\lim\limits_{x\to0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}$

ត. $\lim\limits_{x\to0}\dfrac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x-\sqrt{\cos 3x}}}$

លំហាត់ទី២៦

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to0}\dfrac{1+x\sin x-\cos 2x}{\sin^2 x}$

ខ. $\lim\limits_{x\to0}\dfrac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}$

គ. $\lim\limits_{x\to0}\dfrac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$

ឃ. $\lim\limits_{x\to0}\dfrac{\tan x-\sin x}{x^3}$

ង. $\lim\limits_{x\to0}\dfrac{1+\sin x-\cos x}{1-\sin x-\cos x}$

ច. $\lim\limits_{x\to0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

ឆ. $\lim\limits_{x\to0}\dfrac{1-\cos x.\cos 2x}{x^2}$

ជ. $\lim\limits_{x\to0}\dfrac{1-\cos(1-\cos x)}{x^4}$

ឈ. $\lim\limits_{x\to1}(1-x^2)\tan\dfrac{\pi x}{2}$

ញ. $\lim\limits_{x\to1}\dfrac{\tan \pi x}{1-x}$

លំហាត់ទី២៧

គេមានអនុគមន៍ $y = f(x) = \dfrac{2 - 2\cos ax}{x^2}$ ដែល $a \in \mathbb{R}$ ។

កំណត់តម្លៃនៃ a ដើម្បីឲ្យ $\lim\limits_{x \to 0} f(x) = 100$ ។

លំហាត់ទី២៨

មានអនុគមន៍ f កំណត់ដោយ $f(x) = \dfrac{\tan x - \sin x}{x^n}$ ដែល $n \in \mathbb{N}$ ។

កំណត់គ្រប់ n ដែលធ្វើឲ្យ $\lim\limits_{x \to 0} f(x)$ ជាចំនួនពិត ។

លំហាត់ទី២៩

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\pi - 2x}{\cos x}$

ខ. $\lim\limits_{x \to 1} \dfrac{\sin \pi x}{x - 1}$

គ. $\lim\limits_{x \to 0} \left[\dfrac{1}{x^2} \left(\dfrac{2}{\cos x} + \cos x - 3 \right) \right]$

ឃ. $\lim\limits_{x \to 1} \dfrac{1 - \sin \dfrac{\pi x}{2}}{(1 - x)^2}$

លំហាត់ទី៣០

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim\limits_{x \to 0} \dfrac{\sin^2 x - \sin^2 3x}{1 - \cos 4x}$

ខ) $\lim\limits_{x \to 0} \dfrac{\cos x - \cos 3x}{1 - \cos 2x}$

គ) $\lim\limits_{x \to 0} \dfrac{x^2 - \sin^2 x}{x^2 - x \sin x}$

ឃ) $\lim\limits_{x \to 0} \dfrac{1 - 2\cos 2x + \cos^2 2x}{x^3 \sin 2x}$

ង) $\lim\limits_{x \to 0} \dfrac{\tan 2x - \sin 2x}{x(1 - \cos 4x)}$

ច) $\lim\limits_{x \to 0} \dfrac{3\sin 2x - \sin 6x}{x^3 + \sin^3 x}$

ឆ) $\lim\limits_{x \to 0} \dfrac{2 - \cos 2x - \cos 4x}{1 - \cos^4 5x}$

ជ) $\lim\limits_{x \to 0} \dfrac{\sin^3 (2x^2)}{(\tan x - \sin x)^2}$

ឈ) $\lim_{x\to 0}\dfrac{1-\cos^4(2x^3)}{(3\sin x-\sin 3x)^2}$ ញ) $\lim_{x\to 0}\dfrac{1-\cos^3 2x}{1-\cos^4 3x}$

លំហាត់ទី៣១

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\dfrac{1-\cos 2x\cos 6x}{\sin^2 5x}$ ខ) $\lim_{x\to 0}\dfrac{1-\cos 4x}{1-\cos 6x}$

គ) $\lim_{x\to 0}\dfrac{1-\cos^2 6x}{1-\cos^3 2x}$ ឃ) $\lim_{x\to 0}\dfrac{1-\cos^2 2x\ \cos 4x}{x^2}$

ង) $\lim_{x\to 0}\dfrac{\cos^2 2x-\cos 6x}{3\sin^2 x+\sin^2 2x}$ ច) $\lim_{x\to 0}\dfrac{1-\sqrt{\cos^3 4x}}{3x^2}$

ឆ) $\lim_{x\to 0}\dfrac{\cos 4x-\sqrt{\cos 2x}}{x^2}$ ជ) $\lim_{x\to 0}\dfrac{1-\sqrt[3]{\cos 3x}}{x^2}$

ឈ) $\lim_{x\to 0}\dfrac{\cos 2x-\sqrt[3]{\cos 4x}}{x^2}$ ញ) $\lim_{x\to 0}\dfrac{1-\cos 2x\sqrt{\cos 4x}}{x^2}$

លំហាត់ទី៣២

ចូរគណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0}\dfrac{1-\cos x+\sin x}{x}$ ខ. $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}$

គ. $\lim_{x\to 0}\dfrac{x+\tan x}{x+\tan 3x}$ ឃ. $\lim_{x\to 0}\dfrac{\sin[\sin(\sin x)]}{x}$

ង. $\lim_{x\to 0}\dfrac{\sin x\sin 2x\sin 3x....\sin(nx)}{x^n}$

លំហាត់ទី៣៣

ចូរគណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 1}\dfrac{1-\sin\frac{\pi x}{2}}{(1-x)^2}$ ខ. $\lim_{x\to \frac{\pi}{2}}\dfrac{\cos x}{\pi^2-4x^2}$

គ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{\pi-4x}$

ឃ. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$

ង. $\lim\limits_{x\to1}\dfrac{x^3-1+\tan\pi x}{1-x^2}$

ច. $\lim\limits_{x\to2}(4-x^2)\tan\dfrac{\pi x}{4}$

ឆ. $\lim\limits_{x\to\pi}(\pi-x)\tan\dfrac{x}{2}$

ជ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\tan x}{1-\sqrt{2}\sin x}$

ឈ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\pi-3x}{\sqrt{3}-2\sin x}$

ញ. $\lim\limits_{x\to\pi}\dfrac{\cos\dfrac{\pi x}{x+\pi}}{\pi-x}$

ដ. $\lim\limits_{x\to2}\dfrac{2-x}{\sin\dfrac{2\pi}{x}}$

លំហាត់ទី៣៤

ចូរគណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\pi}\dfrac{1-\sin\dfrac{x}{2}}{(\pi-x)^2}$

ខ. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\pi-2x}{\cos x}$

គ. $\lim\limits_{x\to\frac{1}{2}}\dfrac{\cos(\pi x)}{2x-1}$

ឃ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(\pi-4x)\cos 2x}{1-\sin 2x}$

ង. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\pi-3x}{\sqrt{3}-2\sin x}$

ច. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{2\cos x-\sqrt{2}}{\pi-4x}$

ឆ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\cos x+\sqrt{3}\sin x-2}{(\pi-3x)^2}$

ជ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{2-\sqrt{2}(\cos x+\sin x)}{\cos^2 2x}$

ឈ. $\lim\limits_{x\to1}(1-x^2)\tan\left(\dfrac{\pi x}{2}\right)$

ញ. $\lim\limits_{x\to\frac{\pi}{2}}\left[(\pi-2x)\tan x\right]$

លំហាត់ទី៣៥

ចូរគណនាលីមីតខាងក្រោម៖

1) $\lim\limits_{x\to\infty}\dfrac{4x^3-3x^2+1}{2x^3+x+3}$ 2) $\lim\limits_{x\to\infty}\dfrac{(4x^2+1)(6x^3+x)}{(3x+1)(8x^4+5)}$

3) $\lim\limits_{x\to\infty}\dfrac{4x^3-7x^2+x+3}{6x^3+x^2-5x+1}$ 4) $\lim\limits_{x\to\infty}\dfrac{x^{10}+(x+1)^{10}+\ldots+(x+10)^{10}}{x^{10}+10^{10}}$

5) $\lim\limits_{x\to+\infty}\dfrac{x^{100}+(x+1)^{100}+\ldots.+(x+100)^{100}}{x^{100}+100^{100}}$ 6) $\lim\limits_{x\to+\infty}\dfrac{\sqrt{4x^2+x+3}}{x}$

7) $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{8x^3+4x+1}+x+2}{x}$ 8) $\lim\limits_{x\to\infty}\dfrac{8x+1}{\sqrt[3]{x^3+4x}+\sqrt[3]{27x^3+9x+1}}$

លំហាត់ទី៣៦

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to\infty}\dfrac{(x-1)(2x+3)(2-x)}{(x^2+1)(2x+1)}$ ខ. $\lim\limits_{x\to+\infty}\dfrac{e^x-x}{2e^x+1}$

គ. $\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ ឃ. $\lim\limits_{x\to+\infty}\ln\left(\dfrac{2x+3}{x+1}\right)$

ង. $\lim\limits_{x\to+\infty}\left(\sqrt[4]{x^4+4x^3}+\sqrt[3]{x^3+3x^2}+\sqrt{x^2+2x}-3x\right)$

លំហាត់ទី៣៧

ចូរគណនាលីមីត ៖

ក. $\lim\limits_{x\to+\infty}\left(\sqrt{4x^2+x+2}-2\sqrt{x^2-x+3}\right)$

ខ. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x+2}-\sqrt{x^2-x+3}\right)$

លំហាត់ទី៣៨

ចូរគណនាលីមីត $\lim\limits_{n\to+\infty}\dfrac{1.2+2.3+3.4+\ldots.+(n-1)n}{n^3}$

លំហាត់ទី៣៩

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{n\to+\infty}(1-\frac{1}{2^2})(1-\frac{1}{3^2})....(1-\frac{1}{n^2})$

ខ. $\lim\limits_{n\to+\infty}(\frac{n}{\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+2}}+....+\frac{n}{\sqrt{n^4+n}})$ ។

លំហាត់ទី៤០

ចំពោះគ្រប់ $n\in\mathbb{N}$ គេមាន

$$S_n=\frac{2}{1\times3}+\frac{2}{3\times5}+...+\frac{2}{(2n+1)(2n+3)}=\sum_{p=0}^{n}\frac{2}{(2p+1)(2p+3)}$$

ក.គណនា S_n ជាអនុគមន៍នៃ n ដោយប្រើ $\frac{2}{(2p+1)(2p+3)}$ ជាទម្រង់ $\frac{a}{2p+1}+\frac{b}{2p+3}$ ។

ខ.គណនា $\lim\limits_{n\to+\infty}S_n$ ។

លំហាត់ទី៤១

ដោយប្រើសមភាព $\frac{k}{(k+1)!}=\frac{1}{k!}-\frac{1}{(k+1)!}$ ចូរគណនាផលបូក៖

$S_n=\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+...+\frac{n}{(n+1)!}$ រួចទាញរក $\lim\limits_{n\to+\infty}S_n$ ។

លំហាត់ទី៤២

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to+\infty}\frac{x+2\ln x}{1-2x+\ln x}$

ខ. $\lim\limits_{x\to+\infty}\frac{2e^x+x-1}{e^x+1}$

គ. $\lim\limits_{x\to+\infty}[x-\ln(2e^x+1)]$

ឃ. $\lim\limits_{x\to+\infty}\frac{3-2\ln x}{1+3\ln x}$

ង. $\lim\limits_{x\to+\infty}\dfrac{2x-\ln x}{x+\ln x}$ ច. $\lim\limits_{x\to\infty}\dfrac{2e^x+x-1}{e^x+x}$

លំហាត់ទី៤៣

ចូរគណនាលីមីតនៃអនុគមន៍ខាងក្រោម ៖

ក. $\lim\limits_{x\to0}\dfrac{e^x-e^{-x}}{\sin x}$ ខ. $\lim\limits_{x\to0}\dfrac{e^x+x-1}{x}$

គ. $\lim\limits_{x\to0}\dfrac{e^{-x^2}-\cos 2x}{x^2}$ ឃ. $\lim\limits_{x\to0}\dfrac{e^{\sin x}-e^{\tan x}}{x}$

ង. $\lim\limits_{x\to0}\dfrac{e^{ax}-e^{bx}}{x}$ ច. $\lim\limits_{x\to0}\dfrac{e^x+e^{-x}-2}{x^2}$

ឆ. $\lim\limits_{x\to0}\dfrac{e^x+e^{2x}-2}{\sin x}$ ជ. $\lim\limits_{x\to0}\dfrac{(e^x-1)(e^{3x}-1)}{x^2}$

ឈ. $\lim\limits_{x\to0}\dfrac{e^{x\sin 2x}-\cos 2x}{\tan^2 x}$ ញ. $\lim\limits_{x\to+\infty}x(e^{\frac{2}{x}}-1)$

លំហាត់ទី៤៤

ចូរគណនាលីមីត ៖

ក. $\lim\limits_{x\to0}\dfrac{\ln(1+4x)}{x}$ ខ. $\lim\limits_{x\to0}\dfrac{\ln(1-3x)}{x}$

គ. $\lim\limits_{x\to0}\dfrac{\ln(2-\cos x)}{x^2}$ ឃ. $\lim\limits_{x\to0}\dfrac{\ln(1+x\sin 3x)}{x^2}$

ង. $\lim\limits_{x\to0}\dfrac{\ln(\cos x)}{x^2}$ ច. $\lim\limits_{x\to0}\dfrac{\ln(2\cos x-\cos 2x)}{\sin^2 x}$

ឆ. $\lim\limits_{x\to0}\dfrac{\ln[(1+x)(1+2x)]}{x}$ ជ. $\lim\limits_{x\to0}\dfrac{\ln(1+\tan x)}{x}$

ឈ. $\lim\limits_{x\to0}\dfrac{\ln(2-e^x)}{x}$ ញ. $\lim\limits_{x\to0}\dfrac{\ln(3-2\cos x\sqrt{\cos 2x})}{x^2}$

លំហាត់ទី៤៥

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\frac{\ln(1+x)-\ln(1-x)}{x}$

ខ. $\lim_{x\to 1}\frac{\ln(x^2-2x+2)}{x^3-3x+2}$

គ. $\lim_{x\to \frac{\pi}{4}}\frac{\ln(\tan x)}{\pi-4x}$

ឃ. $\lim_{x\to \frac{\pi}{2}}\frac{\ln(\sin x)}{(\frac{\pi}{2}-x)^2}$

ង. $\lim_{x\to 2}\frac{\ln(x^2-4x+5)}{e^{x-2}+e^{2-x}-2}$

ច. $\lim_{x\to 2}\frac{\ln(4-x)-\ln 2}{x-2}$

ឆ. $\lim_{x\to 1}\frac{\ln x}{x^2-1}$

ជ. $\lim_{x\to e}\frac{\ln^2 x-3\ln x+2}{1-\ln x}$

លំហាត់ទី៤៦

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\frac{e^{2x+\sin x}-1}{x}$

ខ. $\lim_{x\to 0}\frac{e^{-3x}-e^{\sin 4x}}{x}$

គ. $\lim_{x\to 0}\frac{e^{ax}+e^{bx}+e^{cx}-3}{x}$

ឃ. $\lim_{x\to 0}\frac{e^{\sin x}-e^{-\sin x}}{x}$

ង. $\lim_{x\to 0}\frac{e^{2x}+4x-1}{e^{4x}-x-1}$

ច. $\lim_{x\to 0}\frac{e^{2x}+e^{4x}-2}{e^{3x}-1}$

ឆ. $\lim_{x\to 0}\frac{e^{x^3}+e^{x\sin^2 2x}-2}{x^3}$

ជ. $\lim_{x\to 0}\frac{e^{2x^2}-\cos 4x}{x^2}$

ឈ. $\lim_{x\to 0}\frac{e^{x\tan 2x}-2+\cos 2x}{x^2}$

ញ. $\lim_{x\to 0}\frac{e^{\sin x}+e^{-\sin x}-2}{x^2}$

ដ. $\lim_{x\to 0}\frac{2e^{3x}-3e^{2x}+1}{x^2}$

ឋ. $\lim_{x\to 0}\frac{4e^{-x}-5e^{2x}+1}{x}$

លំហាត់ទី៤៧

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{e^{1+x\sin 2x}-e^{\cos 2x}}{\sqrt{1+x^2}-1}$

ខ. $\lim\limits_{x\to 0}\dfrac{e^{\sin 2x}-e^{\tan 2x}}{x^3}$

គ. $\lim\limits_{x\to 0}\dfrac{e^{3\sin x}-e^{\sin 3x}}{1-\sqrt{1-x^3}}$

ឃ. $\lim\limits_{x\to 0}\dfrac{1-e^{x^2}\cos 2x}{\tan^2 x}$

ង. $\lim\limits_{x\to 0}\dfrac{(e^x-1)(e^{2x}-1)...(e^{nx}-1)}{x^n}$

ច. $\lim\limits_{x\to 0}\dfrac{1+x^2-e^{-x^2}\cos 4x}{x^2+1-\cos 2x}$

ឆ. $\lim\limits_{x\to 0}\dfrac{e^{1-x^2}-e^{\cos 4x}}{x^2}$

ជ. $\lim\limits_{x\to 0}\dfrac{e^{(x+1)^2}-e^{(x-1)^2}}{e^{x+1}-e^{1-x}}$

ឈ. $\lim\limits_{x\to 0}\dfrac{e^{2-\cos 2x}-e^{1+\sin^2 3x}}{e^{1+x^2}-e^{1-x^2}}$

ញ. $\lim\limits_{x\to 0}\dfrac{e^{\cos 2x}-e^{\cos 4x}}{e^x+e^{-x}-2}$

ដ. $\lim\limits_{x\to 0}\dfrac{e^{\sqrt{1+x\sin 2x}}-e^{\cos 2x}}{x^2}$

ឋ. $\lim\limits_{x\to 0}\dfrac{e^{\tan^2 x+\cos 2x}-e^{1+\sin^2 3x}}{x^2}$

លំហាត់ទី៤៨

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{e^{x^2+2x}-e^{x^2-2x}}{x}$

ខ. $\lim\limits_{x\to 0}\dfrac{e^{\sqrt{1+x^2}}-e^{\cos 2x}}{x^2}$

គ. $\lim\limits_{x\to 0}\dfrac{1-e^{\sin^2 x}\sqrt{\cos x}}{x^2}$

ឃ. $\lim\limits_{x\to 0}\dfrac{e^{\cos ax}-e^{\cos bx}}{x^2}$

ង. $\lim\limits_{x\to 0}\dfrac{e^{\sin^2 x}-e^{1-\sqrt{\cos 2x}}}{x^2}$

ច. $\lim\limits_{x\to 0}\dfrac{e^{\cos 2x}-e^{\cos x\sqrt{\cos 2x}}}{x^2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{e^{x^3+3x^2}-e^{x^3-3x^2}}{e^{1+x^2}-e^{\cos 2x}}$

ជ. $\lim\limits_{x\to 0}\dfrac{e^{1-\sqrt{\cos 2x}}-e^{\tan^{2x}}}{x^2}$

ឈ. $\lim\limits_{x\to 0}\dfrac{1-e^{\sin^2 x}\cos 2x\cos 4x}{x^2}$

ញ. $\lim\limits_{x\to 0}\dfrac{e^{1+\sin^2 3x}-e^{\cos x\cos 3x}}{x^2}$

ដ. $\lim\limits_{x\to 0}\dfrac{e^{1+3\sin 2x}-e^{2-\cos 4x}}{x^2}$

ឋ. $\lim\limits_{x\to 0}\dfrac{1-(1+x^2)e^{x^2}}{1-e^{-x^2}\cos 2x}$

លំហាត់ទី៤៩

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{2^x-3^x}{x}$

ខ. $\lim\limits_{x\to 0}\dfrac{a^x+b^x-2}{a^x-b^x}$

គ. $\lim\limits_{x\to 0}\dfrac{6^x-2^x}{4^x-2^x}$

ឃ. $\lim\limits_{x\to 0}\dfrac{2^{x+1}-2e^x}{x}$

ង. $\lim\limits_{x\to 0}\dfrac{4-(1+a^x)(1+b^x)}{x}$

ច. $\lim\limits_{x\to 0}\dfrac{2^{\cos x}-2^{x^2}}{x^2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{3^{\tan x}-3^{-\tan x}}{x}$

ជ. $\lim\limits_{x\to 0}\dfrac{2^{\cos 2x}-2^{\cos 4x}}{x^2}$

ឈ. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2^{x+3}}-\sqrt{3^{x+2}}}{x}$

ញ. $\lim\limits_{x\to 0}\dfrac{4^{\cos 2x}-2^{1+\cos 4x}}{x^2}$

ត. $\lim\limits_{x\to 0}\dfrac{a^x+a^{-x}-2}{x^2}$

ថ. $\lim\limits_{x\to 0}\dfrac{1-2^{x^2}\cos 2x}{x^2}$

លំហាត់ទី៥០

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{\ln(1-2x)}{x}$

ខ. $\lim\limits_{x\to 0}\dfrac{\ln(1+2\sin 3x)}{x}$

គ. $\lim\limits_{x\to 0}\dfrac{\ln(1+x\sin 2x)}{x^2}$

ឃ. $\lim\limits_{x\to 0}\dfrac{\ln(3-2\cos 2x)}{x^2}$

ង. $\lim\limits_{x\to 0}\dfrac{\ln(\cos 6x)}{x^2}$

ច. $\lim\limits_{x\to 0}\dfrac{\ln(\cos 2x)+\ln(\cos 4x)}{x^2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{\ln(2-\sqrt{1+x^2})}{x^2}$

ជ. $\lim\limits_{x\to 0}\dfrac{\ln 2-\ln(1+\cos 2x)}{2^{x^2}-1}$

ឈ. $\lim\limits_{x\to 0}\dfrac{\ln(1+4x^2)}{\ln(\cos x)-\ln(\cos 3x)}$

ញ. $\lim\limits_{x\to 0}\dfrac{\ln(\cos x\cos^3 2x)}{x^2}$

ត. $\lim\limits_{x\to 0}\dfrac{\ln\left[(1+x)(1+2x)...(1+nx)\right]}{x}$

ថ. $\lim\limits_{x\to 0}\dfrac{\ln(2-e^x)}{x}$

លំហាត់ទី៥១

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\left(\dfrac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{x}}$

ខ. $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$

គ. $\lim_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{x}{x-\tan x}}$

ឃ. $\lim_{x\to 1}\left(\dfrac{x+1}{2}\right)^{\frac{1}{1-x}}$

ង. $\lim_{x\to 2}\left(\dfrac{x}{2}\right)^{\frac{2}{x^2-2x}}$

ច. $\lim_{x\to \frac{\pi}{4}}(\tan x)^{\tan 2x}$

ឆ. $\lim_{x\to 0}\left(\dfrac{e^x+e^{2x}}{2}\right)^{\frac{1}{\sin x}}$

ជ. $\lim_{x\to \frac{\pi}{2}}(\sin x)^{\frac{1}{(\pi-2x)^2}}$

ឈ. $\lim_{x\to \pi}(\cos 2x)^{\frac{1}{(\pi-x)^2}}$

ញ. $\lim_{x\to 0}\left(2e^x-e^{2x}\right)^{\frac{1}{x^2}}$

ដ. $\lim_{x\to \frac{\pi}{2}}\left(\tan\dfrac{x}{2}\right)^{\frac{1}{\cos x}}$

ឋ. $\lim_{x\to 0}\left(\cos x\cos 2x\ldots\cos(nx)\right)^{\frac{1}{x^2}}$

លំហាត់ទី៨២

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\left(\dfrac{e^x+e^{2x}+\ldots+e^{nx}}{n}\right)^{\frac{1}{x}}$

ខ. $\lim_{x\to 0}\left(\dfrac{a^x+b^x}{2}\right)^{\frac{1}{x}}$

គ. $\lim_{x\to 1}\left(\dfrac{2x+1}{x+2}\right)^{\frac{x}{x-1}}$

ឃ. $\lim_{x\to 2}(x-1)^{\frac{x}{x-2}}$

ង. $\lim_{x\to \infty}\left(\dfrac{x-1}{x+1}\right)^{x}$

ច. $\lim_{n\to +\infty}\left(\cos\dfrac{1}{n}\right)^{n^3+2n^2}$

ឆ. $\lim_{n\to+\infty}\left(\frac{n^2+n+1}{n^2-n+1}\right)^n$ ជ. $\lim_{n\to+\infty}\left(\frac{2n+1}{2n-1}\right)^{n+1}$

ឈ. $\lim_{n\to+\infty}\left(\frac{\sqrt[n]{a}+\sqrt[n]{b}}{2}\right)^n$ ញ. $\lim_{x\to\infty}\left(\frac{x(x+2)}{x^2+1}\right)^x$

ត. $\lim_{x\to\infty}\left(\frac{x^2+1}{x(x+1)}\right)^x$ ប. $\lim_{x\to+\infty}\left(\frac{e^x-1}{e^x+1}\right)^{e^x}$

លំហាត់ទី៥៣

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\frac{a_1^x+a_2^x+\ldots+a_n^x}{n}$

ដែល $a_1,a_2,\ldots,a_n>0$ ។ ចូរគណនាលីមីត $\lim_{x\to0}\left(f(x)\right)^{\frac{1}{x}}$ ។

លំហាត់ទី៥៤

កំណត់ចំនួនពិត a និង b ដើម្បីឲ្យ $\lim_{x\to1}\frac{e^{x^2+ax}-e^{3x^2-ax}}{x-1}=b$ ។

លំហាត់ទី៥៥

គណនាលីមីត $\lim_{x\to0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}$ ។

លំហាត់ទី៥៦

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to+\infty}xe^{x+1}$ ខ. $\lim_{x\to-\infty}x^2e^{4x}$ គ. $\lim_{x\to+\infty}\left(1+\frac{n}{x}\right)^x$ ។

លំហាត់ទី៥៧

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 1}\frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$

ខ. $\lim_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}$, $(a>0\ \&\ b<0)$ ។

លំហាត់ទី៥៨

ចូរគណនាលីមីតខាងក្រោមនេះ

ក. $\lim_{x\to 0}\frac{\sqrt{x^2-x+1}-1}{\sqrt{1+x}-\sqrt{1-x}}$

ខ. $\lim_{x\to +\infty}\frac{(x-1)(2x+3)(2-x)}{(x^2+1)(2x+1)}$

គ. $\lim_{x\to 0^-}\frac{x^2+x}{|x|}$

ឃ. $\lim_{x\to -\infty}\left(\sqrt{x^2+8x-1}-\sqrt{x^2-3}\right)$

លំហាត់ទី៥៩

គណនាលីមីតខាងក្រោម

ក. $\lim_{x\to 0}\frac{x\sin 3x}{\sin^2 5x}$

ខ. $\lim_{x\to 0}\frac{(1-\cos x)^2}{\tan^3 x-\sin^3 x}$

គ. $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

ឃ. $\lim_{x\to \frac{\pi}{2}}\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}$

ង. $\lim_{x\to \pm\infty} x\sin\frac{1}{x}$

ច. $\lim_{x\to +\infty} x^2\left(1-\cos\frac{1}{x}\right)$

លំហាត់ទី៦០

គណនាលីមីតខាងក្រោម

ក. $\lim_{x\to \frac{\pi}{6}}\frac{2\sin^2 x-3\sin x+1}{4\sin^2 x-1}$

ខ. $\lim_{x\to 0}\frac{1+x\sin x-\cos x}{\sin^2 x}$

គ. $\lim_{x\to a}\frac{x(a-b)}{\sin ax-\sin bx}$ $(\ a\neq 0\ ,\ b\neq 0\ ,\ a\neq b\ )$

ឃ. $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$

ង. $\lim_{x\to +\infty}(\sqrt{x^2+3x}-\sqrt{x^2+1}-1)$

ឋ. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}$ ។

លំហាត់ទី៦១

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases}\dfrac{x^3-4x}{x^3-8} & \text{បើ } x\neq 2\\ \dfrac{2}{3} & \text{បើ } x=2\end{cases}$

ចូរសិក្សាភាពជាប់នៃអនុគមន៍ f ត្រង់ $x=2$ ។

លំហាត់ទី៦២

កំណត់តម្លៃ a ដើម្បីឲ្យអនុគមន៍ខាងក្រោមជាប់លើ $\mathbb{R}$ ៖

ក. $f(x)=\begin{cases}-2x+a & \text{បើ } x\leq 1\\ \log_3 x & \text{បើ } x>1\end{cases}$

ខ. $f(x)=\begin{cases}a & \text{បើ } x\leq 0\\ x\sin\dfrac{1}{x} & \text{បើ } x>0\end{cases}$

លំហាត់ទី៦៣

រកតម្លៃ A ដែលធ្វើឱ្យ $f(x)$ ជាប់គ្រប់តម្លៃ x

ក. $f(x)=\begin{cases}Ax-3 & \text{បើ } x<2\\ 3-x+2x^2 & \text{បើ } x\geq 2\end{cases}$

ខ. $f(x)=\begin{cases}1-3x & \text{បើ } x<4\\ Ax^2+2x-3 & \text{បើ } x\geq 4\end{cases}$

លំហាត់ទី៦៤

រកតម្លៃ A និង Bដែលធ្វើឱ្យអនុគមន៍កំណត់ដោយ

$$f(x)=\begin{cases} Ax^2+5x-9 & \text{បើ } x<1 \\ B & \text{បើ } x=1 \\ (3-x)(A-2x) & \text{បើ } x>1 \end{cases}$$

ជាប់លើ $\mathbb{R}$ ។

លំហាត់ទី៦៥

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases} \dfrac{1+x\sin x-\cos 2x}{x^2} & \text{បើ } x\neq 0 \\ 3 & \text{បើ } x=0 \end{cases}$

ចូរសិក្សាភាពជាប់នៃអនុគមន៍ f ត្រង់ $x=0$ ។

លំហាត់ទី៦៦

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases} \dfrac{2\sin x-\sin 2x}{x^3} & \text{បើ } x\neq 0 \\ 1 & \text{បើ } x=0 \end{cases}$

ចូរសិក្សាភាពជាប់នៃអនុគមន៍ f ត្រង់ $x=0$ ។

លំហាត់ទី៦៧

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases} \dfrac{1-\cos ax}{x^2} & \text{បើ } x\neq 0 \\ \dfrac{1}{8} & \text{បើ } x=0 \end{cases}$

ចូរកំណត់ចំនួនពិត a ដើម្បីឲ្យ f ជាអនុគមន៍ជាប់ត្រង់ $x=0$ ។

លំហាត់ទី៦៨

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases}\dfrac{1-x}{1+x} & \text{បើ } x\geq 0\\ \dfrac{2\sqrt{x^2}-\sin x}{x} & \text{បើ } x<0\end{cases}$

តើ f ជាអនុគមន៍ជាប់ត្រង់ $x=0$ ឬទេ ?

