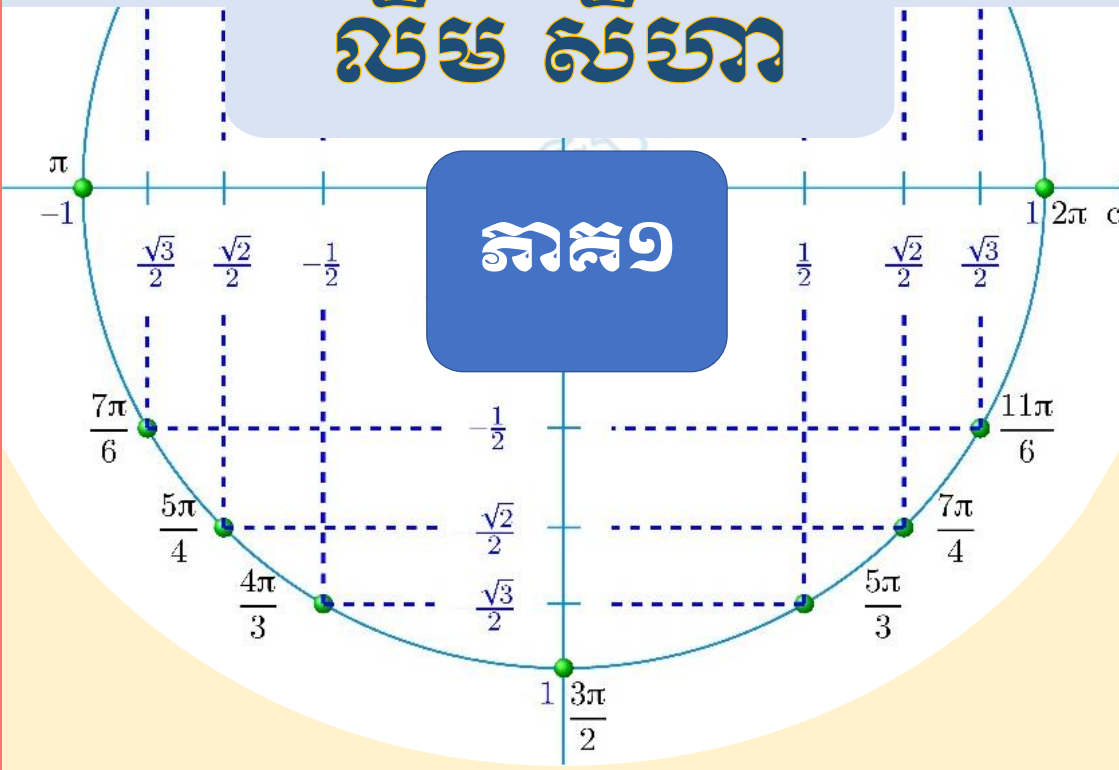
ជម្រើសលំហាត់
អនុគមន៍ត្រីកោណមាត្រ
ថ្នាក់ទី១១

រៀបរៀងដោយ

លឹម សីហា

តាគ១



ស្របតាមកម្មវិធីសិក្សាក្រសួងអប់រំ យុវជន និងកីឡា

មាតិកា

អារម្ភកថា

ការបង្កើតសៀវភៅនេះឡើងក្នុងគោលបំណងទុកជាឯកសារ សម្រាប់សិស្សានុសិស្សកម្រិតវិទ្យាល័យសិក្សាស្វែង យល់ពីលំហាត់និងដំណោះស្រាយនៃអនុគមន៍ត្រីកោណមាត្រថ្នាក់ទី១១ ដែលជាមេរៀនសំខាន់នៃគណិតវិទ្យាសម្រាប់ថ្នាក់វិទ្យាល័យៗ

សៀវភៅនេះ បែងចែកជាពីរភាគ គឺ

☞ ភាគ១ មានដំណោះស្រាយ លំហាត់ទី១ ដល់ទី៤៩

☞ ហើយភាគ២ បន្តពីទី៥០ ដល់លំហាត់ទី១០៣ៗ

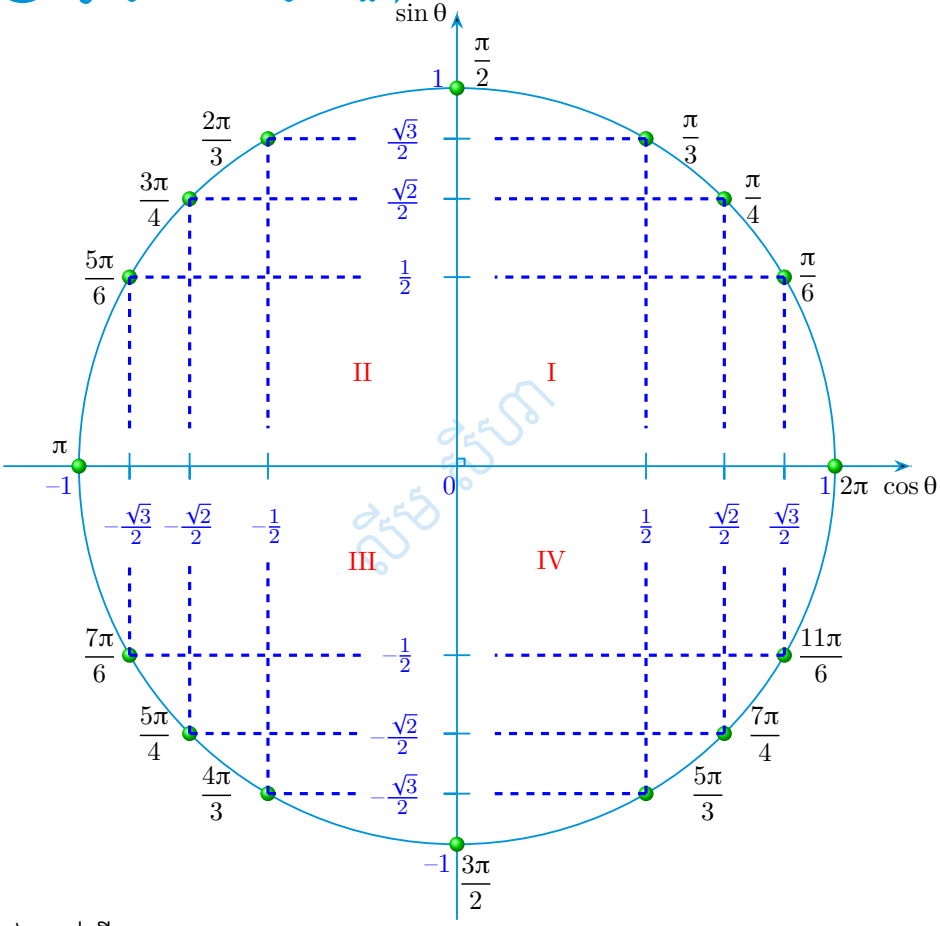
ចំពោះសៀវភៅនេះផងដែរ បើមាននូវចំណុចខ្វះខាត ឬខុសឆ្គងត្រង់ចំណុចណា សូមបងប្អូន សិស្សានុសិស្ស មិត្តអ្នកអាន មេត្តាអធ្យាស្រ័យ ដោយកំហុសអចេតនារបស់ខ្ញុំបាទៗ

សៀមរាប ២៩ កញ្ញា ២០១៨

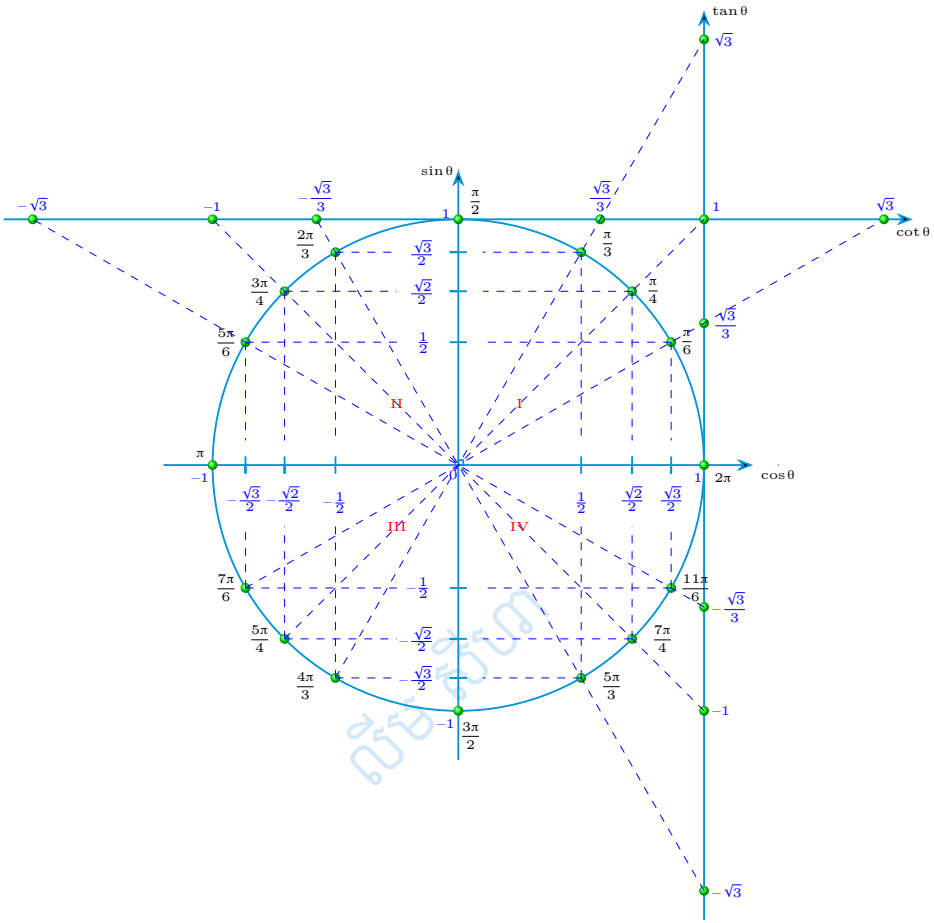
លីម សីហា .

1 រូបមន្តសង្ខេបសំខាន់ៗនៃអនុគមន៍ត្រីកោណមាត្រ

1 រង្វង់ត្រីកោណមាត្រ-តម្លៃមុំពិសេស



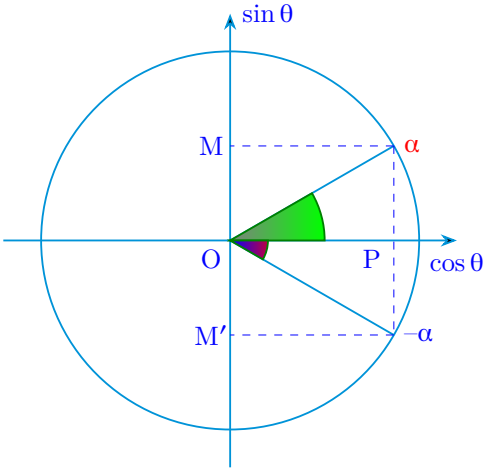
នៅកាជ្រុង ទី I	$\sin\theta > 0,$	$\cos\theta > 0$
នៅកាជ្រុង ទី II	$\sin\theta > 0,$	$\cos\theta < 0$
នៅកាជ្រុង ទី III	$\sin\theta < 0,$	$\cos\theta < 0$
នៅកាជ្រុង ទី IV	$\sin\theta < 0,$	$\cos\theta > 0$



នៅកាជ្រុង ទី I និង កាជ្រុង ទី III $\tan\theta > 0, \quad \cot\theta > 0$

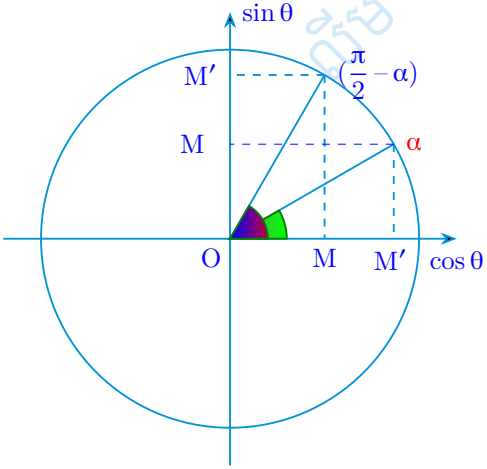
នៅកាជ្រុង ទី II និង កាជ្រុង ទី IV $\tan\theta < 0, \quad \cot\theta < 0$

2 មុំផ្ទុយ



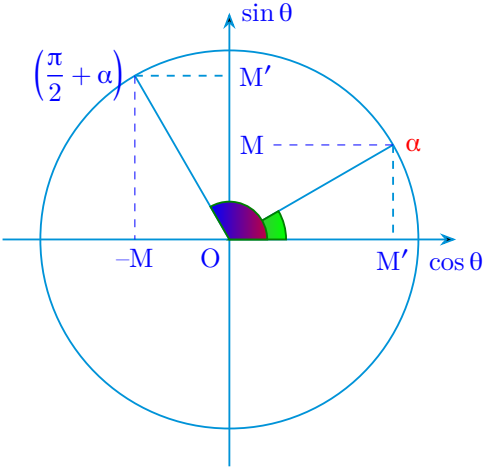
$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$
$$\tan(-\alpha) = -\tan\alpha$$
$$\cot(-\alpha) = -\cot\alpha$$

3 មុំបំពេញ



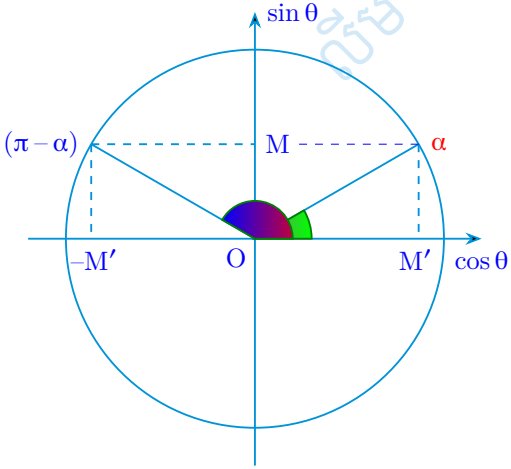
$$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$$
$$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$$
$$\tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha$$
$$\cot\left(\frac{\pi}{2}-\alpha\right) = \tan\alpha$$

4 មុំដែលមានផលសងស្មើ $\frac{\pi}{2}$



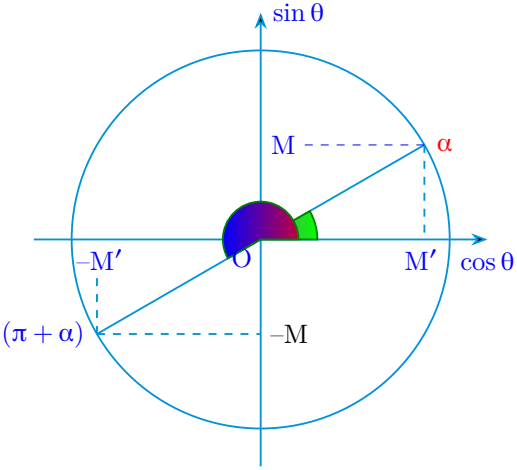
$$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$$
$$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$$
$$\tan\left(\frac{\pi}{2}+\alpha\right) = -\cot\alpha$$
$$\cot\left(\frac{\pi}{2}+\alpha\right) = -\tan\alpha$$

5 មុំបន្ថែម



$$\sin(\pi-\alpha) = \sin\alpha$$
$$\cos(\pi-\alpha) = -\cos\alpha$$
$$\tan(\pi-\alpha) = -\tan\alpha$$
$$\cot(\pi-\alpha) = -\cot\alpha$$

6 ម៉ុំដែលមានផលសងស្មើ π



$$\sin(\pi+\alpha) = -\sin\alpha$$
$$\cos(\pi+\alpha) = -\cos\alpha$$
$$\tan(\pi+\alpha) = \tan\alpha$$
$$\cot(\pi+\alpha) = \cot\alpha$$

7 ទំនាក់ទំនងសំខាន់ៗ

- $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$
- $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$
- $\tan\alpha \cdot \cot\alpha = 1$
- $\tan\alpha = \dfrac{1}{\cot\alpha}$
- $\cot\alpha = \dfrac{1}{\tan\alpha}$
- $\sin^2\alpha + \cos^2\alpha = 1$
- $\sin^2\alpha = 1 - \cos^2\alpha$
- $\cos^2\alpha = 1 - \sin^2\alpha$
- $1 + \tan^2\alpha = \dfrac{1}{\cos^2\alpha}$
- $1 + \cot^2\alpha = \dfrac{1}{\sin^2\alpha}$
- $\sin(\alpha + 2k\pi) = \sin\alpha \;\; ; (k \in \mathbb{Z})$
- $\cos(\alpha + 2k\pi) = \cos\alpha \;\; ; (k \in \mathbb{Z})$
- $\tan(\alpha + k\pi) = \tan\alpha \;\; ; (k \in \mathbb{Z})$
- $\cot(\alpha + k\pi) = \tan\alpha \;\; ; (k \in \mathbb{Z})$
- $\sin(k\pi + \theta) = \begin{cases} \sin\theta & \text{បើ } k \text{ គូ} \\ -\sin\theta & \text{បើ } k \text{ សេស} \end{cases}$

- $\cos(k\pi+\theta)=\begin{cases}\cos\theta & \text{បើ k គូ}\\ -\cos\theta & \text{បើ k សេស}\end{cases}$
- $\tan(k\pi+\theta)=\tan\theta \quad (k \text{ គូឬ } k \text{ សេស})$
- $\cot(k\pi+\theta)=\cot\theta \quad (k \text{ គូឬ } k \text{ សេស})$

8 រូបមន្តផលបូកនិងផលដកមុំ

- $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
- $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
- $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$
- $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$
- $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
- $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

9 រូបមន្តមុំ2ប

- $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$
- $\cos 2\alpha=1-2\sin^2\alpha$
- $\cos 2\alpha=2\cos^2\alpha-1$
- $\sin 2\alpha=2\sin\alpha\cos\alpha$
- $\tan 2\alpha=\dfrac{2\tan\alpha}{1-\tan^2\alpha}$

10 រូបមន្តកន្លះមុំ

- $\cos\alpha=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}$
- $\cos\alpha=1-2\sin^2\frac{\alpha}{2}$
- $\cos\alpha=2\cos^2\frac{\alpha}{2}-1$
- $\sin\alpha=2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$
- $\sin^2\frac{\alpha}{2}=\dfrac{1-\cos\alpha}{2}$
- $\cos^2\frac{\alpha}{2}=\dfrac{1+\cos\alpha}{2}$
- $\tan^2\frac{\alpha}{2}=\dfrac{1-\cos\alpha}{1+\cos\alpha}$
- $\tan\frac{\alpha}{2}=\dfrac{\sin\alpha}{1+\cos\alpha}$

- ចំពោះ $t = \tan\frac{\alpha}{2}$ គេបាន $\cos\alpha = \dfrac{1-t^2}{1+t^2}$; $\sin\alpha = \dfrac{2t}{1+t^2}$; $\tan\alpha = \dfrac{2t}{1-t^2}$

11 រូបមន្តបម្លែងពីផលគុណទៅជាផលបូក

- $\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$
- $\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$
- $\sin\alpha\sin\beta = \dfrac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$
- $\sin\beta\cos\alpha = \dfrac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$

12 រូបមន្តបម្លែងពីផលបូកទៅជាផលគុណ

- $\cos p+\cos q = 2\cos\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$
- $\sin p+\sin q = 2\sin\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$
- $\cos p-\cos q = -2\sin\dfrac{p+q}{2}\sin\dfrac{p-q}{2}$
- $\sin p-\sin q = 2\sin\dfrac{p-q}{2}\cos\dfrac{p+q}{2}$
- $\tan p+\tan q = \dfrac{\sin(p+q)}{\cos p\cos q}$
- $\tan p-\tan q = \dfrac{\sin(p-q)}{\cos p\cos q}$
- $\cot p+\cot q = \dfrac{\sin(p+q)}{\sin p\sin q}$
- $\cot p-\cot q = \dfrac{\sin(p-q)}{\sin p\sin q}$

13 សមីការត្រីកោណមាត្រ

ចំពោះ $k\in\mathbb{Z}$

- $\cos x=\cos\alpha \Rightarrow \begin{cases} x=\alpha+2k\pi \\ x=-\alpha+2k\pi \end{cases}$
- $\sin x=\sin\alpha \Rightarrow \begin{cases} x=\alpha+2k\pi \\ x=\pi-\alpha+2k\pi \end{cases}$
- $\cot x=\cot\alpha \Rightarrow x=\alpha+k\pi$
- $\tan x=\tan\alpha \Rightarrow x=\alpha+k\pi$

- $\cos u(x) = \cos v(x) \Rightarrow \begin{cases} u(x) = v(x) + 2k\pi \\ u(x) = -v(x) + 2k\pi \end{cases} ; k \in \mathbb{Z}$
- $\sin u(x) = \sin v(x) \Rightarrow \begin{cases} u(x) = v(x) + 2k\pi \\ u(x) = \pi - v(x) + 2k\pi \end{cases} ; k \in \mathbb{Z}$
- $\tan u(x) = \tan v(x) \Rightarrow u(x) = v(x) + k\pi ; \left(u(x), v(x) \neq \frac{\pi}{2} + k\pi\right)$
- $\cot u(x) = \cot v(x) \Rightarrow u(x) = v(x) + k\pi ; (u(x), v(x) \neq \pi + k\pi)$