លំហាត់ទី៦៩

គេមានអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases}\pi & \text{បើ } x=0\\ \dfrac{\sin\pi x}{x-x^4} & \text{បើ } 0<x<1\\ \dfrac{\pi}{3} & \text{បើ } x=1\end{cases}$

ចូរសិក្សាភាពជាប់នៃអនុគមន៍ f លើចន្លោះ $[0,1]$ ។

លំហាត់ទី៧០

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\dfrac{\sin x+x^2\ln|x|}{x}$ ដែល $x\neq 0$ ។

តើ f អាចមានអនុគមន៍បន្លាយតាមភាពជាប់ ត្រង់ $x=0$ ឬទេ ?

បើមានចូរកំណត់រកអនុគមន៍នោះ ។

លំហាត់ទី៧១

គេឲ្យសមីការដឺក្រេទីពីរ $ax^2+bx+c=0$ ដែល $a\neq 0$ ហើយលេខមេគុណ a,b,c បំពេញលក្ខខណ្ឌ $2a+3b+6c=0$ ។

បង្ហាញថាសមីការនេះមានឫសយ៉ាងតិចមួយនៅចន្លោះ $\left[0,\dfrac{2}{3}\right]$ ។

លំហាត់ទី៧២

ស្រាយបញ្ជាក់ថាសមីការ $x\tan x=\cos x$ យ៉ាងហោចណាស់មានឬសជាចំនួនពិតមួយនៅចន្លោះ $\left[0\,,\frac{\pi}{4}\right]$ ។

លំហាត់ទី៧៣

គេឲ្យអនុគមន៍ $f(x)=\frac{\cos\pi x}{1-2x}$ ដែល $x\neq\frac{1}{2}$

ក.ចូរគណនាលីមីត $\lim\limits_{x\to\frac{1}{2}}f(x)$ ។

ខ.តើ f អាចមានអនុគមន៍បន្លាយតាមភាពជាប ត្រង់ $x=\frac{1}{2}$ ឬទេ ?

លំហាត់ទី៧៤

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f\left(x\right)=\frac{e^{3\sin x}-e^{\sin 3x}}{x^{3}}$ ដែល $x\neq0$ ។

គណនា $\lim\limits_{x\to0}f\left(x\right)$ រួចទាញរកអនុគមន៍បន្លាយតាមភាពជាប់នៃ f ត្រង់ $x=0$ ។

លំហាត់ទី៧៥

គេឲ្យសមីការ $(E):\ ax^{3}+bx^{2}+cx+d=0$ ដែល $a\neq0,a,b,c,d\in\Re$

បើ $|b+d|<|a+c|$ ចូរស្រាយថាសមីការ (E) មានឬសយ៉ាងតិចមួយស្ថិតនៅចន្លោះ -1 និង 1 ។

លំហាត់ទី៧៦

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f\left(x\right)=\frac{\ln\left(x+\sqrt{1+x^{2}}\right)}{x}$ ដែល $x\neq0$ ។

រកអនុគមន៍បន្លាយតាមភាពជាប់នៃ f ត្រង់ $x=0$ ។

លំហាត់ទី៧៧

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases}(ax+1)^2 & \text{បើ } x\le 2\\ x^2+ax+3 & \text{បើ } x>2\end{cases}$

កំណត់ចំនួនពិត a ដើម្បីឲ្យអនុគមន៍ f ជាប់ត្រង់ $x=2$ ។

លំហាត់ទី៧៨

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases}2\cos 2x & \text{បើ } x\le \frac{\pi}{6}\\ a\sin x+b & \text{បើ } \frac{\pi}{6}<x<\frac{\pi}{2}\\ \cos 2x & \text{បើ } x\ge \frac{\pi}{2}\end{cases}$

កំណត់ចំនួនពិត a និង b ដើម្បីឲ្យអនុគមន៍ f ជាប់លើ $\mathbb{R}$ ។

លំហាត់ទី៧៩

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases}x^2-2x & \text{បើ } x\le 1\\ a\ln x+b & \text{បើ } 1<x\le e\\ 2\ln x & \text{បើ } x>e\end{cases}$

កំណត់ចំនួនពិត a និង b ដើម្បីឲ្យអនុគមន៍ f ជាប់លើ $\mathbb{R}$ ។

លំហាត់ទី៨០

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases}-\frac{\pi}{2} & \text{បើ } x=-1\\ \frac{\sin \pi x}{1-x^2} & \text{បើ } -1<x<1\\ \frac{\pi}{2} & \text{បើ } x=1\end{cases}$

ចូរសិក្សាភាពជាប់នៃអនុគមន៍ f ជាប់លើចន្លោះ $[-1,1]$ ។

លំហាត់ទី៨១

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=\begin{cases} x^2 & \text{បើ } x<-1 \\ x^3+ax^2+bx+a & \text{បើ } -1\le x\le 1 \\ 3+\dfrac{4}{x} & \text{បើ } x>1 \end{cases}$

កំណត់ចំនួនពិត a និង b ដើម្បីឲ្យអនុគមន៍ f ជាប់លើ $\mathbb{R}$ ។

លំហាត់ទី៨២

គេឲ្យសមីការ $(E): ax^2+bx+c=0$ ដែល $a\neq 0$ និង $a,b,c\in\mathbb{R}$ ។

គេដឹងថា $10a+3b+c=0$ ។ ចូរស្រាយថាសមីការ (E) មានឫសយ៉ាងតិចមួយស្ថិត

នៅក្នុងចន្លោះចំនួន 2 និង 4 ។

លំហាត់ទី៨៣

គេឲ្យសមីការ $(E): ax^3+bx^2+cx+d=0$ ដែល $a\neq 0$ និង $a,b,c,d\in\mathbb{R}$ ។

បើ $9a+5b+3c+2d=0$ និង $7a+3b+c\neq 0$ ចូរស្រាយថាសមីការ (E)

មានឫសយ៉ាងតិចមួយស្ថិតនៅក្នុងចន្លោះចំនួន 1 និង 2 ។

លំហាត់ទី៨៤

គេឲ្យសមីការ $(E): x^4+x^3\cos\varphi-(2-\sin 2\varphi)x^2+x\cos\varphi+1=0$

ដែល $\varphi\in\mathbb{R}$ ។

ចូរស្រាយថាសមីការ (E) មានឫសជាចំនួនពិតយ៉ាងតិចមួយនៅចន្លោះ -1 និង 1 ។

លំហាត់ទី៨៥

គេឲ្យ f ជាអនុគមន៍កំណត់និងជាប់លើចន្លោះ $[a,b]$ ហើយ m និង n ជាពីរចំនួនពិតវិជ្ជមានពីរដែលគេឲ្យ ។ ចូរស្រាយបញ្ជាក់ថាសមីការ $f(x)=\dfrac{m\,f(a)+n\,f(b)}{m+n}$ មានឫសយ៉ាងតិចមួយស្ថិតនៅក្នុងចន្លោះ $[a,b]$ ។

លំហាត់ទី៨៦

គេឲ្យសមីការ $(E): x^2+5x-1+3\cos\theta+4\sin\theta=0$ ដែល $\theta\in\mathbb{R}$ ។

ចូរស្រាយបញ្ជាក់ថាសមីការ (E) មានឫសយ៉ាងតិចមួយស្ថិតក្នុងចន្លោះ $[-1,1]$ ។

លំហាត់ទី៨៧

ប្រើទ្រឹស្តីបទតម្លៃកណ្តាល បង្ហាញថា អនុគមន៍ខាងក្រោមមានចំនួន c ក្នុងចន្លោះដែលឱ្យ ។

ក. $f(x)=x^2+x-1$, $[0,5]$, $f(c)=11$ ។

ខ. $f(x)=x^2-6x+8$, $[0\,,\,3\,]$, $f(c)=0$ ។

គ. $f(x)=x^3-x^2+x-2$, $[0,3\,]$, $f(c)=4$ ។

ឃ. $f(x)=\dfrac{x^2+x}{x-1}$, $[\,\dfrac{5}{2}\,,4\,]$, $f(c)=6$

លំហាត់ទី៨៨

ចូរកំណត់ចំនួនថេរ a ដើម្បីឱ្យលីមីតខាងក្រោមជាលីមីតនៃចំនួនថេរហើយកំណត់លីមីតនេះផង ។

ក. $\lim\limits_{x\to1}\dfrac{\sqrt{x+3}-a}{x-1}$ ខ. $\lim\limits_{x\to0}\dfrac{\sqrt{1+3x}+a}{x}$

គ. $\lim_{x\to 2}\frac{\sqrt{x+a}-1}{x-2}$

ឃ. $\lim_{x\to -1}\frac{\sqrt{x^2+ax}-1}{x^2-1}$

លំហាត់ទី៨៩

គេមានអនុគមន៍ $y=f(x)$ កំណត់លើចន្លោះ $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ ដែល ៖

$$f(x)=\begin{cases}\sin x+\dfrac{\sqrt{1-\cos 2x}}{\sin x} & \text{បើ } x\neq 0\\ \sqrt{2} & \text{បើ } x=0\end{cases}$$

តើ $f(x)$ ជាប់ត្រង់ $x=0$ ឬទេ។

លំហាត់ទី៩០

គេមានពហុកោណនិយ័តមាន n ជ្រុងចារឹកក្នុងរង្វង់ដែលមានកាំស្មើនឹង a ។

តាង S_n ជាផ្ទៃក្រឡានៃពហុកោណនេះ។ គណនា S_n រួចកំណត់ $\lim_{n\to+\infty}S_n$ ។

លំហាត់ទី៩១

ចូរគណនាលីមីតខាងក្រោមនេះ

ក. $\lim_{x\to\pm\infty}(x^2+xe^x)$

ខ. $\lim_{x\to+\infty}(1-x)e^x$

គ. $\lim_{x\to+\infty}(x+2)e^{-x}$

ឃ. $\lim_{x\to+\infty}\frac{e^x-x}{2e^x+1}$

ង. $\lim_{x\to 0}\ln\left(\frac{x}{x+1}\right)$

ច. $\lim_{x\to\pm\infty}x\ln(x^2+1)$

ឆ. $\lim_{x\to -4}x\ln\left(4-3x-x^2\right)$

ជ. $\lim_{x\to+\infty}\{x[\ln(x+1)-\ln x]\}$

លំហាត់ទី៩២

កំណត់តម្លៃ a ដែលបំពេញលក្ខខណ្ឌ $\lim_{x\to 0}\frac{1+ax-\sqrt{1+x}}{x}=\frac{1}{8}$ ។

លំហាត់ទី៩៣

ស្រាយបញ្ជាក់ថាសមីការខាងក្រោមមានឫសយ៉ាងតិចមួយនៅក្នុងចន្លោះដែលគេឲ្យ ៖

ក. $\sin x = x - 1 \;\; , x \in (0,\pi)$ ខ. $20\log_{10} x - x = 0 \;\; , x \in (1\,,10)$

លំហាត់ទី៩៤

ចូរគណនាលីមីតខាងក្រោមនេះ

ក. $\lim\limits_{x \to +\infty} (\sqrt{4x^2 + x + 2} - \sqrt{x^2 - x + 3})$ ខ. $\lim\limits_{x \to +\infty} (\sqrt{x^2 + x + 2} - \sqrt{x^2 - x + 3})$

គ. $\lim\limits_{x \to 0} \dfrac{x^2 - x\sin x}{x - \sin^2 x}$ ឃ. $\lim\limits_{x \to 0} \dfrac{2x - \sin x}{\sqrt{1 - \cos x}}$

លំហាត់ទី៩៥

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x) = \dfrac{|x| + 2x^2}{x}$ បើ $x \neq 0$ និង $f(0) = 1$ ។

ក.តើអនុគមន៍ f ជាប់ត្រង់ $x = 0$ ឬទេ ?

ខ.សង់ក្រាបតាងអនុគមន៍ f ។

លំហាត់ទី៩៦

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x \to +\infty} \dfrac{3e^{x-2}}{x^3}$ ខ. $\lim\limits_{x \to +\infty} \left(\dfrac{x+3}{x-1}\right)^{x+2}$ គ. $\lim\limits_{x \to 0} \left(\dfrac{e^x - e^{-x}}{x}\right)$

លំហាត់ទី៩៧

រកដែនកំណត់នៃអនុគមន៍ខាងក្រោម៖

ក. $y = \sqrt{x^2 - 2\sqrt{x^2 - 1}} + \sqrt{x - 3 + 2\sqrt{x - 4}}$

ខ. $y = \lg\left(\dfrac{2^{1-x} - 2x + 1}{2^x - 1}\right)$

លំហាត់ទី៩៨

គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x)=2\sqrt{x}-\ln x$ ដែល $x>0$ ។

ក)បង្ហាញថា $f(x)>0$ ចំពោះគ្រប់ $x>0$

ខ)បង្ហាញថាចំពោះគ្រប់ $x>1$ គេបាន $0<\dfrac{\ln x}{x}<\dfrac{2}{\sqrt{x}}$ រួចទាញថា $\lim\limits_{x\to+\infty}\dfrac{\ln x}{x}=0$ ។

លំហាត់ទី៩៩

គេឲ្យអនុគមន៍ f កំណត់លើ $(0,+\infty)$ ដោយ $f(x)=x\ln(\dfrac{x}{e})$ ដែល $e=2.71828...$

១.ចូរគណនា $f'(x)$ រួចចំពោះគ្រប់ $x\in\left[\dfrac{k}{n},\dfrac{k+1}{n}\right]$ ដែល $n\in\mathbb{N}$ និង $k\in\mathbb{N}$

ចូរស្រាយបញ្ជាក់ថា $\ln\left(\dfrac{k}{n}\right)\le f'(x)\le\ln\left(\dfrac{k+1}{n}\right)$ ។

២.ប្រើទ្រឹស្តីបទវិសមភាពកំណើនមានកំណត់ទៅនិងអនុគមន៍ f គ្រប់ $k,n\in\mathbb{N}$

ចូរទាញបញ្ជាក់ថា $\ln\left(\dfrac{k}{n}\right)\le(k+1)\ln\left(\dfrac{k+1}{ne}\right)-k\ln\left(\dfrac{k}{ne}\right)\le\ln\left(\dfrac{k+1}{n}\right)$ ។

៣.ទាញឲ្យបានថាចំពោះគ្រប់ $k,n\in\mathbb{N}$ គេមាន៖

$$\ln\left(\frac{1}{e}\right)-\frac{1}{n}\ln\left(\frac{1}{e}\right)\le\frac{1}{n}\ln\left(\frac{n!}{n^n}\right)\le\left(\frac{n+1}{n}\right)\ln\left(\frac{n+1}{ne}\right)-\frac{1}{n}\ln\left(\frac{1}{ne}\right)$$

៤.ដោយប្រើលទ្ធផលខាងលើចូរទាញថា $\lim\limits_{n\to+\infty}\left(\dfrac{\sqrt[n]{n!}}{n}\right)=\dfrac{1}{e}$ ។

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ជំពូកទី០៥

ផ្នែកដំណោះស្រាយ

កំណត់ចំណាំ!!!

◈.បើ $f(a)=\ell$ ដែល $\ell\in\mathbb{R}$ នោះគេបាន $\lim\limits_{x\to a} f(x)=f(a)=\ell$ ។

◈.បើ $f(a)=\ell_1$ និង $g(a)=\ell_2$ ដែល $\ell_1\in\mathbb{R}$, $\ell_2\in\mathbb{R}$ នោះគេបាន ៖

$$a)\ \lim_{x\to a}\left[f(x)\pm g(x)\right]=\lim_{x\to a}\left[f(x)\right]\pm\lim_{x\to a}\left[g(x)\right]=f(a)\pm g(a)=\ell_1\pm\ell_2$$ ។

$$b)\ \lim_{x\to a}\left[f(x)g(x)\right]=\lim_{x\to a}\left[f(x)\right]\times\lim_{x\to a}\left[g(x)\right]=f(a)\times g(a)=\ell_1\ell_2$$ ។

◈.បើ $f(a)=\ell_1$ និង $g(a)=\ell_2$ ដែល $\ell_1\in\mathbb{R}$, $\ell_2\in\mathbb{R}^*$ នោះគេបាន ៖

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim\limits_{x\to a}\left[f(x)\right]}{\lim\limits_{x\to a}\left[g(x)\right]}=\frac{f(a)}{g(a)}=\frac{\ell_1}{\ell_2}$$ ។

លំហាត់ទី០១

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 2}\left(x^3+x-5\right)$ ខ. $\lim\limits_{x\to 3}\left(2x^2-5x+1\right)$ គ. $\lim\limits_{x\to -1}\left(x^4+6x^2+1\right)$

ឃ. $\lim\limits_{x\to 2}\left(x+\sqrt{x^3+1}\right)$ ង. $\lim\limits_{x\to 3}\left(\sqrt{x+3+\sqrt{x+6}}\right)$ ច. $\lim\limits_{x\to 3}\left(x^2-\sqrt[3]{x^2-1}\right)$

ឆ. $\lim\limits_{x\to 2}\dfrac{x^4+3x^2-8}{x^2+1}$ ជ. $\lim\limits_{x\to 3}\dfrac{x^2+5x-4}{2x-1}$ ឈ. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+x+2}}{3x+1}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 2}\left(x^3+x-5\right)=2^3+2-5=8+2-5=5$ ។

ខ. $\lim_{x\to 3}\left(2x^2-5x+1\right)=2\left(3\right)^2-5\left(3\right)+1=18-15+1=4$ ។

គ. $\lim_{x\to -1}\left(x^4+6x^2+1\right)=\left(-1\right)^4+6\left(-1\right)^2+1=1+6+1=8$ ។

ឃ. $\lim_{x\to 2}\left(x+\sqrt{x^3+1}\right)=2+\sqrt{(2)^3+1}=2+3=5$ ។

ង. $\lim_{x\to 3}\left(\sqrt{x+3+\sqrt{x+6}}\right)=\sqrt{3+3+\sqrt{6+3}}=\sqrt{6+3}=3$ ។

ច. $\lim_{x\to 3}\left(x^2-\sqrt[3]{x^2-1}\right)=3^2-\sqrt[3]{3^2-1}=9-2=7$ ។

ឆ. $\lim_{x\to 2}\dfrac{x^4+3x^2-8}{x^2+1}=\dfrac{2^4+3(2)^2-8}{2^2+1}=\dfrac{16+12-8}{5}=4$ ។

ជ. $\lim_{x\to 3}\dfrac{x^2+5x-4}{2x-1}=\dfrac{3^2+5(3)-4}{2(3)-1}=\dfrac{9+15-4}{6-1}=4$ ។

ឈ. $\lim_{x\to 1}\dfrac{\sqrt{x^2+x+2}}{3x+1}=\dfrac{\sqrt{1^2+1+2}}{3(1)+1}=\dfrac{2}{4}=\dfrac{1}{2}$ ។

លំហាត់ទី០២

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0}\left(\sin x+\cos x\right)$

ខ. $\lim_{x\to \frac{\pi}{3}}\left(\cos x+\sqrt{3}\sin x\right)$

គ. $\lim_{x\to 0}\dfrac{3+5\cos x}{3-\cos x}$

ឃ. $\lim_{x\to \frac{\pi}{6}}\left(\sin x+2\cos^2 x\right)$

ង. $\lim_{x\to \frac{\pi}{2}}\dfrac{2\sin x-\cos x}{1+\sin x}$

ច. $\lim_{x\to \frac{\pi}{3}}\dfrac{1+4\sin^2 x}{1+2\cos x}$

ឆ. $\lim_{x\to \frac{\pi}{3}}\dfrac{2\cos^2 x+3\cos x+1}{\cos x+\sqrt{3}\sin x}$

ជ. $\lim_{x\to \frac{\pi}{4}}\dfrac{3+\sqrt{2}\sin x}{3-\sqrt{2}\cos x}$

ឈ. $\lim_{x\to \frac{\pi}{3}}\dfrac{1+\sqrt{3}\tan x}{1+4\sin^2 x}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}(\sin x+\cos x)=\sin 0+\cos 0=0+1=1$ ។

ខ. $\lim\limits_{x\to \frac{\pi}{3}}\left(\cos x+\sqrt{3}\sin x\right)=\frac{1}{2}+\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)=\frac{1}{2}+\frac{3}{2}=2$ ។

គ. $\lim\limits_{x\to 0}\frac{3+5\cos x}{3-\cos x}=\frac{3+5}{3-1}=\frac{8}{2}=4$ ។

ឃ. $\lim\limits_{x\to \frac{\pi}{6}}\left(\sin x+2\cos^2 x\right)=\frac{1}{2}+2\left(\frac{\sqrt{3}}{2}\right)^2=\frac{1}{2}+\frac{3}{2}=2$ ។

ង. $\lim\limits_{x\to \frac{\pi}{2}}\frac{2\sin x-\cos x}{1+\sin x}=\frac{2-0}{1+1}=\frac{2}{2}=1$ ។

ច. $\lim\limits_{x\to \frac{\pi}{3}}\frac{1+4\sin^2 x}{1+2\cos x}=\frac{1+4\left(\frac{\sqrt{3}}{2}\right)^2}{1+2\left(\frac{1}{2}\right)}=\frac{1+3}{1+1}=\frac{4}{2}=2$ ។

ឆ. $\lim\limits_{x\to \frac{\pi}{3}}\frac{2\cos^2 x+3\cos x+1}{\cos x+\sqrt{3}\sin x}=\frac{2\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1}{\frac{1}{2}+\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)}=\frac{\frac{1}{2}+\frac{3}{2}+1}{\frac{1}{2}+\frac{3}{2}}=\frac{3}{2}$ ។

ជ. $\lim\limits_{x\to \frac{\pi}{4}}\frac{3+\sqrt{2}\sin x}{3-\sqrt{2}\cos x}=\frac{3+\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)}{3-\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)}=\frac{3+1}{3-1}=\frac{4}{2}=2$ ។

ឈ. $\lim\limits_{x\to \frac{\pi}{3}}\frac{1+\sqrt{3}\tan x}{1+4\sin^2 x}=\frac{1+\sqrt{3}\left(\sqrt{3}\right)}{1+4\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{1+3}{1+3}=\frac{4}{4}=1$ ។

<u>កំណត់ចំណាំ!!!</u>

ឧបមាថាគេមានលីមីត $L=\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}$ ។

បើ $f(a)=0$ និង $g(a)=0$ នោះលីមីត L មានរាងមិនកំណត់ $\dfrac{0}{0}$ ។

ក្នុងករណីនេះដើម្បីគណនាលីមីត L គេត្រូវអនុវត្តដូចតទៅ ៖

- ◈ បំបែកកន្សោម $f(x)$ និង $g(x)$ ជាផលគុណកត្តា ។
- ◈ សម្រួលកត្តាមិនកំណត់ចោល ។
- ◈ ធ្វើលីមីតលើប្រភាគថ្មី ។

<u>លំហាត់ទី០៣</u>

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 1}\dfrac{x-1}{x^2-1}$ ខ. $\lim\limits_{x\to 3}\dfrac{x^2-4x+3}{x-3}$ គ. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^2-3x+2}$

ឃ. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^3-2x^2+2x-4}$ ង. $\lim\limits_{x\to 1}\dfrac{(x-1)^2}{x^3-x^2-x+1}$ ច. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x^3-3x^2-3x+9}$

ឆ. $\lim\limits_{x\to -1}\dfrac{x^4+x^3-x^2+1}{x^3+1}$ ជ. $\lim\limits_{x\to 1}\dfrac{x^3+x-2}{x^3-x^2+x-1}$ ញ. $\lim\limits_{x\to 2}\dfrac{x^4-16}{x^3-2x^2+4x-8}$

<u>ដំណោះស្រាយ</u>

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 1}\dfrac{x-1}{x^2-1}=\lim\limits_{x\to 1}\dfrac{x-1}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{1}{x+1}=\dfrac{1}{1+1}=\dfrac{1}{2}$ ។

ខ. $\lim\limits_{x\to 3}\dfrac{x^2-4x+3}{x-3}=\lim\limits_{x\to 3}\dfrac{(x-1)(x-3)}{x-3}=\lim\limits_{x\to 3}\left(x-1\right)=3-1=2$ ។

គ. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{(x-1)(x+2)}{(x-1)(x-2)}=\lim\limits_{x\to 1}\dfrac{x+2}{x-2}=\dfrac{3}{-1}=-3$ ។

ឃ. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^3-2x^2+2x-4}=\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{x^2(x-2)+2(x-2)}=\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{(x-2)(x^2+2)}$

$=\lim\limits_{x\to 2}\dfrac{x^2+2x+4}{x^2+2}=\dfrac{4+4+4}{4+2}=\dfrac{12}{6}=2$ ។

ង. $\lim\limits_{x\to 1}\dfrac{(x-1)^2}{x^3-x^2-x+1}=\lim\limits_{x\to 1}\dfrac{(x-1)^2}{x^2(x-1)-(x-1)}=\lim\limits_{x\to 1}\dfrac{(x-1)^2}{(x-1)(x^2-1)}$

$=\lim\limits_{x\to 1}\dfrac{x-1}{x^2-1}=\lim\limits_{x\to 1}\dfrac{x-1}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{1}{x+1}=\dfrac{1}{2}$ ។

ច. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x^3-3x^2-3x+9}=\lim\limits_{x\to 3}\dfrac{(x-3)(x^2+3x+9)}{x^2(x-3)-3(x-3)}=\lim\limits_{x\to 3}\dfrac{(x-3)(x^2+3x+9)}{(x-3)(x^2-3)}$

$=\lim\limits_{x\to 3}\dfrac{x^2+3x+9}{x^2-3}=\dfrac{9+9+9}{9-3}=\dfrac{27}{6}=\dfrac{9}{2}$ ។

ឆ. $\lim\limits_{x\to -1}\dfrac{x^4+x^3-x^2+1}{x^3+1}=\lim\limits_{x\to -1}\dfrac{x^3(x+1)-(x-1)(x+1)}{(x+1)(x^2-x+1)}=\lim\limits_{x\to -1}\dfrac{(x+1)(x^3-x+1)}{(x+1)(x^2-x+1)}$

$=\lim\limits_{x\to -1}\dfrac{x^3-x+1}{x^2-x+1}=\dfrac{-1+1+1}{1+1+1}=\dfrac{1}{3}$ ។

ជ. $\lim\limits_{x\to 1}\dfrac{x^3+x-2}{x^3-x^2+x-1}=\lim\limits_{x\to 1}\dfrac{x^2(x-1)+x(x-1)+2(x-1)}{x^2(x-1)+(x-1)}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+x+2)}{(x-1)(x^2+1)}=\lim\limits_{x\to 1}\dfrac{x^2+x+2}{x^2+1}=\dfrac{4}{2}=2$ ។

ញ. $\lim\limits_{x\to 2}\dfrac{x^4-16}{x^3-2x^2+4x-8}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+2)(x^2+4)}{(x-2)(x^2+4)}=\lim\limits_{x\to 2}\left(x+2\right)=4$ ។

លំហាត់ទី០៤

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to1}\dfrac{x^3-1}{x^2+x-2}$

ខ. $\lim_{x\to3}\dfrac{x^3-3x^2+45x-135}{x^3-27}$

គ. $\lim_{x\to2}\dfrac{x^3-8}{x^2-4}$

ឃ. $\lim_{x\to-1}\dfrac{x^4+x^3+4x+4}{x^3+1}$

ង. $\lim_{x\to2}\dfrac{x^4-2x^3+x^2-4}{x^3-2x^2-x+2}$

ច. $\lim_{x\to3}\dfrac{x^2-9}{x^2-3x}$

ឆ. $\lim_{x\to1}\dfrac{x^3+x-2}{x^2-1}$

ជ. $\lim_{x\to2}\dfrac{x^2-6x+8}{x^2-3x+2}$

ឈ. $\lim_{x\to0}\dfrac{(1+x)^4-4x-1}{x^4+x^2}$

ញ. $\lim_{x\to0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}$

ដំណោះស្រាយ

ក. $\lim_{x\to1}\dfrac{x^3-1}{x^2+x-2}$ មានរាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to1}\frac{(x-1)(x^2+x+1)}{(x-1)(x+2)}=\lim_{x\to1}\frac{x^2+x+1}{x+2}=\frac{1+1+1}{1+2}=1$$

ដូចនេះ $\lim_{x\to1}\dfrac{x^3-1}{x^2+x-2}=1$ ។

ខ. $\lim_{x\to3}\dfrac{x^3-3x^2+45x-135}{x^3-27}$ មានរាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to3}\frac{x^3-3x^2+45x-135}{x^3-27}=\lim_{x\to3}\frac{x^2(x-3)+45(x-3)}{(x-3)(x^2+3x+9)}$$

$$=\lim_{x\to3}\frac{(x-3)(x^2+45)}{(x-3)(x^2+3x+9)}=\lim_{x\to3}\frac{x^2+45}{x^2+3x+9}=\frac{9+45}{9+9+9}=\frac{54}{27}=2$$

ដូចនេះ $\lim_{x\to3}\dfrac{x^3-3x^2+45x-135}{x^3-27}=2$ ។

គ. $\lim_{x\to2}\dfrac{x^3-8}{x^2-4}=\lim_{x\to2}\dfrac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}=\lim_{x\to2}\dfrac{x^2+2x+4}{x+2}$

$$= \frac{4+4+4}{2+2} = \frac{12}{4} = 3$$ ។

ដូចនេះ $\lim_{x\to 2} \frac{x^3-8}{x^2-4} = 3$ ។

ឃ. $\lim_{x\to -1} \frac{x^4+x^3+4x+4}{x^3+1} = \lim_{x\to -1} \frac{x^3(x+1)+4(x+1)}{(x+1)(x^2-x+1)}$

$$= \lim_{x\to -1} \frac{(x+1)(x^3+4)}{(x+1)(x^2-x+1)} = \lim_{x\to -1} \frac{x^3+4}{x^2-x+1} = \frac{-1+4}{1+1+1} = \frac{3}{3} = 1$$ ។

ដូចនេះ $\lim_{x\to -1} \frac{x^4+x^3+4x+4}{x^3+1} = 1$ ។

ង. $\lim_{x\to 2} \frac{x^4-2x^3+x^2-4}{x^3-2x^2-x+2}$

$$= \lim_{x\to 2} \frac{x^3(x-2)+(x-2)(x+2)}{x^2(x-2)-(x-2)} = \lim_{x\to 2} \frac{(x-2)(x^3+x+2)}{(x-2)(x^2-2)}$$

$$= \lim_{x\to 2} \frac{x^3+x+2}{x^2-2} = \frac{8+2+2}{4-2} = 6$$

ដូចនេះ $\lim_{x\to 2} \frac{x^4-2x^3+x^2-4}{x^3-2x^2-x+2} = 6$ ។

ច. $\lim_{x\to 3} \frac{x^2-9}{x^2-3x} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x(x-3)} = \lim_{x\to 3} \frac{x+3}{x} = \frac{3+3}{3} = 2$ ។

ឆ. $\lim_{x\to 1} \frac{x^3+x-2}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+2)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x^2+x+2}{x+1} = \frac{4}{2} = 2$ ។

ជ. $\lim_{x\to 2} \frac{x^2-6x+8}{x^2-3x+2} = \lim_{x\to 2} \frac{(x-2)(x-4)}{(x-1)(x-2)} = \lim_{x\to 2} \frac{x-4}{x-1} = \frac{2-4}{2-1} = -2$ ។

ឈ. $\lim_{x\to 0} \frac{(1+x)^4-4x-1}{x^4+x^2} = \lim_{x\to 0} \frac{x^4+4x^3+6x^2}{x^4+x^2} = \lim_{x\to 0} \frac{x^2+4x+6}{x^2+1} = 6$ ។

ញ. $\lim_{x\to 0} \frac{(1+x)(1+2x)(1+3x)-1}{x}$

$$= \lim_{x\to 0} \frac{6x+11x^2+6x^3}{x} = \lim_{x\to 0}\left(6+11x+6x^2\right) = 6$$ ។

លំហាត់ទី០៥

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 4}\dfrac{x^3-64}{x^2-16}$

ខ. $\lim\limits_{x\to 1}\dfrac{x^3-6x^2+9x-4}{x^3-x^2-x+1}$

គ. $\lim\limits_{x\to 2}\dfrac{5x^2-8x-4}{x^3-8}$

ឃ. $\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2-3x}$

ង. $\lim\limits_{x\to 1}\dfrac{x^3+x-2}{x^2-1}$

ច. $\lim\limits_{x\to 2}\dfrac{x^2-6x+8}{x^2-3x+2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{(1+x)^4-4x-1}{x^4+x^2}$

ជ. $\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}$

ឈ. $\lim\limits_{x\to 4}\dfrac{x^3-64}{x^2-16}$

ញ. $\lim\limits_{x\to 2}\dfrac{x^3-8+4(x-2)}{x^2-4}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\dfrac{x^3-64}{x^2-16}=\lim\limits_{x\to 4}\dfrac{x^3-4^3}{x^2-4^2}=\lim\limits_{x\to 4}\dfrac{(x-4)(x^2+4x+16)}{(x-4)(x+4)}=\lim\limits_{x\to 4}\dfrac{x^2+4x+16}{x+4}=6$ ។

ខ. $\lim\limits_{x\to 1}\dfrac{x^3-6x^2+9x-4}{x^3-x^2-x+1}=\lim\limits_{x\to 1}\dfrac{(x-1)^2(x-4)}{(x-1)^2\left(x+1\right)}=\lim\limits_{x\to 1}\dfrac{x-4}{x+1}=-\dfrac{3}{2}$ ។

គ. $\lim\limits_{x\to 2}\dfrac{5x^2-8x-4}{x^3-8}$ យើងសង្កេតឃើញថាលីមីតនេះមានរាងមិនកំណត់ $\dfrac{0}{0}$

យើងមាន $5x^2-8x-4=(5x^2-10x)+(2x-4)=(x-2)(5x-2)$

និង $x^3-8=(x-2)(x^2+2x+4)$

យើងបាន $\lim\limits_{x\to 2}\dfrac{5x^2-8x-4}{x^3-8}=\lim\limits_{x\to 2}\dfrac{(x-2)(5x+2)}{(x-2)(x^2+2x+4)}$

$$=\lim_{x\to 2}\frac{5x+2}{x^2+2x+4}=\frac{5(2)+2}{(2)^2+2(2)+4}=1$$

ដូចនេះ $\lim\limits_{x\to 2}\frac{5x^2-8x-4}{x^3-8}=1$ ។

ឃ. $\lim\limits_{x\to 3}\frac{x^2-9}{x^2-3x}=\lim\limits_{x\to 3}\frac{(x-3)(x+3)}{x(x-3)}=\lim\limits_{x\to 3}\frac{x+3}{x}=\frac{3+3}{3}=2$

ង. $\lim\limits_{x\to 1}\frac{x^3+x-2}{x^2-1}=\lim\limits_{x\to 1}\frac{(x-1)(x^2+x+2)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\frac{x^2+x+2}{x+1}=\frac{4}{2}=2$

ច. $\lim\limits_{x\to 2}\frac{x^2-6x+8}{x^2-3x+2}=\lim\limits_{x\to 2}\frac{(x-2)(x-4)}{(x-1)(x-2)}=\lim\limits_{x\to 2}\frac{x-4}{x-1}=\frac{2-4}{2-1}=-2$

ឆ. $\lim\limits_{x\to 0}\frac{(1+x)^4-4x-1}{x^4+x^2}=\lim\limits_{x\to 0}\frac{x^4+4x^3+6x^2}{x^4+x^2}=\lim\limits_{x\to 0}\frac{x^2+4x+6}{x^2+1}=6$

ជ. $\lim\limits_{x\to 0}\frac{(1+x)(1+2x)(1+3x)-1}{x}$

$$=\lim_{x\to 0}\frac{6x+11x^2+6x^3}{x}=\lim_{x\to 0}\left(6+11x+6x^2\right)=6$$ ។

ឈ. $\lim\limits_{x\to 4}\frac{x^3-64}{x^2-16}=\lim\limits_{x\to 4}\frac{x^3-4^3}{x^2-4^2}$

$$=\lim_{x\to 4}\frac{(x-4)(x^2+4x+16)}{(x-4)(x+4)}=\lim_{x\to 4}\frac{x^2+4x+16}{x+4}=\frac{48}{8}=6$$

ញ. $\lim\limits_{x\to 2}\frac{x^3-8+4(x-2)}{x^2-4}=\lim\limits_{x\to 2}\frac{(x-2)(x^2+2x+4)+4(x-2)}{(x-2)(x+2)}$

$$=\lim_{x\to 2}\frac{(x-2)(x^2+2x+8)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x^2+2x+8}{x+2}=\frac{16}{4}=4$$

ដូចនេះ $\lim\limits_{x\to 2}\frac{x^3-8+4(x-2)}{x^2-4}=4$ ។

លំហាត់ទី០៦

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to3}\frac{x^3-3x^2+x-3}{2x^2-7x+3}$

ខ. $\lim\limits_{x\to1}\frac{3x^4-4x^3+1}{(x-1)^2}$

គ. $\lim\limits_{x\to1}\frac{x^4-4x+3}{x^3-x^2-x+1}$

ឃ. $\lim\limits_{x\to0}\frac{(1+ax)^n-1}{x}$

ង. $\lim\limits_{x\to0}\frac{(1+x)^3-(3x+1)}{x^2}$

ច. $\lim\limits_{x\to0}\frac{1-(1+x)(1+2x)(1+3x)}{x}$

ឆ. $\lim\limits_{x\to1}\frac{x^n-1}{x-1}$ ដែល $n\in\mathbb{N}$

ជ. $\lim\limits_{x\to1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$

ឈ. $\lim\limits_{x\to2}\frac{x^3-3x^2+4}{x^3-4x^2+4x}$

ញ. $\lim\limits_{x\to1}\frac{x^3-x^2+x-1}{x^3-1}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to3}\frac{x^3-3x^2+x-3}{2x^2-7x+3}$ មានរាងមិនកំណត់ $\frac{0}{0}$ ។

$$=\lim_{x\to3}\frac{(x^3-3x^2)+(x-3)}{(2x^2-6x)-(x-3)}=\lim_{x\to3}\frac{x^2(x-3)+(x-3)}{2x(x-3)-(x-3)}$$

$$=\lim_{x\to3}\frac{(x-3)(x^2+1)}{(x-3)(2x-1)}=\lim_{x\to3}\frac{x^2+1}{2x-1}=\frac{3^2+1}{2(3)-1}=2$$

ដូចនេះ $\lim\limits_{x\to3}\frac{x^3-3x^2+x-3}{2x^2-7x+3}=2$ ។

ខ. $\lim\limits_{x\to1}\frac{3x^4-4x^3+1}{(x-1)^2}$

$$=\lim_{x\to1}\frac{(3x^4-3x^3)-(x^3-1)}{(x-1)^2}=\lim_{x\to1}\frac{3x^3(x-1)-(x-1)(x^2+x+1)}{(x-1)^2}$$

$$= \lim_{x\to1}\frac{3x^3-x^2-x-1}{x-1} = \lim_{x\to1}\frac{(x^3-x^2)+(x^3-x)+(x^3-1)}{x-1}$$

$$= \lim_{x\to1}\frac{x^2(x-1)+x(x-1)(x+1)+(x-1)(x^2+x+1)}{x-1}$$

$$= \lim_{x\to1}\left[x^2+x(x+1)+(x^2+x+1)\right] = 1+2+3=6$$

ដូចនេះ $\lim_{x\to1}\frac{3x^4-4x^3+1}{(x-1)^2}=6$ ។

គ. $\lim_{x\to1}\frac{x^4-4x+3}{x^3-x^2-x+1}$

$$= \lim_{x\to1}\frac{(x^4-x)-3(x-1)}{x^2(x-1)-(x-1)} = \lim_{x\to1}\frac{x(x-1)(x^2+x+1)-3(x-1)}{(x-1)(x^2-1)}$$

$$= \lim_{x\to1}\frac{(x^4-x)-3(x-1)}{x^2(x-1)-(x-1)} = \lim_{x\to1}\frac{x(x-1)(x^2+x+1)-3(x-1)}{(x-1)(x^2-1)}$$

$$= \lim_{x\to1}\frac{x^3+x^2+x-3}{x^2-1} = \lim_{x\to1}\frac{(x^3-1)+(x^2-1)+(x-1)}{(x-1)(x+1)}$$

$$= \lim_{x\to1}\frac{(x^2+x+1)+(x+1)+1}{x+1} = \frac{3+2+1}{2} = \frac{6}{2} = 3$$

ដូចនេះ $\lim_{x\to1}\frac{x^4-4x+3}{x^3-x^2-x+1}=3$ ។

ឃ. $\lim_{x\to0}\frac{(1+ax)^n-1}{x}$

$$= \lim_{x\to0}\frac{\left[(1+ax)-1\right]\left[(1+ax)^{n-1}+\ldots.+(1+ax)+1\right]}{x}$$

$$= \lim_{x\to0} a\left[(1+ax)^{n-1}+\ldots.+(1+ax)+1\right]$$

$$= a\left(1+1+\ldots..+1\right) = n.a$$

ដូចនេះ $\lim_{x\to0}\frac{(1+ax)^n-1}{x}=na$ ។

ង. $\lim_{x\to 0}\frac{(1+x)^3-(3x+1)}{x^2}$

$$=\lim_{x\to 0}\frac{1+3x+3x^2+x^3-3x-1}{x^2}=\lim_{x\to 0}\frac{3x^2+x^3}{x^2}=\lim_{x\to 0}(3+x)=3$$

ដូចនេះ: $\lim_{x\to 0}\frac{(1+x)^3-(3x+1)}{x^2}=3$ ។

ច. $\lim_{x\to 0}\frac{1-(1+x)(1+2x)(1+3x)}{x}$

$$=\lim_{x\to 0}\frac{1-(1+x)+(1+x)[1-(1+2x)]+(1+x)(1+2x)[1-(1+3x)]}{x}$$

$$=\lim_{x\to 0}\frac{-x-2x(1+x)-3x(1+x)(1+2x)}{x}$$

$$=-\lim_{x\to 0}\left[1+2(1+x)+3(1+x)(1+2x)\right]=-6$$

ដូចនេះ: $\lim_{x\to 0}\frac{1-(1+x)(1+2x)(1+3x)}{x}=-6$ ។

ឆ. $\lim_{x\to 1}\frac{x^n-1}{x-1}$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-1}+....+x+1)}{x-1}$$

$$=\lim_{x\to 1}(x^{n-1}+....+x+1)=1+1+...+1=n$$

ដូចនេះ: $\lim_{x\to 1}\frac{x^n-1}{x-1}=n$ ។

ជ. $\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$

$$=\lim_{x\to 1}\left[\frac{1}{1-x}-\frac{3}{(1-x)(1+x+x^2)}\right]=\lim_{x\to 1}\left[\frac{(1+x+x^2)-3}{(1-x)(1+x+x^2)}\right]$$

$$=\lim_{x\to 1}\frac{(x-1)+(x^2-1)}{(1-x)(1+x+x^2)}=\lim_{x\to 1}\frac{(x-1)+(x-1)(x+1)}{-(x-1)(x^2+x+1)}$$

$$=\lim_{x\to 1}\frac{1+(x+1)}{-(x^2+x+1)}=\frac{1+2}{-3}=-1$$

ដូចនេះ $\lim\limits_{x\to 1}\left(\dfrac{1}{1-x}-\dfrac{3}{1-x^3}\right)=-1$ ។

ឈ. $\lim\limits_{x\to 2}\dfrac{x^3-3x^2+4}{x^3-4x^2+4x}=\lim\limits_{x\to 2}\dfrac{x^2(x-2)-(x^2-4)}{x(x^2-4x+4)}$

$=\lim\limits_{x\to 2}\dfrac{(x-2)^2(x+1)}{x(x-2)^2}=\lim\limits_{x\to 2}\dfrac{x+1}{x}=\dfrac{3}{2}$ ។

ញ. $\lim\limits_{x\to 1}\dfrac{x^3-x^2+x-1}{x^3-1}=\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+1)}{(x-1)(x^2+x+1)}=\lim\limits_{x\to 1}\dfrac{x^2+1}{x^2+x+1}=\dfrac{2}{3}$ ។

លំហាត់ទី០៧

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to -1}\dfrac{x^2-1}{x+1}$

ខ. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}$

គ. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x-3}$

ឃ. $\lim\limits_{x\to -4}\dfrac{x^3+64}{x+4}$

ង. $\lim\limits_{x\to 2}\left(\dfrac{1}{2-x}-\dfrac{12}{8-x^3}\right)$

ច. $\lim\limits_{x\to 2}\left[\dfrac{x(x+1)}{x-2}-\dfrac{6x^2}{x^2-4}\right]$

ឆ. $\lim\limits_{x\to 1}\dfrac{x^3+x-2}{x^2-1}$

ជ. $\lim\limits_{x\to 2}\dfrac{x^5-32}{x^4-16}$

ឈ. $\lim\limits_{x\to 1}\dfrac{x^5+x^3-1}{x^4-1}$

ញ. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^4-2x^3-x^2+4}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to -1}\dfrac{x^2-1}{x+1}=\lim\limits_{x\to -1}\dfrac{(x-1)(x+1)}{x+1}=\lim\limits_{x\to -1}(x-1)=-2$ ។

ខ. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+2)}{x-2}=\lim\limits_{x\to 2}(x+2)=4$ ។

គ. $\lim_{x\to 3}\frac{x^3-27}{x-3}=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)}{(x-3)}=\lim_{x\to 3}(x^2+3x+9)=27$ ។

ឃ. $\lim_{x\to -4}\frac{x^3+64}{x+4}=\lim_{x\to -4}\frac{(x+4)(x^2-4x+16)}{x+4}=\lim_{x\to -4}(x^2-4x+16)=48$ ។

ង. $\lim_{x\to 2}\left(\frac{1}{2-x}-\frac{12}{8-x^3}\right)=\lim_{x\to 2}\frac{(4+2x+x^2)-12}{(2-x)(4+2x+x^2)}=\lim_{x\to 2}\frac{x^2+2x-8}{(2-x)(4+2x+x^2)}$

$=\lim_{x\to 2}\frac{(x-2)(x+4)}{(2-x)(4+2x+x^2)}=\lim_{x\to 2}\frac{-(x+4)}{4+2x+x^2}=-\frac{6}{12}=-\frac{1}{2}$ ។

ច. $\lim_{x\to 2}\left[\frac{x(x+1)}{x-2}-\frac{6x^2}{x^2-4}\right]=\lim_{x\to 2}\frac{x(x+1)(x+2)-6x^2}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x(x^2-3x+2)}{(x-2)(x+2)}$

$=\lim_{x\to 2}\frac{x(x-1)(x-2)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x(x-1)}{x+2}=\frac{1}{2}$

ឆ. $\lim_{x\to 1}\frac{x^3+x-2}{x^2-1}=\lim_{x\to 1}\frac{(x^3-1)+2(x-1)}{(x-1)(x+1)}=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)+2(x-1)}{(x-1)(x+1)}$

$=\lim_{x\to 1}\frac{(x-1)(x^2+x+3)}{(x-1)(x+1)}=\lim_{x\to 1}\frac{x^2+x+3}{x+1}=\frac{5}{2}$ ។

ជ. $\lim_{x\to 2}\frac{x^5-32}{x^4-16}=\lim_{x\to 2}\frac{(x-2)(x^4+2x^3+4x^2+8x+16)}{(x-2)(x^3+2x^2+4x+8)}$

$=\lim_{x\to 2}\frac{(x^4+2x^3+4x^2+8x+16)}{(x^3+2x^2+4x+8)}$

$=\frac{16+16+16+16+16}{8+8+8+8}=\frac{80}{32}=\frac{5}{2}$

ដូចនេះ $\lim_{x\to 2}\frac{x^5-32}{x^4-16}=\frac{5}{2}$ ។

ឈ. $\lim_{x\to 1}\frac{x^5+x^3-2}{x^4-1}=\lim_{x\to 1}\frac{(x^5-1)+(x^3-1)}{(x^4-1)}$

$$=\lim_{x\to1}\frac{(x-1)(x^4+x^3+x^2+x+1)+(x-1)(x^2+x+1)}{(x-1)\left(x^3+x^2+x+1\right)}$$

$$=\lim_{x\to1}\frac{\left(x^4+x^3+x^2+x+1\right)+\left(x^2+x+1\right)}{x^3+x^2+x+1}=\frac{5+3}{4}=2$$ ។

ដូចនេះ៖ $\lim_{x\to1}\frac{x^5+x^3-1}{x^4-1}=2$ ។

ញ. $\lim_{x\to2}\frac{x^3-8}{x^4-2x^3-x^2+4}=\lim_{x\to2}\frac{(x-2)(x^2+2x+4)}{x^3(x-2)-(x-2)(x+2)}$

$$=\lim_{x\to2}\frac{x^2+2x+4}{x^3-x-2}=\frac{4+4+4}{8-2-2}=\frac{12}{4}=3$$ ។

ដូចនេះ៖ $\lim_{x\to2}\frac{x^3-8}{x^4-2x^3-x^2+4}=3$ ។

លំហាត់ទី០៨

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to1}\frac{x^4-2x^2+1}{x^3-3x+2}$

ខ. $\lim_{x\to3}\frac{x^3-3x^2+7x-21}{x^3-3x^2-x+3}$

គ. $\lim_{x\to2}\frac{x^6-64}{x^4-2x^3+8x-16}$

ឃ. $\lim_{x\to1}\frac{2x^2-5x+3}{x^2-3x+2}$

ង. $\lim_{x\to2}\frac{x^3-3x-2}{x^2-5x+6}$

ច. $\lim_{x\to1}\frac{x^3-3x+2}{2x^3-3x+1}$

ឆ. $\lim_{x\to1}\frac{x^4-4x+3}{(x-1)^2}$

ជ. $\lim_{x\to2}\frac{x^3-2x^2+2x-4}{x^2-6x+8}$

ឈ. $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2x-3\sin x+1}{4\sin^2x-1}$

ញ. $\lim_{x\to\frac{\pi}{6}}\frac{8\sin^3x-1}{2\sin^2x-3\sin x+1}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to1}\frac{x^4-2x^2+1}{x^3-3x+2}=\lim_{x\to1}\frac{\left(x^2-1\right)^2}{(x-1)^2(x+2)}=\lim_{x\to1}\frac{(x+1)^2}{x+2}=\frac{4}{3}$

ខ. $\lim_{x\to3}\frac{x^3-3x^2+7x-21}{x^3-3x^2-x+3}=\lim_{x\to3}\frac{(x-3)(x^2+7)}{(x-3)(x^2-1)}=\lim_{x\to3}\frac{x^2+7}{x^2-1}=\frac{16}{8}=2$ ។

គ. $\lim_{x\to2}\frac{x^6-64}{x^4-2x^3+8x-16}=\lim_{x\to2}\frac{(x-2)(x^2+2x+4)(x^3+8)}{(x-2)(x^3+8)}$