14 វិសមីការត្រីកោណមាត្រ

- $\sin x \leq \sin \alpha \Rightarrow \pi - \alpha + 2k\pi \leq x \leq \alpha + 2k\pi ; k \in \mathbb{Z}$
- $\sin x \geq \sin \alpha \Rightarrow \alpha + 2k\pi \leq x \leq \pi - \alpha + 2k\pi ; k \in \mathbb{Z}$
- $\cos x \leq \cos \alpha \Rightarrow \alpha + 2k\pi \leq x \leq -\alpha + 2k\pi ; k \in \mathbb{Z}$
- $\cos x \geq \cos \alpha \Rightarrow -\alpha + 2k\pi \leq x \leq \alpha + 2k\pi ; k \in \mathbb{Z}$

$\tan x \leq \tan \alpha \Rightarrow -\frac{\pi}{2} + k\pi \leq x \leq \alpha + k\pi$

$\tan x \geq \tan \alpha \Rightarrow \alpha + k\pi \leq x \leq k\pi$

15 ទំនាក់ទំនងរវាងអនុគមន៍អ៊ុចស្ប៉ែរាង់ស្បែល និង ត្រីកោណមាត្រ

- $e^{i\theta} = \cos\theta + i\sin\theta$
- $e^{-i\theta} = \cos\theta - i\sin\theta$
- $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$
- $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$
- $\tan\theta = \frac{e^{i\theta} - e^{-i\theta}}{i\left(e^{i\theta} + e^{-i\theta}\right)}$
- $\cot\theta = i\left(\frac{e^{i\theta} + e^{-i\theta}}{e^{i\theta} - e^{-i\theta}}\right)$

ចិត្តស្ងប់ជាទ្រព្យ ដ៏ល្អប្រសើរ

លីម សីហា

2 ប្រធានលំហាត់

1

១. បម្លែងមុំ 30^{o}; 45^{o}; 135^{o}; 270^{o}; 330^{o}; -100^{o} ; 570^{o}; 630^{o} ទៅជារ៉ាដ្យង់ៗ

២. បម្លែងមុំ $\frac{\pi}{5}$; $\frac{2\pi}{3}$; 3π; $\frac{4\pi}{3}$; $\frac{7\pi}{4}$; $\frac{-7\pi}{2}$; $\frac{5\pi}{4}$; $\frac{5\pi}{7}$; $\frac{\pi}{9}$; $\frac{-11\pi}{6}$ ទៅជាដឺក្រេៗ

2

ចូរគណនាកន្សោមខាងក្រោម៖

$A = 2\sin\frac{\pi}{3} + 4\cos\frac{\pi}{6} - 3\tan\frac{\pi}{3} + 4\cot\frac{\pi}{4}$

$B = \dfrac{5 - 4\tan^2 45^{o} + \cot^2 60^{o}}{2\cos^2 60^{o} - 2\sin^3 90^{o} + 4\tan 45^{o}}$

$C = 2\sin\frac{2\pi}{4} - 3\tan^2\frac{\pi}{6} + 2\cos^4\frac{\pi}{2} + 3\cot^2\frac{\pi}{4}$

3

១. តើមុំ x នៅកាន្ទ្រង់ណាដែលផ្ទៀងផ្ទាត់ចំពោះវិសមីការ $\begin{cases}\sin x < 0\\ \cos x < 0\end{cases}$?

២. តើមុំ x នៅកាន្ទ្រង់ណាដែលផ្ទៀងផ្ទាត់ចំពោះវិសមីការ $\begin{cases}\cos x > 0\\ \tan x > 0\end{cases}$?

៣. តើនៅកាន្ទ្រង់ណាដែល $\tan x$ និង $\cot x$ មានសញ្ញាដូចគ្នា?

4

១. ចូរគណនាតម្លៃ $\cos\theta$; $\tan\theta$; $\cot\theta$ ដោយស្គាល់ $\sin\theta = \frac{8}{17}$ និង θ នៅកាដ្រង់ទី I ។

២. ចូរគណនាតម្លៃ $\cos\alpha$; $\tan\alpha$; $\cot\alpha$ ដោយស្គាល់ $\sin\alpha = -\frac{3}{12}$ និង $\frac{3\pi}{2} < \alpha < 2\pi$ ។

5

១. គណនា $\frac{5\sin\alpha + 7\cos\alpha}{6\cos\alpha - 3\sin\alpha}$ ដោយដឹងថា $\tan\alpha = \frac{4}{15}$ ។

២. គណនា $\frac{\cot\alpha + \tan\alpha}{\cot\alpha - \tan\alpha}$ ដោយស្គាល់ $\sin\alpha = \frac{3}{5}$ និង $0 < \alpha < \frac{\pi}{2}$ ។

6

១. គេដឹងថា $\cos\frac{\pi}{4} = \sin\frac{\pi}{4}$ ។ គណនា $\cos\frac{65\pi}{4}$ និង $\sin\left(-\frac{39\pi}{4}\right)$ ។

២. ចូរគណនាតម្លៃរបស់ $\sin 6\pi$; $\sin\frac{11\pi}{3}$; $\cos\left(-\frac{23\pi}{6}\right)$; $\tan\left(-\frac{17\pi}{4}\right)$ ។

7

គេមាន $\sin\theta + \cos\theta = -\frac{1}{2}$ ហើយ $\pi < \theta < 2\pi$ ។

ក. $\sin\theta\cos\theta$ ខ. $\tan\theta + \frac{1}{\tan\theta}$ គ. $\sin^3\theta - \cos^3\theta$

8

ចូរគណនាតម្លៃនៃកន្សោមខាងក្រោម៖

$A = \cos(-\theta) + \sin\left(\frac{\pi}{2} - \theta\right) + \cos(\pi - \theta) + \sin\left(\frac{3}{2}\pi + \theta\right)$

$B = \sin\frac{5\pi}{6} + \cos\frac{3\pi}{4} + \sin\left(-\frac{\pi}{6}\right)$

$C = \sin(-x) + \sin(\pi - x) + \cos(\pi - x) + \sin\left(\frac{\pi}{2} - x\right)$

$D = \sin\left(\frac{5\pi}{2} - x\right) + \cos(3\pi - x)$

$E = \tan\frac{5\pi}{6} + \cot\frac{7\pi}{4} + \frac{1}{\tan\frac{\pi}{3}}$

$F = 2\cos(\pi - 2x) + \sin(\pi + y) - 2\cos(\pi - 2x) - \sin(\pi + y)$

9

១. ចូរគណនាកន្សោម $\frac{\cos(-288^\circ)\cot 72^\circ}{\tan(-162^\circ)\sin 108^\circ} - \tan 18^\circ$ ។

២. ចូរគណនាកន្សោម $\frac{(\cot 44^\circ + \tan 226^\circ)\cos 406^\circ}{\cos 316^\circ} - \cot 72^\circ \cot 18^\circ$ ។

10

ចូររកតម្លៃនៃកន្សោមខាងក្រោម៖

$A = \frac{8\cos^3\alpha - 2\sin^3\alpha + \cos\alpha}{2\cos\alpha - \sin^3\alpha}$ ដោយស្គាល់ $\tan\alpha = 2$ ។

$B = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$ ដោយស្គាល់ $\tan\alpha = -2$ ។

11

ចូររកតម្លៃនៃកន្សោម $A = \dfrac{\sin^2\alpha - \cos^2\alpha}{\sin^2\alpha - 2\cos^2\alpha}$ ដោយស្គាល់ $\cot\alpha = 3$ ។

12

ចូរសម្រួលកន្សោមខាងក្រោម៖

ក. $\sqrt{\tan^2\alpha + \cot^2\alpha + 2}$

ខ. $\sqrt{\sin^2\alpha(1+\cot\alpha) + \cos^2\alpha(1+\tan\alpha)}$

13

ចូរសម្រួលកន្សោមខាងក្រោម៖

ក. $\dfrac{\sin^2\alpha - \tan^2\alpha}{\cos^2\alpha - \cot^2\alpha}$

ខ. $\dfrac{\cos^2\alpha + \cos^2\alpha\cot^2\alpha}{\sin^2\alpha + \sin^2\alpha\tan^2\alpha}$

14

ចូរធ្វើឯផ្ទាត់សមភាពខាងក្រោម៖

ក. $\dfrac{\sin x + \cos x - 1}{\sin x - \cos x + 1} = \dfrac{\cos x}{1+\sin x}$

ខ. $\dfrac{\cos x\cot x - \sin x\tan x}{\dfrac{1}{\sin x} - \dfrac{1}{\cos x}} = 1 + \sin x\cos x$

15

ចូរធ្វើឯផ្ទាត់សមភាពខាងក្រោម៖

ក. $(\tan\theta - \sin\theta)^2 + (1-\cos\theta)^2 = \left(\dfrac{1}{\cos\theta} - 1\right)^2$

ខ. $\dfrac{2\sin\theta\cos\theta-\cos\theta}{1-\sin\theta+\sin^2\theta-\cos^2\theta}=\dfrac{1}{\tan\theta}$

16

ចូរគណនាតម្លៃ

ក. $\cos 15^\circ$ គ. $\cos 75^\circ$ ង. $\sin\dfrac{5\pi}{12}$ ឆ. $\sin\dfrac{\pi}{12}$

ខ. $\sin 15^\circ$ ឃ. $\sin 75^\circ$ ច. $\cos\dfrac{5\pi}{12}$ ជ. $\cos\dfrac{\pi}{12}$

17

ក. បង្ហាញថា $\sin a+\cos a=\sqrt{2}\sin\left(a+\dfrac{\pi}{4}\right)$ ។

ខ. បង្ហាញថា $\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}+\dfrac{\sin(\beta-\theta)}{\cos\beta\cos\theta}+\dfrac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha}=0$ ។

18

១. បង្ហាញថា $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ ។

២. ចូរគណនាតម្លៃ

ក. $\tan 105^\circ$ ខ. $\tan\dfrac{\pi}{12}$ គ. $\sin 105^\circ$ ឃ. $\cos 105^\circ$

19

១. គេមាន $0<\alpha<\frac{\pi}{2}$; $-\frac{\pi}{2}<\beta<0$ និង $\tan\alpha=2$; $\tan\beta=-3$ ។

ក. គណនា $\tan(\alpha-\beta)$ ខ. គណនាមុំ $(\alpha-\beta)$

២. ចូរសម្រួលកន្សោម $\dfrac{\tan 3\theta-\tan\theta}{1+\tan\theta\tan 3\theta}+\cot\left(\dfrac{\pi}{2}+2\theta\right)$ ។

20

១. គណនា $\tan(a+b+c)$ ដោយប្រើរូបមន្ត $\tan(\alpha+\beta)$ ។

២. ផ្ទៀងផ្ទាត់សមភាព $\dfrac{1-\tan\theta}{1+\tan\theta} = \tan\left(\dfrac{\pi}{4}-\theta\right)$ ។

21

១. រក $\sin 2\alpha;\ \cos 2\alpha$ និង $\tan 2\alpha$ បើ $\cos\alpha = -\dfrac{4}{5}$ និង $\dfrac{\pi}{2} < \alpha < \pi$ ។

២. គណនា $\sin 3\alpha$ និង $\cos 3\alpha$ ។

៣. បង្ហាញថា $\dfrac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)} = 1$ ។

22

១. គណនា $\sin 22^{\circ}30';\ \sin\dfrac{\pi}{12}$ និង $\tan\dfrac{3\pi}{8}$ ។

២. ផ្ទៀងផ្ទាត់សមភាព $\dfrac{1-\cos x}{\sin x} = \tan\dfrac{x}{2}$ ។

23

១. គេមាន $\cos\alpha = -\dfrac{3}{5}$ ហើយ $\dfrac{\pi}{2} < \alpha < \pi$ ។ គណនា $\sin\dfrac{\alpha}{2};\quad \cos\dfrac{\alpha}{2}$ ។

២. គណនា $\tan\dfrac{\alpha}{2}$ ដោយស្គាល់ $\tan\alpha = \dfrac{24}{7}$ និង $180^{\circ} < \alpha < 270^{\circ}$ ។

៣. ចូរគណនាកន្សោមខាងក្រោម៖

ក. $\cos 75^{\circ}\cos 45^{\circ}$

ខ. $\sin\dfrac{5\pi}{12}\sin\dfrac{\pi}{4}$

24

ក. ចូរសរសេរកន្សោម $1+\cos a+\cos 2a$ ជាផលគុណកត្តាៗ

ខ. ចូរផ្ទៀងផ្ទាត់សមភាព $2\left(\frac{1}{\sin 2\alpha}+\cot 2\alpha\right)=\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}$ ៗ

25

១. គេមាន A, B, C ជាមុំក្នុងត្រីកោណមួយៗ

ចូរបង្ហាញថា $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ ៗ

២. A, B, C ជាមុំក្នុងត្រីកោណមួយៗ

ចូរបង្ហាញថា $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ ៗ

26

គេមាន $\sin\alpha+\cos\beta=\frac{5}{4}$ និង $\cos\alpha+\sin\beta=\frac{5}{4}$ ហើយ $\left(0\le\alpha\le\frac{\pi}{2}\right);\left(0\le\beta\le\frac{\pi}{2}\right)$ៗ

គណនា $\sin(\alpha+\beta)$ និង $\tan(\alpha+\beta)$ ៗ

27

១. គេមាន $\sin\alpha=\frac{1}{2}$; $\left(0<\alpha<\frac{\pi}{2}\right)$ និង $\sin\beta=\frac{1}{3}$; $\left(\frac{\pi}{2}<\beta<\pi\right)$ៗ

ចូរគណនា ៖

ក. $\sin(\alpha+\beta)$ គ. $\tan(\alpha-\beta)$

ខ. $\cos(\alpha-\beta)$ ឃ. $\cot(\alpha-\beta)$ ៗ

២. គេមាន $\cos\theta=-\frac{2}{3}$, $\left(\frac{\pi}{2}<\theta<\pi\right)$ ៗ គណនា $\cos 2\theta$, $\sin\frac{\theta}{2}$, $\sin 3\theta$ ៗ

28

គេមាន $t = \tan\frac{\theta}{2}$ $(t \neq \pm 1)$; $\sin\theta = \frac{2t}{1+t^2}$; $\cos\theta = \frac{1-t^2}{1+t^2}$ និង $\tan\theta = \frac{2t}{1-t^2}$ ។

តាមសមភាព $2\cos 2\theta - \cos\theta + 2 = 0$ ។ ចូរគណនា $\tan\frac{\theta}{2}$ ដែល $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ ។

29

គេមាន $\theta = 36^\circ$ និង $2\theta = 180^\circ - 3\theta$ ។ តាមលក្ខណៈនេះ គណនា $\cos 36^\circ$ ។

30

ចូរបង្ហាញសមភាពខាងក្រោម៖

ក. $\cot(a \pm b) = \frac{\cot a \cot b \mp 1}{\cot b \pm \cot a}$

ខ. $\tan(a+b) - \tan a - \tan b = \tan a \tan b \tan(a+b)$

គ. $\frac{\sin^4\alpha + 2\sin\alpha\cos\alpha - \cos^4\alpha}{\tan 2\alpha - 1} = \cos 2\alpha$

31

ចូរសម្រួលកន្សោមខាងក្រោម៖

ក. $\sin 4x - 4\sin 3x + 6\sin 2x - 4\sin x$

ខ. $\cos^2(\alpha+\beta) + \cos^2(\alpha-\beta) - \cos 2\alpha\cos 2\beta$

គ. $\frac{\sin\alpha + \sin 3\alpha + \sin 5\alpha}{\cos\alpha + \cos 3\alpha + \cos 5\alpha}$

ឃ. $\frac{\sin\alpha + \sin 3\alpha + \sin 5\alpha + \sin 7\alpha}{\cos\alpha + \cos 3\alpha + \cos 5\alpha + \cos 7\alpha}$

32

ចូរបង្ហាញសមភាពខាងក្រោម៖

ក. $\sin 3a = 4\sin a \sin\left(\frac{\pi}{3}+a\right)\sin\left(\frac{\pi}{3}-a\right)$

ខ. $\cos 3a = 4\cos a \cos\left(\frac{\pi}{3}+a\right)\cos\left(\frac{\pi}{3}-a\right)$

គ. $4\sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha-\frac{\pi}{3}\right) = 4\sin^2\alpha - 3$

ឃ. $\sin a - \cos a = \sqrt{2}\sin\left(a-\frac{\pi}{4}\right) = -\sqrt{2}\cos\left(a+\frac{\pi}{4}\right)$

ង. $\frac{\cos^3\alpha - \cos 3\alpha}{\cos\alpha} + \frac{\sin^3\alpha + \sin 3\alpha}{\sin\alpha} = 3$

33

ចូរបង្ហាញថាត្រីកោណដែលផ្ទៀងផ្ទាត់សមភាពខាងក្រោម៖

ក. $\sin A = \frac{\sin B + \sin C}{\cos B + \cos C}$

ខ. $\frac{\sin C}{\cos B} = \sin A + \cos A \cot C$ ជាត្រីកោណកែង ។

34

គេមាន ΔABC និង $A,\ B,\ C$ ជាមុំក្នុងត្រីកោណនេះ។

ក. បើគេដឹងថា $\sin A + \sin B + \sin C = 1$ គណនា $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ ។

ខ. បង្ហាញថា $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$ ។

35

ចូរគណនារង្វាស់មុំនៃត្រីកោណ ABC បើគេដឹងថា៖

ក. $\sin 3A + \sin 3B + \sin 3C = 0$

ខ. $\sin 5A + \sin 5B + \sin 5C = 0$

គ. $\sin 6A + \sin 6B + \sin 6C = 0$ ។

36

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos x = \frac{1}{2}$

ខ. $\cos\left(3x - \frac{\pi}{6}\right) = -1$

37

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos\left(x - \frac{\pi}{4}\right) = \sin 3x$

ខ. $\sin^4 x - \cos^4 x = \frac{1}{2}$

គ. $\cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = \cos\frac{\pi}{4}$

ឃ. $1 + 3\cos x + \cos 2x = \cos 3x + 2\sin x \cdot \sin 2x$

38

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin x = \frac{\sqrt{2}}{2}$

ខ. $\cos\left(3x + \frac{\pi}{3}\right) = \cos\left(x - \frac{\pi}{6}\right)$

គ. $\sin\left(2x + \frac{\pi}{4}\right) = 1$

ឃ. $\sin 2x = \sin\left(\frac{\pi}{3} - x\right)$

39

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin\frac{x}{2}\cos\frac{\pi}{3}-\cos\frac{x}{2}\sin\frac{\pi}{3}=\frac{1}{2}$

ខ. $2\sin x\cos x-3\sin 2x=0$

គ. $2\sin x\cos x+\sqrt{3}-2\cos x-\sqrt{3}\sin x=0$

40

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\tan 3x=\tan\left(\frac{\pi}{3}-2x\right)$

ខ. $\tan x=\frac{1}{\sqrt{3}}$

គ. $\tan 3x=\sqrt{3}$

ឃ. $\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan x\tan\frac{\pi}{4}}=\sqrt{3}$

41

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\frac{2\tan x}{1-\tan^2 x}=\sqrt{3}$

ខ. $2\tan x\cos x+1=2\cos x+\tan x$

42

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cot\left(2x+\frac{\pi}{3}\right)=\sqrt{3}$

ខ. $\cot x=-\frac{1}{\sqrt{3}}$

គ. $\cot 3x=\sqrt{3}$

ឃ. $\cot\left(\frac{x}{2}-3\right)=-1$

43

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $3\cot x - \sqrt{3} = 0$

ខ. $2(\cot 2x - \cot 3x) = \tan 2x + \cot 3x$

44

ចូរដោះស្រាយវិសមីការខាងក្រោម៖

ក. $2\cos x + 1 < 0$

ខ. $\tan x \geq -1$

គ. $2\cos x \geq -\sqrt{2}$

ឃ. $\cos 2x > \cos \frac{2\pi}{3}$

ង. $\sqrt{2}\cos x - 1 < 0$

ច. $\cot x < \frac{\sqrt{3}}{3}$

ឆ. $2\sin^2 x + 3\sin x - 2 \geq 0$

45

ចូរដោះស្រាយវិសមីការខាងក្រោម៖

ក. $\sin\left(\frac{\pi}{2} - x\right) < \cos\left(\frac{\pi}{2} + \frac{x}{2}\right)$

ខ. $\sin\left(x - \frac{\pi}{3}\right) > \sin x$

46

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin\theta = \frac{\sqrt{3}}{2}$

ខ. $\cos\theta = -\frac{1}{2}$

គ. $\cos^2 x = 1$

ឃ. $\sin\sqrt{x} = -1$

ង. $\cot x = 1$

47

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\frac{1}{\cos 2x} = \sqrt{2}$

ខ. $2\sin\left(2x - \frac{\pi}{3}\right) = 1$

48

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos\left(2x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