$=\lim_{x\to2}\left(x^2+2x+4\right)=12$ ។

ឃ. $\lim_{x\to1}\frac{2x^2-5x+3}{x^2-3x+2}$ រាងមិនកំណត់ $\frac{0}{0}$

$=\lim_{x\to1}\frac{\left(2x^2-2x\right)-\left(3x-3\right)}{\left(x^2-x\right)-\left(2x-2\right)}=\lim_{x\to1}\frac{2x\left(x-1\right)-3\left(x-1\right)}{x\left(x-1\right)-2\left(x-1\right)}$

$=\lim_{x\to1}\frac{\left(x-1\right)\left(2x-3\right)}{\left(x-1\right)\left(x-2\right)}=\lim_{x\to1}\frac{2x-3}{x-2}=\frac{-1}{-1}=1$

ដូចនេះ $\lim_{x\to1}\frac{2x^2-5x+3}{x^2-3x+2}=1$ ។

ង. $\lim_{x\to2}\frac{x^3-3x-2}{x^2-5x+6}$ រាងមិនកំណត់ $\frac{0}{0}$

$=\lim_{x\to2}\frac{\left(x^3-8\right)-\left(3x-6\right)}{\left(x^2-2x\right)-\left(3x-6\right)}=\lim_{x\to2}\frac{\left(x-2\right)\left(x^2+2x+4\right)-3\left(x-2\right)}{x\left(x-2\right)-3\left(x-2\right)}$

$=\lim_{x\to2}\frac{\left(x-2\right)\left(x^2+2x+1\right)}{\left(x-2\right)\left(x-3\right)}=\lim_{x\to2}\frac{x^2+2x+1}{x-3}=\frac{4+4+1}{4-3}=9$

ដូចនេះ $\lim_{x\to2}\frac{x^3-3x-2}{x^2-5x+6}=9$ ។

ប. $\lim_{x\to1}\frac{x^3-3x+2}{2x^3-3x+1}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to1}\frac{(x^3-1)-(3x-3)}{(2x^3-2)-(3x-3)}=\lim_{x\to1}\frac{(x-1)(x^2+x+1)-3(x-1)}{2(x-1)(x^2+x+1)-3(x-1)}$$

$$=\lim_{x\to1}\frac{(x-1)(x^2+x-2)}{(x-1)(2x^2+2x-1)}=\lim_{x\to1}\frac{x^2+x-2}{2x^2+2x-1}=0$$

ដូចនេះ $\lim_{x\to1}\frac{x^3-3x+2}{2x^3-3x+1}=0$ ។

ឃ. $\lim_{x\to1}\frac{x^4-4x+3}{(x-1)^2}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to1}\frac{(x^4-1)-4(x-1)}{(x-1)^2}=\lim_{x\to1}\frac{(x-1)(x^3+x^2+x+1)-4(x-1)}{(x-1)^2}$$

$$=\lim_{x\to1}\frac{(x-1)(x^3+x^2+x-3)}{(x-1)^2}=\lim_{x\to1}\frac{(x^3-1)+(x^2-1)+(x-1)}{x-1}$$

$$=\lim_{x\to1}\frac{(x-1)(x^2+x+1)+(x-1)(x+1)+(x-1)}{x-1}$$

$$=\lim_{x\to1}\left[(x^2+x+1)+(x+1)+1\right]=3+2+1=6$$

ដូចនេះ $\lim_{x\to1}\frac{x^4-4x+3}{(x-1)^2}=6$ ។

ង. $\lim_{x\to2}\frac{x^3-2x^2+2x-4}{x^2-6x+8}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to2}\frac{(x^3-2x^2)+(2x-4)}{(x^2-2x)-(4x-8)}=\lim_{x\to2}\frac{x^2(x-2)+2(x-2)}{x(x-2)-4(x-2)}$$

$$=\lim_{x\to2}\frac{(x-2)(x^2+2)}{(x-2)(x-4)}=\lim_{x\to2}\frac{x^2+2}{x-4}=\frac{4+2}{2-4}=-3$$

ដូចនេះ $\lim_{x\to 2}\frac{x^3-2x^2+2x-4}{x^2-6x+8}=-3$ ។

ច. $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2x-3\sin x+1}{4\sin^2x-1}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\left(2\sin^2x-\sin x\right)-\left(2\sin x-1\right)}{\left(2\sin x\right)^2-1^2}=\lim_{x\to\frac{\pi}{6}}\frac{\sin x\left(2\sin x-1\right)-\left(2\sin x-1\right)}{\left(2\sin x-1\right)\left(2\sin x+1\right)}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\left(2\sin x-1\right)\left(\sin x-1\right)}{\left(2\sin x-1\right)\left(2\sin x+1\right)}=\lim_{x\to\frac{\pi}{6}}\frac{\sin x-1}{2\sin x+1}=\frac{\frac{1}{2}-1}{2\left(\frac{1}{2}\right)+1}=-\frac{1}{4}$$

ដូចនេះ $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2x-3\sin x+1}{4\sin^2x-1}=-\frac{1}{4}$ ។

ញ. $\lim_{x\to\frac{\pi}{6}}\frac{8\sin^3x-1}{2\sin^2x-3\sin x+1}=\lim_{x\to\frac{\pi}{6}}\frac{(2\sin x-1)(4\sin^2x+2\sin x+1)}{(2\sin x-1)(\sin x-1)}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{4\sin^2x+2\sin x+1}{\sin x-1}=\frac{1+1+1}{\frac{1}{2}-1}=-6 \text{ ។}$$

ដូចនេះ $\lim_{x\to\frac{\pi}{6}}\frac{8\sin^3x-1}{2\sin^2x-3\sin x+1}=-6$ ។

លំហាត់ទី០៩

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 1}\left(\frac{2}{1-x^2}-\frac{3}{1-x^3}\right)$

ខ. $\lim_{x\to 0}\frac{(1+x)^4-4x-1}{x^3+x^2}$

គ. $\lim_{x\to 2}\frac{x+x^2+x^3-12}{x-2}$

ឃ. $\lim_{x\to 1}\frac{x+2x^2+3x^3-6}{x-1}$

ង. $\lim_{x\to 1}\frac{x^3-3x^2+3x-1}{(x^3-x)^3}$

ច. $\lim_{x\to -1}\frac{x^2+3x+2}{x^2-2x-3}$

ឆ. $\lim_{x\to\frac{1}{2}}\frac{2x^2-x}{2x^2-3x+1}$

ជ. $\lim_{x\to a}\frac{x^n-a^n}{x-a}$ $(\ n\in\mathbb{N}\ )$ ។

ឈ. $\lim_{x\to1}\frac{x+x^2+x^3-3}{x-1}$

ញ. $\lim_{x\to1}\frac{x+x^2+x^3+\ldots+x^n-n}{x-1}$ ។

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to1}\left(\frac{2}{1-x^2}-\frac{3}{1-x^3}\right)=\lim_{x\to1}\frac{2\left(1+x+x^2\right)-3\left(1+x\right)}{(1-x)(1+x)(1+x+x^2)}$

$=\lim_{x\to1}\frac{2x^2-x-1}{(1-x)(1+x)(1+x+x^2)}=\lim_{x\to1}\frac{\left(x-1\right)\left(2x+1\right)}{(1-x)(1+x)(1+x+x^2)}$

$=\lim_{x\to1}\frac{\left(2x+1\right)}{(1+x)(1+x+x^2)}=\frac{3}{2\times3}=\frac{1}{2}$

ខ. $\lim_{x\to0}\frac{(1+x)^4-4x-1}{x^3+x^2}=\lim_{x\to0}\frac{1+4x+6x^2+4x^3+x^4-4x-1}{x^2\left(x+1\right)}$

$=\lim_{x\to0}\frac{6x^2+4x^3+x^4}{x^2\left(x+1\right)}=\lim_{x\to0}\frac{6+4x+x^2}{x+1}=6$

គ. $\lim_{x\to2}\frac{x+x^2+x^3-12}{x-2}=\lim_{x\to2}\frac{\left(x-2\right)+\left(x^2-4\right)+\left(x^3-8\right)}{x-2}$

$=\lim_{x\to2}\left[1+\left(x+2\right)+\left(x^2+2x+4\right)\right]=17$

ឃ. $\lim_{x\to1}\frac{x+2x^2+3x^3-6}{x-1}=\lim_{x\to1}\frac{\left(x-1\right)+2\left(x^2-1\right)+3\left(x^3-1\right)}{x-1}$

$=\lim_{x\to1}\left[1+2\left(x+1\right)+3\left(x^2+x+1\right)\right]=14$

ង. $\lim_{x\to1}\frac{x^3-3x^2+3x-1}{(x^3-x)^3}=\lim_{x\to1}\frac{\left(x-1\right)^3}{x^3\left(x-1\right)^3\left(x+1\right)^3}=\lim_{x\to1}\frac{1}{x^3\left(x+1\right)^3}=\frac{1}{8}$

ប. $\lim_{x\to -1}\frac{x^2+3x+2}{x^2-2x-3}=\lim_{x\to -1}\frac{(x+1)(x+2)}{(x+1)(x-3)}=\lim_{x\to -1}\frac{x+2}{x-3}=-\frac{1}{4}$ ។

ផ. $\lim_{x\to\frac{1}{2}}\frac{2x^2-x}{2x^2-3x+1}=\lim_{x\to\frac{1}{2}}\frac{x(2x-1)}{(2x-1)(x-1)}=\lim_{x\to\frac{1}{2}}\frac{x}{x-1}=\frac{\frac{1}{2}}{\frac{1}{2}-1}=-1$ ។

ព. $\lim_{x\to a}\frac{x^n-a^n}{x-a}=\lim_{x\to a}\frac{(x-a)\left(x^{n-1}+ax^{n-2}+\ldots+a^{n-1}\right)}{x-a}$

$=\lim_{x\to a}\left(x^{n-1}+ax^{n-2}+\ldots+a^{n-1}\right)=a^{n-1}+a^{n-1}+\ldots+a^{n-1}=na^{n-1}$

ភ. $\lim_{x\to 1}\frac{x+x^2+x^3-3}{x-1}=\lim_{x\to 1}\frac{(x-1)+\left(x^2-1\right)+\left(x^3-1\right)}{(x-1)}$

$=\lim_{x\to 1}\frac{(x-1)+(x-1)(x+1)+(x-1)\left(x^2+x+1\right)}{(x-1)}$

$=\lim_{x\to 1}\left[1+(x+1)+\left(x^2+x+1\right)\right]=1+2+3=6$

ម. $\lim_{x\to 1}\frac{x+x^2+x^3+\ldots+x^n-n}{x-1}$

$=\lim_{x\to 1}\frac{(x-1)+\left(x^2-1\right)+\left(x^3-1\right)+\ldots+\left(x^n-1\right)}{(x-1)}$

$=\lim_{x\to 1}\frac{(x-1)+(x-1)(x+1)+\ldots+(x-1)\left(x^{n-1}+x^{n-2}+\ldots+1\right)}{(x-1)}$

$=\lim_{x\to 1}\left[1+(x+1)+\left(x^2+x+1\right)+\ldots+\left(x^{n-1}+x^{n-2}+\ldots+1\right)\right]$

$=1+2+3+\ldots.+n=\frac{n(n+1)}{2}$

ដូចនេះ $\lim_{x\to 1}\frac{x+x^2+x^3+\ldots+x^n-n}{x-1}=\frac{n(n+1)}{2}$ ។

កំណត់ចំណាំ!!!

រូបមន្តអនុគមន៍ត្រីកោណមាត្រ

◈ ទំនាក់ទំនងគ្រឹះ

$a)\ \tan u=\dfrac{\sin u}{\cos u}$ $b)\ \cot u=\dfrac{\cos u}{\sin u}$

$c)\ \sin^2 u+\cos^2 u=1$ $d)\ 1+\tan^2 u=\dfrac{1}{\cos^2 u}$

$e) 1+\cot^2 u=\dfrac{1}{\sin^2 u}$

◈ រូបមន្តមុំឌុប

$a)\ \sin 2u=2\sin u\cos u$ $b) \cos 2u=\cos^2 u-\sin^2 u$

$c) \tan 2u=\dfrac{2\tan u}{1-\tan^2 u}$ $d) \cot 2u=\dfrac{\cot^2 u-1}{2\cot u}$

◈ រូបមន្តមុំទ្រីប

$a) \sin 3u=3\sin u-4\sin^3 u$ $b) \cos 3u=4\cos^3 u-3\cos u$

$c) \tan 3u=\dfrac{3\tan u-\tan^3 u}{1-3\tan^2 u}$

◈ រូបមន្តមុំផលបូកនិងផលដកមុំពីរ

$a)\ \sin(u+v)=\sin u\cos v+\sin v\cos u$ $b) \sin(u-v)=\sin u\cos v-\sin v\cos u$

$c)\ \cos(u+v)=\cos u\cos v-\sin u\sin v$ $d)\ \cos(u-v)=\cos u\cos v+\sin u\sin v$

$e)\ \tan(u+v)=\dfrac{\tan u+\tan v}{1-\tan u\tan v}$ $f) \tan(u-v)=\dfrac{\tan u-\tan v}{1+\tan u\tan v}$

លំហាត់ទី១០

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x \to \frac{\pi}{3}} \frac{\tan^2 x - (1+\sqrt{3})\tan x + \sqrt{3}}{\sin x - \sqrt{3}\cos x}$

ខ. $\lim_{x \to \frac{\pi}{4}} \frac{\tan^2 x - 3\tan x + 2}{\sin x - \cos x}$

គ. $\lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{\cos^3 x - \sin^3 x}$

ឃ. $\lim_{x \to \frac{\pi}{3}} \frac{4\cos^2 x - 1}{\sin x - \sin 2x}$

ង. $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan^3 x}{\cos 2x}$

ច. $\lim_{x \to \frac{\pi}{6}} \frac{\cos 3x}{\cos x - \sin 2x}$

ឆ. $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \tan^3 x}$

ជ. $\lim_{x \to 0} \frac{1 - \cos 2x}{\sin^2 x + \sin^3 x}$

ឈ. $\lim_{x \to \frac{\pi}{3}} \frac{3 - 4\sin^2 x}{2\cos x - 1}$

ញ. $\lim_{x \to 0} \frac{2x - x^2 - \sin^2 x}{\cos x + x - 1}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x \to \frac{\pi}{3}} \frac{\tan^2 x - (1+\sqrt{3})\tan x + \sqrt{3}}{\sin x - \sqrt{3}\cos x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$= \lim_{x \to \frac{\pi}{3}} \frac{(\tan x - 1)\left(\tan x - \sqrt{3}\right)}{\cos x\left(\tan x - \sqrt{3}\right)} = \lim_{x \to \frac{\pi}{3}} \frac{\tan x - 1}{\cos x} = \frac{\sqrt{3} - 1}{\frac{\sqrt{3}}{2}} = \frac{2\left(3 - \sqrt{3}\right)}{3}$$

ដូចនេះ $\lim_{x \to \frac{\pi}{3}} \frac{\tan^2 x - (1+\sqrt{3})\tan x + \sqrt{3}}{\sin x - \sqrt{3}\cos x} = \frac{2\left(3 - \sqrt{3}\right)}{3}$ ។

ខ. $\lim_{x \to \frac{\pi}{4}} \frac{\tan^2 x - 3\tan x + 2}{\sin x - \cos x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{(\tan x-1)(\tan x-2)}{\cos x(\tan x-1)}=\lim_{x\to\frac{\pi}{4}}\frac{\tan x-2}{\cos x}=\frac{1-2}{\frac{1}{\sqrt{2}}}=-\sqrt{2}$$

ដូចនេះ: $\lim_{x\to\frac{\pi}{4}}\frac{\tan^2 x-3\tan x+2}{\sin x-\cos x}=-\sqrt{2}$ ។

គ. $\lim_{x\to\frac{\pi}{4}}\frac{\cos 2x}{\cos^3 x-\sin^3 x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos^2 x+\cos x\sin x+\sin^2 x)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\cos x+\sin x}{1+\sin x\cos x}=\frac{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}}=\frac{2\sqrt{2}}{3}$$

ដូចនេះ: $\lim_{x\to\frac{\pi}{4}}\frac{\cos 2x}{\cos^3 x-\sin^3 x}=\frac{2\sqrt{2}}{3}$ ។

ឃ. $\lim_{x\to\frac{\pi}{3}}\frac{4\cos^2 x-1}{\sin x-\sin 2x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{3}}\frac{(2\cos x-1)(2\cos x+1)}{\sin x-2\sin x\cos x}=\lim_{x\to\frac{\pi}{3}}\frac{(2\cos x+1)(2\cos x-1)}{\sin x(1-2\cos x)}$$

$$=-\lim_{x\to\frac{\pi}{3}}\frac{2\cos x+1}{\sin x}=-\frac{1+1}{\frac{\sqrt{3}}{2}}=-\frac{4\sqrt{3}}{3}$$

ដូចនេះ: $\lim_{x\to\frac{\pi}{3}}\frac{4\cos^2 x-1}{\sin x-\sin 2x}=-\frac{4\sqrt{3}}{3}$ ។

ង. $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan^3 x}{\cos 2x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{1-\tan^3 x}{\cos^2 x-\sin^2 x}=\lim_{x\to\frac{\pi}{4}}\frac{1-\tan^3 x}{\cos^2 x\left(1-\tan^2 x\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\left(1-\tan x\right)\left(1+\tan x+\tan^2 x\right)}{\cos^2 x\left(1-\tan x\right)\left(1+\tan x\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{1+\tan x+\tan^2 x}{\cos^2 x\left(1+\tan x\right)}=\frac{1+1+1}{\left(\frac{\sqrt{2}}{2}\right)^2\left(1+1\right)}=3$$

ដូចនេះ $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan^3 x}{\cos 2x}=3$ ។

ឆ. $\lim_{x\to\frac{\pi}{6}}\frac{\cos 3x}{\cos x-\sin 2x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{4\cos^3 x-3\cos x}{\cos x-2\sin x\cos x}=\lim_{x\to\frac{\pi}{6}}\frac{\cos x\left(4\cos^2 x-3\right)}{\cos x\left(1-2\sin x\right)}=\lim_{x\to\frac{\pi}{6}}\frac{4\left(1-\sin^2 x\right)-3}{1-2\sin x}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{1-4\sin^2 x}{1-2\sin x}=\lim_{x\to\frac{\pi}{6}}\frac{\left(1-2\sin x\right)\left(1+2\sin x\right)}{1-2\sin x}$$

$$=\lim_{x\to\frac{\pi}{6}}\left(1+2\sin x\right)=1+2\left(\frac{1}{2}\right)=1+1=2$$

ដូចនេះ $\lim_{x\to\frac{\pi}{6}}\frac{\cos 3x}{\cos x-\sin 2x}=2$ ។

ឆ. $\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{1-\tan^3 x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\cos x\left(\tan x-1\right)}{1-\tan^3 x}=-\lim_{x\to\frac{\pi}{4}}\frac{\cos x\left(1-\tan x\right)}{\left(1-\tan x\right)\left(1+\tan x+\tan^2 x\right)}$$

$$=-\lim_{x\to\frac{\pi}{4}}\frac{\cos x}{1+\tan x+\tan^2 x}=-\frac{\frac{\sqrt{2}}{2}}{1+1+1}=-\frac{\sqrt{2}}{6}$$

ដូចនេះ: $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\tan^3 x}=-\dfrac{\sqrt{2}}{6}$ ។

ង. $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{\sin^2 x+\sin^3 x}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{\sin^2 x\left(1+\sin x\right)}=\lim_{x\to 0}\frac{2}{1+\sin x}=\frac{2}{1+0}=2$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{\sin^2 x+\sin^3 x}=2$ ។

ឈ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{3-4\sin^2 x}{2\cos x-1}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to\frac{\pi}{3}}\frac{3-4\left(1-\cos^2 x\right)}{2\cos x-1}=\lim_{x\to\frac{\pi}{3}}\frac{4\cos^2 x-1}{2\cos x-1}=\lim_{x\to\frac{\pi}{3}}\frac{\left(2\cos x-1\right)\left(2\cos x+1\right)}{2\cos x-1}$$

$$=\lim_{x\to\frac{\pi}{3}}\left(2\cos x+1\right)=1+1=2$$

ដូចនេះ: $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{3-4\sin^2 x}{2\cos x-1}=2$ ។

ញ. $\lim\limits_{x\to 0}\dfrac{2x-x^2-\sin^2 x}{\cos x+x-1}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{2x-x^2-\left(1-\cos^2 x\right)}{\cos x+x-1}=\lim_{x\to 0}\frac{\cos^2 x-\left(x-1\right)^2}{\cos x+x-1}$$

$$=\lim_{x\to 0}\frac{\left(\cos x+x-1\right)\left(\cos x-x+1\right)}{\cos x+x-1}=\lim_{x\to 0}\left(\cos x-x+1\right)=2$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{2x-x^2-\sin^2 x}{\cos x+x-1}=2$ ។

ដ. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\cos x-\sin 2x}{4\cos^2 x-3}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$= \lim_{x\to\frac{\pi}{6}} \frac{\cos x - 2\sin x\cos x}{4\left(1-\sin^2 x\right)-3} = \lim_{x\to\frac{\pi}{6}} \frac{\cos x\left(1-2\sin x\right)}{1-4\sin^2 x}$$

$$= \lim_{x\to\frac{\pi}{6}} \frac{\cos x\left(1-2\sin x\right)}{\left(1-2\sin x\right)\left(1+2\sin x\right)} = \lim_{x\to\frac{\pi}{6}} \frac{\cos x}{1+2\sin x} = \frac{\frac{\sqrt{3}}{2}}{1+1} = \frac{\sqrt{3}}{4}$$

ដូចនេះ $\lim_{x\to\frac{\pi}{6}} \frac{\cos x - \sin 2x}{4\cos^2 x - 3} = \frac{\sqrt{3}}{4}$ ។

លំហាត់ទី១១

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to\frac{\pi}{6}} \frac{\cos x - \sin 2x}{4\cos^2 x - 3}$

ខ. $\lim_{x\to 0} \frac{\cos x - \sqrt{x^2+1}}{x^2 + \sin^2 x}$

គ. $\lim_{x\to\frac{\pi}{4}} \frac{\tan^4 x + \cot^4 x - 2}{1 - \sin 2x}$

ឃ. $\lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \sqrt[3]{\tan x}}$

ង. $\lim_{x\to\frac{\pi}{6}} \frac{1 - 8\sin^3 x}{4\cos^2 x - 3}$

ច. $\lim_{x\to 0} \frac{x^4 - \sin^4 x}{\sqrt{x^2+1} - \cos x}$

ឆ. $\lim_{x\to\frac{\pi}{4}} \frac{\cos^3 x - \sin^3 x}{1 - \tan^2 x}$

ជ. $\lim_{x\to\frac{\pi}{3}} \frac{3 - 4\sin^2 x}{2\cos x - 1}$

ឈ. $\lim_{x\to\frac{\pi}{6}} \frac{4\cos^2 x - 3}{1 - 2\sin x}$

ញ. $\lim_{x\to\frac{\pi}{2}} \frac{2 - \sin x - \sin^2 x}{1 - \sin x}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to 0} \frac{\cos x - \sqrt{x^2+1}}{x^2 + \sin^2 x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to 0}\frac{\cos^2 x - x^2 - 1}{\left(x^2+\sin^2 x\right)\left(\cos x+\sqrt{x^2+1}\right)}=\lim_{x\to 0}\frac{-x^2-\sin^2 x}{\left(x^2+\sin^2 x\right)\left(\cos x+\sqrt{x^2+1}\right)}$$

$$=\lim_{x\to 0}\frac{-1}{\cos x+\sqrt{x^2+1}}=-\frac{1}{2}$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{\cos x-\sqrt{x^2+1}}{x^2+\sin^2 x}=-\dfrac{1}{2}$ ។

ខ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\tan^4 x+\cot^4 x-2}{1-\sin 2x}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\left(\cot^2 x-\tan^2 x\right)^2}{\left(\cos x-\sin x\right)^2}=\lim_{x\to\frac{\pi}{4}}\frac{\left(1-\tan^4 x\right)^2}{\tan^4 x\cos^2 x\left(1-\tan x\right)^2}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\left(1-\tan x\right)^2\left(1+\tan x\right)^2\left(1+\tan^2 x\right)^2}{\tan^4 x\cos^2 x\left(1-\tan x\right)^2}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\left(1+\tan x\right)^2\left(1+\tan^2 x\right)^2}{\tan^4 x\cos^2 x}=\frac{4\times 4}{\frac{1}{2}}=32$$

ដូចនេះ: $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\tan^4 x+\cot^4 x-2}{1-\sin 2x}=32$ ។

គ. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\sqrt[3]{\tan x}}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\cos x\left(\tan x-1\right)\left(1+\sqrt[3]{\tan x}+\sqrt[3]{\tan^2 x}\right)}{\left(1-\tan x\right)}$$

$$=-\lim_{x\to\frac{\pi}{4}}\left[\cos x\left(1+\sqrt[3]{\tan x}+\sqrt[3]{\tan^2 x}\right)\right]=-\frac{\sqrt{2}}{2}\times 3=-\frac{3\sqrt{2}}{2}$$

ដូចនេះ: $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\sqrt[3]{\tan x}}=-\dfrac{3\sqrt{2}}{2}$ ។

ឃ. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{1-8\sin^3 x}{4\cos^2 x-3}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{1-8\sin^3 x}{4\left(1-\sin^2 x\right)-3}=\lim_{x\to\frac{\pi}{6}}\frac{1-\left(2\sin x\right)^3}{1-4\sin^2 x}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\left(1-2\sin x\right)\left(1+2\sin x+4\sin^2 x\right)}{\left(1-2\sin x\right)\left(1+2\sin x\right)}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{1+2\sin x+4\sin^2 x}{1+2\sin x}=\frac{1+1+1}{1+1}=\frac{3}{2}$$

ដូចនេះ: $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{1-8\sin^3 x}{4\cos^2 x-3}=\dfrac{3}{2}$ ។

ង. $\lim\limits_{x\to 0}\dfrac{x^4-\sin^4 x}{\sqrt{x^2+1}-\cos x}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{\left(x^2+\sin^2 x\right)\left(x^2-\sin^2 x\right)\left(\sqrt{1+x^2}+\cos x\right)}{x^2+1-\cos^2 x}$$

$$=\lim_{x\to 0}\frac{\left(x^2+\sin^2 x\right)\left(x^2-\sin^2 x\right)\left(\sqrt{1+x^2}+\cos x\right)}{x^2+\sin^2 x}$$

$$=\lim_{x\to 0}\left(x^2-\sin^2 x\right)\left(\sqrt{1+x^2}+\cos x\right)=0$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{x^4-\sin^4 x}{\sqrt{x^2+1}-\cos x}=0$ ។

ច. $\lim\limits_{x\to 0}\dfrac{x^4-\sin^4 x}{\sqrt{x^2+1}-\cos x}=\lim\limits_{x\to 0}(x^2-\sin^2 x)(\sqrt{x^2+1}+\cos x)=0$

ឆ. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos^3 x-\sin^3 x}{1-\tan^2 x}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\cos^3 x\left(1-\tan^3 x\right)}{1-\tan^2 x}=\lim_{x\to\frac{\pi}{4}}\frac{\cos^3 x\left(1-\tan x\right)\left(1+\tan x+\tan^2 x\right)}{\left(1-\tan x\right)\left(1+\tan x\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\cos^3 x\left(1+\tan x+\tan^2 x\right)}{1+\tan x}=\left(\frac{\sqrt{2}}{2}\right)^3\times\frac{3}{2}=\frac{3\sqrt{2}}{8}$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos^3 x-\sin^3 x}{1-\tan^2 x}=\frac{3\sqrt{2}}{8}$ ។