ខ. $\tan^3\left(x - \frac{\pi}{4}\right) = \tan x - 1$

គ. $\cot\left(2x + \frac{\pi}{3}\right) = 2$

ឃ. $\frac{2\tan x}{1 - \tan^2 x} = 5$

49

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\tan^3 3x - 2\sin^3 3x = 0$

ខ. $\cos\left(\frac{3\pi}{2} + x\right) = \sqrt{2}\sin x \cos x$

គ. $\sin 2x = (\cos x - \sin x)^2$

ឃ. $\sqrt{3}\sin x + \sin x = \frac{1}{\cos x}$

50

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin 2x + \tan x = 2$

ខ. $\frac{\sqrt{3}}{\cos^2 x} = 4\tan x$

គ. $\frac{1 + \sin x}{1 + \cos x} = \frac{1}{2}$

ឃ. $\cos\left(4x + \frac{\pi}{3}\right) = \cos\left(x - \frac{\pi}{6}\right)$

51

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x$

ខ. $\sin 2(x-\pi) - \sin(3x-\pi) = \sin x$

52

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\tan x + \cot x = 2(\sin 2x + \cos 2x)$

ខ. $\dfrac{\sin x + \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x} = \sqrt{3}$

53

ចូរដោះស្រាយវិសមីការខាងក្រោម៖

ក. $2\cos\theta \le -\sqrt{2}$ ខ. $-\sqrt{2}\sin\theta + 1 \ge 0$ គ. $\sqrt{3}\tan\theta - 1 < 0$

54

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin\left(2\theta + \dfrac{\pi}{3}\right) = -\dfrac{\sqrt{3}}{2} \quad (-\pi \le \theta \le \pi)$

ខ. $\cos\left(\dfrac{1}{2}\theta - \dfrac{\pi}{3}\right) \le \dfrac{1}{\sqrt{2}} \quad (0 \le \theta \le 2\pi)$

55

រកគ្រប់មុំដែលនៅចន្លោះ 0° និង 360° ដែលផ្ទៀងផ្ទាត់សមីការ៖

ក. $2\tan y = 5\sin y$

ខ. $8\sin x\cos x = \sin x$

គ. $4\sin^2 x = 6 - 9\cos x$

ឃ. $3\cos y + \cot y = 0$

ង. $2\sin 2x + 1 = 0$

ច. $6\sin x - 8\sin^2 x = 5\cos^2 x$

56

រកគ្រប់មុំដែលនៅចន្លោះ 0° និង 360° ដែលផ្ទៀងផ្ទាត់សមីការ៖

ក. $2\cot 2x = 5$

ខ. $3\sin y\tan y + 8 = 0$

គ. $3\cos^2 y = 7\sin y + 5$

ឃ. $\cot\left(\frac{z}{2}\right) - 2\cos\left(\frac{z}{2}\right) = 0$

ង. $\tan(x - 30^\circ)\tan 50^\circ = 0$

57

គេឲ្យ $x = 3\sin\theta - 2\cos\theta$ និង $y = 3\cos\theta + 2\sin\theta$ ។

ក. រកតម្លៃនៃមុំស្រួច θ ចំពោះ $x = y$ ។

ខ. បង្ហាញថា $x^2 + y^2$ ជាចំនួនថេរចំពោះគ្រប់តម្លៃនៃ θ ។

58

១. គេឲ្យត្រីកោណ ABC មាន $\dfrac{\sin A+\cos B}{\sin B+\cos A}=\tan A$ ។
បង្ហាញថា Δ ជាត្រីកោណកែង។

២. គេឲ្យត្រីកោណ ABC មាន $\dfrac{\sin C}{\sin B}=2\cos A$ ។ រកប្រភេទនៃ ΔABC ។

៣. គេឲ្យ ΔABC មានមុំបំពេញលក្ខខណ្ឌ $\tan B+\tan C=2\cos\dfrac{A}{2}$ ។
បង្ហាញថា ΔABC ជាត្រីកោណសមបាត។

59

គណនា

ក. $\sin 960^{\circ}$ ខ. $\cos 1215^{\circ}$ គ. $\sin 2940^{\circ}$

60

គណនា

ក. $\cos\dfrac{13\pi}{6}$ ខ. $\tan\left(-\dfrac{19}{4}\pi\right)$ គ. $\sin\left(-\dfrac{17}{\pi}\right)$

61

ចូរគណនាកន្សោមខាងក្រោម៖

$A=\sin\theta+\sin\left(\theta+\dfrac{\pi}{2}\right)+\sin(\theta+\pi)+\sin\left(\theta+\dfrac{3}{2}\pi\right)$

$B=\cos\dfrac{20}{3}\pi\tan\dfrac{7}{4}\pi+\sin\dfrac{5}{2}\pi\cos(-3\pi)$

$C=\sin\left(\dfrac{\pi}{2}-\theta\right)+\cos\left(\dfrac{\pi}{2}+2\theta\right)+\sin(\pi-2\theta)+\cos(\pi-\theta)$

62

១. គេមាន $\tan\alpha+\tan\beta+\tan\gamma=\tan\alpha\tan\beta\tan\gamma \quad \left(-\frac{\pi}{2}<\alpha,\beta,\gamma<\frac{\pi}{2}\right)$

គណនាមុំ $(\alpha+\beta+\gamma)$ ។

២. គេមាន $\alpha=\cos 10^{\circ},\ \beta=\cos 50^{\circ},\ \gamma=\cos 70^{\circ}$ ។

គណនា $\alpha-\beta-\gamma,\ \alpha^2-\beta\gamma$ និង $\alpha^2+\beta^2+\gamma^2$ ។

63

ចូរបង្ហាញថា

ក. $(1-\sin A+\cos A)^2=2(1-\sin A)(1+\cos A)$

ខ. $\cot A+\tan A=\dfrac{1}{\sin A\cos A}$

64

ចូរបង្ហាញថា

ក. $(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)=\tan^2\theta\sin^2\theta$

ខ. $\left(1+\dfrac{1}{\sin\theta}\right)(1-\sin\theta)=\cos\theta\cot\theta$

65

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $4\cos^3 x-\cos 2x-4\cos x+1=0$

ខ. $1+\sin x+\cos x+\sin 2x+\cos 2x=0$

គ. $\sin^{2007}x+\cos^{2007}x=1$

66

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin^2 2x - \cos^2 8x = 0.5\cos 10x$

ខ. $\sin^4 x + \cos^4 x - \cos 2x + \frac{1}{4}\sin^2 2x - 2 = 0$

គ. $\tan x = \cot x + 2\cot^2 2x$

67

ក. គេឲ្យ $2\sin A\cos A + (\cos A + \sin A)^2 - (2\cos A + \sin A)^2 = p\sin^2 A + q$ រកតម្លៃនៃចំនួនថេរ p និង q ។

ខ. គេឲ្យ $\sin x\cos x(5\tan x + 2\cot x) = a + b\sin^2 x$ ។ រកតម្លៃនៃចំនួនថេរ a និង b ។

68

ចូរដោះស្រាយវិសមីការខាងក្រោម ដោយដឹងថា $0 \le x \le 2\pi$៖

ក. $\cos\left(x + \frac{\pi}{3}\right) \ge -\frac{1}{2}$

ខ. $\tan\left(x - \frac{\pi}{6}\right) \ge -1$

69

ចូរដោះស្រាយសមីការ និង វិសមីការខាងក្រោម៖

ក. $2\sin^2\theta - \sqrt{2}\cos\theta = 0$

ខ. $2\cos^2\theta + \sqrt{3}\sin\theta + 1 > 0$

70

គេឲ្យ ΔABC មានមុំ 3 ជាមុំស្រួចៗ

ក. បង្ហាញថា $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$ៗ

ខ. តាង $T = \tan A + \tan B + \tan C$ ។ បង្ហាញថា $T \ge 3\sqrt{3}$ ។

71

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin x + \sqrt{3}\cos x = 0$

ខ. $\sin x + 7\cos x = 5$

គ. $\sqrt{3}\sin 2x + \cos 2x = \sqrt{2}$

72

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin x - 3\cos x = 3$

ខ. $\cos 2x - \sin 2x = -1$

គ. $\cos x + \sqrt{3}\sin x = 1$

ឃ. $\cos x - \sqrt{3}\sin x = 3$

73

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\tan^2\frac{x}{2} - \left(1 - \sqrt{3}\right)\tan\frac{x}{2} - \sqrt{3} = 0$

ខ. $\dfrac{\cos x\left(2\sin x + 3\sqrt{2}\right) - 2\cos^2 x - 1}{1 + \sin 2x} = 1$

គ. $5\tan^2 y + 5\tan y = 2\left(1 + \tan^2 y\right)$

74

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $2\cos^2 x - 3\sqrt{2}\cos x + 2 = 0$

ខ. $\dfrac{1}{\sin^2 x} = \cot x + 3$

គ. $8\sin^2 x - 6\sin x = 5$

ឃ. $4 - \cos 2x - 7\sin x = 0$

75

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\frac{\sin^4\frac{x}{2}+\cos^4\frac{x}{2}}{1-\sin x}-\tan^2 x\sin x=\frac{1+\sin x}{2}+\tan^2 x$ ។

ខ. $3\sin^2 x+2\sin x\cos x=2$

គ. $7\cos^2 x+6\sqrt{3}\sin x\cos x-\sin^2 x+2=0$

76

១. រកតម្លៃ m ដែលនាំឲ្យសមីការ $1+m\cos x=m^2-\cos^2 x$ មានឫស ។

២. រកតម្លៃ m ដើម្បីឲ្យសមីការ $\sin 4x=m\tan x$ មានឫស $x\neq k\pi$ ។

77

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $2\cos^2 x+\cot^2 x=\frac{\sin^3 x+1}{\sin^2 x}$

ខ. $\tan^3 x+\tan^2 x-3\tan x=3$

គ. $\sin^2 x+3\cos^2 x-2\sin 2x=0$

78

ចូរដោះស្រាយប្រព័ន្ធសមីការខាងក្រោម៖

ក. $\begin{cases} x+y=\frac{\pi}{3} \\ \sin x+\sin y=1 \end{cases}$

ខ. $\begin{cases} x+y=\frac{\pi}{2} \\ \tan x+\tan y=\sqrt{3}+\frac{\sqrt{3}}{3} \end{cases}$

79

ចូររកតម្លៃ m ដែលនាំឲ្យប្រព័ន្ធសមីការខាងក្រោមមានឫស៖

$$\begin{cases} x - y = m \\ 2(\cos 2x + \cos 2y) - 1 - 4\cos^2 m = 0 \end{cases}$$

80

ចូរដោះស្រាយប្រព័ន្ធសមីការខាងក្រោម៖

ក. $\begin{cases} \cos^2 x + \cos^2 y = \dfrac{1}{4} \\ x + y = \dfrac{5\pi}{6} \end{cases}$

ខ. $\begin{cases} \sin x \cos y = \dfrac{1}{4} \\ 3\tan x = \tan y \end{cases}$

81

គេឲ្យប្រព័ន្ធសមីការ $\begin{cases} \sin x \cos 2y = m^2 + 1 \\ \cos x \sin 2y = m \end{cases}$

១. បញ្ជាក់តម្លៃ m ដែលនាំឲ្យប្រព័ន្ធសមីការមានឫស។

២. ដោះស្រាយប្រព័ន្ធសមីការតាមតម្លៃ m ដែលរកបាននោះ។

82

ចូរដោះស្រាយវិសមីការខាងក្រោម៖

ក. $\sin x - \cos x > 0$

ខ. $\sin x + \dfrac{1}{\sin x} \geq \dfrac{5}{2}$

83

1. ចូរដោះស្រាយ $2\cos 2x + \sin^2 x \cos x + \sin x \cos^2 x > 2(\sin x + \cos x)$
2. បង្ហាញថាក្នុងត្រីកោណ ABC គេបាន $\cos A + \cos B + \cos C \le \frac{3}{2}$ ។
3. បង្ហាញថាក្នុងត្រីកោណ ABC គេបាន $\sin^2 A + \sin^2 B + \sin^2 C \le \frac{9}{4}$ ។

84

ចូរដោះស្រាយវិសមីការខាងក្រោម៖

ក. $\sin x + \cos x > \cos \frac{\pi}{6}$

ខ. $\frac{2\cos x - 5\sin x}{\cos x} > 0$

គ. $\frac{\cos x}{1-3\cos x} < \frac{1-\cos x}{1-9\cos^2 x}$

ឃ. $2\sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3}\cos 2x > 0$

85

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos x + \sqrt{3}\sin x = \cos 3x$

ខ. $\sin 3x + 2\cos x - 2 = 0$

គ. $\sin 2x + \tan x = 2$

ឃ. $\sin 5x + \cos 5x = \sqrt{2}\cos 13x$

86

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $6\sin x - 2\cos^3 x = 5\sin 2x \cos x$

ខ. $\sqrt{5\cos x - \cos 2x} + 2\sin x = 0$

87

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos^6 x + \sin^6 x = \frac{7}{16}$

ខ. $\cos^4 x + \sin^6 x = \cos 2x$

គ. $1 + \cot 2x = \frac{1 - \cos 2x}{\sin^2 2x}$

88

ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $3\sin 3x - \sqrt{3}\cos 9x = 1 + 4\sin^3 3x$

ខ. $1 + 3\cos x + \cos 2x = \cos 3x + 2\sin x \sin 2x$

គ. $(1 - \tan x)(1 + \sin 2x) = 1 + \tan x$

89

គេឲ្យសមីការ $\cos 2x - (2m + 1)\cos x + m + 1 = 0 \quad (1)$ ។

ក. ដោះស្រាយសមីការ (1) កាលណា $m = \frac{3}{2}$ ។

ខ. រកតម្លៃ m ដែលធ្វើឲ្យសមីការមានឫស x នៅចន្លោះ $\frac{\pi}{2} < x < \frac{3\pi}{2}$ ។

90

ចូរដោះស្រាយប្រព័ន្ធសមីការខាងក្រោម៖

ក. $\begin{cases} \sin(x + y) = \frac{1}{2} \\ \cos(x - y) = \frac{\sqrt{2}}{2} \end{cases}$

ខ. $\begin{cases} \sin x + \sin y = \sqrt{2} \\ \cos x + \sin y = \sqrt{2} \end{cases}$

91

ចូរដោះស្រាយប្រព័ន្ធសមីការខាងក្រោម៖

ក. $\begin{cases} \sin(x+y)=\cos(x-y) \\ \tan x-\tan y=1 \end{cases}$

ខ. $\begin{cases} \sin x+\sin y=\dfrac{3}{2} \\ \sin^2 x+\sin^2 y=\dfrac{5}{4} \end{cases}$

92

ចូរដោះស្រាយវិសមីការខាងក្រោម៖

ក. $\sin^2\left(\dfrac{x}{2}-\dfrac{\pi}{4}\right)<\cos^2\dfrac{x}{2}$

ខ. $\dfrac{\cos x}{1+2\cos x}>\dfrac{1-\cos x}{1-2\cos x}$

គ. $\dfrac{1-\sin x}{1-3\sin x}<\dfrac{1+\sin x}{1-9\sin^2 x}$

93

ចូរដោះស្រាយវិសមីការ $6\left(\sin^2 x-\sin x\cos x-\cos^2 x\right)>2$

94

ចូរបង្ហាញថា

ក. $\sin^4\alpha+\cos^4\alpha\geq\dfrac{1}{2}$

ខ. $\sin^6\alpha+\cos^6\alpha\geq\dfrac{1}{4}$

គ. $\sin^8\alpha+\cos^8\alpha\geq\dfrac{1}{8}$

95

១. បង្ហាញថាក្នុង ΔABC គេបាន៖ $\sin\dfrac{A}{2}\cdot\sin\dfrac{B}{2}\cdot\sin\dfrac{C}{2}\leq\dfrac{1}{8}$ ។

២. បង្ហាញថា ΔABC ដែលបំពេញលក្ខខណ្ឌ៖ $\tan A\tan B\tan^2\dfrac{C}{2}=1$ ជាត្រីកោណកែងសមបាត។

96

ដោះស្រាយសមីការខាងក្រោម៖

ក. $2\cos 3x + \sqrt{3}\sin x + \cos x = 0$

ខ. $\sin^6 x + \cos^3 x = 2\left(\sin^6 x + \cos^8 x\right)$

គ. $\dfrac{\sin 5x}{5\sin x} = 1$

ឃ. $\sin^2 x + \sin^2 3x = \cos^2 2x + \cos^2 4x$

ង. $\cos^3 x + \dfrac{3\sqrt{2}}{4}\sin 2x - 2\cos x = 0$

ច. $\sin x + \sin 2x + \sin 3x + \sin 4x + \sin 5x + \sin 6x = 0$

97

ដោះស្រាយវិសមីការខាងក្រោម៖

ក. $2\cos^2 x - \cos x + 1 \le 0$ ក្នុង $[0, \pi]$ ។

ខ. $\dfrac{2\sin^2 x - \sin x - 1}{\sin x} > 0$ ក្នុង $[0, \pi]$ ។

គ. $\dfrac{\sin x - \cos x + 1}{\sin x + \cos x - 1} > 0$

98

១. ចូរពន្លាតកន្សោម $\left(x + \dfrac{1}{2}\right)(x-8)(x-1)$ ។

២. ដោះស្រាយសមីការ $2\sin^3 x - 17\sin^2 x + 7\sin x + 8 = 0$ ។

99

គេឲ្យសមីការ $\cos 2x-(2m+1)\cos x+m+1=0 \quad (1)$

១. ដោះស្រាយសមីការ (1) កាលណា $m=1$ ។

២. រកតម្លៃ m ដែលធ្វើឲ្យសមីការមានឫសនៅចន្លោះ $[0,\pi]$ ។

100

ដោះស្រាយប្រព័ន្ធសមីការខាងក្រោម៖

ក. $\begin{cases}\cos^3 x-\cos x+\sin y=0\\ \sin^3 x-\sin y+\cos x=0\end{cases}$

ខ. $\begin{cases}\dfrac{1-\tan x}{1+\tan x}=\tan y\\ x-y=\dfrac{\pi}{6}\end{cases}$

101

គេមាន ΔABC ដែលមានមុំ និងជ្រុងបំពេញលក្ខខណ្ឌ៖ $\dfrac{1+\cos A}{1+\cos B}=\dfrac{2a+c}{2a-c}$ ។
បង្ហាញថា ΔABC ជាត្រីកោណសមបាត។

102

មុំ A, B, C នៃ ΔABC មួយមាន $\dfrac{A+C}{2}=B$ ។ រកវង្វាស់មុំនៃត្រីកោណនោះបើគេដឹងថា $\sin A+\sin B+\sin C=\dfrac{3+\sqrt{3}}{2}$ ។

103

គេមានវិសមីការ (E) : $2\sin^2 x - 5\sin x + 2 > 0$ ។ គេតាង $X = \sin x$
វិសមីការសរសេរជា $2X^2 - 5X + 2 > 0$ ។

ក. ដក់ជាផលគុណកត្តានូវកន្សោម $2x^2 - 5x + 2$ ។

ខ. បង្ហាញថាវិសមីការ (E) សរសេរជារាង $2(\sin x - 2)\left(\sin x - \frac{1}{2}\right) > 0$ ។

គ. សិក្សាសញ្ញា $(\sin x - 2)\left(\sin x - \frac{1}{2}\right)$ នៅលើចន្លោះ $[0, 2\pi]$ ។

ឃ. រកសំណុំចម្លើយនៃវិសមីការ (E) ។

3 ផ្នែកដំណោះស្រាយ

1

១. បម្លែង 30^o; 45^o; 135^o; 270^o; 330^o; -100^o ; 570^o; 630^o ទៅជារ៉ាដ្យង់ៗ

២. បម្លែង $\frac{\pi}{5}$; $\frac{2\pi}{3}$; 3π; $\frac{4\pi}{3}$; $\frac{7\pi}{4}$; $\frac{-7\pi}{2}$; $\frac{5\pi}{4}$; $\frac{5\pi}{7}$; $\frac{\pi}{9}$; $\frac{-11\pi}{6}$ ទៅជាដឺក្រេៗ

សម្រាយ.

១. បម្លែងពីដឺក្រេទៅជារ៉ាដ្យង់

- $30^o = \frac{30^o \times \pi}{180^o} = \frac{30^o \times \pi}{6 \times 30^o} = \frac{\pi}{6}$; $30^o = \frac{\pi}{6}$
- $45^o = \frac{45^o \times \pi}{180^o} = \frac{45^o \times \pi}{4 \times 45^o} = \frac{\pi}{4}$; $45^o = \frac{\pi}{4}$
- $135^o = \frac{135^o \times \pi}{180^o} = \frac{3 \times 45^o \times \pi}{4 \times 45^o} = \frac{3\pi}{4}$; $135^o = \frac{3\pi}{4}$
- $270^o = \frac{270^o \times \pi}{180^o} = \frac{3 \times 90^o \times \pi}{2 \times 90^o} = \frac{3\pi}{2}$; $270^o = \frac{3\pi}{2}$
- $330^o = \frac{330 \times \pi}{180^0} = \frac{11 \times 30^o \pi}{6 \times 30^o} = \frac{11\pi}{6}$; $330^o = \frac{11\pi}{6}$
- $-100^o = \frac{-100^o \times \pi}{180^o} = \frac{-5 \times 2 \times 10^o \pi}{9 \times 2 \times 10^o} = -\frac{5\pi}{9}$; $-100^o = -\frac{5\pi}{9}$
- $570^o = \frac{570^o \times \pi}{180^o} = \frac{57 \times 10^o \times \pi}{18 \times 10^o} = \frac{57\pi}{18}$; $570^o = \frac{57\pi}{18}$
- $630^o = \frac{630^o \times \pi}{180^o} = \frac{9 \times 7 \times 10^o \times \pi}{9 \times 2 \times 10^o} = \frac{7\pi}{2}$; $630^o = \frac{7\pi}{2}$

២. ប្តូរមុំពីរ៉ាដ្យង់ទៅជាដឺក្រេ

- $\frac{\pi}{5} = \frac{180^0}{5} = 36^o$ — $\boxed{\frac{\pi}{5} = 36^o}$
- $\frac{2\pi}{3} = \frac{2\times 180^o}{3} = 120^o$ — $\boxed{\frac{2\pi}{3} = 120^o}$
- $3\pi = 3\times 180^o = 540^o$ — $\boxed{3\pi = 540^o}$
- $\frac{4\pi}{3} = \frac{4\times 180^o}{3} = 240^o$ — $\boxed{\frac{4\pi}{3} = 240^o}$
- $\frac{7\pi}{4} = \frac{7\times 180^o}{4} = 315^o$ — $\boxed{\frac{7\pi}{4} = 315^o}$
- $\frac{-7\pi}{2} = \frac{-7\times 180^o}{2} = -630^o$ — $\boxed{\frac{-7\pi}{2} = -630^o}$
- $\frac{5\pi}{4} = \frac{5\times 180^o}{4} = 225^o$ — $\boxed{\frac{5\pi}{4} = 225^o}$
- $\frac{5\pi}{7} = \frac{5\times 180^o}{7} = 128.5^o$ — $\boxed{\frac{5\pi}{7} = 128^o30'}$
- $\frac{\pi}{9} = \frac{180^o}{9} = 20^o$ — $\boxed{\frac{\pi}{9} = 20^o}$
- $\frac{-11\pi}{6} = -\frac{11\times 180^o}{6} = -330^o$ — $\boxed{\frac{-11\pi}{6} = -330^o}$

2

គណនាកន្សោម៖

$A=2\sin\frac{\pi}{3}+4\cos\frac{\pi}{6}-3\tan\frac{\pi}{3}+4\cot\frac{\pi}{4}$

$B=\dfrac{5-4\tan^2 45^o+\cot^2 60^o}{2\cos^2 60^o-2\sin^3 90^o+4\tan 45^o}$

$C=2\sin\frac{2\pi}{4}-3\tan^2\frac{\pi}{6}+2\cos^4\frac{\pi}{2}+3\cot^2\frac{\pi}{4}$

សម្រាយ.

គណនាកន្សោម៖

$$A=2\sin\frac{\pi}{3}+4\cos\frac{\pi}{6}-3\tan\frac{\pi}{3}+4\cot\frac{\pi}{4}=2\left(\frac{\sqrt{3}}{2}\right)+4\left(\frac{\sqrt{3}}{2}\right)-3\left(\sqrt{3}\right)+4(1)$$

$=\sqrt{3}+2\sqrt{3}-3\sqrt{3}+4=4$ ដូចនេះ $\boxed{A=4}$

$$B=\frac{5-4\tan^2 45^o+\cot^2 60^o}{2\cos^2 60^o-2\sin^3 90^o+4\tan 45^o}=\frac{5-4(1)^2+\left(\frac{\sqrt{3}}{3}\right)^2}{2\left(\frac{1}{2}\right)^2-2(1)^3+4(1)}=\frac{1+\frac{3}{9}}{\frac{1}{2}+2}$$

$=\dfrac{\frac{9+3}{9}}{\frac{1+4}{2}}=\frac{12}{9}\times\frac{2}{5}=\frac{8}{15}$ ដូចនេះ $\boxed{B=\frac{8}{15}}$

$$C=2\sin\frac{2\pi}{4}-3\tan^2\frac{\pi}{6}+2\cos^4\frac{\pi}{2}+3\cot^2\frac{\pi}{4}=2(1)-3\left(\frac{\sqrt{3}}{3}\right)^2+2(0)^4+3(1)^2$$

$=2-1+3=4$ ដូចនេះ $\boxed{C=4}$

3

១. តើមុំ x នៅកាន់ត្រង់ណាដែលផ្ទៀងផ្ទាត់ចំពោះវិសមីការ $\begin{cases}\sin x<0\\ \cos x<0\end{cases}$?

២. តើមុំ x នៅកាន់ត្រង់ណាដែលផ្ទៀងផ្ទាត់ចំពោះវិសមីការ $\begin{cases}\cos x>0\\ \tan x>0\end{cases}$?

៣. តើនៅកាន់ត្រង់ណាដែល $\tan x$ និង $\cot x$ មានសញ្ញាដូចគ្នា?

សម្រាយ.

១. ចំពោះ $\begin{cases}\sin x<0\\ \cos x<0\end{cases}$ លុះត្រាតែ មុំx ស្ថិតនៅកាន់ត្រង់ទីIII

២. ចំពោះ $\begin{cases}\cos x>0\\ \tan x>0\end{cases}$ លុះត្រាតែ មុំx ស្ថិតនៅកាន់ត្រង់ទីI

៣. កាន់ត្រង់ដែល $\tan x$ និង $\cot x$ មានសញ្ញាដូចគ្នា គឺ

- កាន់ត្រង់ទីI និងទីIII $(\tan x>0,\ \cot>0)$
- កាន់ត្រង់ទីII និងទីIV $(\tan x<0,\ \cot<0)$

4

១. ចូរគណនាតម្លៃ $\cos\theta$; $\tan\theta$; $\cot\theta$ ដោយស្គាល់ $\sin\theta=\frac{8}{17}$ និងមុំ θ នៅកាន់ត្រង់ទីI។

២. ចូរគណនាតម្លៃ $\cos\alpha$; $\tan\alpha$; $\cot\alpha$ ដោយស្គាល់ $\sin\alpha=-\frac{3}{12}$ និង $\frac{3\pi}{2}<\alpha<2\pi$។

សម្រាយ.

១. គណនាតម្លៃ $\cos\theta$; $\tan\theta$; $\cot\theta$ ដោយស្គាល់ $\sin\theta=\frac{8}{17}$ និងមុំ θ នៅកាង្រ្តង់ទី I

តាម $\cos^2\theta=1-\sin^2\theta=1-\left(\frac{8}{17}\right)^2=1-\frac{64}{289}=\frac{289-64}{289}=\frac{225}{289}$

$\Rightarrow \cos\theta=\sqrt{\frac{225}{289}}=\frac{15}{17}$ (កាង្រ្តង់ទី I $\cos\theta>0$)

ដូចនេះ $\boxed{\cos\theta=\frac{15}{17}}$

តាមទំនាក់ទំនង $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{8}{17}}{\frac{15}{17}}=\frac{8}{15}$ ដូចនេះ $\boxed{\tan\theta=\frac{8}{15}}$

តាមទំនាក់ទំនង $\cot\theta=\frac{\cos\theta}{\sin\theta}=\frac{\frac{15}{17}}{\frac{8}{17}}=\frac{15}{8}$ ដូចនេះ $\boxed{\cot\theta=\frac{15}{8}}$

២. គណនាតម្លៃ $\cos\alpha$; $\tan\alpha$; $\cot\alpha$ ដោយស្គាល់ $\sin\alpha=-\frac{3}{12}$ និង $\frac{3\pi}{2}<\alpha<2\pi$

តាមទំនាក់ទំនង $\cos^2\alpha=1-\sin^2\alpha=1-\left(-\frac{3}{12}\right)^2=1-\frac{9}{144}=\frac{135}{144}$

$\Rightarrow \cos\alpha=\sqrt{\frac{135}{144}}=\frac{\sqrt{135}}{12}$ $\boxed{\cos\alpha=\frac{\sqrt{135}}{12}}$ $\left(\cos\alpha>0 \text{ ព្រោះ } \frac{3\pi}{2}<\alpha<2\pi\right)$

តាមទំនាក់ទំនង $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{-3}{12}}{\frac{\sqrt{135}}{12}}=\frac{-3\sqrt{135}}{135}$ $\boxed{\tan\alpha=\frac{-3\sqrt{135}}{135}}$

តាមទំនាក់ទំនង $\cot\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{\frac{\sqrt{135}}{12}}{\frac{-3}{12}}=-\frac{\sqrt{135}}{3}$ $\boxed{\cot\alpha=-\frac{\sqrt{135}}{3}}$

5

១. គណនា $\dfrac{5\sin\alpha+7\cos\alpha}{6\cos\alpha-3\sin\alpha}$ ដោយដឹងថា $\tan\alpha=\dfrac{4}{15}$ ។

២. គណនា $\dfrac{\cot\alpha+\tan\alpha}{\cot\alpha-\tan\alpha}$ ដោយស្គាល់ $\sin\alpha=\dfrac{3}{5}$ និង $0<\alpha<\dfrac{\pi}{2}$ ។

សម្រាយ.

១. គណនា $\dfrac{5\sin\alpha+7\cos\alpha}{6\cos\alpha-3\sin\alpha}$ ដោយដឹងថា $\tan\alpha=\dfrac{4}{15}$

$$\frac{5\sin\alpha+7\cos\alpha}{6\cos\alpha-3\sin\alpha}=\frac{5\dfrac{\sin\alpha}{\cos\alpha}+7\dfrac{\cos\alpha}{\cos\alpha}}{6\dfrac{\cos\alpha}{\cos\alpha}-3\dfrac{\sin\alpha}{\cos\alpha}}$$

$$=\frac{5\tan\alpha+7}{6-3\tan\alpha}=\frac{5\left(\dfrac{4}{15}\right)+7}{6-3\left(\dfrac{4}{15}\right)}=\frac{\dfrac{4+21}{3}}{\dfrac{30-4}{5}}=\frac{25\times5}{3\times26}=\frac{125}{78}$$

ដូចនេះ $\boxed{\dfrac{5\sin\alpha+7\cos\alpha}{6\cos\alpha-3\sin\alpha}=\dfrac{125}{78}}$

២. គណនា $\dfrac{\cot\alpha+\tan\alpha}{\cot\alpha-\tan\alpha}$ ដោយស្គាល់ $\sin\alpha=\dfrac{3}{5}$ និង $0<\alpha<\frac{\pi}{2}$

$$\frac{\cot\alpha+\tan\alpha}{\cot\alpha-\tan\alpha}=\frac{\dfrac{\cos\alpha}{\sin\alpha}+\dfrac{\sin\alpha}{\cos\alpha}}{\dfrac{\cos\alpha}{\sin\alpha}-\dfrac{\sin\alpha}{\cos\alpha}}=\frac{\dfrac{\cos^2\alpha+\sin^2\alpha}{\sin\alpha\cos\alpha}}{\dfrac{\cos^2\alpha-\sin^2\alpha}{\sin\alpha\cos\alpha}}=\frac{1}{\cos^2\alpha-\sin^2\alpha}$$

$$=\frac{1}{1-\sin^2\alpha-\sin^2\alpha}=\frac{1}{1-2\sin^2\alpha}=\frac{1}{1-2\left(\dfrac{3}{5}\right)^2}=\frac{1}{\dfrac{25-18}{25}}=\frac{25}{7}$$

ដូចនេះ $\boxed{\dfrac{\cot\alpha+\tan\alpha}{\cot\alpha-\tan\alpha}=\dfrac{25}{7}}$

6

១. គេដឹងថា $\cos\frac{\pi}{4}=\sin\frac{\pi}{4}$ ។ គណនា $\cos\frac{65\pi}{4}$ និង $\sin\left(-\frac{39\pi}{4}\right)$ ។

២. ចូរគណនាតម្លៃរបស់ $\sin 6\pi$; $\sin\frac{11\pi}{3}$; $\cos\left(-\frac{23\pi}{6}\right)$; $\tan\left(-\frac{17\pi}{4}\right)$ ។

សម្រាយ.

១. គណនា $\cos\frac{65\pi}{4}$ និង $\sin\left(-\frac{39\pi}{4}\right)$

$$\cos\left(\frac{65\pi}{4}\right)=\cos\left(\frac{64\pi+\pi}{4}\right)=\cos\left(16\pi+\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

ដូចនេះ $\boxed{\cos\frac{65\pi}{4}=\frac{\sqrt{2}}{2}}$

$$\sin\left(-\frac{39\pi}{4}\right)=\sin\left(\frac{-39\pi}{4}\right)=\sin\left(\frac{-40\pi+\pi}{4}\right)=\sin\left(-10\pi+\frac{\pi}{4}\right)$$
$$=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

ដូចនេះ $\boxed{\sin\left(-\frac{39\pi}{4}\right)=\frac{\sqrt{2}}{2}}$

២. គណនាតម្លៃរបស់ $\sin 6\pi$; $\sin\frac{11\pi}{3}$; $\cos\left(-\frac{23\pi}{6}\right)$; $\tan\left(-\frac{17\pi}{4}\right)$

- $\sin 6\pi=\sin(6\pi+0)=\sin 0=0$ ដូចនេះ $\boxed{\sin 6\pi}$
- $\sin\frac{11\pi}{3}=\sin\left(3\pi+\frac{2\pi}{3}\right)=-\sin\frac{2\pi}{3}=-\frac{\sqrt{3}}{2}$ ដូចនេះ $\boxed{\sin\frac{11\pi}{3}=-\frac{\sqrt{3}}{2}}$
- $\cos\left(-\frac{23\pi}{6}\right)=\cos\left(\frac{23\pi}{6}\right)=\cos\left(3\pi+\frac{5\pi}{6}\right)=-\cos\frac{5\pi}{6}=-\left(-\frac{\sqrt{3}}{2}\right)$

  ដូចនេះ $\boxed{\cos\left(-\frac{23\pi}{6}\right)=\frac{\sqrt{3}}{2}}$
- $\tan\left(-\frac{17\pi}{4}\right)=-\tan\left(4\pi+\frac{\pi}{4}\right)=-\tan\frac{\pi}{4}=-1$ ដូចនេះ $\boxed{\tan\left(-\frac{17\pi}{4}\right)=-1}$

7

គេមាន $\sin\theta+\cos\theta=-\dfrac{1}{2}$ ហើយ $\pi<\theta<2\pi$ ។

ក. $\sin\theta\cos\theta$ ខ. $\tan\theta+\dfrac{1}{\tan\theta}$ គ. $\sin^3\theta-\cos^3\theta$

សម្រាយ.

ក. $\sin\theta\cos\theta$

យើងមាន $(\sin\theta+\cos\theta)^2=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$

$$\Leftrightarrow 2\sin\theta\cos\theta=(\sin\theta+\cos\theta)^2-1=\left(-\frac{1}{2}\right)^2-1=\frac{1}{4}-1=\frac{1-4}{4}=-\frac{3}{4}$$

$$\Rightarrow \sin\theta\cos\theta=\frac{-\frac{3}{4}}{2}=-\frac{3}{8}$$ ដូចនេះ $\boxed{\sin\theta\cos\theta=-\dfrac{3}{8}}$

ខ. $$\tan\theta+\frac{1}{\tan\theta}=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{1}{-\frac{3}{8}}=-\frac{8}{3}$$

ដូចនេះ $\boxed{\tan\theta+\dfrac{1}{\tan\theta}=-\dfrac{8}{3}}$

គ. $$\sin^3\theta-\cos^3\theta=(\sin\theta-\cos\theta)\left(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta\right)$$
$$=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$$

តែ $(\sin\theta-\cos\theta)^2=\sin^2\theta-2\sin\theta\cos\theta+\sin^2\theta$

$$\Rightarrow \sin\theta-\cos\theta=\pm\sqrt{1-2\sin\theta\cos\theta}$$

ដោយ $\pi<\theta<2\pi\Rightarrow\ \sin\theta<0;\ \sin\theta\cos\theta=-\dfrac{3}{8}<0\ \Rightarrow\ \cos\theta>0$

$\sin\theta<0,\ \cos\theta>0$ គេបាន $\sin\theta-\cos\theta<0$

$$\Rightarrow \sin^3\theta-\cos^3\theta=-\left(\sqrt{1-2\left(-\frac{3}{8}\right)}\right)\left(1+\left(-\frac{3}{8}\right)\right)=-\frac{\sqrt{7}}{2}\times\frac{5}{8}$$

ដូចនេះ $\boxed{\sin^3\theta-\cos^3\theta=-\dfrac{5\sqrt{7}}{16}}$

8

គណនាតម្លៃនៃកន្សោម៖

$A = \cos(-\theta) + \sin\left(\frac{\pi}{2} - \theta\right) + \cos(\pi - \theta) + \sin\left(\frac{3}{2}\pi + \theta\right)$

$B = \sin\frac{5\pi}{6} + \cos\frac{3\pi}{4} + \sin\left(-\frac{\pi}{6}\right)$

$C = \sin(-x) + \sin(\pi - x) + \cos(\pi - x) + \sin\left(\frac{\pi}{2} - x\right)$

$D = \sin\left(\frac{5\pi}{2} - x\right) + \cos(3\pi - x)$

$E = \tan\frac{5\pi}{6} + \cot\frac{7\pi}{4} + \frac{1}{\tan\frac{\pi}{3}}$

$F = 2\cos(\pi - 2x) + \sin(\pi + y) - 2\cos(\pi - 2x) - \sin(\pi + y)$

សម្រាយ.

$A = \cos(-\theta) + \sin\left(\frac{\pi}{2} - \theta\right) + \cos(\pi - \theta) + \sin\left(\frac{3}{2}\pi + \theta\right)$

$= \cos\theta + \cos\theta - \cos\theta + \sin\left(\pi + \frac{\pi}{2} + \theta\right)$

$= \cos\theta - \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta - \cos\theta = 0$ ដូចនេះ $\boxed{A = 0}$

$B = \sin\frac{5\pi}{6} + \cos\frac{3\pi}{4} + \sin\left(-\frac{\pi}{6}\right) = \frac{1}{2} - \frac{\sqrt{2}}{2} - \frac{1}{2} = -\frac{\sqrt{2}}{2}$ ដូចនេះ $\boxed{B = -\frac{\sqrt{2}}{2}}$

$C = \sin(-x) + \sin(\pi - x) + \cos(\pi - x) + \sin\left(\frac{\pi}{2} - x\right)$

$= -\sin x + \sin x - \cos x + \cos x = 0$ ដូចនេះ $\boxed{C = 0}$

$D = \sin\left(\frac{5\pi}{2} - x\right) + \cos(3\pi - x) = \sin\left(2\pi + \frac{\pi}{2} - x\right) - \cos(-x)$

$= \sin\left(\frac{\pi}{2} - x\right) - \cos x = \cos x - \cos x = 0$ **ដូចនេះ** $\boxed{D = 0}$

$$E = \tan\frac{5\pi}{6} + \cot\frac{7\pi}{4} + \frac{1}{\tan\frac{\pi}{3}} = -\frac{\sqrt{3}}{3} - 1 + \frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} - 1 + \frac{\sqrt{3}}{3} = -1$$

ដូចនេះ $\boxed{E = -1}$

$F = 2\cos(\pi - 2x) + \sin(x + y) - 2\cos(\pi - 2x) - \sin(\pi + y) = 0$ ដូចនេះ $\boxed{F = 0}$

9

១. ចូរគណនាកន្សោម $\dfrac{\cos(-288^\circ)\cot 72^\circ}{\tan(-162^\circ)\sin 108^\circ} - \tan 18^\circ$ ។

២. ចូរគណនាកន្សោម $\dfrac{(\cot 44^\circ + \tan 226^\circ)\cos 406^\circ}{\cos 316^\circ} - \cot 72^\circ \cot 18^\circ$ ។

សម្រាយ.

១. គណនាកន្សោម $\dfrac{\cos(-288^\circ)\cot 72^\circ}{\tan(-162^\circ)\sin 108^\circ} - \tan 18^\circ$

$$\begin{aligned}
\frac{\cos(-288^\circ)\cot 72^\circ}{\tan(-162^\circ)\sin 108^\circ} - \tan 18^\circ &= \frac{\cos(-360^\circ + 72^\circ)\cot 72^\circ}{\tan(-180^\circ + 18^\circ)\sin(90^\circ + 18^\circ)} - \tan 18^\circ \\
&= \frac{\cos 72^\circ \cot 72^\circ}{\tan 18^\circ \cos 18^\circ} - \tan 18^\circ \\
&= \frac{\cos(90^\circ - 18^\circ)\cot(90^\circ - 18^\circ)}{\tan 18^\circ \cos 18^\circ} - \tan 18^\circ \\
&= \frac{\sin 18^\circ \tan 18^\circ}{\tan 18^\circ \cos 18^\circ} - \tan 18^\circ = \tan 18^\circ - \tan 18^\circ = 0
\end{aligned}$$

ដូចនេះ $\boxed{\dfrac{\cos(-288^\circ)\cot 72^\circ}{\tan(-162^\circ)\sin 108^\circ} - \tan 18^\circ = 0}$

២. គណនាកន្សោម $\dfrac{(\cot 44^\circ + \tan 226^\circ)\cos 406^\circ}{\cos 316^\circ} - \cot 72^\circ \cot 18^\circ$

$$\begin{aligned}
&\frac{(\cot 44^\circ + \tan 226^\circ)\cos 406^\circ}{\cos 316^\circ} - \cot 72^\circ \cot 18^\circ \\
&= \frac{(\cot(90^\circ - 46^\circ) + \tan(180^\circ + 46^\circ))\cos(360^\circ + 46^\circ)}{\cos(360^\circ - 44^\circ)} - \cot(72^\circ)\cot 18^\circ \\
&= \frac{(\tan 46^\circ + \tan 46^\circ)\cos 46^\circ}{\cos(-44^\circ)} - \cot(90^\circ - 18^\circ)\cot 18^\circ
\end{aligned}$$

$$= \frac{2\tan 46^\circ \cos 46^\circ}{\cos 44^\circ} - \tan 18^\circ \cot 18^\circ = \frac{2\tan 46^\circ \cos 46^\circ}{\cos(90^\circ - 46^\circ)} - 1$$

$$= \frac{2\tan 46^\circ \cos 46^\circ}{\sin 46^\circ} - 1 = 2\tan 46^\circ \cot 46^\circ - 1 = 2 - 1 = 1$$

ដូចនេះ $\boxed{\dfrac{(\cot 44^\circ + \tan 226^\circ)\cos 406^\circ}{\cos 316^\circ} - \cot 72^\circ \cot 18^\circ = 1}$

10

ចូររកតម្លៃនៃកន្សោមខាងក្រោម៖

$A = \dfrac{8\cos^3\alpha - 2\sin^3\alpha + \cos\alpha}{2\cos\alpha - \sin^3\alpha}$ ដោយស្គាល់ $\tan\alpha = 2$ ។

$B = \dfrac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$ ដោយស្គាល់ $\tan\alpha = -2$ ។

សម្រាយ.