ជ. $\lim\limits_{x\to\frac{\pi}{3}}\frac{3-4\sin^2 x}{2\cos x-1}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{3}}\frac{3-4\left(1-\cos^2 x\right)}{2\cos x-1}=\lim_{x\to\frac{\pi}{3}}\frac{4\cos^2 x-1}{2\cos x-1}=\lim_{x\to\frac{\pi}{3}}\left(2\cos x+1\right)=1+1=2$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{3}}\frac{3-4\sin^2 x}{2\cos x-1}=2$ ។

ឈ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{4\cos^2 x-3}{1-2\sin x}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{4\left(1-\sin^2 x\right)-3}{1-2\sin x}=\lim_{x\to\frac{\pi}{6}}\frac{1-4\sin^2 x}{1-2\sin x}=\lim_{x\to\frac{\pi}{6}}\left(1+2\sin x\right)=1+1=2$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{6}}\frac{4\cos^2 x-3}{1-2\sin x}=2$ ។

ញ. $\lim\limits_{x\to\frac{\pi}{2}}\frac{2-\sin x-\sin^2 x}{1-\sin x}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to\frac{\pi}{2}}\frac{\left(1-\sin x\right)\left(2+\sin x\right)}{\left(1-\sin x\right)}=\lim_{x\to\frac{\pi}{2}}\left(2+\sin x\right)=2+1=3$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{2}}\frac{2-\sin x-\sin^2 x}{1-\sin x}=3$ ។

លំហាត់ទី១២

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{\sin x(1-\cos^3 x)}$

ខ. $\lim\limits_{x\to \frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{2\sin x-1}$

គ. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{1-\tan^3 x}{\cos x-\sin x}$

ឃ. $\lim\limits_{x\to \frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{\cos x-\sin 2x}$

ង. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{2\cos x-1}{3-4\sin^2 x}$

ង. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\cos^4 x-\sin^4 x}{1-\tan x}$

ប. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{\tan^2 x-(2+\sqrt{3})\tan x+2\sqrt{3}}{\sin x-\sqrt{3}\cos x}$

ឆ. $\lim\limits_{x\to \frac{\pi}{6}}\dfrac{2\sin^2 x-5\sin x+2}{3-4\cos^2 x}$

ដំណោះស្រាយ

គណនាលីមីត

ក. $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{\sin x(1-\cos^3 x)}$ មានរាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{2\sin x-2\sin x\cos x}{1-\cos^3 x}=\lim_{x\to 0}\frac{2\sin x\left(1-\cos x\right)}{\sin x\left(1-\cos x\right)\left(1+\cos x+\cos^2 x\right)}$$

$$=\lim_{x\to 0}\frac{2}{1+\cos x+\cos^2 x}=\frac{2}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{\sin x(1-\cos^3 x)}=\dfrac{2}{3}$ ។

ខ. $\lim\limits_{x\to \frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{2\sin x-1}$ មានរាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to \frac{\pi}{6}}\frac{\left(2\sin x-1\right)\left(\sin x-1\right)}{2\sin x-1}=\lim_{x\to \frac{\pi}{6}}\left(\sin x-1\right)=\frac{1}{2}-1=-\frac{1}{2}$$ ។

ដូចនេះ $\lim\limits_{x\to \frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{2\sin x-1}=-\dfrac{1}{2}$ ។

គ. $\dfrac{1-\tan^3 x}{\cos x-\sin x}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(1-\tan x)(1+\tan x+\tan^2 x)}{\cos x(1-\tan x)}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{1+\tan x+\tan^2 x}{\cos x}=\frac{3}{\frac{\sqrt{2}}{2}}=3\sqrt{2}$$

ឃ. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{\cos x-\sin 2x}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{(2\sin x-1)(\sin x-1)}{\cos x(1-2\sin x)}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\sin x-1}{-\cos x}=\frac{\frac{1}{2}-1}{-\frac{\sqrt{3}}{2}}=\frac{\sqrt{3}}{3}$$ ។

ង. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\cos x-1}{3-4\sin^2 x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\cos x-1}{3-4(1-\cos^2 x)}$

$$=\lim_{x\to\frac{\pi}{3}}\frac{(2\cos x-1)}{(2\cos x-1)(2\cos x+1)}=\lim_{x\to\frac{\pi}{3}}\frac{1}{2\cos x+1}=\frac{1}{2(\frac{1}{2})+1}=\frac{1}{2}$$

ច. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos^4 x-\sin^4 x}{1-\tan x}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(\cos^2 x+\sin^2 x)(\cos^2 x-\sin^2 x)}{1-\tan x}$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\cos^2 x(1-\tan^2 x)}{1-\tan x}$$

$$=\lim_{x\to\frac{\pi}{4}}\left[\cos^2 x(1+\tan x)\right]=\frac{1}{2}(1+1)=1$$

ឆ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\tan^2 x-(2+\sqrt{3})\tan x+2\sqrt{3}}{\sin x-\sqrt{3}\cos x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{(\tan x-\sqrt{3})(\tan x-2)}{\cos x(\tan x-\sqrt{3})}$

$$=\lim_{x\to\frac{\pi}{3}}\frac{\tan x-2}{\cos x}=2(\sqrt{3}-2)$$ ។

ជ. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x-5\sin x+2}{3-4\cos^2 x}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin x-2}{2\sin x+1}=-\dfrac{3}{4}$ ។

កំណត់ចំណាំ!!!

រូបមន្តបម្លែងកន្សោមអសនិទាន

$$a)\ \sqrt{A}-\sqrt{B}=\frac{A-B}{\sqrt{A}+\sqrt{B}}$$

$$b)\ \sqrt[3]{A}-\sqrt[3]{B}=\frac{A-B}{\sqrt[3]{A^2}+\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}$$

$$c)\ \sqrt[3]{A}+\sqrt[3]{B}=\frac{A+B}{\sqrt[3]{A^2}-\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}$$

$$d)\ \sqrt[4]{A}-\sqrt[4]{B}=\frac{A-B}{(\sqrt[4]{A}+\sqrt[4]{B})(\sqrt{A}+\sqrt{B})}$$

លំហាត់ទី១៣

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-1}{x^2}$

ខ. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3}-2}{x-1}$

គ. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

ឃ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2-2}-\sqrt{2}}{x-2}$

ង. $\lim\limits_{x\to 0}\dfrac{1+x-\sqrt{2x+1}}{x^2}$

ច. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-\sqrt{x+7}}{x-2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{\sqrt{(1+x)(1+2x)}-1}{x}$

ជ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^3+1}-3}{x-2}$

ឈ. $\lim\limits_{x\to 1}\dfrac{\sqrt{2x^2+3x+4}-\sqrt{x^2+3x+5}}{x-1}$

ញ. $\lim\limits_{x\to 3}\dfrac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

$$ក.\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{x^2}=\lim_{x\to 0}\frac{x^2+1-1}{x^2\left(\sqrt{x^2+1}+1\right)}=\lim_{x\to 0}\frac{1}{\sqrt{x^2+1}+1}=\frac{1}{2}$$

$$ខ.\lim_{x\to 1}\frac{\sqrt{x^2+3}-2}{x-1}=\lim_{x\to 1}\frac{x^2+3-4}{(x-1)\left(\sqrt{x^2+3}+2\right)}=\lim_{x\to 1}\frac{x^2-1}{(x-1)\left(\sqrt{x^2+3}+2\right)}$$

$$=\lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)\left(\sqrt{x^2+3}+2\right)}=\lim_{x\to 1}\frac{x+1}{\sqrt{x^2+3}+2}=\frac{1}{2}$$

$$គ.\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=\lim_{x\to 0}\frac{1+x-1+x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}=\lim_{x\to 0}\frac{2}{\sqrt{1+x}+\sqrt{1-x}}=1$$

$$ឃ.\lim_{x\to 2}\frac{\sqrt{x^2-2}-\sqrt{2}}{x-2}=\lim_{x\to 2}\frac{x^2-2-2}{(x-2)\left(\sqrt{x^2-2}+\sqrt{2}\right)}=\lim_{x\to 2}\frac{x^2-4}{(x-2)\left(\sqrt{x^2-2}+\sqrt{2}\right)}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)\left(\sqrt{x^2-2}+\sqrt{2}\right)}=\lim_{x\to 2}\frac{x+2}{\sqrt{x^2-2}+\sqrt{2}}=\sqrt{2}$$

$$ង.\lim_{x\to 0}\frac{1+x-\sqrt{2x+1}}{x^2}=\lim_{x\to 0}\frac{(1+x)^2-(2x+1)}{x^2\left(1+x+\sqrt{2x+1}\right)}=\lim_{x\to 0}\frac{x^2}{x^2\left(1+x+\sqrt{2x+1}\right)}$$

$$=\lim_{x\to 0}\frac{1}{1+x+\sqrt{2x+1}}=\frac{1}{2}$$ ។

$$ច.\lim_{x\to 2}\frac{\sqrt{x^2+5}-\sqrt{x+7}}{x-2}=\lim_{x\to 2}\frac{x^2+5-x-7}{(x-2)\left(\sqrt{x^2+5}+\sqrt{x+7}\right)}$$

$$=\lim_{x\to 2}\frac{x^2-x-2}{(x-2)\left(\sqrt{x^2+5}+\sqrt{x+7}\right)}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+1)}{(x-2)\left(\sqrt{x^2+5}+\sqrt{x+7}\right)}=\lim_{x\to 2}\frac{x+1}{\sqrt{x^2+5}+\sqrt{x+7}}=\frac{3}{6}=\frac{1}{2}$$

ដូចនេះ $\lim_{x\to 2}\frac{\sqrt{x^2+5}-\sqrt{x+7}}{x-2}=\frac{1}{2}$ ។

ឆ. $\lim_{x\to 0}\frac{\sqrt{(1+x)(1+2x)}-1}{x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to 0}\frac{(1+x)(1+2x)-1}{x\left[\sqrt{(1+x)(1+2x)}+1\right]}=\lim_{x\to 0}\frac{1+3x+2x^2-1}{x\left[\sqrt{(1+x)(1+2x)}+1\right]}$$

$$=\lim_{x\to 0}\frac{3x+2x^2}{x\left[\sqrt{(1+x)(1+2x)}+1\right]}$$

$$=\lim_{x\to 0}\frac{3+2x}{\sqrt{(1+x)(1+2x)}+1}=\frac{3}{2}$$

ដូចនេះ $\lim_{x\to 0}\frac{\sqrt{(1+x)(1+2x)}-1}{x}=\frac{3}{2}$ ។

ជ. $\lim_{x\to 2}\frac{\sqrt{x^3+1}-3}{x-2}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to 2}\frac{x^3+1-9}{(x-2)\left(\sqrt{x^3+1}+3\right)}=\lim_{x\to 2}\frac{x^3-8}{(x-2)\left(\sqrt{x^3+1}+3\right)}$$

$$=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)\left(\sqrt{x^3+1}+3\right)}=\lim_{x\to 2}\frac{x^2+2x+4}{\sqrt{x^3+1}+3}=\frac{12}{6}=2$$

ដូចនេះ $\lim_{x\to 2}\frac{\sqrt{x^3+1}-3}{x-2}=2$ ។

ឈ. $\lim_{x\to 1}\frac{\sqrt{2x^2+3x+4}-\sqrt{x^2+3x+5}}{x-1}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to1}\frac{2x^2+3x+4-x^2-3x-5}{(x-1)\left(\sqrt{2x^2+3x+4}+\sqrt{x^2+3x+5}\right)}$$

$$=\lim_{x\to1}\frac{(x-1)(x+1)}{(x-1)\left(\sqrt{2x^2+3x+4}+\sqrt{x^2+3x+5}\right)}$$

$$=\lim_{x\to1}\frac{x+1}{\sqrt{2x^2+3x+4}+\sqrt{x^2+3x+5}}=\frac{2}{6}=\frac{1}{3}$$

ដូចនេះ. $\lim_{x\to1}\frac{\sqrt{2x^2+3x+4}-\sqrt{x^2+3x+5}}{x-1}=\frac{1}{3}$ ។

ញ. $\lim_{x\to3}\frac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to3}\frac{\left(x^2-3x-2x+6\right)\left(\sqrt{2x+3}+\sqrt{x+6}\right)}{2x+3-x-6}$$

$$=\lim_{x\to3}\frac{(x-3)(x-2)\left(\sqrt{2x+3}+\sqrt{x+6}\right)}{x-3}$$

$$=\lim_{x\to3}\left[(x-2)\left(\sqrt{2x+3}+\sqrt{x+6}\right)\right]=3+3=6$$

ដូចនេះ $\lim_{x\to3}\frac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}=6$ ។

លំហាត់ទី១៤

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to2}\frac{\sqrt{2+\sqrt{2+x}}-2}{x-2}$

ខ. $\lim_{x\to1}\frac{x\sqrt{x}-\sqrt{3x-2}}{\sqrt{2x-1}-x}$

គ. $\lim_{x\to2}\frac{\sqrt{x^3+1}-3}{x^2-2x}$

ឃ. $\lim_{x\to1}\frac{\sqrt{2x^2-2x+1}-x}{(x-1)^2}$

ង. $\lim_{x\to2}\frac{2x+1-\sqrt{3x^2+8x-3}}{(x-2)^2}$

ច. $\lim_{x\to2}\frac{\sqrt{x^3+1}-\sqrt{6x^2-12x+9}}{(x-2)^3}$

ឆ. $\lim\limits_{x\to 3}\dfrac{x^3-27}{\sqrt{x^2-5}-2}$

ជ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}$

ញ. $\lim\limits_{x\to 3}\dfrac{x-\sqrt{2x+3}}{x-3}$

ញ. $\lim\limits_{x\to 1}\dfrac{(x-1)^2}{x-\sqrt{2x-1}}$

ដំណោះស្រាយ

គណនាលីមីត

ក. $\lim\limits_{x\to 2}\dfrac{\sqrt{2+\sqrt{2+x}}-2}{x-2}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 2}\frac{2+\sqrt{2+x}-4}{(x-2)\left(\sqrt{2+\sqrt{2+x}}+2\right)}=\lim_{x\to 2}\frac{\sqrt{2+x}-2}{(x-2)\left(\sqrt{2+\sqrt{2+x}}+2\right)}$$

$$=\lim_{x\to 2}\frac{2+x-4}{(x-2)\left(\sqrt{2+\sqrt{2+x}}+2\right)\left(\sqrt{2+x}+2\right)}$$

$$=\lim_{x\to 2}\frac{x-2}{(x-2)\left(\sqrt{2+\sqrt{2+x}}+2\right)\left(\sqrt{2+x}+2\right)}$$

$$=\lim_{x\to 2}\frac{1}{\left(\sqrt{2+\sqrt{2+x}}+2\right)\left(\sqrt{2+x}+2\right)}=\frac{1}{4\times 4}=\frac{1}{16}$$

ដូចនេះ: $\lim\limits_{x\to 2}\dfrac{\sqrt{2+\sqrt{2+x}}-2}{x-2}=\dfrac{1}{16}$ ។

ខ. $\lim\limits_{x\to 1}\dfrac{x\sqrt{x}-\sqrt{3x-2}}{\sqrt{2x-1}-x}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 1}\frac{\left(x^3-3x+2\right)\left(\sqrt{2x-1}+x\right)}{\left(2x-1-x^2\right)\left(x\sqrt{x}+\sqrt{3x-2}\right)}=\lim_{x\to 1}\frac{(x-1)^2(x+2)\left(\sqrt{2x-1}+x\right)}{-(x-1)^2\left(x\sqrt{x}+\sqrt{3x-2}\right)}$$

$$=-\lim_{x\to 1}\frac{(x+2)\left(\sqrt{2x-1}+x\right)}{x\sqrt{x}+\sqrt{3x-2}}=-\frac{3\times 2}{2}=-3\ ។$$

ដូចនេះ: $\lim\limits_{x\to1}\dfrac{x\sqrt{x}-\sqrt{3x-2}}{\sqrt{2x-1}-x}=-3$ ។

គ. $\lim\limits_{x\to2}\dfrac{\sqrt{x^3+1}-3}{x^2-2x}$

គេមាន $\sqrt{x^3+1}-3=\dfrac{(x^3+1)-3^2}{\sqrt{x^3+1}+3}=\dfrac{x^3-8}{\sqrt{x^3+1}+3}=\dfrac{(x-2)(x^2+2x+4)}{\sqrt{x^3+1}+3}$

$$\lim_{x\to2}\frac{\sqrt{x^3+1}-3}{x^2-2x}=\lim_{x\to2}\frac{(x-2)(x^2+2x+4)}{x(x-2)(\sqrt{x^3+1}+3)}=\lim_{x\to2}\frac{x^2+2x+4}{x(\sqrt{x^3+1}+3)}=\frac{12}{12}=1$$

ដូចនេះ: $\lim\limits_{x\to2}\dfrac{\sqrt{x^3+1}-3}{x^2-2x}=1$ ។

ឃ. $\lim\limits_{x\to1}\dfrac{\sqrt{2x^2-2x+1}-x}{(x-1)^2}=\lim\limits_{x\to1}\dfrac{2x^2-2x+1-x^2}{(x-1)^2(\sqrt{2x^2-2x+1}+x)}$

$$=\lim_{x\to1}\frac{(x-1)^2}{(x-1)^2(\sqrt{2x^2-2x+1}+x)}=\lim_{x\to1}\frac{1}{\sqrt{2x^2-2x+1}+x}=\frac{1}{1+1}=\frac{1}{2}$$

ដូចនេះ: $\lim\limits_{x\to1}\dfrac{\sqrt{2x^2-2x+1}-x}{(x-1)^2}=\dfrac{1}{2}$ ។

ង. $\lim\limits_{x\to2}\dfrac{2x+1-\sqrt{3x^2+8x-3}}{(x-2)^2}=\lim\limits_{x\to2}\dfrac{(2x+1)^2-(3x^2+8x-3)}{(x-2)^2(2x+1+\sqrt{3x^2+8x-3})}$

$$=\lim_{x\to2}\frac{4x^2+4x+1-3x^2-8x+3}{(x-2)^2(2x+1+\sqrt{3x^2+8x+3})}$$

$$=\lim_{x\to2}\frac{(x-2)^2}{(x-2)^2(2x+1+\sqrt{3x^2+8x+3})}$$

$$=\lim_{x\to2}\frac{1}{2x+1+\sqrt{3x^2+8x-3}}=\frac{1}{10}$$

ដូចនេះ: $\lim\limits_{x\to2}\dfrac{2x+1-\sqrt{3x^2+8x-3}}{(x-2)^2}=\dfrac{1}{10}$ ។

ប. $\lim_{x\to 2}\frac{\sqrt{x^3+1}-\sqrt{6x^2-12x+9}}{(x-2)^3}=\lim_{x\to 2}\frac{(x^3+1)-(6x^2-12x+9)}{(x-2)^3(\sqrt{x^3+1}+\sqrt{6x^2-12x+9})}$

$$=\lim_{x\to 2}\frac{(x-2)^3}{(x-2)^3(\sqrt{x^3+1}+\sqrt{6x^2-12x+9})}$$

$$=\lim_{x\to 2}\frac{1}{(\sqrt{x^3+1}+\sqrt{6x^2-12x+9})}=\frac{1}{6}$$

ដូចនេះ $\lim_{x\to 2}\frac{\sqrt{x^3+1}-\sqrt{6x^2-12x+9}}{(x-2)^3}=\frac{1}{6}$ ។

ផ. $\lim_{x\to 3}\frac{x^3-27}{\sqrt{x^2-5}-2}$

$$=\lim_{x\to 3}\frac{(x^3-27)(\sqrt{x^2-5}+2)}{x^2-5-4}=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)(\sqrt{x^2-5}+2)}{(x-3)(x+3)}$$

$$=\lim_{x\to 3}\frac{(x^2+3x+9)(\sqrt{x^2-5}+2)}{x+3}=\frac{27\times 4}{6}=18$$

ដូចនេះ $\lim_{x\to 3}\frac{x^3-27}{\sqrt{x^2-5}-2}=18$ ។

ព. $\lim_{x\to 2}\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}$

$$=\lim_{x\to 2}\frac{(x^2-3-1)(\sqrt{x+2}+2)}{(\sqrt{x^2-3}+1)(x+2-4)}=\lim_{x\to 2}\frac{(x-2)(x+2)(\sqrt{x+2}+2)}{(\sqrt{x^2-3}+1)(x-2)}$$

$$=\lim_{x\to 2}\frac{(x+2)(\sqrt{x+2}+2)}{\sqrt{x^2-3x}+1}=\frac{4\times 4}{2}=8$$

ដូចនេះ $\lim_{x\to 2}\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}=8$ ។

ភ. $\lim_{x\to 3}\frac{x-\sqrt{2x+3}}{x-3}$

$$= \lim_{x\to3}\frac{x^2-(2x+3)}{(x-3)(x+\sqrt{2x+3})} = \lim_{x\to3}\frac{x^2-2x-3}{(x-3)(x+\sqrt{2x+3})}$$

$$= \lim_{x\to3}\frac{(x+1)(x-3)}{(x-3)(x+\sqrt{2x+3})} = \lim_{x\to3}\frac{x+1}{x+\sqrt{2x+3}} = \frac{4}{6} = \frac{2}{3}$$

ដូចនេះ $\lim_{x\to3}\frac{x-\sqrt{2x+3}}{x-3} = \frac{2}{3}$ ។

ញ. $\lim_{x\to1}\frac{(x-1)^2}{x-\sqrt{2x-1}} = \lim_{x\to1}\frac{(x-1)^2(x+\sqrt{2x-1})}{x^2-(2x-1)}$

$$= \lim_{x\to1}\frac{(x-1)^2(x+\sqrt{2x-1})}{x^2-2x+1} = \lim_{x\to1}\frac{(x-1)^2(x+\sqrt{2x-1})}{(x-1)^2}$$

$$= \lim_{x\to1}(x+\sqrt{2x-1}) = 1+1 = 2$$

ដូចនេះ $\lim_{x\to1}\frac{(x-1)^2}{x-\sqrt{2x-1}} = 2$ ។

លំហាត់ទី១៥

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to4}\frac{\sqrt{x^2+9}-5}{\sqrt{x}-2}$

ខ. $\lim_{x\to0}\frac{\sqrt{x^2-x+1}-\sqrt{x^2+x+1}}{x}$

គ. $\lim_{x\to2}\frac{x-2}{\sqrt{x^2+12}-4}$

ឃ. $\lim_{x\to1}\frac{(x-2)^2}{x-2\sqrt{x-1}}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to4}\frac{\sqrt{x^2+9}-5}{\sqrt{x}-2}$

$$= \lim_{x\to4}\frac{(x^2+9-25)(\sqrt{x}+2)}{(x-4)(\sqrt{x^2+9}+5)} = \lim_{x\to4}\frac{(x-4)(x+4)(\sqrt{x}+2)}{(x-4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x\to4}\frac{(x+4)(\sqrt{x}+2)}{\sqrt{x^2+9}+5} = \frac{8\times4}{10} = \frac{16}{5}$$

ដូចនេះ $\lim\limits_{x\to4}\dfrac{\sqrt{x^2+9}-5}{\sqrt{x}-2}=\dfrac{16}{5}$ ។

ខ. $\lim\limits_{x\to0}\dfrac{\sqrt{x^2-x+1}-\sqrt{x^2+x+1}}{x}$

$$=\lim_{x\to0}\frac{(x^2-x+1)-(x^2+x+1)}{x(\sqrt{x^2-x+1}+\sqrt{x^2+x+1})}=\lim_{x\to0}\frac{-2x}{x(\sqrt{x^2-x+1}+\sqrt{x^2+x+1})}$$

$$=\lim_{x\to0}\frac{-2}{\sqrt{x^2-x+1}+\sqrt{x^2+x+1}}=\frac{-2}{2}=-1$$

ដូចនេះ $\lim\limits_{x\to0}\dfrac{\sqrt{x^2-x+1}-\sqrt{x^2+x+1}}{x}=-1$ ។

គ. $\lim\limits_{x\to2}\dfrac{x-2}{\sqrt{x^2+12}-4}=\lim\limits_{x\to2}\dfrac{(x-2)(\sqrt{x^2+12}+4)}{(x-2)(x+2)}=\lim\limits_{x\to2}\dfrac{\sqrt{x^2+12}+4}{x+2}=2$ ។

ដូចនេះ $\lim\limits_{x\to0}\dfrac{\sqrt{x^2-x+1}-\sqrt{x^2+x+1}}{x}=2$ ។

ឃ. $\lim\limits_{x\to2}\dfrac{(x-2)^2}{x-2\sqrt{x-1}}=\lim\limits_{x\to2}\dfrac{(x-2)^2\left(x+2\sqrt{x-1}\right)}{x^2-4x+4}=\lim\limits_{x\to2}\left(x+2\sqrt{x-1}\right)=4$ ។

ដូចនេះ $\lim\limits_{x\to2}\dfrac{(x-2)^2}{x-2\sqrt{x-1}}=4$ ។

លំហាត់ទី១៦

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x}-2}{x-2}$

ខ. $\lim\limits_{x\to0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}$

គ. $\lim\limits_{x\to2}\dfrac{\sqrt[3]{3x+2}-\sqrt[3]{x+6}}{x-2}$

ឃ. $\lim\limits_{x\to0}\dfrac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$

ង. $\lim\limits_{x\to2}\dfrac{\sqrt[3]{x-1}-x+1}{x-2}$

ច. $\lim\limits_{x\to2}\dfrac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

ឆ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2-2}-\sqrt{2}}{x-2}$

ជ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-\sqrt{4x+1}}{(x-2)^2}$

ឈ. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+x+2}-\sqrt{5x-1}}{x^2-1}$

ញ. $\lim\limits_{x\to 3}\dfrac{\sqrt{x^2+x+4}-4}{x-3}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x}-2}{x-2}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to8}\frac{x-2}{(x-2)\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)}=\lim_{x\to8}\frac{1}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}=\frac{1}{4+4+4}=\frac{1}{12}$$

ដូចនេះ. $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x}-2}{x-2}=\dfrac{1}{12}$ ។

ខ. $\lim\limits_{x\to0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to0}\frac{1+x^2-1}{x^2\left[\sqrt[3]{\left(1+x^2\right)^2}+\sqrt[3]{1+x^2}+1\right]}=\lim_{x\to0}\frac{1}{\sqrt[3]{\left(1+x^2\right)^2}+\sqrt[3]{1+x^2}+1}=\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}=\dfrac{1}{3}$ ។

គ. $\lim\limits_{x\to2}\dfrac{\sqrt[3]{3x+2}-\sqrt[3]{x+6}}{x-2}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to2}\frac{3x+2-x-6}{(x-2)\left[\sqrt[3]{(3x+2)^2}+\sqrt[3]{(3x+2)(x+6)}+\sqrt[3]{(x+6)^2}\right]}$$

$$=\lim_{x\to2}\frac{2(x-2)}{(x-2)\left[\sqrt[3]{(3x+2)^2}+\sqrt[3]{(3x+2)(x+6)}+\sqrt[3]{(x+6)^2}\right]}$$

$$=\lim_{x\to 2}\frac{2}{\sqrt[3]{(3x+2)^2}+\sqrt[3]{(3x+2)(x+6)}+\sqrt[3]{(x+6)^2}}=\frac{2}{4+4+4}=\frac{1}{6}$$ ។

ដូចនេះ $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt[3]{x+6}}{x-2}=\dfrac{1}{6}$ ។

ឃ. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{(x^2+1)+(x^2-1)}{x^2\left[\sqrt[3]{(x^2+1)^2}-\sqrt[3]{x^2+1}\,\sqrt[3]{x^2-1}+\sqrt[3]{(x^2-1)^2}\right]}$$

$$=\lim_{x\to 0}\frac{2x^2}{x^2\left[\sqrt[3]{(x^2+1)^2}-\sqrt[3]{x^2+1}\,\sqrt[3]{x^2-1}+\sqrt[3]{(x^2-1)^2}\right]}$$

$$=\lim_{x\to 0}\frac{2}{\sqrt[3]{(x^2+1)^2}-\sqrt[3]{x^2+1}\,\sqrt[3]{x^2-1}+\sqrt[3]{(x^2-1)^2}}=\frac{2}{1+1+1}=\frac{2}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}=\dfrac{2}{3}$ ។

ង. $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x-1}-x+1}{x-2}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 2}\frac{x-1-x^3+3x^2-3x+1}{(x-2)\left[\sqrt[3]{(x-1)^2}+(x-1)\sqrt[3]{x-1}+(x-1)^2\right]}$$

$$=\lim_{x\to 2}\frac{-x^3+3x^2-2x}{(x-2)\left[\sqrt[3]{(x-1)^2}+(x-1)\sqrt[3]{x-1}+(x-1)^2\right]}$$

$$=\lim_{x\to 2}\frac{-x(x-1)(x-2)}{(x-2)\left[\sqrt[3]{(x-1)^2}+(x-1)\sqrt[3]{x-1}+(x-1)^2\right]}$$

$$=\lim_{x\to 2}\frac{-x(x-1)}{\sqrt[3]{(x-1)^2}+(x-1)\sqrt[3]{x-1}+(x-1)^2}=\frac{-2}{1+1+1}=-\frac{2}{3}$$

ដូចនេះ: $\lim\limits_{x\to2}\dfrac{\sqrt[3]{x-1}-x+1}{x-2}=-\dfrac{2}{3}$ ។

ច. $\lim\limits_{x\to2}\dfrac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to2}\frac{\left(x^6-x^2-60\right)\left(x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{\left(x^2+60\right)^2}\right)}{\left(x^6-x^2-60\right)\left(x^3+\sqrt{x^2+60}\right)}$$

$$=\lim_{x\to2}\frac{x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{\left(x^2+60\right)^2}}{x^3+\sqrt{x^2+60}}=\frac{16+16+16}{16}=3$$

ដូច្នេះ: $\lim\limits_{x\to2}\dfrac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}=3$ ។

ឆ. $$\lim_{x\to2}\frac{\sqrt{x^2-2}-\sqrt{2}}{x-2}=\lim_{x\to2}\frac{(\sqrt{x^2-2})^2-\sqrt{2}^2}{(x-2)(\sqrt{x^2-2}+\sqrt{2})}$$

$$=\lim_{x\to2}\frac{(x-2)(x+2)}{(x-2)(\sqrt{x^2-2}+\sqrt{2})}=\lim_{x\to2}\frac{x+2}{\sqrt{x^2-2}+\sqrt{2}}=\frac{4}{2\sqrt{2}}=\sqrt{2}$$

ជ. $$\lim_{x\to2}\frac{\sqrt{x^2+5}-\sqrt{4x+1}}{(x-2)^2}=\lim_{x\to2}\frac{(x^2+5)-(4x+1)}{(x-2)^2(\sqrt{x^2+5}+\sqrt{4x+1})}$$

$$=\lim_{x\to2}\frac{(x-2)^2}{(x-2)^2(\sqrt{x^2+5}+\sqrt{4x+1})}=\lim_{x\to2}\frac{1}{\sqrt{x^2+5}+\sqrt{4x+1}}=\frac{1}{3+3}=\frac{1}{6}$$

ឈ. $$\lim_{x\to1}\frac{\sqrt{x^2+x+2}-\sqrt{5x-1}}{x^2-1}=\lim_{x\to1}\frac{(x^2+x+2)-(5x-1)}{(x^2-1)(\sqrt{x^2+x+2}+\sqrt{5x-1})}$$

$$=\lim_{x\to1}\frac{(x-1)(x-3)}{(x-1)(x+1)(\sqrt{x^2+x+2}+\sqrt{5x-1})}$$

$$= \lim_{x\to 1}\frac{x-3}{(x+1)(\sqrt{x^2+x+2}+\sqrt{5x-1})} = -\frac{1}{2+2} = -\frac{1}{4}$$ ។

ដូចនេះ: $\lim_{x\to 1}\frac{\sqrt{x^2+x+2}-\sqrt{5x-1}}{x^2-1} = -\frac{1}{4}$ ។

ញ. $\lim_{x\to 3}\frac{\sqrt{x^2+x+4}-4}{x-3} = \lim_{x\to 3}\frac{(x^2+x+4)-16}{(x-3)(\sqrt{x^2+x+4}+4)}$

$$= \lim_{x\to 3}\frac{(x-3)(x+4)}{(x-3)(\sqrt{x^2+x+4}+4)} = \lim_{x\to 3}\frac{x+4}{\sqrt{x^2+x+4}+4} = \frac{3+4}{4+4} = \frac{7}{8}$$

ដូចនេះ: $\lim_{x\to 3}\frac{\sqrt{x^2+x+4}-4}{x-3} = \frac{7}{8}$ ។

លំហាត់ទី១៧

គណនាលីមីតខាងក្រោម ៖

ក. $\lim_{x\to3}\frac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}$

ខ. $\lim_{x\to2}\frac{\sqrt{x^2-2}-\sqrt{4x-6}}{(\sqrt{x}-\sqrt{2})^2}$

គ. $\lim_{x\to1}\frac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}$

ឃ. $\lim_{x\to1}\frac{\sqrt[3]{2x^3+6x}-x-1}{(x-1)^3}$

ង. $\lim_{x\to8}\frac{\sqrt[3]{x}-2}{x-8}$

ច. $\lim_{x\to2}\frac{x-1-\sqrt[3]{3x^2-9x+7}}{(x-2)^3}$

ឆ. $\lim_{x\to0}\frac{\sqrt[3]{1+x^2}-1}{x^2}$

ជ. $\lim_{x\to2}\frac{x-\sqrt[3]{x^2+4}}{x-2}$

ឈ. $\lim_{x\to1}\frac{\sqrt[3]{x^3-2x^2}-\sqrt[3]{x^2-3x+1}}{(x-1)^3}$

ញ. $\lim_{x\to2}\frac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

ដំណោះស្រាយ

គណនាលីមីត

ក. $\lim_{x\to 3}\dfrac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}$

$=\lim_{x\to 3}\dfrac{(x^2-5x+6)(\sqrt{2x+3}+\sqrt{x+6})}{(2x+3)-(x+6)}=\lim_{x\to 3}\dfrac{(x-2)(x-3)(\sqrt{2x+3}+\sqrt{x+6})}{x-3}$

$=\lim_{x\to 3}\left[(x-2)(\sqrt{2x+3}+\sqrt{x+6})\right]=6$

ដូចនេះ $\lim_{x\to 3}\dfrac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}=6$ ។

ខ. $\lim_{x\to 2}\dfrac{\sqrt{x^2-2}-\sqrt{4x-6}}{(\sqrt{x}-\sqrt{2})^2}$

$=\lim_{x\to 2}\dfrac{(x^2-2-4x+6)(\sqrt{x}+\sqrt{2})^2}{(x-2)^2(\sqrt{x^2-2}+\sqrt{4x-6})}=\lim_{x\to 2}\dfrac{(x-2)^2(\sqrt{x}+\sqrt{2})^2}{(x-2)^2(\sqrt{x^2-2}+\sqrt{4x-6})}$

$=\lim_{x\to 2}\dfrac{(\sqrt{x}+\sqrt{2})^2}{\sqrt{x^2-2}+\sqrt{4x-6}}=\dfrac{(2\sqrt{2})^2}{2\sqrt{2}}=2\sqrt{2}$

ដូចនេះ $\lim_{x\to 2}\dfrac{\sqrt{x^2-2}-\sqrt{4x-6}}{(\sqrt{x}-\sqrt{2})^2}=2\sqrt{2}$ ។

គ. $\lim_{x\to 1}\dfrac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}$

$=\lim_{x\to 1}\dfrac{\left(x^2+3x-3x-1\right)}{x-1}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x^2+3x}+\sqrt{3x+1}}$

$=\lim_{x\to 1}\dfrac{(x-1)(x+1)}{x-1}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x^2+3x}+\sqrt{3x+1}}=\lim_{x\to 1}\dfrac{(x+1)(\sqrt{x}+1)}{\sqrt{x^2+3x}+\sqrt{3x+1}}=\dfrac{2.2}{2+2}=1$

ដូចនេះ $\lim_{x\to 1}\dfrac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}=1$ ។

ឃ. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x^3+6x}-x-1}{(x-1)^3}$

$=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x^3+6x}-(x+1)}{(x-1)^3}$

$=\lim\limits_{x\to 1}\dfrac{(2x^3+6x)-(x+1)^3}{(x-1)^3\left[\sqrt[3]{(2x^3+6x)^2}+(x+1)\sqrt[3]{2x^3+6x}+(x+1)^2\right]}$

$=\lim\limits_{x\to 1}\dfrac{2x^3+6x-x^3-3x^2-3x-1}{(x-1)^3\left[\sqrt[3]{(2x^3+6x)^2}+(x+1)\sqrt[3]{2x^3+6x}+(x+1)^2\right]}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)^3}{(x-1)^3\left[\sqrt[3]{(2x^3+6x)^2}+(x+1)\sqrt[3]{2x^3+6x}+(x+1)^2\right]}$

$=\lim\limits_{x\to 1}\dfrac{1}{\sqrt[3]{(2x^3+6x)^2}+(x+1)\sqrt[3]{2x^3+6x}+(x+1)^2}=\dfrac{1}{12}$

ដូចនេះ $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x^3+6x}-x-1}{(x-1)^3}=\dfrac{1}{12}$ ។

ង. $\lim\limits_{x\to 8}\dfrac{\sqrt[3]{x}-2}{x-8}$ តាមរូបមន្ត $(a-b)(a^2+ab+b^2)=a^3-b^3$

$=\lim\limits_{x\to 2}\dfrac{x-8}{(x-8)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}=\lim\limits_{x\to 2}\dfrac{1}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}=\dfrac{1}{4+4+4}=\dfrac{1}{12}$

ដូចនេះ $\lim\limits_{x\to 8}\dfrac{\sqrt[3]{x}-2}{x-8}=\dfrac{1}{12}$ ។

ច. $\lim\limits_{x\to 2}\dfrac{x-1-\sqrt[3]{3x^2-9x+7}}{(x-2)^3}$

$=\lim\limits_{x\to 2}\dfrac{(x-1)^3-(3x^2-9x+7)}{(x-2)^3\left[(x-1)^2+(x-1)\sqrt[3]{3x^2-9x+7}+\sqrt[3]{(3x^2-9x+7)^2}\right]}$

$=\lim\limits_{x\to 2}\dfrac{(x-2)^3}{(x-2)^3\left[(x-1)^2+(x-1)\sqrt[3]{3x^2-9x+7}+\sqrt[3]{(3x^2-9x+7)^2}\right]}$

$$=\lim_{x\to 2}\frac{1}{\left[(x-1)^2+(x-1)\sqrt[3]{3x^2-9x+7}+\sqrt[3]{(3x^2-9x+7)^2}\right]}=\frac{1}{3}$$

ដូចនេះ $\lim_{x\to 2}\frac{x-1-\sqrt[3]{3x^2-9x+7}}{(x-2)^3}=\frac{1}{3}$ ។

ឆ. $\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}$

$$=\lim_{x\to 0}\frac{(1+x^2)-1}{x^2\left(\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1\right)}=\lim_{x\to 0}\frac{1}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1}=\frac{1}{3}$$

ដូចនេះ $\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}=\frac{1}{3}$ ។

ជ. $\lim_{x\to 2}\frac{x-\sqrt[3]{x^2+4}}{x-2}$

$$=\lim_{x\to 2}\frac{x^3-(x^2+4)}{(x-2)(x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2})}=\lim_{x\to 2}\frac{x^3-x^2-4}{(x-2)(x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2})}$$

$$=\lim_{x\to 2}\frac{(x^3-2x^2)+(x^2-4)}{(x-2)(x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2})}=\lim_{x\to 2}\frac{(x-2)(x^2+x+2)}{(x-2)(x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2})}$$

$$=\lim_{x\to 2}\frac{x^2+x+2}{x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2}}=\frac{8}{12}=\frac{2}{3}$$

ដូចនេះ $\lim_{x\to 2}\frac{x-\sqrt[3]{x^2+4}}{x-2}=\frac{2}{3}$ ។

ឈ. $\lim_{x\to 1}\frac{\sqrt[3]{x^3-2x^2}-\sqrt[3]{x^2-3x+1}}{(x-1)^3}$

$$=\lim_{x\to 1}\frac{(x^3-2x^2)-(x^2-3x+1)}{(x-1)^3\left[\sqrt[3]{(x^3-2x^2)^2}+\sqrt[3]{(x^3-2x^2)(x^2-3x+1)}+\sqrt[3]{(x^2-3x+1)^2}\right]}$$

$$=\lim_{x\to 1}\frac{(x-1)^3}{(x-1)^3\left[\sqrt[3]{(x^3-2x^2)^2}+\sqrt[3]{(x^3-2x^2)(x^2-3x+1)}+\sqrt[3]{(x^2-3x+1)^2}\right]}$$

$$=\lim_{x\to1}\frac{1}{\left[\sqrt[3]{(x^3-2x^2)^2}+\sqrt[3]{(x^3-2x^2)(x^2-3x+1)}+\sqrt[3]{(x^2-3x+1)^2}\right]}=\frac{1}{3}$$

ដូចនេះ $\lim_{x\to1}\frac{\sqrt[3]{x^3-2x^2}-\sqrt[3]{x^2-3x+1}}{(x-1)^3}=\frac{1}{3}$ ។

ញ. $\lim_{x\to2}\frac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

$$=\lim_{x\to2}\frac{x^6-x^2-60}{x^6-x^2-60}\cdot\frac{x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{(x^2+60)^2}}{x^3+\sqrt{x^2+60}}$$

$$=\lim_{x\to2}\frac{x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{(x^2+60)^2}}{x^3+\sqrt{x^2+60}}=\frac{16+16+16}{8+8}=\frac{48}{16}=3$$

ដូចនេះ $\lim_{x\to2}\frac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}=3$ ។

លំហាត់ទី១៨

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to1}\frac{\sqrt{x^2+3}-2}{x^3-1}$

ខ. $\lim_{x\to2}\frac{x\sqrt{x}-2\sqrt{2}}{x-2}$

គ. $\lim_{x\to8}\frac{\sqrt[3]{x^2}-4}{x-8}$

ឃ. $\lim_{x\to a}\frac{\sqrt[n]{x}-\sqrt[n]{a}}{x-a}$

ង. $\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

ច. $\lim_{x\to0}\frac{1+x-\sqrt[2006]{1+2006x}}{x^2}$

ឆ. $\lim_{x\to0}\frac{1-\sqrt{(1+x)(1+2x)(1+3x)...(1+nx)}}{x}$

ជ. $\lim_{x\to2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$

ឈ. $\lim_{x\to1}\frac{\sqrt{2x^2+2x}-\sqrt{2x+2}}{x^3-1}$

ញ. $\lim_{x\to2}\frac{\sqrt{x+2}+\sqrt[3]{x-1}-3}{x-2}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to1}\dfrac{\sqrt{x^2+3}-2}{x^3-1}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to1}\frac{(x^2+3)-4}{(x^3-1)\left(\sqrt{x^2+3}+2\right)}=\lim_{x\to1}\frac{(x-1)(x+1)}{(x-1)(x^2+x+1)\left(\sqrt{x^2+3}+2\right)}$$

$$=\lim_{x\to1}\frac{x+1}{(x^2+x+1)\left(\sqrt{x^2+3}+2\right)}=\frac{2}{3\times4}=\frac{1}{6}$$

ដូចនេះ: $\lim\limits_{x\to1}\dfrac{\sqrt{x^2+3}-2}{x^3-1}=\dfrac{1}{6}$ ។

ខ. $\lim\limits_{x\to2}\dfrac{x\sqrt{x}-2\sqrt{2}}{x-2}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to2}\frac{\left(\sqrt{x}\right)^3-\left(\sqrt{2}\right)^3}{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}=\lim_{x\to2}\frac{\left(\sqrt{x}-\sqrt{2}\right)\left(x+\sqrt{x}\sqrt{2}+2\right)}{\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right)}$$

$$=\lim_{x\to2}\frac{x+\sqrt{2x}+2}{\sqrt{x}+\sqrt{2}}=\frac{2+2+2}{2\sqrt{2}}=\frac{3\sqrt{2}}{2}$$

ដូចនេះ: $\lim\limits_{x\to2}\dfrac{x\sqrt{x}-2\sqrt{2}}{x-2}=\dfrac{3\sqrt{2}}{2}$ ។

គ. $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x^2}-4}{x-8}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to8}\frac{\left(\sqrt[3]{x}-2\right)\left(\sqrt[3]{x}+2\right)}{x-8}=\lim_{x\to8}\frac{(x-8)\left(\sqrt[3]{x}+2\right)}{(x-8)\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)}$$

$$=\lim_{x\to8}\frac{\sqrt[3]{x}+2}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}=\frac{2+2}{4+4+4}=\frac{4}{12}=\frac{1}{3}$$

ដូចនេះ: $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x^2}-4}{x-8}=\dfrac{1}{3}$ ។

ឃ. $\lim_{x\to a}\frac{\sqrt[n]{x}-\sqrt[n]{a}}{x-a}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to a}\frac{x-a}{(x-a)\left(\sqrt[n]{x^{n-1}}+\sqrt[n]{x^{n-2}}\sqrt[n]{a}+\ldots+\sqrt[n]{a^{n-1}}\right)}$$

$$=\lim_{x\to a}\frac{1}{\sqrt[n]{x^{n-1}}+\sqrt[n]{x^{n-2}}\sqrt[n]{a}+\ldots+\sqrt[n]{a^{n-1}}}=\frac{1}{n\sqrt[n]{a^{n-1}}}$$

ដូចនេះ $\lim_{x\to a}\frac{\sqrt[n]{x}-\sqrt[n]{a}}{x-a}=\frac{1}{n\sqrt[n]{a^{n-1}}}$ ។

ង. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to 0}\frac{(1+x)-(1-x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}=\lim_{x\to 0}\frac{2x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}=\lim_{x\to 0}\frac{2}{\sqrt{1+x}+\sqrt{1-x}}=\frac{2}{2}=1$$

ដូចនេះ $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=1$ ។

ច. $\lim_{x\to 0}\frac{1+x-\sqrt[2006]{1+2006x}}{x^2}$ រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to 0}\frac{(1+x)^{2006}-(1+2006x)}{x^2\left[(1+x)^{2005}+\ldots+\sqrt[2006]{(1+2006x)^{2005}}\right]}$$

$$=\lim_{x\to 0}\frac{C_{2006}^{2}+C_{2006}^{3}x+\ldots+C_{2006}^{2006}x^{2005}}{(1+x)^{2005}+\ldots+\sqrt[2006]{(1+2006x)^{2005}}}=1003$$

ដូចនេះ $\lim_{x\to 0}\frac{1+x-\sqrt[2006]{1+2006x}}{x^2}=1003$ ។

ឆ. $\lim_{x\to 0}\frac{1-\sqrt{(1+x)(1+2x)(1+3x)\ldots(1+nx)}}{x}$ រាងមិនកំណត់ $\frac{0}{0}$

$$= \lim_{x\to 0} \frac{1-(1+x)(1+2x)(1+3x)\dots(1+nx)}{x\left[1+\sqrt{(1+x)(1+2x)(1+3x)\dots(1+nx)}\right]}$$

$$= \lim_{x\to 0} \frac{1-\left[1+\frac{n(n+1)}{2}x+\dots+n!x^n\right]}{x\left[1+\sqrt{(1+x)(1+2x)(1+3x)\dots(1+nx)}\right]}$$

$$= \lim_{x\to 0} \frac{-\frac{n(n+1)}{2}-\dots-n!x^n}{1+\sqrt{(1+x)(1+2x)(1+3x)\dots(1+nx)}} = -\frac{n(n+1)}{4}$$

ដូចនេះ $\lim_{x\to 0} \frac{1-\sqrt{(1+x)(1+2x)(1+3x)\dots(1+nx)}}{x} = -\frac{n(n+1)}{4}$ ។

ឆ. $\lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$ រាងមិនកំណត់ $\frac{0}{0}$

$$= \lim_{x\to 2} \frac{(2x+5)-(x+7)}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)} = \lim_{x\to 2} \frac{x-2}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$= \lim_{x\to 2} \frac{1}{\sqrt{2x+5}+\sqrt{x+7}} = \frac{1}{6}$$

ដូចនេះ $\lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} = \frac{1}{6}$ ។

ឈ. $\lim_{x\to 1} \frac{\sqrt{2x^2+2x}-\sqrt{2x+2}}{x^3-1}$ រាងមិនកំណត់ $\frac{0}{0}$

$$= \lim_{x\to 1} \frac{2x^2+2x-2x-2}{\left(x^3-1\right)} = \lim_{x\to 1} \frac{2(x-1)(x+1)}{(x-1)\left(x^2+x+1\right)\left(\sqrt{2x^2+2x}+\sqrt{2x+2}\right)}$$

$$= \lim_{x\to 1} \frac{2(x+1)}{\left(x^2+x+1\right)\left(\sqrt{2x^2+2x}+\sqrt{2x+2}\right)} = \frac{4}{3\times 4} = \frac{1}{3}$$

ដូចនេះ $\lim_{x\to 1} \frac{\sqrt{2x^2+2x}-\sqrt{2x+2}}{x^3-1} = \frac{1}{3}$ ។

ញ. $\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}+\sqrt[3]{x-1}-3}{x-2}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to 2}\frac{\left(\sqrt{x+2}-2\right)+\left(\sqrt[3]{x-1}-1\right)}{x-2}=\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2}+\lim_{x\to 2}\frac{\sqrt[3]{x-1}-1}{x-2}$$

$$=\lim_{x\to 2}\frac{x-2}{(x-2)\left(\sqrt{x+2}+2\right)}+\lim_{x\to 2}\frac{x-2}{(x-2)\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1\right]}$$

$$=\lim_{x\to 2}\frac{1}{\sqrt{x+2}+2}+\lim_{x\to 2}\frac{1}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1}=\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$$

ដូចនេះ $\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}+\sqrt[3]{x-1}-3}{x-2}=\dfrac{7}{12}$ ។

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កំណត់ចំណាំ!!!

រូបមន្តលីមីតនៃអនុគមន៍ត្រីកោណមាត្រសំខាន់ៗ

$$a)\lim_{u\to 0}\frac{\sin u}{u}=\lim_{u\to 0}\frac{u}{\sin u}=1$$

$$b)\lim_{u\to 0}\frac{\tan u}{u}=\lim_{u\to 0}\frac{u}{\tan u}=1$$

$$c)\lim_{u\to 0}\frac{1-\cos u}{u}=1$$

$$d)\lim_{u\to 0}\frac{\sin\lambda u}{u}=\lim_{u\to 0}\frac{\sin\lambda u}{\lambda u}\times\lambda=\lambda$$

$$e)\lim_{u\to 0}\frac{\tan\lambda u}{u}=\lim_{u\to 0}\frac{\tan\lambda u}{\lambda u}\times\lambda=\lambda\quad,\ (\ \lambda\neq 0\ )$$

លំហាត់ទី១៩

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 6x}$

ខ. $\lim\limits_{x\to 0}\dfrac{\sin^2 3x}{6x^2}$

គ. $\lim\limits_{x\to 0}\dfrac{\sin^3 2x}{x^2\sin 6x}$

ឃ. $\lim\limits_{x\to 0}\dfrac{2x+\sin 4x}{3x}$

ង. $\lim\limits_{x\to 0}\dfrac{x-\sin 3x}{x+\sin 3x}$

ច. $\lim\limits_{x\to 0}\dfrac{5x^2+\sin^2 2x}{x\sin 3x}$

ឆ. $\lim\limits_{x\to 0}\dfrac{x^2\sin 4x}{x^3+\sin^3 x}$

ជ. $\lim\limits_{x\to 0}\dfrac{3x\sin 4x}{\sin^2 6x}$

ឈ. $\lim\limits_{x\to 0}\dfrac{\sin 2x\sin 6x}{\sin^2 3x}$

ញ. $\lim\limits_{x\to 0}\dfrac{x^2\sin 2x}{\sin^3 x}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 6x}=\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}\times\dfrac{x}{\sin 6x}=\lim\limits_{x\to 0}\left(\dfrac{\sin 2x}{2x}\times 2\right)\lim\limits_{x\to 0}\left(\dfrac{6x}{\sin 6x}\times\dfrac{1}{6}\right)$

$=2\times\dfrac{1}{6}=\dfrac{1}{3}$ ។

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 6x}=\dfrac{1}{3}$ ។

ខ. $\lim\limits_{x\to 0}\dfrac{\sin^2 3x}{6x^2}=\dfrac{1}{6}\lim\limits_{x\to 0}\left(\dfrac{\sin 3x}{x}\right)^2=\dfrac{1}{6}\lim\limits_{x\to 0}\left(\dfrac{\sin 3x}{3x}\times 3\right)^2=\dfrac{1}{6}\times 3^2=\dfrac{3}{2}$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\sin^2 3x}{6x^2}=\dfrac{3}{2}$ ។

គ. $\lim_{x\to 0}\frac{\sin^3 2x}{x^2\sin 6x}=\lim_{x\to 0}\frac{\sin^3 2x}{x^3}\times\frac{x}{\sin 6x}$

$=\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\times 2\right)^3\lim_{x\to 0}\left(\frac{6x}{\sin 6x}\times\frac{1}{6}\right)=\frac{2^3}{6}=\frac{4}{3}$

ដូចនេះ: $\lim_{x\to 0}\frac{\sin^3 2x}{x^2\sin 6x}=\frac{4}{3}$ ។

ឃ. $\lim_{x\to 0}\frac{2x+\sin 4x}{3x}=\lim_{x\to 0}\left(\frac{2x}{3x}+\frac{\sin 4x}{3x}\right)=\lim_{x\to 0}\left(\frac{2}{3}+\frac{\sin 4x}{4x}\times\frac{4}{3}\right)=\frac{2}{3}+\frac{4}{3}=2$ ។

ដូចនេះ: $\lim_{x\to 0}\frac{2x+\sin 4x}{3x}=2$ ។

ង. $\lim_{x\to 0}\frac{x-\sin 3x}{x+\sin 3x}=\lim_{x\to 0}\frac{x(1-\frac{\sin 3x}{x})}{x(1+\frac{\sin 3x}{x})}=\lim_{x\to 0}\frac{1-\frac{\sin 3x}{3x}\times 3}{1+\frac{\sin 3x}{3x}\times 3}=\frac{1-3}{1+3}=-\frac{1}{2}$ ។