រកតម្លៃនៃកន្សោម៖

$A = \dfrac{8\cos^3\alpha - 2\sin^3\alpha + \cos\alpha}{2\cos\alpha - \sin^3\alpha}$ ដោយស្គាល់ $\tan\alpha = 2$

$$= \frac{\dfrac{8\cos^3\alpha}{\cos^3\alpha} - \dfrac{2\sin^3\alpha}{\cos^3\alpha} + \dfrac{\cos\alpha}{\cos^3\alpha}}{\dfrac{2\cos\alpha}{\cos^3\alpha} - \dfrac{\sin^3\alpha}{\cos^3\alpha}} = \frac{8 - 2\tan^3\alpha + \dfrac{1}{\cos^2\alpha}}{2\dfrac{1}{\cos^2\alpha} - \tan^3\alpha}$$

$$= \frac{8 - 2\tan^3\alpha + 1 + \tan^2\alpha}{2(1 + \tan^2\alpha) - \tan^3\alpha} = \frac{8 - 2(2)^3 + 1 + (2)^2}{2(1 + 2^2) - 2^3} = \frac{-3}{2}$$

ដូចនេះ $\boxed{A = -\dfrac{3}{2}}$

$B = \dfrac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$ ដោយស្គាល់ $\tan\alpha = -2$

$$= \frac{\dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\cos\alpha}{\cos\alpha}}{\dfrac{\cos\alpha}{\cos\alpha} - \dfrac{\sin\alpha}{\cos\alpha}} = \frac{\tan\alpha + 1}{1 - \tan\alpha} = \frac{-2 + 1}{1 - (-2)} = \frac{-1}{3}$$

ដូចនេះ $\boxed{\dfrac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = -\dfrac{1}{3}}$

11

ចូររកតម្លៃនៃកន្សោម $A=\dfrac{\sin^2\alpha-\cos^2\alpha}{\sin^2\alpha-2\cos^2\alpha}$ ដោយស្គាល់ $\cot\alpha=3$ ។

សម្រាយ.

គណនា $A=\dfrac{\sin^2\alpha-\cos^2\alpha}{\sin^2\alpha-2\cos^2\alpha}$ ដោយស្គាល់ $\cot\alpha=3$

$$A=\frac{\sin^2\alpha-\cos^2\alpha}{\sin^2\alpha-2\cos^2\alpha}=\frac{\dfrac{\sin^2\alpha}{\sin^2\alpha}-\dfrac{\cos^2\alpha}{\sin^2\alpha}}{\dfrac{\sin^2\alpha}{\sin^2\alpha}-\dfrac{2\cos^2\alpha}{\sin^2\alpha}}=\frac{1-\cot^2\alpha}{1-2\cot^2\alpha}=\frac{1-3^2}{1-2(3)^2}=\frac{8}{17}$$

ដូចនេះ $\boxed{A=\dfrac{8}{17}}$

12

ចូរសម្រួលកន្សោមខាងក្រោម៖

ក. $\sqrt{\tan^2\alpha+\cot^2\alpha+2}$

ខ. $\sqrt{\sin^2\alpha(1+\cot\alpha)+\cos^2\alpha(1+\tan\alpha)}$

សម្រាយ.

សម្រួលកន្សោម៖

ក. $\sqrt{\tan^2\alpha+\cot^2\alpha+2}$

$=\sqrt{\tan^2\alpha+2\tan\alpha\cot\alpha+\cot^2\alpha}=\sqrt{(\tan\alpha+\cot\alpha)^2}=|\tan\alpha+\cot\alpha|$

ដូចនេះ $\boxed{\sqrt{\tan^2\alpha+\cot^2\alpha+2}=|\tan\alpha+\cot\alpha|}$

ខ. $\sqrt{\sin^2\alpha(1+\cot\alpha)+\cos^2\alpha(1+\tan\alpha)}$

$=\sqrt{\sin^2\alpha+\sin^2\alpha\cot\alpha+\cos^2\alpha+\cos^2\alpha\tan\alpha}$

$$= \sqrt{\sin^2\alpha + \sin^2\alpha\frac{\cos\alpha}{\sin\alpha} + \cos^2\alpha\frac{\sin\alpha}{\cos\alpha} + \cos^2\alpha}$$

$$= \sqrt{\sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha} = \sqrt{(\sin\alpha + \cos\alpha)^2} = |\sin\alpha + \cos\alpha|$$

ដូចនេះ $\boxed{\sqrt{\sin^2\alpha(1+\cot\alpha) + \cos^2\alpha(1+\tan\alpha)} = |\sin\alpha + \cos\alpha|}$

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ចូរសម្រួលកន្សោមខាងក្រោម៖

ក. $\dfrac{\sin^2\alpha - \tan^2\alpha}{\cos^2\alpha - \cot^2\alpha}$

ខ. $\dfrac{\cos^2\alpha + \cos^2\alpha\cot\alpha}{\sin^2\alpha + \sin^2\alpha\tan^2\alpha}$

សម្រាយ.

សម្រួលកន្សោម៖

ក. $\dfrac{\sin^2\alpha - \tan^2\alpha}{\cos^2\alpha - \cot^2\alpha} = \dfrac{\sin^2\alpha - \dfrac{\sin^2\alpha}{\cos^2\alpha}}{\cos^2\alpha - \dfrac{\cos^2\alpha}{\sin^2\alpha}} = \dfrac{\dfrac{\sin^2\alpha\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha}}{\dfrac{\sin^2\alpha\cos^2\alpha - \cos^2\alpha}{\sin^2\alpha}}$

$$= \frac{-\sin^2\alpha\left(1-\cos^2\alpha\right)\left(\sin^2\alpha\right)}{-\cos^2\alpha\left(1-\sin^2\alpha\right)\left(\cos^2\alpha\right)} = \frac{\sin^2\alpha\sin^2\alpha\sin^2\alpha}{\cos^2\alpha\cos^2\alpha\cos^2\alpha}$$

$$= \frac{\sin^6\alpha}{\cos^6\alpha} = \tan^6\alpha$$

ដូចនេះ $\boxed{\dfrac{\sin^2\alpha - \tan^2\alpha}{\cos^2\alpha - \cot^2\alpha} = \tan^6\alpha}$

ខ. $\dfrac{\cos^2\alpha + \cos^2\alpha\cot^2\alpha}{\sin^2\alpha + \sin^2\alpha\tan^2\alpha} = \dfrac{\cos^2\alpha\left(1+\cot^2\alpha\right)}{\sin^2\alpha\left(1+\tan^2\alpha\right)} = \cot^2\alpha \cdot \dfrac{\dfrac{1}{\sin^2\alpha}}{\dfrac{1}{\cos^2\alpha}}$

$$= \cot^2\alpha \cdot \frac{\cos^2\alpha}{\sin^2\alpha} = \cot^2\alpha \cdot \cot^2\alpha = \cot^4\alpha$$

ដូចនេះ $\boxed{\dfrac{\cos^2\alpha + \cos^2\alpha\cot^2\alpha}{\sin^2\alpha + \sin^2\alpha\tan^2\alpha} = \cot^4\alpha}$

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ចូរធ្វើបង្ហាញផ្ទៀងផ្ទាត់សមភាពខាងក្រោម៖

ក. $\dfrac{\sin x+\cos x-1}{\sin x-\cos x+1}=\dfrac{\cos x}{1+\sin x}$

ខ. $\dfrac{\cos x\cot x-\sin x\tan x}{\dfrac{1}{\sin x}-\dfrac{1}{\cos x}}=1+\sin x\cos x$

សម្រាយ.

ក. $\dfrac{\sin x+\cos x-1}{\sin x-\cos x+1}=\dfrac{\cos x}{1+\sin x}$

ដោយ $\dfrac{\sin x+\cos x-1}{\sin x-\cos x+1}=\dfrac{\cos x}{1+\sin x}$

$\Leftrightarrow (1+\sin x)(\sin x+\cos x-1)=\cos x(\sin x-\cos x+1)$

$\Leftrightarrow \sin x+\cos x-1+\sin^2 x+\sin x\cos x-\sin x=\sin x\cos x-\cos^2 x+\cos x$

$\Leftrightarrow \cos x-1+\sin^2 x+\sin x\cos x-\sin x\cos x+\cos^2 x-\cos x=0$

$\Leftrightarrow -1+1=0 \quad \Leftrightarrow 0=0$ **ពិត** ដូចនេះ $\boxed{\dfrac{\sin x+\cos x-1}{\sin x-\cos x+1}=\dfrac{\cos x}{1+\sin x}}$

ខ. $\dfrac{\cos x\cot x-\sin x\tan x}{\dfrac{1}{\sin x}-\dfrac{1}{\cos x}}=1+\sin x\cos x$

$$\frac{\cos x\cot x-\sin x\tan x}{\dfrac{1}{\sin x}-\dfrac{1}{\cos x}}=\frac{\cos x\cdot\dfrac{\cos x}{\sin x}-\sin x\cdot\dfrac{\sin x}{\cos x}}{\dfrac{\cos x-\sin x}{\sin x\cos x}}=\frac{\dfrac{\cos^3 x-\sin^3 x}{\sin x\cos x}}{\dfrac{\cos x-\sin x}{\sin x\cos x}}$$

$$=\frac{(\cos x-\sin x)\left(\cos^2 x+\sin x\cos x+\sin^2 x\right)}{\cos x-\sin x}$$

$$=1+\sin x\cos x \quad \textbf{ពិត}$$

ដូចនេះ $\boxed{\dfrac{\cos x\cot x-\sin x\tan x}{\dfrac{1}{\sin x}-\dfrac{1}{\cos x}}=1+\sin x\cos x}$

15

ចូរធ្វើឯងផ្ទាត់សមភាពខាងក្រោម៖

ក. $(\tan\theta-\sin\theta)^2+(1-\cos\theta)^2=\left(\dfrac{1}{\cos\theta}-1\right)^2$

ខ. $\dfrac{2\sin\theta\cos\theta-\cos\theta}{1-\sin\theta+\sin^2\theta-\cos^2\theta}=\dfrac{1}{\tan\theta}$

សម្រាយ.

ក. $(\tan\theta-\sin\theta)^2+(1-\cos\theta)^2=\left(\dfrac{1}{\cos\theta}-1\right)^2$

$$\begin{aligned}(\tan\theta-\sin\theta)^2+(1-\cos\theta)^2&=\tan^2\theta-2\tan\theta\sin\theta+\sin^2\theta+1-2\cos\theta+\cos^2\theta\\&=1+\tan^2\theta-\frac{2\sin\theta\cdot\sin\theta}{\cos\theta}+1-2\cos\theta\\&=\frac{1}{\cos^2\theta}-\frac{2\sin^2\theta}{\cos\theta}+1-2\cos\theta\\&=\frac{1}{\cos^2\theta}-2\left(\frac{\sin^2\theta}{\cos\theta}+\cos\theta\right)+1\\&=\frac{1}{\cos^2\theta}-2\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta}\right)+1\\&=\frac{1}{\cos^2\theta}-2\frac{1}{\cos\theta}+1=\left(\frac{1}{\cos\theta}-1\right)^2\quad\textbf{ពិត}\end{aligned}$$

ដូចនេះ $\boxed{(\tan\theta-\sin\theta)^2+(1-\cos\theta)^2=\left(\dfrac{1}{\cos\theta}-1\right)^2}$

ខ. $\dfrac{2\sin\theta\cos\theta-\cos\theta}{1-\sin\theta+\sin^2\theta-\cos^2\theta}=\dfrac{1}{\tan\theta}$

$$\frac{2\sin\theta\cos\theta-\cos\theta}{1-\sin\theta+\sin^2\theta-\cos^2\theta}=\frac{\cos\theta(2\sin\theta-1)}{-\sin\theta+\sin^2\theta+\sin^2\theta}=\frac{\cos\theta(2\sin\theta-1)}{\sin\theta(2\sin\theta-1)}$$

$$=\cot\theta=\frac{1}{\tan\theta}\quad\textbf{ពិត}$$

ដូចនេះ $\boxed{\dfrac{2\sin\theta\cos\theta-\cos\theta}{1-\sin\theta+\sin^2\theta-\cos^2\theta}=\dfrac{1}{\tan\theta}}$

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ចូរគណនាតម្លៃ

ក. $\cos 15^\circ$ គ. $\cos 75^\circ$ ង. $\sin \frac{5\pi}{12}$ ឆ. $\sin \frac{\pi}{12}$

ខ. $\sin 15^\circ$ ឃ. $\sin 75^\circ$ ច. $\cos \frac{5\pi}{12}$ ជ. $\cos \frac{\pi}{12}$

សម្រាយ.

គណនាតម្លៃ រៀបទី១ (ប្រើរូបមន្តផលបូក ផលដកមុំ)

ក. $\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$ **ដូចនេះ** $\boxed{\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}}$

ខ. $\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$ **ដូចនេះ** $\boxed{\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}}$

គ. $\cos 75^\circ = \cos(30^\circ + 45^\circ) = \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$ **ដូចនេះ** $\boxed{\cos 75^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}}$

ឃ. $\sin 75^\circ = \sin(30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$ **ដូចនេះ** $\boxed{\sin 75^\circ = \frac{\sqrt{2}+\sqrt{6}}{4}}$

ង. $\sin \frac{5\pi}{12} = \sin\left(\frac{2\pi+3\pi}{12}\right) = \sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \sin \frac{\pi}{6} \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos \frac{\pi}{6}$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$ **ដូចនេះ** $\boxed{\sin \frac{5\pi}{12} = \frac{\sqrt{2}+\sqrt{6}}{4}}$

ច. $\cos\dfrac{5\pi}{12}=\cos\left(\dfrac{2\pi+3\pi}{12}\right)=\cos\left(\dfrac{\pi}{6}+\dfrac{\pi}{4}\right)=\cos\dfrac{\pi}{6}\cos\dfrac{\pi}{4}-\sin\dfrac{\pi}{6}\sin\dfrac{\pi}{4}$

$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}-\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$ **ដូចនេះ** $\boxed{\cos\dfrac{5\pi}{12}=\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

ឆ. $\sin\dfrac{\pi}{12}=\sin\left(\dfrac{4\pi-3\pi}{12}\right)=\sin\left(\dfrac{\pi}{3}-\dfrac{\pi}{4}\right)=\sin\dfrac{\pi}{3}\cos\dfrac{\pi}{4}-\sin\dfrac{\pi}{4}\cos\dfrac{\pi}{3}$

$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$ **ដូចនេះ** $\boxed{\sin\dfrac{\pi}{12}=\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

ជ. $\cos\dfrac{\pi}{12}=\cos\left(\dfrac{4\pi-3\pi}{12}\right)=\cos\left(\dfrac{\pi}{3}-\dfrac{\pi}{4}\right)=\cos\dfrac{\pi}{3}\cos\dfrac{\pi}{4}+\sin\dfrac{\pi}{4}\sin\dfrac{\pi}{3}$

$=\dfrac{\sqrt{1}}{2}\cdot\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **ដូចនេះ** $\boxed{\cos\dfrac{\pi}{12}=\dfrac{\sqrt{6}+\sqrt{2}}{4}}$

របៀបទី២ (ប្រើរូបមន្ត មុំឌុប)

ក. $\cos 15^{\circ}$

តាមទំនាក់ទំនង $\cos 2\alpha=2\cos^2\alpha-1 \Rightarrow \cos^2\alpha=\dfrac{1}{2}(\cos 2\alpha+1)$

គេបាន $\cos^2 15^{\circ}=\dfrac{1}{2}(\cos 30^{\circ}+1)=\dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}+1\right)=\dfrac{2+\sqrt{3}}{4}$

$\Rightarrow \quad \cos 15^{\circ}=\sqrt{\dfrac{2+\sqrt{3}}{4}}=\dfrac{\sqrt{2+\sqrt{3}}}{2}$ **ដូចនេះ** $\boxed{\cos 15^{\circ}=\dfrac{\sqrt{2+\sqrt{3}}}{2}}$

ខ. $\sin 15^{\circ}$

តាមទំនាក់ទំនង $\cos 2\alpha=1-2\sin^2\alpha \Rightarrow \sin^2\alpha=\dfrac{1}{2}(1-\cos 2\alpha)$

គេបាន $\sin^2 15^{\circ}=\dfrac{1}{2}(1-\cos 30^{\circ})=\dfrac{1}{2}\left(1-\dfrac{\sqrt{3}}{2}\right)=\dfrac{2-\sqrt{3}}{4}$

$\Rightarrow \quad \sin 15^{\circ}=\sqrt{\dfrac{2-\sqrt{3}}{4}}=\dfrac{\sqrt{2-\sqrt{3}}}{2}$

ដូចនេះ $\boxed{\sin 15^{\circ}=\dfrac{\sqrt{2-\sqrt{3}}}{2}}$

គ. $\cos 75^{\circ}$

តាមទំនាក់ទំនង $\cos 2\alpha = 2\cos^2\alpha - 1 \quad \Rightarrow \cos^2\alpha = \frac{1}{2}(\cos 2\alpha + 1)$

គេបាន $\cos^2 75^{\circ} = \frac{1}{2}(\cos 150^{\circ} + 1) = \frac{1}{2}\left(\frac{-\sqrt{3}}{2} + 1\right) = \frac{2-\sqrt{3}}{4}$

$$\Rightarrow \quad \cos 75^{\circ} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

ដូចនេះ $\boxed{\cos 75^{\circ} = \frac{\sqrt{2-\sqrt{3}}}{2}}$

ឃ. $\sin 75^{\circ}$

តាមទំនាក់ទំនង $\cos 2\alpha = 1 - 2\sin^2\alpha \quad \Rightarrow \sin^2\alpha = \frac{1}{2}(1 - \cos 2\alpha)$

គេបាន $\sin^2 75^{\circ} = \frac{1}{2}(1 - \cos 150^{\circ}) = \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\right) = \frac{2+\sqrt{3}}{4}$

$$\Rightarrow \quad \sin 75^{\circ} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

ដូចនេះ $\boxed{\sin 75^{\circ} = \frac{\sqrt{2+\sqrt{3}}}{2}}$

ង. $\sin\frac{5\pi}{12}$

តាមទំនាក់ទំនង $\cos 2\alpha = 1 - 2\sin^2\alpha \quad \Rightarrow \sin^2\alpha = \frac{1}{2}(1 - \cos 2\alpha)$

គេបាន $\sin^2\frac{5\pi}{12} = \frac{1}{2}\left(1 - \cos\frac{2\times 5\pi}{12}\right) = \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\right) = \frac{2+\sqrt{3}}{4}$

$$\Rightarrow \quad \sin\frac{5\pi}{6} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

ដូចនេះ $\boxed{\sin\frac{5\pi}{6} = \frac{\sqrt{2+\sqrt{3}}}{2}}$

ច. $\cos\dfrac{5\pi}{12}$

តាមទំនាក់ទំនង $\cos 2\alpha = 2\cos^2\alpha - 1 \Rightarrow \cos^2\alpha = \dfrac{1}{2}(\cos 2\alpha + 1)$

គេបាន $\cos^2\dfrac{5\pi}{12} = \dfrac{1}{2}\left(\cos\dfrac{2\times 5\pi}{12}+1\right) = \dfrac{1}{2}\left(\dfrac{-\sqrt{3}}{2}+1\right) = \dfrac{2-\sqrt{3}}{4}$

$\Rightarrow \quad \cos\dfrac{5\pi}{12} = \sqrt{\dfrac{2-\sqrt{3}}{4}} = \dfrac{\sqrt{2-\sqrt{3}}}{2}$

ដូចនេះ $\boxed{\cos\dfrac{5\pi}{12} = \dfrac{\sqrt{2-\sqrt{3}}}{2}}$

ឆ. $\sin\dfrac{\pi}{12}$

តាមទំនាក់ទំនង $\cos 2\alpha = 1 - 2\sin^2\alpha \Rightarrow \sin^2\alpha = \dfrac{1}{2}(1-\cos 2\alpha)$

គេបាន $\sin^2\dfrac{\pi}{12} = \dfrac{1}{2}\left(1-\cos\dfrac{2\times\pi}{12}\right) = \dfrac{1}{2}\left(1-\dfrac{\sqrt{3}}{2}\right) = \dfrac{2-\sqrt{3}}{4}$

$\Rightarrow \quad \sin\dfrac{\pi}{12} = \sqrt{\dfrac{2-\sqrt{3}}{4}} = \dfrac{\sqrt{2-\sqrt{3}}}{2}$

ដូចនេះ $\boxed{\sin\dfrac{\pi}{12} = \dfrac{\sqrt{2-\sqrt{3}}}{2}}$

ជ. $\cos\dfrac{\pi}{12}$

តាមទំនាក់ទំនង $\cos 2\alpha = 2\cos^2\alpha - 1 \Rightarrow \cos^2\alpha = \dfrac{1}{2}(\cos 2\alpha + 1)$

គេបាន $\cos^2\dfrac{\pi}{12} = \dfrac{1}{2}\left(\cos\dfrac{2\times\pi}{12}+1\right) = \dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}+1\right) = \dfrac{2+\sqrt{3}}{4}$

$\Rightarrow \quad \cos\dfrac{\pi}{12} = \sqrt{\dfrac{2+\sqrt{3}}{4}} = \dfrac{\sqrt{2+\sqrt{3}}}{2}$

ដូចនេះ $\boxed{\cos\dfrac{\pi}{12} = \dfrac{\sqrt{2+\sqrt{3}}}{2}}$

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ក. បង្ហាញថា $\sin a + \cos a = \sqrt{2}\sin\left(a + \frac{\pi}{4}\right)$ ។

ខ. បង្ហាញថា $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} + \frac{\sin(\beta-\theta)}{\cos\beta\cos\theta} + \frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha} = 0$ ។

សម្រាយ.