ដូចនេះ: $\lim_{x\to 0}\frac{x-\sin 3x}{x+\sin 3x}=-\frac{1}{2}$ ។

ច. $\lim_{x\to 0}\frac{5x^2+\sin^2 2x}{x\sin 3x}=\lim_{x\to 0}\frac{\frac{5x^2+\sin^2 2x}{x^2}}{\frac{x\sin 3x}{x^2}}=\lim_{x\to 0}\frac{5+\frac{\sin^2 2x}{(2x)^2}\times 4}{\frac{\sin 3x}{3x}\times 3}=\frac{5+4}{3}=3$

ដូចនេះ: $\lim_{x\to 0}\frac{5x^2+\sin^2 2x}{x\sin 3x}=3$ ។

ឆ. $\lim_{x\to 0}\frac{x^2\sin 4x}{x^3+\sin^3 x}=\lim_{x\to 0}\frac{\frac{x^2\sin 4x}{x^3}}{\frac{x^3+\sin^3 x}{x^3}}=\lim_{x\to 0}\frac{\frac{\sin 4x}{4x}\times 4}{1+\left(\frac{\sin x}{x}\right)^3}=\frac{4}{1+1}=2$ ។

ដូចនេះ: $\lim_{x\to 0}\frac{x^2\sin 4x}{x^3+\sin^3 x}=2$ ។

ជ. $\lim_{x\to 0}\frac{3x\sin 4x}{\sin^2 6x}=3\lim_{x\to 0}\frac{x\sin 4x}{x^2}\times\frac{x^2}{\sin^2 6x}=3\lim_{x\to 0}\frac{\sin 4x}{4x}\times 4\ \lim_{x\to 0}\frac{(6x)^2}{\sin^2 6x}\times\frac{1}{36}$

$$=\frac{12}{36}=\frac{1}{3}$$ ។

ដូចនេះ $\lim_{x\to 0}\frac{3x\sin 4x}{\sin^2 6x}=\frac{1}{3}$ ។

ឈ. $\lim_{x\to 0}\frac{\sin 2x\sin 6x}{\sin^2 3x}=\lim_{x\to 0}\frac{\frac{\sin 2x\sin 6x}{x^2}}{\frac{\sin^2 3x}{x^2}}$

$$=\lim_{x\to 0}\frac{\frac{\sin 2x}{2x}\times 2\times\frac{\sin 6x}{6x}\times 6}{\left(\frac{\sin 3x}{3x}\times 3\right)^2}=\frac{2\times 6}{3^2}=\frac{4}{3}$$ ។

ដូចនេះ $\lim_{x\to 0}\frac{\sin 2x\sin 6x}{\sin^2 3x}=\frac{4}{3}$ ។

ញ. $\lim_{x\to 0}\frac{x^2\sin 2x}{\sin^3 x}=\lim_{x\to 0}\frac{x^3}{\sin^3 x}\times\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2=2$ ។

ដូចនេះ $\lim_{x\to 0}\frac{x^2\sin 2x}{\sin^3 x}=2$ ។

លំហាត់ទី២០

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\frac{1-\cos 2x}{4x^2}$

ខ) $\lim_{x\to 0}\frac{\sin^2 6x}{3x\sin 4x}$

គ) $\lim_{x\to 0}\frac{\sin 3x\sin 4x}{6x^2}$

ឃ) $\lim_{x\to 0}\frac{2\sin 3x-\sin 4x}{x}$

ង) $\lim_{x\to 0}\frac{1-\cos^4 x}{4x^2}$

ច) $\lim_{x\to 0}\frac{2x}{x+\sin 3x}$

ឆ) $\lim\limits_{x\to 0}\dfrac{x\sin 2x+\sin^2 4x}{9x^2}$

ជ) $\lim\limits_{x\to 0}\dfrac{1+2x\sin 4x-\cos^2 2x}{6x^2}$

ឈ) $\lim\limits_{x\to 0}\dfrac{x\sin^3 2x}{(1-\cos 2x)^2}$

ញ) $\lim\limits_{x\to 0}\dfrac{1+x\sin 3x-\cos 6x}{3x^2+\sin^2 2x}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក) $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{4x^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2 x}{4x^2}=\dfrac{1}{2}\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^2=\dfrac{1}{2}$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{4x^2}=\dfrac{1}{2}$ ។

ខ) $\lim\limits_{x\to 0}\dfrac{\sin^2 6x}{3x\sin 4x}=\dfrac{1}{3}\lim\limits_{x\to 0}\dfrac{\dfrac{\sin^2 6x}{x^2}}{\dfrac{x\sin 4x}{x^2}}=\dfrac{1}{3}\lim\limits_{x\to 0}\dfrac{\left(\dfrac{\sin 6x}{6x}\times 6\right)^2}{\dfrac{\sin 4x}{4x}\times 4}=\dfrac{36}{12}=3$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{\sin^2 6x}{3x\sin 4x}=3$ ។

គ) $\lim\limits_{x\to 0}\dfrac{\sin 3x\sin 4x}{6x^2}=\dfrac{1}{6}\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}\dfrac{\sin 4x}{x}$

$=\dfrac{1}{6}\lim\limits_{x\to 0}\left(\dfrac{\sin 3x}{3x}\times 3\right)\lim\limits_{x\to 0}\left(\dfrac{\sin 4x}{4x}\times 4\right)=\dfrac{12}{6}=2$ ។

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{\sin 3x\sin 4x}{6x^2}=2$ ។

ឃ) $\lim\limits_{x\to 0}\dfrac{2\sin 3x-\sin 4x}{x}=\lim\limits_{x\to 0}\left(\dfrac{2\sin 3x}{x}-\dfrac{\sin 4x}{x}\right)$

$=\lim\limits_{x\to 0}\left(6\times\dfrac{\sin 3x}{3x}-4\times\dfrac{\sin 4x}{4x}\right)=2$ ។

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{2\sin 3x-\sin 4x}{x}=2$ ។

ង) $\lim_{x\to0}\frac{1-\cos^4 x}{4x^2}=\frac{1}{4}\lim_{x\to0}\frac{(1-\cos^2 x)(1+\cos^2 x)}{x^2}$

$$=\frac{1}{4}\lim_{x\to0}\left[\left(\frac{\sin x}{x}\right)^2\left(1+\cos^2 x\right)\right]=\frac{1}{2}$$ ។

ដូចនេះ: $\lim_{x\to0}\frac{1-\cos^4 x}{4x^2}=\frac{1}{2}$ ។

ប) $\lim_{x\to0}\frac{2x}{x+\sin 3x}=\lim_{x\to0}\frac{2x}{x\left(1+\frac{\sin 3x}{x}\right)}=\lim_{x\to0}\frac{2}{1+\frac{\sin 3x}{3x}\times3}=\frac{2}{1+3}=\frac{1}{2}$ ។

ផ) $\lim_{x\to0}\frac{x\sin 2x+\sin^2 4x}{9x^2}=\frac{1}{9}\lim_{x\to0}\left(\frac{x\sin 2x}{x^2}+\frac{\sin^2 4x}{x^2}\right)$

$$=\frac{1}{9}\lim_{x\to0}\left[\frac{\sin 2x}{2x}\times2+\left(\frac{\sin 4x}{4x}\times4\right)^2\right]=\frac{1}{9}(2+16)=2$$

ដូចនេះ: $\lim_{x\to0}\frac{x\sin 2x+\sin^2 4x}{9x^2}=2$ ។

ព) $\lim_{x\to0}\frac{1+2x\sin 4x-\cos^2 2x}{6x^2}=\lim_{x\to0}\frac{\sin^2 2x+2x\sin 4x}{6x^2}$

$$=\frac{1}{6}\lim_{x\to0}\left[\left(\frac{\sin 2x}{2x}\times2\right)^2+8\times\frac{\sin 4x}{4x}\right]=\frac{4+8}{6}=2$$

ដូចនេះ: $\lim_{x\to0}\frac{1+2x\sin 4x-\cos^2 2x}{6x^2}=2$ ។

ឈ) $\lim_{x\to0}\frac{x\sin^3 2x}{(1-\cos 2x)^2}=\lim_{x\to0}\frac{x\sin^3 2x}{4\sin^4 x}=\frac{1}{4}\lim_{x\to0}\frac{\left(\frac{\sin 2x}{2x}\times2\right)^3}{\left(\frac{\sin x}{x}\right)^4}=\frac{1}{4}\times2^3=2$ ។

ដូចនេះ: $\lim_{x\to0}\frac{x\sin^3 2x}{(1-\cos 2x)^2}=2$ ។

ញ) $\lim_{x\to 0}\dfrac{1+x\sin 3x-\cos 6x}{3x^2+\sin^2 2x}=\lim_{x\to 0}\dfrac{2\sin^2 3x+x\sin 3x}{3x^2+\sin^2 2x}$

$$=\lim_{x\to 0}\dfrac{\dfrac{2\sin^2 3x+x\sin 3x}{x^2}}{\dfrac{3x^2+\sin^2 2x}{x^2}}$$

$$=\lim_{x\to 0}\dfrac{2\left(\dfrac{\sin 3x}{3x}\times 3\right)^2+\dfrac{\sin 3x}{3x}\times 3}{3+\left(\dfrac{\sin 2x}{2x}\times 2\right)^2}=\dfrac{18+3}{3+4}=\dfrac{21}{7}=3$$

ដូចនេះ $\lim_{x\to 0}\dfrac{1+x\sin 3x-\cos 6x}{3x^2+\sin^2 2x}=3$ ។

លំហាត់ទី២១

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\dfrac{\sin^2 x-\sin^2 3x}{1-\cos 4x}$

ខ) $\lim_{x\to 0}\dfrac{\cos x-\cos 3x}{1-\cos 2x}$

គ) $\lim_{x\to 0}\dfrac{x^2-\sin^2 x}{x^2-x\sin x}$

ឃ) $\lim_{x\to 0}\dfrac{1-2\cos 2x+\cos^2 2x}{x^3\sin 2x}$

ង) $\lim_{x\to 0}\dfrac{\tan 2x-\sin 2x}{x(1-\cos 4x)}$

ច) $\lim_{x\to 0}\dfrac{3\sin 2x-\sin 6x}{x^3+\sin^3 x}$

ឆ) $\lim_{x\to 0}\dfrac{2-\cos 2x-\cos 4x}{1-\cos^4 5x}$

ជ) $\lim_{x\to 0}\dfrac{\sin^3(2x^2)}{(\tan x-\sin x)^2}$

ឈ) $\lim_{x\to 0}\dfrac{1-\cos^4(2x^3)}{(3\sin x-\sin 3x)^2}$

ញ) $\lim_{x\to 0}\dfrac{1-\cos^3 2x}{1-\cos^4 3x}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

$$\text{ក. } \lim_{x\to 0}\frac{\sin^2 x-\sin^2 3x}{1-\cos 4x}=\lim_{x\to 0}\frac{\sin^2 x-\sin^2 3x}{2\sin^2 2x}=\frac{1}{2}\lim_{x\to 0}\frac{\dfrac{\sin^2 x-\sin^2 3x}{x^2}}{\dfrac{\sin^2 2x}{x^2}}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{\left(\dfrac{\sin x}{x}\right)^2-\left(\dfrac{\sin 3x}{3x}\times 3\right)^2}{\left(\dfrac{\sin 2x}{2x}\times 2\right)^2}=\frac{1}{2}\times\frac{1-9}{4}=-1 \text{ ។}$$

ដូចនេះ $\lim_{x\to 0}\dfrac{\sin^2 x-\sin^2 3x}{1-\cos 4x}=-1$ ។

$$\text{ខ. } \lim_{x\to 0}\frac{\cos x-\cos 3x}{1-\cos 2x}=\lim_{x\to 0}\frac{\left(1-2\sin^2\dfrac{x}{2}\right)-\left(1-2\sin^2\dfrac{3x}{2}\right)}{2\sin^2 x}$$

$$=\lim_{x\to 0}\frac{-2\sin^2\dfrac{x}{2}+2\sin^2\dfrac{3x}{2}}{2\sin^2 x}=\lim_{x\to 0}\frac{-\sin^2\dfrac{x}{2}+\sin^2\dfrac{3x}{2}}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{\dfrac{-\sin^2\dfrac{x}{2}+\sin^2\dfrac{3x}{2}}{x^2}}{\dfrac{\sin^2 x}{x^2}}=\lim_{x\to 0}\frac{-\left(\dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\times\dfrac{1}{2}\right)+\left(\dfrac{\sin\dfrac{3x}{2}}{\dfrac{3x}{2}}\times\dfrac{3}{2}\right)}{\left(\dfrac{\sin x}{x}\right)^2}=-\frac{1}{4}+\frac{9}{4}=2$$

ដូចនេះ $\lim_{x\to 0}\dfrac{\cos x-\cos 3x}{1-\cos 2x}=2$ ។

គ) $\lim_{x\to0}\frac{x^2-\sin^2 x}{x^2-x\sin x}=\lim_{x\to0}\frac{(x-\sin x)(x+\sin x)}{x(x-\sin x)}=\lim_{x\to0}\frac{x+\sin x}{x}$

$$=\lim_{x\to0}\left(1+\frac{\sin x}{x}\right)=1+1=2$$ ។

ដូចនេះ: $\lim_{x\to0}\frac{x^2-\sin^2 x}{x^2-x\sin x}=2$ ។

ឃ) $\lim_{x\to0}\frac{1-2\cos2x+\cos^2 2x}{x^3\sin2x}=\lim_{x\to0}\frac{(1-\cos2x)^2}{x^3\sin2x}=\lim_{x\to0}\frac{4\sin^4 x}{2x^3\sin x\cos x}$

$$=2\lim_{x\to0}\frac{\sin^3 x}{x^3\cos x}=2\lim_{x\to0}\left[\left(\frac{\sin x}{x}\right)^3\frac{1}{\cos x}\right]=2$$

ដូចនេះ: $\lim_{x\to0}\frac{1-2\cos2x+\cos^2 2x}{x^3\sin2x}=2$ ។

ង) $\lim_{x\to0}\frac{\tan2x-\sin2x}{x(1-\cos4x)}=\lim_{x\to0}\frac{\frac{\sin2x}{\cos2x}-\sin2x}{2x\sin^2 2x}=\lim_{x\to0}\frac{\sin2x(1-\cos2x)}{2x\sin^2 2x\cos2x}$

$$=\lim_{x\to0}\frac{2\sin^2 x}{2x\sin2x\cos2x}=\lim_{x\to0}\frac{\sin^2 x}{2x\sin x\cos x\cos2x}$$

$$=\lim_{x\to0}\frac{\sin x}{x}\times\frac{1}{2\cos x\cos2x}=\frac{1}{2}$$

ដូចនេះ: $\lim_{x\to0}\frac{\tan2x-\sin2x}{x(1-\cos4x)}=\frac{1}{2}$ ។

ច) $\lim_{x\to0}\frac{3\sin2x-\sin6x}{x^3+\sin^3 x}=\lim_{x\to0}\frac{3\sin2x-(3\sin2x-4\sin^3 2x)}{x^3+\sin^3 x}$

$$=\lim_{x\to0}\frac{4\sin^3 2x}{x^3+\sin^3 x}=4\lim_{x\to0}\frac{\frac{\sin^3 2x}{x^3}}{\frac{x^3+\sin^3 x}{x^3}}$$

$$=4\lim_{x\to0}\frac{\left(\frac{\sin2x}{2x}\times2\right)^3}{1+\left(\frac{\sin x}{x}\right)^3}=\frac{4\times8}{2}=16$$

ដូចនេះ $\lim_{x\to0}\frac{3\sin2x-\sin6x}{x^3+\sin^3x}=16$ ។

ឆ) $\lim_{x\to0}\frac{2-\cos2x-\cos4x}{1-\cos^4 5x}=\lim_{x\to0}\frac{(1-\cos2x)+(1-\cos4x)}{(1-\cos^2 5x)(1+\cos^2 5x)}$

$$=\lim_{x\to0}\frac{2\sin^2x+2\sin^2 2x}{\sin^2 5x(1+\cos^2 5x)}$$

$$=2\lim_{x\to0}\frac{\frac{\sin^2x+\sin^2 2x}{x^2}}{\frac{\sin^2 5x}{x^2}(1+\cos^2 5x)}$$

$$=2\lim_{x\to0}\frac{\left(\frac{\sin x}{x}\right)^2+\left(\frac{\sin2x}{2x}\times2\right)^2}{\left(\frac{\sin5x}{5x}\times5\right)^2(1+\cos^2 5x)}=2\times\frac{1+4}{25\times2}=\frac{1}{5}$$

ដូចនេះ $\lim_{x\to0}\frac{2-\cos2x-\cos4x}{1-\cos^4 5x}=\frac{1}{5}$ ។

ជ) $\lim_{x\to0}\frac{\sin^3(2x^2)}{(\tan x-\sin x)^2}=\lim_{x\to0}\frac{\sin^3(2x^2)}{(\tan x-\cos x\tan x)^2}=\lim_{x\to0}\frac{\sin^3(2x^2)}{(1-\cos x)^2\tan^2x}$

$$=\lim_{x\to0}\frac{\sin^3(2x^2)}{4\sin^4\frac{x}{2}\tan^2x}=\frac{1}{4}\lim_{x\to0}\frac{\left[\frac{\sin(2x^2)}{2x^2}\times2\right]^3}{\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\frac{1}{2}\right)^4\left(\frac{\tan x}{x}\right)^2}=\frac{1}{4}\times\frac{2^3}{\frac{1}{16}}=32$$ ។

ដូចនេះ $\lim_{x\to0}\frac{\sin^3(2x^2)}{(\tan x-\sin x)^2}=32$ ។

ឈ) $\lim_{x\to0}\frac{1-\cos^4(2x^3)}{(3\sin x-\sin3x)^2}=\lim_{x\to0}\frac{[1-\cos^2(2x^3)][1+\cos^2(2x^3)]}{[3\sin x-(3\sin x-4\sin^3x)]^2}$

$$= \lim_{x\to 0}\frac{\sin^2(2x^3)\left[1+\cos^2(2x^3)\right]}{16\sin^6 x}$$

$$= \frac{1}{16}\lim_{x\to 0}\left[\frac{\sin(2x^3)}{2x^3}\times 2\right]^2\left(\frac{x}{\sin x}\right)^6\left[1+\cos^2(2x^3)\right] = \frac{1}{16}\times 4\times 2 = \frac{1}{2}$$ ។

ដូចនេះ: $\lim_{x\to 0}\frac{1-\cos^4(2x^3)}{(3\sin x-\sin 3x)^2} = \frac{1}{2}$ ។

ញ) $\lim_{x\to 0}\frac{1-\cos^3 2x}{1-\cos^4 3x} = \lim_{x\to 0}\frac{(1-\cos 2x)(1+\cos 2x+\cos^2 2x)}{(1-\cos^2 3x)(1+\cos^2 3x)}$

$$= \lim_{x\to 0}\frac{2\sin^2 x(1+\cos 2x+\cos^2 2x)}{\sin^2 3x(1+\cos^2 3x)}$$

$$= 2\lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^2}{\left(\frac{\sin 3x}{3x}\times 3\right)}\frac{1+\cos 2x+\cos^2 2x}{1+\cos^2 3x}$$

$$= 2\times\frac{1}{9}\times\frac{3}{2} = \frac{1}{3}$$

ដូចនេះ: $\lim_{x\to 0}\frac{1-\cos^3 2x}{1-\cos^4 3x} = \frac{1}{3}$ ។

លំហាត់ទី២២

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\frac{1-\cos 2x\cos 6x}{\sin^2 5x}$

ខ) $\lim_{x\to 0}\frac{1-\cos 4x}{1-\cos 6x}$

គ) $\lim_{x\to 0}\frac{1-\cos^2 6x}{1-\cos^3 2x}$

ឃ) $\lim_{x\to 0}\frac{1-\cos^2 2x\ \cos 4x}{x^2}$

ង) $\lim_{x\to 0}\frac{\cos^2 2x-\cos 6x}{3\sin^2 x+\sin^2 2x}$

ច) $\lim_{x\to 0}\frac{1-\sqrt{\cos^3 4x}}{3x^2}$

ឆ) $\lim\limits_{x\to0}\dfrac{\cos 4x-\sqrt{\cos 2x}}{x^2}$

ជ) $\lim\limits_{x\to0}\dfrac{1-\sqrt[3]{\cos 3x}}{x^2}$

ឈ) $\lim\limits_{x\to0}\dfrac{\cos 2x-\sqrt[3]{\cos 4x}}{x^2}$

ញ) $\lim\limits_{x\to0}\dfrac{1-\cos 2x\sqrt{\cos 4x}}{x^2}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក) $$\lim_{x\to0}\frac{1-\cos 2x\cos 6x}{\sin^2 5x}=\lim_{x\to0}\frac{(1-\cos 2x)+\cos 2x(1-\cos 6x)}{\sin^2 5x}$$

$$=\lim_{x\to0}\frac{2\sin^2 x+2\cos 2x\sin^2 3x}{\sin^2 5x}$$

$$=2\lim_{x\to0}\frac{\dfrac{\sin^2 x+\cos 2x\sin^2 3x}{x^2}}{\dfrac{\sin^2 5x}{x^2}}$$

$$=2\lim_{x\to0}\frac{\left(\dfrac{\sin x}{x}\right)^2+\cos 2x\left(\dfrac{\sin 3x}{3x}\times 3\right)^2}{\left(\dfrac{\sin 5x}{5x}\times 5\right)^2}=2\times\frac{1+9}{25}=\frac{4}{5}$$

ដូចនេះ $\lim\limits_{x\to0}\dfrac{1-\cos 2x\cos 6x}{\sin^2 5x}=\dfrac{4}{5}$ ។

ខ) $$\lim_{x\to0}\frac{1-\cos 4x}{1-\cos 6x}=\lim_{x\to0}\frac{2\sin^2 2x}{2\sin^2 3x}=\lim_{x\to0}\frac{\sin^2 2x}{\sin^2 3x}=\lim_{x\to0}\frac{\left(\dfrac{\sin 2x}{2x}\times 2\right)^2}{\left(\dfrac{\sin 3x}{3x}\times 3\right)^2}=\frac{4}{9}$$

ដូចនេះ $\lim\limits_{x\to0}\dfrac{1-\cos 4x}{1-\cos 6x}=\dfrac{4}{9}$ ។

គ) $$\lim_{x\to0}\frac{1-\cos^2 6x}{1-\cos^3 2x}=\lim_{x\to0}\frac{\sin^2 6x}{(1-\cos 2x)(1+\cos 2x+\cos^2 2x)}$$

$$= \lim_{x \to 0} \frac{\sin^2 6x}{2\sin^2 x\left(1+\cos 2x+\cos^2 2x\right)}$$

$$= \frac{1}{2}\lim_{x \to 0} \frac{\left(\frac{\sin 6x}{6x}\times 6\right)^2}{\left(\frac{\sin x}{x}\right)^2\left(1+\cos 2x+\cos^2 2x\right)} = \frac{1}{2}\times\frac{36}{3} = 6$$

ដូចនេះ $\lim_{x \to 0} \frac{1-\cos^2 6x}{1-\cos^3 2x} = 6$ ។

ឃ) $\lim_{x \to 0} \frac{1-\cos^2 2x\ \cos 4x}{x^2} = \lim_{x \to 0} \frac{\left(1-\cos^2 2x\right)+\cos^2 2x\left(1-\cos 4x\right)}{x^2}$

$$= \lim_{x \to 0} \frac{\sin^2 2x+2\cos^2 2x\sin^2 2x}{x^2} = \lim_{x \to 0} \frac{\sin^2 2x\left(1+2\cos^2 2x\right)}{x^2}$$

$$= \lim_{x \to 0}\left(\frac{\sin 2x}{2x}\times 2\right)^2\left(1+2\cos^2 2x\right) = 4\times 3 = 12$$

ដូចនេះ $\lim_{x \to 0} \frac{1-\cos^2 2x\ \cos 4x}{x^2} = 12$ ។

ង) $\lim_{x \to 0} \frac{\cos^2 2x-\cos 6x}{3\sin^2 x+\sin^2 2x} = \lim_{x \to 0} \frac{1-\sin^2 2x-(1-2\sin^2 3x)}{3\sin^2 x+\sin^2 2x}$

$$= \lim_{x \to 0} \frac{2\sin^2 3x-\sin^2 2x}{3\sin^2 x+\sin^2 2x} = \lim_{x \to 0} \frac{\frac{2\sin^2 3x-\sin^2 2x}{x^2}}{\frac{3\sin^2 x+\sin^2 2x}{x^2}}$$

$$= \lim_{x \to 0} \frac{2\left(\frac{\sin 3x}{3x}\times 3\right)^2-\left(\frac{\sin 2x}{2x}\times 2\right)^2}{3\left(\frac{\sin x}{x}\right)^2+\left(\frac{\sin 2x}{2x}\times 2\right)^2} = \frac{14}{7} = 2$$

ដូចនេះ $\lim_{x \to 0} \frac{\cos^2 2x-\cos 6x}{3\sin^2 x+\sin^2 2x} = 2$ ។

ប) $\lim_{x\to 0}\frac{1-\sqrt{\cos^3 4x}}{3x^2}=\lim_{x\to 0}\frac{1-\cos^3 4x}{3x^2\left(1+\sqrt{\cos^3 4x}\right)}$

$$=\lim_{x\to 0}\frac{(1-\cos 4x)\left(1+\cos 4x+\cos^2 4x\right)}{3x^2\left(1+\sqrt{\cos^3 4x}\right)}$$

$$=\lim_{x\to 0}\frac{2\sin^2 2x\left(1+\cos 4x+\cos^2 4x\right)}{3x^2\left(1+\sqrt{\cos^3 4x}\right)}$$

$$=\frac{2}{3}\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\times 2\right)^2\frac{1+\cos 4x+\cos^2 4x}{1+\sqrt{\cos^3 4x}}=\frac{2}{3}\times 4\times\frac{3}{2}=4$$