ក. បង្ហាញថា $\sin a + \cos a = \sqrt{2}\sin\left(a + \frac{\pi}{4}\right)$

ដោយ $\sqrt{2}\sin\left(a + \frac{\pi}{4}\right) = \sqrt{2}\left(\sin a\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos a\right)$

$= \sqrt{2}\left(\frac{\sqrt{2}}{2}\sin a + \frac{\sqrt{2}}{2}\cos a\right)$

$= \sin a + \cos a$ ពិត

ដូចនេះ $\boxed{\sin a + \cos a = \sqrt{2}\sin\left(a + \frac{\pi}{4}\right)}$

ខ. បង្ហាញថា $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} + \frac{\sin(\beta-\theta)}{\cos\beta\cos\theta} + \frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha} = 0$

$\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} + \frac{\sin(\beta-\theta)}{\cos\beta\cos\theta} + \frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha}$

$= \frac{\sin\alpha\cos\beta - \sin\beta\cos\alpha}{\cos\alpha\cos\beta} + \frac{\sin\beta\cos\theta - \sin\theta\cos\beta}{\cos\beta\cos\theta} + \frac{\sin\theta\cos\alpha - \sin\alpha\cos\theta}{\cos\theta\cos\alpha}$

$= \frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta} + \frac{\sin\beta}{\cos\beta} - \frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta} - \frac{\sin\alpha}{\cos\alpha} = 0$ ពិត

ដូចនេះ $\boxed{\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} + \frac{\sin(\beta-\theta)}{\cos\beta\cos\theta} + \frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha} = 0}$

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១. បង្ហាញថា $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ ។

២. ចូរគណនាតម្លៃ

ក. $\tan 105^\circ$ ខ. $\tan\dfrac{\pi}{12}$ គ. $\sin 105^\circ$ ឃ. $\cos 105^\circ$

សម្រាយ.

១. បង្ហាញថា $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

ដោយ $\tan(\alpha+\beta)=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\dfrac{\sin\alpha\cos\beta+\sin\beta\cos\alpha}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$

$$=\frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\sin\beta\cos\alpha}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

ដូចនេះ $\boxed{\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}}$

២. ចូរគណនាតម្លៃ

ក. $\tan 105^\circ$

តាមរូបមន្ត $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

គេបាន $\tan 105^\circ=\tan(60^\circ+45^\circ)=\dfrac{\tan 60^\circ+\tan 45^\circ}{1-\tan 60^\circ\tan 45^\circ}=\dfrac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}$

$$=\frac{\sqrt{3}+1}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}}=\frac{1+2\sqrt{3}+3}{1-3}=-2-\sqrt{3}$$

ដូចនេះ $\boxed{\tan 105^\circ=-2-\sqrt{3}}$

ខ. $\tan\dfrac{\pi}{12}$

តាមរូបមន្ត $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

គេបាន $\tan\dfrac{\pi}{12}=\tan(\dfrac{\pi}{3}-\dfrac{\pi}{4})=\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}=\dfrac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1}$

$=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}=\dfrac{3-2\sqrt{3}+1}{3-1}=2-\sqrt{3}$

ដូចនេះ $\boxed{\tan\dfrac{\pi}{12}=2-\sqrt{3}}$

គ. $\sin 105^{\circ}$

តាមរូបមន្ត $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$

គេបាន $\sin(105^{\circ})=\sin(60^{\circ}+45^{\circ})=\sin 60^{\circ}\cos 45^{\circ}+\sin 45^{\circ}\cos 60^{\circ}$

$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

ដូចនេះ $\boxed{\sin 105^{\circ}=\dfrac{\sqrt{6}+\sqrt{2}}{4}}$

ឃ. $\cos 105^{\circ}$

តាមរូបមន្ត $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

គេបាន $\cos 105^{\circ}=\cos(60^{\circ}+45^{\circ})=\cos 60^{\circ}\cos 45^{\circ}-\sin 60^{\circ}\sin 45^{\circ}$

$=\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{2}-\sqrt{6}}{4}$

ដូចនេះ $\boxed{\cos 105^{\circ}=\dfrac{\sqrt{2}-\sqrt{6}}{4}}$

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១. គេមាន $0<\alpha<\frac{\pi}{2}$; $-\frac{\pi}{2}<\beta<0$ និង $\tan\alpha=2$; $\tan\beta=-3$ ។

ក. គណនា $\tan(\alpha-\beta)$ ខ. គណនាមុំ $(\alpha-\beta)$

២. ចូរសម្រួលកន្សោម $\dfrac{\tan 3\theta-\tan\theta}{1+\tan\theta\tan 3\theta}+\cot\left(\dfrac{\pi}{2}+2\theta\right)$ ។

សម្រាយ.

១. ក. គណនា $\tan(\alpha-\beta)$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\frac{2-(-3)}{1+2(-3)}=\frac{5}{-5}=-1$$

ដូចនេះ $\boxed{\tan(\alpha-\beta)=-1}$

ខ. គណនាមុំ $(\alpha-\beta)$

ដោយ $0<\alpha<\frac{\pi}{2}$ (1) ; $-\frac{\pi}{2}<\beta<0 \Leftrightarrow 0<-\beta<\frac{\pi}{2}$ (2)

$$\begin{cases}0<\alpha<\frac{\pi}{2}\\ 0<-\beta<\frac{\pi}{2}\end{cases}$$

$\overline{0<\alpha-\beta<\pi}$ មានន័យថា $(\alpha-\beta)$នៅកាន្ទ្រង់ទី I; II

គេបាន $\tan(\alpha-\beta)=-1$ លុះត្រាតែ $(\alpha-\beta)=\dfrac{3\pi}{4}$ ដូចនេះ $\boxed{(\alpha-\beta)=\dfrac{3\pi}{4}}$

២. ចូរសម្រួលកន្សោម $\dfrac{\tan 3\theta-\tan\theta}{1+\tan\theta\tan 3\theta}+\cot\left(\dfrac{\pi}{2}+2\theta\right)$

$$\frac{\tan 3\theta-\tan\theta}{1+\tan\theta\tan 3\theta}+\cot\left(\frac{\pi}{2}+2\theta\right)=\tan(3\theta-\theta)-\tan 2\theta=\tan 2\theta-\tan 2\theta=0$$

ដូចនេះ $\boxed{\dfrac{\tan 3\theta-\tan\theta}{1+\tan\theta\tan 3\theta}+\cot\left(\dfrac{\pi}{2}+2\theta\right)=0}$

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១. គណនា $\tan(a+b+c)$ ដោយប្រើរូបមន្ត $\tan(\alpha+\beta)$ ។

២. ផ្ទៀងផ្ទាត់សមភាព $\dfrac{1-\tan\theta}{1+\tan\theta}=\tan\left(\dfrac{\pi}{4}-\theta\right)$ ។

សម្រាយ.

១. គណនា $\tan(a+b+c)$ ដោយប្រើរូបមន្ត $\tan(\alpha+\beta)$

តាមរូបមន្ត $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ បើ $\alpha=a+b;\ \beta=c$

គេបាន $\tan(a+b+c)=\dfrac{\tan(a+b)+\tan c}{1-\tan(a+b)\tan c}=\dfrac{\dfrac{\tan a+\tan b}{1-\tan a\tan b}+\tan c}{1-\dfrac{\tan a+\tan b}{1-\tan a\tan b}\cdot\tan c}$

$$=\frac{\dfrac{\tan a+\tan b+\tan c(1-\tan a\tan b)}{1-\tan a\tan b}}{\dfrac{1-\tan a\tan b-(\tan a+\tan b)\tan c}{1-\tan a\tan b}}$$

$$=\frac{\tan a+\tan b+\tan c-\tan a\tan b\tan c}{1-\tan a\tan b-\tan a\tan c-\tan b\tan c}$$

ដូចនេះ $\boxed{\tan(a+b+c)=\dfrac{\tan a+\tan b+\tan c-\tan a\tan b\tan c}{1-\tan a\tan b-\tan a\tan c-\tan b\tan c}}$

២. ផ្ទៀងផ្ទាត់សមភាព $\dfrac{1-\tan\theta}{1+\tan\theta}=\tan\left(\dfrac{\pi}{4}-\theta\right)$

ដោយ $\tan\left(\dfrac{\pi}{4}-\theta\right)=\dfrac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}=\dfrac{1-\tan\theta}{1+\tan\theta}$ ពិត

ដូចនេះ $\boxed{\dfrac{1-\tan\theta}{1+\tan\theta}=\tan\left(\dfrac{\pi}{4}-\theta\right)}$

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១. រក $\sin 2\alpha;\ \cos 2\alpha$ និង $\tan 2\alpha$ បើ $\cos\alpha = -\frac{4}{5}$ និង $\frac{\pi}{2} < \alpha < \pi$ ។

២. គណនា $\sin 3\alpha$ និង $\cos 3\alpha$ ។

៣. បង្ហាញថា $\dfrac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)} = 1$ ។

សម្រាយ.

១. រក $\sin 2\alpha;\ \cos 2\alpha$ និង $\tan 2\alpha$ បើ $\cos\alpha = -\frac{4}{5}$ និង $\frac{\pi}{2} < \alpha < \pi$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\sin\alpha\left(-\frac{4}{5}\right) = -\frac{8}{5}\sin\alpha$$

តាមរូបមន្ត $\sin^2\alpha = 1-\cos^2\alpha = 1-\left(-\frac{4}{5}\right) = 1-\frac{16}{25} = \frac{9}{25}$

$$\Longrightarrow \sin\alpha = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5} \quad \text{តែ } \frac{\pi}{2} < \alpha < \pi;\ \sin\alpha > 0 \text{ គេបាន } \sin\alpha = +\frac{3}{5}$$

$\sin 2\alpha = -\frac{8}{5}\sin\alpha = -\frac{8}{5}\cdot\frac{3}{5} = -\frac{24}{25}$ **ដូចនេះ** $\boxed{\sin 2\alpha = -\frac{24}{25}}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$ **ដូចនេះ** $\boxed{\cos 2\alpha = \frac{7}{25}}$

$\tan 2\alpha = \dfrac{\sin 2\alpha}{\cos 2\alpha} = \dfrac{\frac{-24}{25}}{\frac{7}{25}} = -\frac{24}{7}$ **ដូចនេះ** $\boxed{\tan 2\alpha = -\frac{24}{7}}$

២. គណនា $\sin 3\alpha$ និង $\cos 3\alpha$

$$\sin 3\alpha = \sin(2\alpha+\alpha) = \sin 2\alpha\cos\alpha + \sin\alpha\cos 2\alpha = \left(-\frac{24}{25}\right)\left(-\frac{4}{5}\right) + \frac{3}{5}\left(\frac{7}{25}\right)$$

$$= \frac{96}{125} + \frac{21}{125} = \frac{117}{125} \quad \textbf{ដូចនេះ} \quad \boxed{\sin 3\alpha = \frac{117}{125}}$$

$$\cos 3\alpha = \cos(2\alpha+\alpha) = \cos 2\alpha\cos\alpha - \sin 2\alpha\sin\alpha = \frac{7}{25}\left(-\frac{4}{5}\right) - \left(-\frac{24}{25}\right)\left(\frac{3}{5}\right)$$

$$= \frac{-28}{125} + \frac{72}{125} = \frac{44}{125} \quad \textbf{ដូចនេះ} \quad \boxed{\cos 3\alpha = \frac{44}{125}}$$

៣. បង្ហាញថា $\dfrac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)} = 1$

$$\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)} = \frac{1-2\sin^2\alpha}{\frac{2\cos\left(\frac{\pi}{4}+\alpha\right)}{\sin\left(\frac{\pi}{4}+\alpha\right)}\left(\cos\frac{\pi}{4}\cos\alpha+\sin\frac{\pi}{4}\sin\alpha\right)^2}$$

$$= \frac{\cos^2\alpha+\sin^2\alpha-2\sin^2\alpha}{\frac{2\left(\cos\frac{\pi}{4}\cos\alpha-\sin\frac{\pi}{4}\sin\alpha\right)}{\sin\frac{\pi}{4}\cos\alpha+\sin\alpha\cos\frac{\pi}{4}}\left(\frac{\sqrt{2}}{2}\cos\alpha+\frac{\sqrt{2}}{2}\sin\alpha\right)^2}$$

$$= \frac{\cos^2\alpha-\sin^2\alpha}{\frac{2\left(\frac{\sqrt{2}}{2}\cos\alpha-\frac{\sqrt{2}}{2}\sin\alpha\right)}{\frac{\sqrt{2}}{2}\cos\alpha+\frac{\sqrt{2}}{2}\sin\alpha}\left(\frac{\sqrt{2}}{2}\cos\alpha+\frac{\sqrt{2}}{2}\sin\alpha\right)^2}$$

$$= \frac{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}{\sqrt{2}(\cos\alpha-\sin\alpha)\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)}$$

$$= \frac{1}{\sqrt{2}\cdot\frac{\sqrt{2}}{2}} = 1 \quad \text{ពិត}$$

ដូចនេះ $\boxed{\dfrac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)} = 1}$

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១. គណនា $\sin 22^\circ 30'$; $\sin\dfrac{\pi}{12}$ និង $\tan\dfrac{3\pi}{8}$

២. ធ្វើឱ្យផ្ទាត់សមភាព $\dfrac{1-\cos x}{\sin x} = \tan\dfrac{x}{2}$

សម្រាយ.

១. គណនា $\sin 22^{\circ}30'$; $\sin\dfrac{\pi}{12}$ និង $\tan\dfrac{3\pi}{8}$

$(30' = 30' \cdot \dfrac{1^{\circ}}{60'} = \dfrac{1^{\circ}}{2})$ គេបាន

$$\sin 22^{\circ}30' = \sin(22^{\circ} + 30') = \sin(22^{\circ} + \frac{1^{\circ}}{2}) = \sin\left(\frac{45^{\circ}}{2}\right)$$

ដោយ $\cos\alpha = 1 - 2\sin^2\dfrac{\alpha}{2}$ គេបាន $\cos 45^{\circ} = 1 - 2\sin^2\dfrac{45^{\circ}}{2}$

$$\Rightarrow \sin^2\frac{45^{\circ}}{2} = \frac{1-\cos 45^{\circ}}{2} = \frac{1-\dfrac{\sqrt{2}}{2}}{2} = \frac{2-\sqrt{2}}{4}$$

$$\Rightarrow \sin\frac{45^{\circ}}{2} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

ដូចនេះ $\boxed{\sin 22^{\circ}30' = \sin\dfrac{45^{\circ}}{2} = \dfrac{\sqrt{2-\sqrt{2}}}{2}}$

- $\sin^2\dfrac{\pi}{12} = \sin^2\left(\dfrac{1}{2}\cdot\dfrac{\pi}{6}\right) = \dfrac{1-\cos\frac{\pi}{6}}{2} = \dfrac{1-\frac{\sqrt{3}}{2}}{2} = \dfrac{2-\sqrt{3}}{4} \Rightarrow \boxed{\sin\dfrac{\pi}{12} = \dfrac{\sqrt{2-\sqrt{3}}}{2}}$
- $\tan\dfrac{3\pi}{8} = \tan\left(\dfrac{1}{2}\cdot\dfrac{3\pi}{4}\right) = \dfrac{\sin\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}} = \dfrac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \dfrac{\sqrt{2}}{2-\sqrt{2}} = \boxed{\sqrt{2}+1}$

២. ធ្វើឱ្យផ្ទៀងផ្ទាត់សមភាព $\dfrac{1-\cos x}{\sin x} = \tan\dfrac{x}{2}$

$$\frac{1-\cos x}{\sin x} = \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \tan\frac{x}{2} \quad \text{ពិត}$$

ដូចនេះ $\boxed{\dfrac{1-\cos x}{\sin x} = \tan\dfrac{x}{2}}$

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១. គេមាន $\cos\alpha = -\frac{3}{5}$ ហើយ $\frac{\pi}{2} < \alpha < \pi$ ។ គណនា $\sin\frac{\alpha}{2}$; $\cos\frac{\alpha}{2}$ ។

២. គណនា $\tan\frac{\alpha}{2}$ ដោយស្គាល់ $\tan\alpha = \frac{24}{7}$ និង $180^\circ < \alpha < 270^\circ$ ។

៣. ចូរគណនាកន្សោមខាងក្រោម៖

ក. $\cos 75^\circ \cos 45^\circ$ ខ. $\sin\frac{5\pi}{12}\sin\frac{\pi}{4}$

សម្រាយ.

១. គណនា $\sin\frac{\alpha}{2}$; $\cos\frac{\alpha}{2}$ $\left(\cos\alpha = -\frac{3}{5} \;;\; \frac{\pi}{2} < \alpha < \pi\right)$

តាមរូបមន្ត $\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2}$ $\Rightarrow \sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} = \frac{1-\left(-\frac{3}{5}\right)}{2} = \frac{8}{10} = \frac{4}{5}$

$\Rightarrow \sin\frac{\alpha}{2} = \pm\sqrt{\frac{4}{5}} = \pm\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \pm\frac{2\sqrt{5}}{5}$

តែ $\frac{\pi}{2} < \alpha < \pi \Leftrightarrow \frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$; $\sin\frac{\alpha}{2} > 0$ នាំឲ្យ $\sin\frac{\alpha}{2} = \frac{2\sqrt{5}}{5}$

ដូចនេះ $\boxed{\sin\frac{\alpha}{2} = \frac{2\sqrt{5}}{5}}$

តាមរូបមន្ត $\cos^2\frac{\alpha}{2} = 1 - \sin^2\frac{\alpha}{2} = 1 - \left(\frac{2\sqrt{5}}{5}\right)^2 = 1 - \frac{4}{5} = \frac{1}{5}$

$\Longrightarrow \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1}{5}} = \pm\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \pm\frac{\sqrt{5}}{5}$

តែ $\frac{\pi}{2} < \alpha < \pi \Leftrightarrow \frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$; $\cos\frac{\alpha}{2} > 0$ នាំឲ្យ $\cos\frac{\alpha}{2} = \frac{\sqrt{5}}{5}$

ដូចនេះ $\boxed{\cos\frac{\alpha}{2} = \frac{\sqrt{5}}{5}}$

២. គណនា $\tan\frac{\alpha}{2}$ $\left(\tan\alpha=\frac{24}{7};\quad 180^{\circ}<\alpha<270^{\circ}\right)$

តាមរូបមន្ត $\tan\alpha=\frac{2t}{1-t^2}$ ដែល $t=\tan\frac{\alpha}{2}$

គេបាន $1-t^2=\frac{2t}{\tan\alpha}\Rightarrow 1-t^2=\frac{2t}{\frac{24}{7}}\Leftrightarrow -t^2-\frac{7}{12}t+1=0\Leftrightarrow 12t^2+7t-12=0$

$\Delta=b^2-4ac=(7)^2-4(12)(-12)=49+576=625$

$t_1=\frac{-b-\sqrt{\Delta}}{2a}=\frac{-7-\sqrt{625}}{2(12)}=\frac{-7-25}{24}=\frac{-32}{24}=\frac{-4}{3}$

$t_2=\frac{-b+\sqrt{\Delta}}{2a}=\frac{-7+25}{24}=\frac{18}{24}=\frac{3}{4}$

គេបាន $\tan\frac{\alpha}{2}=-\frac{4}{3};\quad \tan\frac{\alpha}{2}=\frac{3}{4}$

តែដោយ $180^{\circ}<\alpha<270^{\circ}\Leftrightarrow\frac{180^{\circ}}{2}<\frac{\alpha}{2}<\frac{270^{\circ}}{2}\Leftrightarrow 90^{\circ}<\frac{\alpha}{2}<135^{\circ}$

គេបាន $\frac{\alpha}{2}$ នៅកាន្ត្រង់ទី II គឺ $\tan\frac{\alpha}{2}<0$ **ដូចនេះ** $\boxed{\tan\frac{\alpha}{2}=-\frac{4}{3}}$

៣. គណនាកន្សោម៖

- រូបមន្ត $\cos\alpha\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$
- រូបមន្ត $\sin\alpha\sin\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

ក. $\cos 75^{\circ}\cos 45^{\circ}=\frac{1}{2}[\cos(75^{\circ}+45^{\circ})+\cos(75^{\circ}-45^{\circ})]=\frac{1}{2}(\cos 120^{\circ}+\cos 30^{\circ})$

$=\frac{1}{2}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}-1}{4}$

ដូចនេះ $\boxed{\cos 75^{\circ}\cos 45^{\circ}=\frac{\sqrt{3}-1}{4}}$

ខ. $\sin\frac{5\pi}{12}\sin\frac{\pi}{4}=\frac{1}{2}\left[\cos\left(\frac{5\pi}{12}-\frac{\pi}{4}\right)-\cos\left(\frac{5\pi}{12}+\frac{\pi}{4}\right)\right]$

$=\frac{1}{2}\left(\cos\frac{\pi}{6}-\cos\frac{2\pi}{3}\right)=\frac{1}{2}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)=\frac{\sqrt{3}+1}{4}$

ដូចនេះ $\boxed{\sin\frac{5\pi}{12}\sin\frac{\pi}{4}=\frac{\sqrt{3}+1}{2}}$

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ក. ចូរសរសេរកន្សោម $1+\cos a+\cos 2a$ ជាផលគុណកត្តា។

ខ. ចូរផ្ទៀងផ្ទាត់សមភាព $2\left(\frac{1}{\sin 2\alpha}+\cot 2\alpha\right)=\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}$ ។

សម្រាយ.

ក. សរសេរកន្សោម $1+\cos a+\cos 2a$ ជាផលគុណកត្តា

$$1+\cos a+\cos 2a=1+\cos a+2\cos^2 a-1=\cos a(1+2\cos a)$$

ដូចនេះ $\boxed{1+\cos a+\cos 2a=\cos a(1+2\cos a)}$

ខ. ផ្ទៀងផ្ទាត់សមភាព $2\left(\frac{1}{\sin 2\alpha}+\cot 2\alpha\right)=\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}$

$$2\left(\frac{1}{\sin 2\alpha}+\cot 2\alpha\right)=2\left(\frac{1}{\sin 2\alpha}+\frac{\cos 2\alpha}{\sin 2\alpha}\right)=2\left(\frac{2\cos^2\alpha}{2\sin\alpha\cos\alpha}\right)=2\left(\frac{\cos\alpha}{\sin\alpha}\right)$$
$$=2\left(\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right)=\frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}-\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}=\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}$$

ដូចនេះ $\boxed{2\left(\frac{1}{\sin 2\alpha}+\cot 2\alpha\right)=\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}}$

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១. គេមាន A, B, C ជាមុំក្នុងត្រីកោណមួយ។

ចូរបង្ហាញថា $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ ។

២. A, B, C ជាមុំក្នុងត្រីកោណមួយ។

ចូរបង្ហាញថា $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ ។

សម្រាយ.

១. បង្ហាញថា $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

- (រូបមន្ត $\sin p+\sin q=2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$)
- (រូបមន្ត $\cos p+\cos q=2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$)
- (រូបមន្ត $\cos p-\cos q=-2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$)
- A,B,C ជាមុំក្នុងត្រីកោណ គេបាន $A+B+C=180^o \Rightarrow C=180^o-(A+B)$

$$\Rightarrow A+B=180^o-C$$

$$\begin{aligned}
\sin A+\sin B+\sin C &= 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+\sin(180^o-(A+B))\\
&= 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+\sin(A+B)\\
&= 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+\sin\frac{2(A+B)}{2}\\
&= 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+2\sin\frac{A+B}{2}\cos\frac{A+B}{2}\\
&= 2\sin\frac{A+B}{2}\left(\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right)\\
&= 2\sin\frac{180^o-C}{2}\left(2\cos\frac{\frac{A-B}{2}+\frac{A+B}{2}}{2}\cos\frac{\frac{A-B}{2}-\frac{A+B}{2}}{2}\right)\\
&= 4\sin\left(90^o-\frac{C}{2}\right)\left(\cos\frac{A}{2}\cos\left(\frac{-B}{2}\right)\right)\\
&= 4\cos\frac{C}{2}\left(\cos\frac{A}{2}\cos\frac{B}{2}\right)=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} \quad \text{ពិត}
\end{aligned}$$

ដូចនេះ $\boxed{\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$

២. បង្ហាញថា $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$$\begin{aligned}
\cos A+\cos B+\cos C &= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}+\cos(180^o-(A+B))\\
&= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}-\cos(A+B)\\
&= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}-\cos\frac{2(A+B)}{2}\\
&= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}-\left(2\cos^2\frac{A+B}{2}-1\right)\\
&= 2\cos\frac{A+B}{2}\left(\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right)+1\\
&= 2\cos\frac{180^o-C}{2}\left(-2\sin\frac{\frac{A-B}{2}+\frac{A+B}{2}}{2}\sin\frac{\frac{A-B}{2}-\frac{A+B}{2}}{2}\right)+1\\
&= -4\cos\left(90^o-\frac{C}{2}\right)\sin\frac{A}{2}\sin\left(-\frac{B}{2}\right)+1\\
&= 4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}+1 \quad \text{ពិត}
\end{aligned}$$

ដូចនេះ $\boxed{\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$

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គេមាន $\sin\alpha+\cos\beta=\frac{5}{4}$ និង $\cos\alpha+\sin\beta=\frac{5}{4}$ ហើយ $\left(0\le\alpha\le\frac{\pi}{2}\right);\left(0\le\beta\le\frac{\pi}{2}\right)$។
គណនា $\sin(\alpha+\beta)$ និង $\tan(\alpha+\beta)$ ។

សម្រាយ.

គណនា $\sin(\alpha+\beta)$ និង $\tan(\alpha+\beta)$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$ យើងមាន $\sin\alpha+\cos\beta=\frac{5}{4}$ និង $\cos\alpha+\sin\beta=\frac{5}{4}$

$$\text{គេបាន}\quad (\sin\alpha+\cos\beta)^2=\left(\tfrac{5}{4}\right)^2\Leftrightarrow \sin^2\alpha+2\sin\alpha\cos\beta+\cos^2\beta=\frac{25}{16}\quad(1)$$

$$(\cos\alpha+\sin\beta)^2=\left(\tfrac{5}{4}\right)^2\Leftrightarrow \cos^2\alpha+2\cos\alpha\sin\beta+\sin^2\beta=\frac{25}{16}\quad(2)$$

យក $(1)+(2)$ គេបាន $1+2(\sin\alpha\cos\beta+\cos\alpha\sin\beta)+1=\frac{50}{16}$

$$\Longrightarrow 2(\sin\alpha\cos\beta+\cos\alpha\sin\beta)=\frac{25}{8}-2$$

$$\Longrightarrow \sin\alpha\cos\beta+\cos\alpha\sin\beta=\frac{9}{16}$$

ដូចនេះ $\boxed{\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha=\frac{9}{16}}$

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

ដោយ $\cos^2(\alpha+\beta)=1-\sin^2(\alpha+\beta)=1-\left(\frac{9}{16}\right)^2=1-\frac{81}{256}=\frac{175}{256}$

$$\Longrightarrow \cos(\alpha+\beta)=\pm\sqrt{\frac{175}{256}}=\pm\frac{5\sqrt{7}}{16}$$

ដោយ $\sin\alpha+\cos\beta=\cos\alpha+\sin\beta=\frac{5}{4}$ គេបាន $\alpha=\beta$

- បើ $0\leq\alpha\leq\frac{\pi}{4} \Rightarrow 0\leq\beta\leq\frac{\pi}{4} \Rightarrow 0\leq\alpha+\beta\leq\frac{\pi}{2} \Rightarrow \cos(\alpha+\beta)>0$
- បើ $\frac{\pi}{4}\leq\alpha\leq\frac{\pi}{2} \Rightarrow \frac{\pi}{4}\leq\beta\leq\frac{\pi}{2} \Rightarrow \frac{\pi}{2}\leq\alpha+\beta\leq\pi \Rightarrow \cos(\alpha+\beta)<0$

គេបាន $\cos(\alpha+\beta)=\pm\frac{5\sqrt{7}}{16}$

នាំឲ្យ $\tan(\alpha+\beta)=\dfrac{\frac{9}{16}}{\pm\frac{5\sqrt{7}}{16}}=\pm\frac{9}{5\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\pm\frac{9\sqrt{7}}{35}$

ដូច្នេះ $\boxed{\tan(\alpha+\beta)=\pm\frac{9\sqrt{7}}{35}}$

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១. គេមាន $\sin\alpha=\frac{1}{2}$; $\left(0<\alpha<\frac{\pi}{2}\right)$ និង $\sin\beta=\frac{1}{3}$; $\left(\frac{\pi}{2}<\beta<\pi\right)$ ។

ចូរគណនា ៖

ក. $\sin(\alpha+\beta)$ គ. $\tan(\alpha-\beta)$

ខ. $\cos(\alpha-\beta)$ ឃ. $\cot(\alpha-\beta)$ ។

២. គេមាន $\cos\theta=-\frac{2}{3}$, $\left(\frac{\pi}{2}<\theta<\pi\right)$ ។ គណនា $\cos 2\theta$, $\sin\frac{\theta}{2}$, $\sin 3\theta$ ។

សម្រាយ.

១. ក. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha=\frac{1}{2}\cos\beta+\frac{1}{3}\cos\alpha$

រក $\cos\alpha;\ \cos\beta$

$$\cos^2\alpha=1-\sin\alpha=1-\left(\frac{1}{2}\right)^2=1-\frac{1}{4}=\frac{3}{4}$$

$$\Rightarrow\ \cos\alpha=\pm\sqrt{\frac{3}{4}}=\pm\frac{\sqrt{3}}{2}\qquad \text{ដោយ } 0<\alpha<\frac{\pi}{2};\quad \cos\alpha>0$$

$$\Rightarrow\ \cos\alpha=\frac{\sqrt{3}}{2}$$

$$\cos^2\beta=1-\sin^2\beta=1-\left(\frac{1}{3}\right)^2=1-\frac{1}{9}=\frac{8}{9}$$

$$\Rightarrow\ \cos\beta=\pm\sqrt{\frac{8}{9}}=\pm\frac{2\sqrt{2}}{3}\qquad \text{ដោយ } \frac{\pi}{2}<\beta<\pi\ ;\ \cos\beta<0$$

$$\Rightarrow\ \cos\beta=-\frac{2\sqrt{2}}{3}$$

គេបាន $\sin(\alpha+\beta)=\frac{1}{2}\cdot\left(-\frac{2\sqrt{2}}{3}\right)+\frac{1}{3}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{3}-2\sqrt{2}}{6}$

ដូចនេះ $\boxed{\sin(\alpha+\beta)=\frac{\sqrt{3}-2\sqrt{2}}{6}}$

ខ. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=\frac{\sqrt{3}}{2}\cdot\frac{-2\sqrt{2}}{3}+\frac{1}{2}\cdot\frac{1}{3}=\frac{1-2\sqrt{6}}{6}$

ដូចនេះ $\boxed{\cos(\alpha-\beta)=\frac{1-2\sqrt{6}}{6}}$

គ. $\tan(\alpha-\beta)=\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin\alpha\cos\beta-\sin\beta\cos\alpha}{\frac{1-2\sqrt{6}}{6}}=\frac{\frac{-\sqrt{3}-2\sqrt{2}}{6}}{\frac{1-2\sqrt{6}}{6}}$

$=\frac{-\sqrt{3}-2\sqrt{2}}{1-2\sqrt{6}}\times\frac{1+2\sqrt{6}}{1+2\sqrt{6}}=\frac{-\sqrt{3}-2\sqrt{18}-2\sqrt{2}-4\sqrt{12}}{1-4(6)}$

$=\frac{-\sqrt{3}-6\sqrt{2}-2\sqrt{2}-8\sqrt{3}}{-23}=\frac{9\sqrt{3}+8\sqrt{2}}{23}$

ដូចនេះ $\boxed{\tan(\alpha-\beta)=\frac{9\sqrt{3}+8\sqrt{2}}{23}}$

ឃ. $\cot(\alpha-\beta)=\frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)}=\frac{\frac{1-2\sqrt{6}}{6}}{\frac{-\sqrt{3}-2\sqrt{2}}{6}}=\frac{1-2\sqrt{6}}{-\sqrt{3}-2\sqrt{2}}\times\frac{-\sqrt{3}+2\sqrt{2}}{-\sqrt{3}+2\sqrt{2}}$

$=\frac{-\sqrt{3}+2\sqrt{2}+2\sqrt{18}-4\sqrt{12}}{3-4(2)}=\frac{-\sqrt{3}+2\sqrt{2}+6\sqrt{2}-8\sqrt{3}}{-5}$

$=\frac{9\sqrt{3}-8\sqrt{2}}{5}$

ដូចនេះ $\boxed{\cot(\alpha-\beta)=\frac{9\sqrt{3}-8\sqrt{2}}{5}}$

២. គណនា $\cos 2\theta$

$$\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{-2}{3}\right)^2 - 1 = \frac{8}{9} - 1 = \frac{-1}{9} \quad \text{ដូចនេះ} \quad \boxed{\cos 2\theta = -\frac{1}{9}}$$

គណនា $\sin\dfrac{\theta}{2}$

$$\cos\theta = 1 - 2\sin^2\frac{\theta}{2} \quad \Rightarrow \sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2} = \frac{1-\left(-\frac{2}{3}\right)}{2} = \frac{5}{6}$$

$$\Rightarrow \sin\frac{\theta}{2} = \pm\sqrt{\frac{5}{6}} = \pm\frac{\sqrt{30}}{6} \quad \text{តែ } \frac{\pi}{2} < \theta < \pi \Leftrightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2} \; ; \sin\frac{\theta}{2} > 0$$

$$\Rightarrow \sin\frac{\theta}{2} = \frac{\sqrt{30}}{6} \quad \text{ដូចនេះ} \quad \boxed{\sin\frac{\theta}{2} = \frac{\sqrt{30}}{6}}$$

គណនា $\sin 3\theta$

$$\sin 3\theta = \sin(\theta + 2\theta) = \sin\theta\cos 2\theta + \sin 2\theta\cos\theta = -\frac{1}{9}\cdot\sin\theta - \frac{2}{3}\sin 2\theta$$

រក $\sin\theta \; ; \; \sin 2\theta$

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\frac{2}{3}\right)^2 = 1 - \frac{4}{9} = \frac{5}{9} \quad \Rightarrow \sin\theta = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{3}$$

តែ $\dfrac{\pi}{2} < \theta < \pi \; ; \; \sin\theta > 0$

$$\Rightarrow \sin\theta = \frac{\sqrt{5}}{3}$$

$$\sin^2 2\theta = 1 - \cos^2 2\theta = 1 - \left(-\frac{1}{9}\right)^2 = \frac{80}{81} \quad \Rightarrow \sin 2\theta = \pm\sqrt{\frac{80}{81}} = \pm\frac{4\sqrt{5}}{9}$$

តែ $\dfrac{\pi}{2} < \theta < \pi \Leftrightarrow \pi < 2\theta < 2\pi \; ; \; \sin 2\theta < 0$

$$\Rightarrow \sin 2\theta = -\frac{4\sqrt{5}}{9}$$

គេបាន $\sin 3\theta = -\dfrac{1}{9}\cdot\dfrac{\sqrt{5}}{3} - \dfrac{2}{3}\cdot\left(-\dfrac{4\sqrt{5}}{9}\right) = -\dfrac{\sqrt{5}}{27} + \dfrac{8\sqrt{5}}{27} = \dfrac{7\sqrt{5}}{27}$

ដូចនេះ $\boxed{\sin 3\theta = \dfrac{7\sqrt{5}}{27}}$

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គេមាន $t=\tan\frac{\theta}{2}$ $(t\neq\pm1)$; $\sin\theta=\frac{2t}{1+t^2}$; $\cos\theta=\frac{1-t^2}{1+t^2}$ និង $\tan\theta=\frac{2t}{1-t^2}$ ។

តាមសមភាព $2\cos2\theta-\cos\theta+2=0$ ។ ចូរគណនា $\tan\frac{\theta}{2}$ ដែល $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$ ។

សម្រាយ.

គណនា $\tan\frac{\theta}{2}$ ដែល $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

យើងមាន $2\cos\theta-\cos\theta+2=0\Leftrightarrow 2\left(2\cos^2\theta-1\right)-\cos\theta+2=0$

$$\Leftrightarrow 4\cos^2\theta-\cos\theta=0$$

$$\Leftrightarrow \cos\theta(4\cos\theta-1)=0$$

$$\Rightarrow \left[\begin{array}{l}\cos\theta=0\\ 4\cos\theta-1=0\end{array}\right. \Rightarrow \left[\begin{array}{l}\cos\theta=0\\ \cos\theta=\frac{1}{4}\end{array}\right.$$

ដោយ $\cos\theta=\frac{1-t^2}{1+t^2}$; ហើយ $t=\tan\frac{\theta}{2}$ $(t\neq\pm1)$ គេបាន

- ចំពោះ $\cos\theta=0\Rightarrow\frac{1-t^2}{1+t^2}=0\Leftrightarrow 1-t^2=0\Rightarrow t=\pm1$ មិនយក
- ចំពោះ $\cos\theta=\frac{1}{4}\Rightarrow\frac{1-t^2}{1+t^2}=\frac{1}{4}\Leftrightarrow 4-4t^2=1+t^2\Leftrightarrow 5t^2=3$

$$\Rightarrow t=\pm\sqrt{\frac{3}{5}}$$

គេបាន $\tan\frac{\theta}{2}=t=\pm\sqrt{\frac{3}{5}}=\pm\frac{\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\pm\frac{\sqrt{15}}{5}$

ដូចនេះ $\boxed{\tan\frac{\theta}{2}=\pm\frac{\sqrt{15}}{5}}$

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គេមាន $\theta = 36^o$ និង $2\theta = 180^o - 3\theta$ ។ តាមលក្ខណៈនេះ គណនា $\cos 36^o$ ។

សម្រាយ.

គេមាន $\theta = 36^o$ និង $2\theta = 180^o - 3\theta$ តាមលក្ខណៈនេះ គណនា $\cos 36^o$

- $\theta = 36 \quad \Rightarrow \quad \cos\theta = \cos 36^o$
- $2\theta = 180^o - 3\theta \quad \Rightarrow \quad \sin 2\theta = \sin(180^o - 3\theta) = \sin 3\theta$

$$\Leftrightarrow \quad 2\sin\theta\cos\theta = 3\sin\theta - 4\sin^3\theta$$

$$\Leftrightarrow \quad 2\cos\theta = 3 - 4\sin^2\theta$$

$$\Leftrightarrow \quad 2\cos\theta = 3 - 4\left(1 - \cos^2\theta\right)$$

$$\Leftrightarrow \quad 4\cos^2\theta - 2\cos\theta - 1 = 0$$

តាង $t = \cos\theta \quad ; \quad t > 0$ ព្រោះ $\cos\theta = \cos 36^o > 0$

$4t^2 - 2t - 1 = 0 \quad \Rightarrow \Delta = b^2 - 4ac = (-2)^2 - 4(4)(-1) = 20 \qquad \sqrt{\Delta} = \sqrt{20} = 2\sqrt{5}$

$$\Rightarrow t_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-(-2) - 2\sqrt{5}}{2(4)} = \frac{1 - \sqrt{5}}{4} < 0 \quad \text{មិនយក}$$

$$t_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-(-2) + 2\sqrt{5}}{2(4)} = \frac{1 + \sqrt{5}}{4} > 0 \quad \text{យក}$$

ដូចនេះ $\boxed{\cos 36^o = \cos\theta = \frac{1 + \sqrt{5}}{4}}$

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ចូរបង្ហាញសមភាពខាងក្រោម៖

ក. $\cot(a \pm b) = \dfrac{\cot a \cot b \mp 1}{\cot b \pm \cot a}$

ខ. $\tan(a+b) - \tan a - \tan b = \tan a \tan b \tan(a+b)$

គ. $\dfrac{\sin^4 \alpha + 2\sin\alpha\cos\alpha - \cos^4 \alpha}{\tan 2\alpha - 1} = \cos 2\alpha$

សម្រាយ.