ដូចនេះ: $\lim_{x\to 0}\frac{1-\sqrt{\cos^3 4x}}{3x^2}=4$ ។

ផ) $\lim_{x\to 0}\frac{\cos 4x-\sqrt{\cos 2x}}{x^2}=\lim_{x\to 0}\frac{\left(1-\sqrt{\cos 2x}\right)-(1-\cos 4x)}{x^2}$

$$=\lim_{x\to 0}\frac{1-\sqrt{\cos 2x}}{x^2}-\lim_{x\to 0}\frac{1-\cos 4x}{x^2}$$

$$=\lim_{x\to 0}\frac{1-\cos 2x}{x^2\left(1+\sqrt{\cos 2x}\right)}-\lim_{x\to 0}\frac{2\sin^2 2x}{x^2}$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{x^2}\frac{2}{1+\sqrt{\cos 2x}}-2\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\times 2\right)^2$$

$$=1-8=-7$$

ដូចនេះ: $\lim_{x\to 0}\frac{\cos 4x-\sqrt{\cos 2x}}{x^2}=-7$ ។

ព) $\lim_{x\to 0}\frac{1-\sqrt[3]{\cos 3x}}{x^2}=\lim_{x\to 0}\frac{1-\cos 3x}{x^2\left(1+\sqrt[3]{\cos 3x}+\sqrt[3]{\cos^2 3x}\right)}$

$$= \lim_{x\to 0} \frac{2\sin^2 \frac{3x}{2}}{x^2\left(1+\sqrt[3]{\cos 3x}+\sqrt[3]{\cos^2 3x}\right)}$$

$$= 2\lim_{x\to 0}\left(\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\times\frac{3}{2}\right)^2 \frac{1}{\left(1+\sqrt[3]{\cos 3x}+\sqrt[3]{\cos^2 3x}\right)}$$

$$= 2\times\frac{9}{4}\times\frac{1}{3}=\frac{3}{2}$$

ដូចនេះ $\lim_{x\to 0}\frac{1-\sqrt[3]{\cos 3x}}{x^2}=\frac{3}{2}$ ។

ឈ) $\lim_{x\to 0}\frac{\cos 2x-\sqrt[3]{\cos 4x}}{x^2}=\lim_{x\to 0}\frac{\left(1-\sqrt[3]{\cos 4x}\right)-\left(1-\cos 2x\right)}{x^2}$

$$= \lim_{x\to 0}\frac{1-\cos 4x}{x^2(1+\sqrt[3]{\cos 4x}+\sqrt[3]{\cos^2 4x})}-\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$$

$$= \lim_{x\to 0}\frac{2\sin^2 2x}{x^2\left(1+\sqrt[3]{\cos 4x}+\sqrt[3]{\cos^2 4x}\right)}-\lim_{x\to 0}\frac{2\sin^2 x}{x^2}$$

$$= 2\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\times 2\right)^2\frac{1}{\left(1+\sqrt[3]{\cos 4x}+\sqrt[3]{\cos^2 4x}\right)}-2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2=\frac{8}{3}-2=\frac{2}{3}$$

ដូចនេះ $\lim_{x\to 0}\frac{\cos 2x-\sqrt[3]{\cos 4x}}{x^2}=\frac{2}{3}$ ។

ញ) $\lim_{x\to 0}\frac{1-\cos 2x\sqrt{\cos 4x}}{x^2}=\lim_{x\to 0}\frac{\left(1-\cos 2x\right)+\cos 2x\left(1-\sqrt{\cos 4x}\right)}{x^2}$

$$= \lim_{x\to 0}\frac{1-\cos 2x}{x^2}+\lim_{x\to 0}\frac{\cos 2x\left(1-\sqrt{\cos 4x}\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x}{x^2}+\lim_{x\to 0}\frac{2\cos 2x\sin^2 2x}{x^2\left(1+\sqrt{\cos 4x}\right)}$$

$$=2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2+2\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\times 2\right)^2\frac{\cos 2x}{1+\sqrt{\cos 4x}}$$

$$=2+2\times 4\times\frac{1}{2}=2+4=6$$

ដូចនេះ $\lim_{x\to 0}\frac{1-\cos 2x\sqrt{\cos 4x}}{x^2}=6$ ។

លំហាត់ទី២៣

ចូរគណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\frac{x\sin 3x}{\sin^2 5x}$

ខ) $\lim_{x\to 0}\frac{x(1-\cos 2x)^2}{\tan^3 2x-\sin^3 2x}$

គ) $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

ឃ) $\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

ង) $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

ច) $\lim_{x\to 0}\frac{1-\cos x\cos 2x}{x^2}$

ឆ) $\lim_{x\to 0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}$

ជ) $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$

ឈ) $\lim_{x\to 0}\frac{x(a-b)}{\sin ax-\sin bx}$

ញ) $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x-3\sin x+1}{4\sin^2 x-1}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក) $\lim_{x\to 0}\frac{x\sin 3x}{\sin^2 5x}$ មានរាងមិនកំណត់ $\frac{0}{0}$

ចែកភាគយកនិងភាគបែងនឹង x^2 គេបាន ៖

$$\lim_{x\to 0}\frac{\frac{x\sin 3x}{x^2}}{\frac{\sin^2 5x}{x^2}}=\lim_{x\to 0}\frac{\frac{\sin 3x}{3x}\times 3}{\left(\frac{\sin 5x}{5x}\times 5\right)^2}=\frac{3}{25}$$

ដូចនេះ $\lim_{x\to 0}\frac{x\sin 3x}{\sin^2 5x}=\frac{3}{25}$ ។

ខ) $\lim\limits_{x\to 0}\dfrac{x(1-\cos 2x)^2}{\tan^3 2x-\sin^3 2x}$ ដោយ $\tan 2x=\dfrac{\sin 2x}{\cos 2x}$

$$=\lim_{x\to 0}\frac{x(1-\cos 2x)^2}{\dfrac{\sin^3 2x}{\cos^2 2x}-\sin^3 2x}=\lim_{x\to 0}\frac{x\cos^3 2x(1-\cos 2x)^2}{\sin^3 2x(1-\cos^3 2x)}$$

$$=\lim_{x\to 0}\frac{x\cos^3 2x(1-\cos 2x)^2}{\sin^3 2x(1-\cos 2x)(1+\cos 2x+\cos^2 2x)}$$

$$=\lim_{x\to 0}\frac{x\cos^3 2x(1-\cos 2x)}{\sin^3 2x(1+\cos 2x+\cos^2 2x)}=\lim_{x\to 0}\frac{2x\cos^3 2x\sin^2 x}{\sin^3 2x(1+\cos 2x+\cos^2 2x)}$$

$$=2\lim_{x\to 0}\frac{\dfrac{\sin^2 x}{x^2}}{\left(\dfrac{\sin 2x}{2x}\times 2\right)^3}\times\frac{\cos^3 2x}{(1+\cos 2x+\cos^2 2x)}=2\times\frac{1}{8}\times\frac{1}{3}=\frac{1}{12}$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{x(1-\cos 2x)^2}{\tan^3 2x-\sin^3 2x}=\dfrac{1}{12}$ ។

គ) $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}=\lim\limits_{x\to 0}\dfrac{2-(1+\cos x)}{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$

$$=\lim_{x\to 0}\frac{1-\cos x}{(1-\cos^2 x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{(1-\cos x)(1+\cos x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$=\lim_{x\to 0}\frac{1}{(1+\cos x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$=\frac{1}{(1+1)\left(\sqrt{2}+\sqrt{2}\right)}=\frac{1}{4\sqrt{2}}=\frac{\sqrt{2}}{8}$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}=\dfrac{\sqrt{2}}{8}$

ឃ) $\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=\lim_{x\to 0}\frac{\frac{1+\sin x-\cos x}{x}}{\frac{1-\sin x-\cos x}{x}}$

$$=\lim_{x\to 0}\frac{\frac{1-\cos x}{2}+\frac{\sin x}{x}}{\frac{1-\cos x}{x}-\frac{\sin x}{x}}=\frac{0+1}{0-1}=-1$$

ដូចនេះ $\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=-1$ ។

ង) $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\lim_{x\to 0}\frac{(1-\cos x)+\cos x(1-\sqrt{\cos 2x})}{x^2}$

$$=\lim_{x\to 0}\frac{1-\cos x}{x^2}+\lim_{x\to 0}\frac{\cos x\left(1-\sqrt{\cos 2x}\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}+\lim_{x\to 0}\frac{\cos x\left(1-\cos 2x\right)}{x^2\left(1+\sqrt{\cos 2x}\right)}$$

$$=2\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\frac{1}{2}\right)^2+2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\frac{\cos x}{1+\sqrt{\cos 2x}}$$

$$=2\times\left(\frac{1}{2}\right)^2+2\times\frac{1}{2}=\frac{1}{2}+1=\frac{3}{2}$$

ដូចនេះ $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\frac{3}{2}$ ។

ច) $\lim_{x\to 0}\frac{1-\cos x\cos 2x}{x^2}=\lim_{x\to 0}\frac{\left(1-\cos x\right)+\cos x\left(1-\cos 2x\right)}{x^2}$

$$=\lim_{x\to 0}\frac{1-\cos x}{x^2}+\lim_{x\to 0}\frac{\cos x\left(1-\cos 2x\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}+\lim_{x\to 0}\frac{2\cos x\sin^2 x}{x^2}$$

$$= 2\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\frac{1}{2}\right)^2 + 2\lim_{x\to 0}\left[\cos x\left(\frac{\sin x}{x}\right)^2\right] = 2\times\frac{1}{4}+2=\frac{5}{2}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{1-\cos x\cos 2x}{x^2}=\frac{5}{2}$ ។

ឆ) $\lim\limits_{x\to 0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}=\lim\limits_{x\to 0}\frac{x\sin x+(1-\cos 2x)}{\sin^2 x}$

$$=\lim_{x\to 0}\frac{x\sin x+2\sin^2 x}{\sin^2 x}=\lim_{x\to 0}\left(\frac{x}{\sin x}+2\right)=1+2=3$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}=3$ ។

ជ) $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}=\lim\limits_{x\to 0}\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{1+x\sin x-\cos^2 x}$

$$=\lim_{x\to 0}\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{x\sin x+\sin^2 x}=\lim_{x\to 0}\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{\sin^2 x\left(\frac{x}{\sin x}+1\right)}$$

$$=\lim_{x\to 0}\frac{\sqrt{1+x\sin x}+\cos x}{\frac{x}{\sin x}+1}=\frac{1+1}{1+1}=1$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}=1$ ។

ឈ) $\lim\limits_{x\to 0}\frac{x(a-b)}{\sin ax-\sin bx}$; $(a,b\neq 0, a\neq b)$

$$=\lim_{x\to 0}\frac{x(a-b)}{2\sin\frac{(a-b)x}{2}\cos\frac{(a+b)x}{x}}=\lim_{x\to 0}\frac{\frac{(a-b)x}{2}}{\sin\frac{(a-b)x}{x}}\times\frac{1}{\cos\frac{(a+b)x}{2}}=1\times 1=1$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{x(a-b)}{\sin ax-\sin bx}=1$ ។

ញ) $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x-3\sin x+1}{4\sin^2 x-1}=\lim_{x\to\frac{\pi}{6}}\frac{(2\sin^2 x-\sin x)-(2\sin x-1)}{(2\sin x)^2-1^2}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\sin x(2\sin x-1)-(2\sin x-1)}{(2\sin x-1)(2\sin x+1)}=\lim_{x\to\frac{\pi}{6}}\frac{(\sin x-1)(2\sin x-1)}{(2\sin x-1)(2\sin x+1)}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\sin x-1}{2\sin x+1}=\frac{\frac{1}{2}-1}{2\left(\frac{1}{2}\right)+1}=\frac{-\frac{1}{2}}{2}=-\frac{1}{4}$$

ដូចនេះ $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x-3\sin x+1}{4\sin^2 x-1}=-\frac{1}{4}$ ។

លំហាត់ទី២៤

ចូរគណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{3\sin x-\sin 3x}$

ខ. $\lim_{x\to 0}\frac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}$

គ. $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$

ឃ. $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}$

ង. $\lim_{x\to 0}\frac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x-\sqrt{\cos 3x}}}$

ច. $\lim_{x\to 0}\frac{1-\sqrt[3]{\cos 3x}}{x(1-\cos\sqrt{x})}$ ។

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{3\sin x-\sin 3x}$ ដោយ $\begin{cases}\sin 2x=2\sin x\cos x\\ \sin 3x=3\sin x-4\sin^3 x\end{cases}$

$$=\lim_{x\to 0}\frac{2\sin x-2\sin x\cos x}{3\sin x-(3\sin x-4\sin^3 x)}$$

$$=\lim_{x\to 0}\frac{2\sin x(1-\cos x)}{4\sin^3 x}$$

$$= \lim_{x\to 0}\frac{4\sin x\sin^2\frac{x}{2}}{4\sin^3 x} = \lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\sin^2 x} = \lim_{x\to 0}\left[(\frac{\sin\frac{x}{2}}{\frac{x}{2}})^2.\frac{1}{4}.\frac{x^2}{\sin^2 x}\right] = 1^2.\frac{1}{4}.1^2 = \frac{1}{4}$$

ដូចនេះ $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{3\sin x-\sin 3x} = \frac{1}{4}$ ។

ខ. $\lim_{x\to 0}\frac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}$

$$= \lim_{x\to 0}\frac{(1-\cos 4x)-(1-\cos 2x)}{(1-\cos 2x)+\cos 2x(1-\cos 4x)} = \lim_{x\to 0}\frac{2\sin^2 2x-2\sin^2 x}{2\sin^2 x+2\cos 2x\sin^2 2x}$$

$$= \lim_{x\to 0}\frac{\sin^2 2x-\sin^2 x}{\sin^2 x+\cos 2x\sin^2 2x} = \lim_{x\to 0}\frac{\frac{\sin^2 2x}{x^2}-\frac{\sin^2 x}{x^2}}{\frac{\sin^2 x}{x^2}+\cos 2x.\frac{\sin^2 2x}{x^2}} = \frac{4-1}{1+4} = \frac{3}{5}$$

គ. $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$

$$= \lim_{x\to 0}\frac{1-\cos(2\sin^2\frac{x}{2})}{x^4} = \lim_{x\to 0}\frac{2\sin^2(\sin^2\frac{x}{2})}{x^4}$$

$$= 2\lim_{x\to 0}\frac{\sin^2(\sin^2\frac{x}{2})}{(\sin^2\frac{x}{2})^2}.\frac{\sin^4\frac{x}{2}}{(\frac{x}{2})^4}.\frac{1}{16} = 2.\frac{1}{16} = \frac{1}{8}$$

ដូចនេះ $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4} = \frac{1}{8}$ ។

ឃ. $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$= \lim_{x\to 0}\frac{2-1-\cos x}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}$$

$$= \lim_{x\to 0}\frac{1-\cos x}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}$$

$$= \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}$$

$$= 2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{(\frac{x}{2})^2}\cdot\frac{x}{\tan x}\cdot\frac{1}{4}\cdot\frac{1}{\sqrt{2}+\sqrt{1+\cos x}} = \frac{\sqrt{2}}{8}$$

ដូចនេះ $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x} = \frac{\sqrt{2}}{8}$ ។

ង. $\lim_{x\to 0}\frac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x}-\sqrt{\cos 3x}}$

$$= \lim_{x\to 0}\frac{1+\sin^2 x-\cos^2 x}{\cos x-\cos 3x}\cdot\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$= \lim_{x\to 0}\frac{\sin^2 x+(1-\cos^2 x)}{2\sin\frac{x+3x}{2}\sin\frac{x-3x}{2}}\cdot\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x}{2\sin 2x.\sin(-x)}\cdot\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$= -\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{2x}{\sin 2x}\cdot\frac{1}{2}\cdot\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x} = -\frac{1}{2}$$

ដូចនេះ. $\lim_{x\to 0}\frac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x}-\sqrt{\cos 3x}} = -\frac{1}{2}$ ។

ច. $\lim_{x\to 0}\frac{1-\sqrt[3]{\cos 3x}}{x(1-\cos\sqrt{x})}$ ។

$$= \lim_{x\to 0}\frac{1-\cos 3x}{x(1-\cos\sqrt{x})(1+\sqrt[3]{\cos 3x}+\sqrt[3]{\cos^2 3x})}$$

$$= \lim_{x\to 0}\frac{2\sin^2\frac{3x}{2}}{2x\sin^2(\frac{\sqrt{x}}{2})(1+\sqrt[3]{\cos 3x}+\sqrt[3]{\cos^2 3x})}$$

$$= \lim_{x\to 0}\frac{\sin^2\frac{3x}{2}}{\left(\frac{3x}{2}\right)^2}\times\frac{9}{4}\times\frac{\left(\frac{\sqrt{x}}{2}\right)^2}{\sin^2\left(\frac{\sqrt{x}}{2}\right)}\times 2\times\frac{1}{1+\sqrt[3]{\cos 3x}+\sqrt[3]{\cos^2 3x}} = \frac{9}{4}\times 2\times\frac{1}{3} = \frac{3}{2}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{\cos 3x}}{x(1-\cos\sqrt{x})}=\dfrac{3}{2}$ ។

លំហាត់ទី២៥

ចូរគណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 0}\dfrac{\sin x\sin 2x\sin 3x}{x^3}$

ខ. $\lim\limits_{x\to 0}\dfrac{\sin x+\sin 2x+\sin 3x}{x}$

គ. $\lim\limits_{x\to 0}\dfrac{x+\sin 3x}{x}$

ឃ. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 6x}$

ង. $\lim\limits_{x\to 0}\dfrac{x+\sin 3x}{x+\tan x}$

ច. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{1-\cos 4x}$

ជ. $\lim\limits_{x\to 0}\dfrac{1-\cos^3 2x}{x\sin x}$

ឈ. $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^3}$

ញ. $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{3\sin x-\sin 3x}$

ដ. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}$

ឋ. $\lim\limits_{x\to 0}\dfrac{1-\cos(1-\cos x)}{x^4}$

ឌ. $\lim\limits_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

ឍ. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{1-\sqrt{\cos 2x}}$

ណ. $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}$

ត. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x-\sqrt{\cos 3x}}}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 0}\dfrac{\sin x\sin 2x\sin 3x}{x^3}=\lim\limits_{x\to 0}\dfrac{\sin x}{x}.\dfrac{\sin 2x}{x}.\dfrac{\sin 3x}{x}$

$$=\lim_{x\to 0}\left(\frac{\sin x}{x}\right).\lim_{x\to 0}\left(2.\frac{\sin 2x}{2x}\right).\lim_{x\to 0}\left(3.\frac{\sin 3x}{3x}\right)$$

$$=1\times 2\times 3=6$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\sin x\sin 2x\sin 3x}{x^3}=6$

ខ. $\lim\limits_{x\to 0}\frac{\sin x+\sin 2x+\sin 3x}{x}=\lim\limits_{x\to 0}\left(\frac{\sin x}{x}+\frac{\sin 2x}{x}+\frac{\sin 3x}{x}\right)$

$$=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)+\lim_{x\to 0}\left(2.\frac{\sin 2x}{2x}\right)+\lim_{x\to 0}\left(3.\frac{\sin 3x}{3x}\right)$$

$$=1+2+3=6$$

ដូចនេះ: $\lim\limits_{x\to 0}\frac{\sin x+\sin 2x+\sin 3x}{x}=6$ ។

គ. $\lim\limits_{x\to 0}\frac{x+\sin 3x}{x}=\lim\limits_{x\to 0}\left(1+3.\frac{\sin 3x}{3x}\right)=1+3=4$

ឃ. $\lim\limits_{x\to 0}\frac{\sin 2x}{\sin 6x}=\lim\limits_{x\to 0}\left(\frac{\sin 2x}{x}.\frac{x}{\sin 6x}\right)$

$$=\lim_{x\to 0}\left(2.\frac{\sin 2x}{2x}\right).\lim_{x\to 0}\left(\frac{1}{6}.\frac{6x}{\sin 6x}\right)=2\times\frac{1}{6}=\frac{1}{3}$$

ដូចនេះ: $\lim\limits_{x\to 0}\frac{\sin 2x}{\sin 6x}=\frac{1}{3}$ ។

ង. $\lim\limits_{x\to 0}\frac{x+\sin 3x}{x+\tan x}$

$$=\lim_{x\to 0}\frac{x\left(1+\frac{\sin 3x}{x}\right)}{x\left(1+\frac{\tan x}{x}\right)}=\lim_{x\to 0}\frac{1+3.\frac{\sin 3x}{3x}}{1+\frac{\tan x}{x}}=\frac{1+3}{1+1}=2$$ ។

ដូចនេះ: $\lim\limits_{x\to 0}\frac{x+\sin 3x}{x+\tan x}=2$

ច. $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\lim\limits_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}=2\lim\limits_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{1}{4}=\frac{1}{2}$ ។

ដូចនេះ: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

ឆ. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{1-\cos 4x}$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{2\sin^2 2x}=\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}.\frac{x^2}{\sin^2 2x}\right)=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\lim_{x\to 0}\left(\frac{2x}{\sin 2x}\right)^2\times\frac{1}{4}=\frac{1}{4}$$ ។

ដូចនេះ $\lim_{x\to0}\frac{1-\cos 2x}{1-\cos 4x}=\frac{1}{4}$

ង. $\lim_{x\to0}\frac{1-\cos^3 2x}{x\sin x}$

$$=\lim_{x\to0}\frac{(1-\cos 2x)(1+\cos 2x+\cos^2 2x)}{x\sin x}=\lim_{x\to0}\frac{2\sin^2 x(1+\cos 2x+\cos^2 2x)}{x\sin x}$$

$$=2\lim_{x\to0}\frac{\sin x}{x}.\lim_{x\to0}(1+\cos 2x+\cos^2 2x)=2.1.3=6$$

ដូចនេះ $\lim_{x\to0}\frac{1-\cos^3 2x}{x\sin x}=6$ ។

ឈ. $\lim_{x\to0}\frac{\tan x-\sin x}{x^3}$ ដោយ $\tan x=\frac{\sin x}{\cos x}\Rightarrow \sin x=\cos x.\tan x$

$$=\lim_{x\to0}\frac{\tan x-\cos x.\tan x}{x^3}=\lim_{x\to0}\frac{\tan x(1-\cos x)}{x^3}=\lim_{x\to0}\frac{2\tan x\sin^2\frac{x}{2}}{x^3}$$

$$=2\lim_{x\to0}\frac{\tan x}{x}.\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{x^2}=2.1.\frac{1}{4}=\frac{1}{2}$$

ដូចនេះ $\lim_{x\to0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2}$ ។

ញ. $\lim_{x\to0}\frac{2\sin x-\sin 2x}{3\sin x-\sin 3x}$

$$=\lim_{x\to0}\frac{2\sin x-2\sin x\cos x}{3\sin x-(3\sin x-4\sin^3 x)}$$

$$=\lim_{x\to0}\frac{2\sin x(1-\cos x)}{4\sin^3 x}=\lim_{x\to0}\frac{4\sin x\sin^2\frac{x}{2}}{4\sin^3 x}=\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{\sin^2 x}$$

$$=\lim_{x\to0}\left[\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2.\frac{1}{4}.\frac{x^2}{\sin^2 x}\right]=1^2.\frac{1}{4}.1^2=\frac{1}{4}$$

ដូចនេះ $\lim_{x\to0}\frac{2\sin x-\sin 2x}{3\sin x-\sin 3x}=\frac{1}{4}$ ។

ង. $\lim\limits_{x\to0}\dfrac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}$

$$=\lim_{x\to0}\frac{(1-\cos 4x)-(1-\cos 2x)}{(1-\cos 2x)+\cos 2x(1-\cos 4x)}=\lim_{x\to0}\frac{2\sin^2 2x-2\sin^2 x}{2\sin^2 x+2\cos 2x\sin^2 2x}$$

$$=\lim_{x\to0}\frac{(1-\cos 4x)-(1-\cos 2x)}{(1-\cos 2x)+\cos 2x(1-\cos 4x)}=\lim_{x\to0}\frac{2\sin^2 2x-2\sin^2 x}{2\sin^2 x+2\cos 2x\sin^2 2x}$$

$$=\lim_{x\to0}\frac{\sin^2 2x-\sin^2 x}{\sin^2 x+\cos 2x\sin^2 2x}=\lim_{x\to0}\frac{\frac{\sin^2 2x}{x^2}-\frac{\sin^2 x}{x^2}}{\frac{\sin^2 x}{x^2}+\cos 2x.\frac{\sin^2 2x}{x^2}}=\frac{4-1}{1+4}=\frac{3}{5}$$

ដូចនេះ $\lim\limits_{x\to0}\dfrac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}=\dfrac{3}{5}$

ច. $\lim\limits_{x\to0}\dfrac{1-\cos(1-\cos x)}{x^4}=\lim\limits_{x\to0}\dfrac{1-\cos(2\sin^2\frac{x}{2})}{x^4}=\lim\limits_{x\to0}\dfrac{2\sin^2(\sin^2\frac{x}{2})}{x^4}$

$$=2\lim_{x\to0}\frac{\sin^2(\sin^2\frac{x}{2})}{(\sin^2\frac{x}{2})^2}.\frac{\sin^4\frac{x}{2}}{(\frac{x}{2})^4}.\frac{1}{16}=2.\frac{1}{16}=\frac{1}{8}$$

ដូចនេះ $\lim\limits_{x\to0}\dfrac{1-\cos(1-\cos x)}{x^4}=\dfrac{1}{8}$ ។

ឆ. $\lim\limits_{x\to0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$$=\lim_{x\to0}\frac{(1-\cos x)+\cos x(1-\sqrt{\cos 2x})}{x^2}=\lim_{x\to0}\frac{1-\cos x}{x^2}+\lim_{x\to0}\frac{\cos x(1-\cos 2x)}{x^2(1+\sqrt{\cos 2x})}$$

$$=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}}{x^2}+\lim_{x\to0}\frac{2\cos x\sin^2 x}{x^2(1+\sqrt{\cos 2x})}$$

$$=2\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{(\frac{x}{2})^2}.\frac{1}{4}+2\lim_{x\to0}\frac{\sin^2 x}{x^2}.\frac{\cos x}{1+\sqrt{\cos 2x}}=2.\frac{1}{2}+2.\frac{1}{2}=2$$

ដូចនេះ $\lim\limits_{x\to0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}=2$ ។

ឍ. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{1-\sqrt{\cos 2x}}=\lim\limits_{x\to 0}\dfrac{1-\cos x}{1-\cos 2x}.\dfrac{1+\sqrt{\cos 2x}}{1+\sqrt{\cos x}}=\lim\limits_{x\to 0}\dfrac{2\sin^2\dfrac{x}{2}}{2\sin^2 x}.\dfrac{1+\sqrt{\cos 2x}}{1+\sqrt{\cos x}}$

$$=\lim_{x\to 0}\frac{2\sin^2\dfrac{x}{2}}{2\sin^2 x}.\frac{1+\sqrt{\cos 2x}}{1+\sqrt{\cos x}}=\lim_{x\to 0}\frac{\sin^2\dfrac{x}{2}}{(\dfrac{x}{2})^2}.\frac{x^2}{\sin^2 x}.\frac{1}{4}.\frac{1+\sqrt{\cos 2x}}{1+\sqrt{\cos x}}=\frac{1}{4}$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{1-\sqrt{\cos 2x}}=\dfrac{1}{4}$ ។

ណ. $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}$

$$=\lim_{x\to 0}\frac{2-1-\cos x}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}=\lim_{x\to 0}\frac{1-\cos x}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}$$

$$=\lim_{x\to 0}\frac{2\sin^2\dfrac{x}{2}}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}=2\lim_{x\to 0}\frac{\sin^2\dfrac{x}{2}}{(\dfrac{x}{2})^2}.\frac{x}{\tan x}.\frac{1}{4}.\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}=\frac{\sqrt{2}}{8}$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}=\dfrac{\sqrt{2}}{8}$ ។

១៦) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x}-\sqrt{\cos 3x}}$

$$=\lim_{x\to 0}\frac{1+\sin^2 x-\cos^2 x}{\cos x-\cos 3x}.\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$=\lim_{x\to 0}\frac{\sin^2 x+(1-\cos^2 x)}{2\sin\dfrac{x+3x}{2}\sin\dfrac{x-3x}{2}}.\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{2\sin 2x.\sin(-x)}.\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$=-\lim_{x\to 0}\frac{\sin x}{x}.\frac{2x}{\sin 2x}.\frac{1}{2}.\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}=-\frac{1}{2}$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x}-\sqrt{\cos 3x}}=-\dfrac{1}{2}$ ។