បង្ហាញសមភាព

ក. $\cot(a \pm b) = \dfrac{\cot a \cot b \mp 1}{\cot b \pm \cot a}$

$$\cot(a \pm b) = \frac{\cos(a \pm b)}{\sin(a \pm b)} = \frac{\cos a \cos b \mp \sin a \sin b}{\sin a \cos b \pm \sin b \cos a} = \frac{\dfrac{\cos a \cos b}{\sin a \sin b} \mp \dfrac{\sin a \sin b}{\sin a \sin b}}{\dfrac{\sin a \cos b}{\sin a \sin b} \pm \dfrac{\sin b \cos a}{\sin a \sin b}}$$

$$= \frac{\cot a \cot b \mp 1}{\cot b \pm \cot a} \quad \text{ពិត}$$

ដូចនេះ $\boxed{\cot(a \pm b) = \dfrac{\cot a \cot b \mp 1}{\cot b \pm \cot a}}$

ខ. $\tan(a+b) - \tan a - \tan b = \tan a \tan b \tan(a+b)$

$$\tan(a+b) - \tan a - \tan b = \frac{\tan a + \tan b}{1 - \tan a \tan b} - \tan a - \tan b$$

$$= \frac{\tan a + \tan b - \tan a(1 - \tan a \tan b) - \tan b(1 - \tan a \tan b)}{1 - \tan a \tan b}$$

$$= \frac{\tan a + \tan b - \tan a + \tan^2 a \tan b - \tan b + \tan a \tan^2 b}{1 + \tan a \tan b}$$

$$= \frac{\tan a \tan b (\tan a + \tan b)}{1 - \tan a \tan b}$$

$$= \tan a \tan b \tan(a+b) \quad \text{ពិត}$$

ដូចនេះ $\boxed{\tan(a+b) - \tan a - \tan b = \tan a \tan b \tan(a+b)}$

គ. $\dfrac{\sin^4\alpha+2\sin\alpha\cos\alpha-\cos^4\alpha}{\tan 2\alpha-1}=\cos 2\alpha$

$$\frac{\sin^4\alpha+2\sin\alpha\cos\alpha-\cos^4\alpha}{\tan 2\alpha-1}=\frac{\left(\sin^2\alpha-\cos^2\alpha\right)\left(\sin^2\alpha+\cos^2\alpha\right)+2\sin\alpha\cos\alpha}{\tan 2\alpha-1}$$

$$=\frac{-\left(\cos^2\alpha-\sin^2\alpha\right)\cdot 1+\sin 2\alpha}{\tan 2\alpha-1}$$

$$=\frac{-\cos 2\alpha+\sin 2\alpha}{\frac{\sin 2\alpha-\cos 2\alpha}{\cos 2\alpha}}$$

$$=\cos 2\alpha \quad \text{ពិត}$$

ដូចនេះ $\boxed{\dfrac{\sin^4\alpha+2\sin\alpha\cos\alpha-\cos^4\alpha}{\tan 2\alpha-1}=\cos 2\alpha}$

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ចូរសម្រួលកន្សោមខាងក្រោម៖

ក. $\sin 4x-4\sin 3x+6\sin 2x-4\sin x$

ខ. $\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta)-\cos 2\alpha\cos 2\beta$

គ. $\dfrac{\sin\alpha+\sin 3\alpha+\sin 5\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha}$

ឃ. $\dfrac{\sin\alpha+\sin 3\alpha+\sin 5\alpha+\sin 7\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha+\cos 7\alpha}$

សម្រាយ.

ក. $\sin 4x-4\sin 3x+6\sin 2x-4\sin x$

$$\sin 4x-4\sin 3x+6\sin 2x-4\sin x=\sin 4x+6\sin 2x-4(\sin 3x+\sin x)$$

$$=2\sin 2x\cos 2x+6\sin 2x-4\cdot 2\cdot\sin 2x\cos x$$

$$=2\sin 2x(\cos 2x+3-4\cos x)$$

$$= 2\sin 2x\left(2\cos^2 x - 1 + 3 - 4\cos x\right)$$
$$= 2\sin 2x\left(2\cos^2 x + 2 - 4\cos x\right)$$
$$= 4\sin 2x\left(\cos^2 x - 2\cos x + 1\right)$$
$$= 4\sin 2x(\cos x - 1)^2$$
$$= 4\sin 2x\left(1 - 2\sin^2\frac{x}{2} - 1\right)^2$$
$$= 16\sin 2x\sin^2\frac{x}{2}$$

ដូចនេះ $\boxed{\sin 4x - 4\sin 3x + 6\sin 2x - 4\sin x = 16\sin 2x\sin^2\frac{x}{2}}$

ខ. $\cos^2(\alpha+\beta) + \cos^2(\alpha-\beta) - \cos 2\alpha\cos 2\beta$

តាមរូបមន្ត $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\Rightarrow \cos^2(\alpha+\beta) = (\cos\alpha\cos\beta - \sin\alpha\sin\beta)^2$

$= \cos^2\alpha\cos^2\beta - 2\cos\alpha\cos\beta\sin\alpha\sin\beta + \sin^2\alpha\sin^2\beta$

តាមរូបមន្ត $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\Rightarrow \cos^2(\alpha-\beta) = (\cos\alpha\cos\beta + \sin\alpha\sin\beta)^2$

$= \cos^2\alpha\cos^2\beta + 2\cos\alpha\cos\beta\sin\alpha\sin\beta + \sin^2\alpha\sin^2\beta$

យើងបាន $\cos^2(\alpha+\beta) + \cos^2(\alpha-\beta) - \cos 2\alpha\cos 2\beta$

$= 2\cos^2\alpha\cos^2\beta + 2\sin^2\alpha\sin^2\beta - \cos 2\alpha\cos 2\beta$

$= 2\left(\frac{1+\cos 2\alpha}{2}\right)\left(\frac{1+\cos 2\beta}{2}\right) + 2\left(\frac{1-\cos 2\alpha}{2}\right)\left(\frac{1-\cos 2\beta}{2}\right) - \cos 2\alpha\cos 2\beta$

$= \frac{2 + 2\cos 2\alpha\cos 2\beta}{2} - \cos 2\alpha\cos 2\beta$

$= 1 + \cos 2\alpha\cos 2\beta - \cos 2\alpha\cos 2\beta$

$= 1$

ដូចនេះ $\boxed{\cos^2(\alpha+\beta) + \cos^2(\alpha-\beta) - \cos 2\alpha\cos 2\beta = 1}$

គ. $\dfrac{\sin\alpha+\sin 3\alpha+\sin 5\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha}=\dfrac{\sin\alpha+\sin 5\alpha+\sin 3\alpha}{\cos\alpha+\cos 5\alpha+\cos 3\alpha}$

$=\dfrac{2\sin 3\alpha\cos 2\alpha+\sin 3\alpha}{2\cos 3\alpha\cos 2\alpha+\cos 3\alpha}$

$=\dfrac{\sin 3\alpha(2\cos 2\alpha+1)}{\cos 3\alpha(2\cos 2\alpha+1)}$

$=\tan 3\alpha$

ដូចនេះ $\boxed{\dfrac{\sin\alpha+\sin 3\alpha+\sin 5\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha}=\tan 3\alpha}$

ឃ. $\dfrac{\sin\alpha+\sin 3\alpha+\sin 5\alpha+\sin 7\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha+\cos 7\alpha}=\dfrac{\sin\alpha+\sin 7\alpha+\sin 3\alpha+\sin 5\alpha}{\cos\alpha+\cos 7\alpha+\cos 3\alpha+\cos 5\alpha}$

$=\dfrac{2\sin 4\alpha\cos 3\alpha+2\sin 4\alpha\cos\alpha}{2\cos 4\alpha\cos 3\alpha+2\cos 4\alpha\cos\alpha}$

$=\dfrac{\sin 4\alpha(2\cos 3\alpha+2\cos\alpha)}{\cos 4\alpha(2\cos 3\alpha+2\cos\alpha)}$

$=\tan 4\alpha$

ដូចនេះ $\boxed{\dfrac{\sin\alpha+\sin 3\alpha+\sin 5\alpha+\sin 7\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha+\cos 7\alpha}=\tan 4\alpha}$

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ចូរបង្ហាញសមភាពខាងក្រោម៖

ក. $\sin 3a=4\sin a\sin\left(\dfrac{\pi}{3}+a\right)\sin\left(\dfrac{\pi}{3}-a\right)$

ខ. $\cos 3a=4\cos a\cos\left(\dfrac{\pi}{3}+a\right)\cos\left(\dfrac{\pi}{3}-a\right)$

គ. $4\sin\left(\alpha+\dfrac{\pi}{3}\right)\sin\left(\alpha-\dfrac{\pi}{3}\right)=4\sin^2\alpha-3$

ឃ. $\sin a-\cos a=\sqrt{2}\sin\left(a-\dfrac{\pi}{4}\right)=-\sqrt{2}\cos\left(a+\dfrac{\pi}{4}\right)$

ឯ. $\dfrac{\cos^3\alpha-\cos 3\alpha}{\cos\alpha}+\dfrac{\sin^3\alpha+\sin 3\alpha}{\sin\alpha}=3$

បង្ហាញសមភាពខាងក្រោម៖

ក. $\sin 3a = 4\sin a\sin\left(\dfrac{\pi}{3}+a\right)\sin\left(\dfrac{\pi}{3}-a\right)$

យើងមាន $4\sin a\sin\left(\dfrac{\pi}{3}+a\right)\sin\left(\dfrac{\pi}{3}-a\right)$

$= 4\sin a\left(\sin\dfrac{\pi}{3}\cos a+\sin a\cos\dfrac{\pi}{3}\right)\left(\sin\dfrac{\pi}{3}\cos a-\sin a\cos\dfrac{\pi}{3}\right)$

$= 4\sin a\left(\sin^2\dfrac{\pi}{3}\cos^2 a-\sin^2 a\cos^2\dfrac{\pi}{3}\right)$

$= 4\sin a\left(\dfrac{3}{4}\cos^2 a-\dfrac{1}{4}\sin^2 a\right)$

$= 3\sin a\cos^2 a-\sin^3 a$

$= 3\sin a\left(1-\sin^2 a\right)-\sin^3 a$

$= 3\sin a-4\sin^3 a \quad (1)$

ដោយ $\sin 3a=\sin(a+2a) = \sin a\cos 2a+\sin 2a\cos a$

$= \sin a\left(1-2\sin^2 a\right)+2\sin a\cos a\cos a$

$= \sin a-2\sin^3 a+2\sin a\left(1-\sin^2 a\right)$

$= \sin a-2\sin^3 a+2\sin a-2\sin^3 a$

$= 3\sin a-4\sin^3 a \quad (2)$

តាម (1) និង (2) ដូចនេះ $\boxed{\sin 3a = 4\sin a\sin\left(\dfrac{\pi}{3}+a\right)\sin\left(\dfrac{\pi}{3}-a\right)}$

ខ. $\cos 3a = 4\cos a\cos\left(\dfrac{\pi}{3}+a\right)\cos\left(\dfrac{\pi}{3}-a\right)$

យើងមាន $4\cos a\cos\left(\dfrac{\pi}{3}+a\right)\cos\left(\dfrac{\pi}{3}-a\right)$

$= 4\cos a\left(\cos\dfrac{\pi}{3}\cos a-\sin\dfrac{\pi}{3}\sin a\right)\left(\cos\dfrac{\pi}{3}\cos a+\sin\dfrac{\pi}{3}\sin a\right)$

$$= 4\cos a\left(\cos^2\frac{\pi}{3}\cos^2 a - \sin^2\frac{\pi}{3}\sin^2 a\right)$$
$$= 4\cos a\left(\frac{1}{4}\cos^2 a - \frac{3}{4}\sin^2 a\right)$$
$$= \cos^3 a - 3\cos a\sin^2 a$$
$$= \cos^3 a - 3\cos a\left(1-\cos^2 a\right)$$
$$= \cos^3 a - 3\cos a + 3\cos^3 a$$
$$= 4\cos^3 a - 3\cos a \quad (1)$$

ដោយ $\cos 3a = \cos(a+2a) = \cos a\cos 2a - \sin a\sin 2a$
$$= \cos a\left(2\cos^2 a - 1\right) - 2\sin^2 a\cos a$$
$$= 2\cos^3 a - \cos a - 2\left(1-\cos^2 a\right)\cos a$$
$$= 2\cos^3 a - \cos a - 2\cos a + 2\cos^3 a$$
$$= 4\cos^3 a - 3\cos a \quad (2)$$

តាម (1) និង (2) ដូចនេះ $\boxed{\cos 3a = 4\cos a\cos\left(\frac{\pi}{3}+a\right)\cos\left(\frac{\pi}{3}-a\right)}$

គ. $4\sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha-\frac{\pi}{3}\right) = 4\sin^2\alpha - 3$

យើងមាន $4\sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha-\frac{\pi}{3}\right)$
$$= 4\left(\sin\alpha\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\alpha\right)\left(\sin\alpha\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\alpha\right)$$
$$= 4\left(\sin^2\alpha\cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}\cos^2\alpha\right)$$
$$= 4\left(\frac{1}{4}\sin^2\alpha - \frac{3}{4}\cos^2\alpha\right)$$
$$= \sin^2\alpha - 3\left(1-\sin^2\alpha\right)$$
$$= 4\sin^2\alpha - 3$$

ដូចនេះ $\boxed{4\sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha-\frac{\pi}{3}\right) = 4\sin^2\alpha - 3}$

ឃ. $\sin a - \cos a = \sqrt{2}\sin\left(a - \frac{\pi}{4}\right) = -\sqrt{2}\cos\left(a + \frac{\pi}{4}\right)$

- $\sqrt{2}\sin\left(a - \frac{\pi}{4}\right) = \sqrt{2}\left(\sin a\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos a\right)$

$= \sqrt{2}\left(\frac{\sqrt{2}}{2}\sin a - \frac{\sqrt{2}}{2}\cos a\right)$

$= \sin a - \cos a \quad (1)$

- $-\sqrt{2}\cos\left(a + \frac{\pi}{4}\right) = -\sqrt{2}\left(\cos a\cos\frac{\pi}{4} - \sin a\sin\frac{\pi}{4}\right)$

$= -\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos a - \frac{\sqrt{2}}{2}\sin a\right)$

$= \sin a - \cos a \quad (2)$

តាម (1) និង (2) ដូចនេះ $\boxed{\sin a - \cos a = \sqrt{2}\sin\left(a - \frac{\pi}{4}\right) = -\sqrt{2}\cos\left(a + \frac{\pi}{4}\right)}$

ង. $\frac{\cos^3\alpha - \cos 3\alpha}{\cos\alpha} + \frac{\sin^3\alpha + \sin 3\alpha}{\sin\alpha} = 3$

យើងមាន

- $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$
- $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

យើងបាន $\frac{\cos^3\alpha - \cos 3\alpha}{\cos\alpha} + \frac{\sin^3\alpha + \sin 3\alpha}{\sin\alpha}$

$= \frac{\cos^3\alpha - 4\cos^3\alpha + 3\cos\alpha}{\cos\alpha} + \frac{\sin^3\alpha + 3\sin\alpha - 4\sin^3\alpha}{\sin\alpha}$

$= \frac{3\cos\alpha\left(1 - \cos^2\alpha\right)}{\cos\alpha} + \frac{3\sin\alpha\left(1 - \sin^2\alpha\right)}{\sin\alpha}$

$= 3 - 3\cos^2\alpha + 3 - 3\sin^2\alpha$

$= 6 - 3\left(\cos^2\alpha + \sin^2\alpha\right) = 6 - 3 = 3$

ដូចនេះ $\boxed{\frac{\cos^3\alpha - \cos 3\alpha}{\cos\alpha} + \frac{\sin^3\alpha + \sin 3\alpha}{\sin\alpha} = 3}$

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ចូរបង្ហាញថាត្រីកោណដែលផ្ទៀងផ្ទាត់សមភាពខាងក្រោម៖

ក. $\sin A = \dfrac{\sin B + \sin C}{\cos B + \cos C}$

ខ. $\dfrac{\sin C}{\cos B} = \sin A + \cos A \cot C$ ជាត្រីកោណកែង ។

សម្រាយ.

ក. បង្ហាញថា ΔABC ផ្ទៀងផ្ទាត់ $\sin A = \dfrac{\sin B + \sin C}{\cos B + \cos C}$ ជាត្រីកោណកែង

$$\sin A = \frac{\sin B + \sin C}{\cos B + \cos C} = \frac{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{2\cos\frac{B+C}{2}\cos\frac{B-C}{2}}$$

ដោយ $A + B + C = 180^o \quad \Rightarrow B + C = 180^o - A \quad \Rightarrow \dfrac{B+C}{2} = 90^o - \dfrac{A}{2}$

$$\Rightarrow \quad \sin A = \frac{2\sin\left(90^o - \frac{A}{2}\right)\cos\frac{B-C}{2}}{2\cos\left(90^o - \frac{A}{2}\right)\cos\frac{B-C}{2}} = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \cot\frac{A}{2}$$

បើយក $t = \tan\dfrac{A}{2} > 0 \quad \Rightarrow \quad \begin{cases} \sin A = \dfrac{2t}{1+t^2} \\ \cot\dfrac{A}{2} = \dfrac{1}{t} \end{cases}$

$$\sin A = \cot\frac{A}{2} \quad \Leftrightarrow \quad \frac{2t}{1+t^2} = \frac{1}{t}$$

$$2t(t) = 1(1+t^2)$$

$$2t^2 = 1 + t^2$$

$$t^2 = 1 \quad \Rightarrow \quad t = \pm 1; \quad t = -1 \text{ មិនយក}; \; t = 1 \text{ យក}$$

$$t = \tan\frac{A}{2} \quad ; \quad t = 1 \quad \Rightarrow \tan\frac{A}{2} = 1$$

$$\tan\frac{A}{2} = \tan\frac{\pi}{4} \Rightarrow \quad \frac{A}{2} = \frac{\pi}{4} \quad \Rightarrow \quad A = \frac{\pi}{2}$$

ដោយ មុំ $A = \dfrac{\pi}{2} = 90^o$ ដូចនេះ $\boxed{\Delta ABC \text{ ជាត្រីកោណកែងត្រង់ } A}$

២. បង្ហាញថា ΔABC ដែលផ្ទៀងផ្ទាត់ $\frac{\sin C}{\cos B} = \sin A + \cos A \cot C$ ជាត្រីកោណកែង

$$\frac{\sin C}{\cos B} = \sin A + \cos A \cot C \quad \Leftrightarrow \quad \frac{\sin C}{\cos B} - \sin A = \cos A \cdot \cot C$$

តាមទ្រឹស្តីបទស៊ីនុស និងកូស៊ីនុសក្នុងត្រីកោណ ABC មាន៖

$$\sin A = \frac{a}{2R} \quad ; \quad \sin C = \frac{c}{2R} \quad ; \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

យើងបាន

$$\frac{\sin C}{\cos B} - \sin A = \cos A \cot C \quad \Leftrightarrow \quad \frac{\frac{c}{2R}}{\frac{a^2+c^2-b^2}{2ac}} - \frac{a}{2R} = \cos A \cot C$$

$$\Leftrightarrow \quad \frac{2ac^2}{2R\left(a^2 + c^2 - b^2\right)} - \frac{a}{2R} = \cos A \cot C$$

$$\Leftrightarrow \quad \frac{2ac^2 - a\left(a^2 + c^2 - b^2\right)}{2R\left(a^2 + c^2 - b^2\right)} = \cos A \cot C$$

$$\Leftrightarrow \quad \frac{a\left(2c^2 - a^2 - c^2 + b^2\right)}{2R\left(a^2 + c^2 - b^2\right)} = \cos A \cot C$$

$$\Leftrightarrow \quad \frac{a\left(b^2 + c^2 - a^2\right)}{2R\left(a^2 + c^2 - b^2\right)} = \cos A \cot C$$

$$\Leftrightarrow \quad \sin A \cdot \frac{2bc \cos A}{2ac \cos B} = \cos A \cot C$$

$$\Leftrightarrow \quad \frac{b}{a} \cdot \frac{\sin A \cos A}{\cos B} = \cos A \cot C \quad (1)$$

ដោយ $\frac{a}{\sin A} = \frac{b}{\sin B} \quad \Rightarrow \quad \frac{b}{a} = \frac{\sin B}{\sin A}$

$$(1) \Rightarrow \quad \frac{\sin B}{\sin A} \cdot \frac{\sin A \cos A}{\cos B} = \cos A \cot C$$

$$\Leftrightarrow \quad \cos A \cot B = \cos A \cot C$$

$$\Leftrightarrow \quad \cos A\left(\cot B - \cot C\right) = 0 \Rightarrow \left[\begin{array}{ll} \cos A = 0 & (*) \\ \cot B - \cot C = 0 & \end{array}\right.$$

$(*) \Rightarrow \quad \cos A = 0 \quad \Leftrightarrow \quad \cos A = \cos\frac{\pi}{2} \quad \Rightarrow \quad A = \frac{\pi}{2}$

ដោយ មុំ $A = \frac{\pi}{2} = 90^o$ ដូចនេះ $\boxed{\Delta ABC \text{ ជាត្រីកោណកែងត្រង់ } A}$

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គេមាន ΔABC និង $A,\ B,\ C$ ជាមុំក្នុងត្រីកោណនេះ។

ក. បើគេដឹងថា $\sin A + \sin B + \sin C = 1$ គណនា $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ ។

ខ. បង្ហាញថា $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$ ។

សម្រាយ.

ក. គណនា $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

ដោយ $\sin A + \sin B + \sin C = 1 \Leftrightarrow \quad 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} + \sin C = 1 \quad (1)$

ដោយ $A + B + C = 180^o \quad \Rightarrow \frac{A+B}{2} = 90^o - \frac{C}{2}$

$C = 180^o - (A + B)$

$$
\begin{aligned}
(1) \Rightarrow \quad & 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} + \sin C = 1 \\
\Leftrightarrow \quad & 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} + \sin(180^o - (A+B)) = 1 \\
\Leftrightarrow \quad & 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} + \sin(A+B) = 1 \\
\Leftrightarrow \quad & 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} + 2\sin\frac{A+B}{2}\cos\frac{A+B}{2} = 1 \\
\Leftrightarrow \quad & 2\sin\frac{A+B}{2}\left(\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right) = 1 \\
\Leftrightarrow \quad & 2\sin\left(90^o - \frac{C}{2}\right)\left(2\cos\frac{\frac{A-B}{2} + \frac{A+B}{2}}{2}\cos\frac{\frac{A-B}{2} - \frac{A+B}{2}}{2}\right) = 1
\end{aligned}
$$

$$\Leftrightarrow \quad 4\cos\frac{C}{2}\cos\frac{A}{2}\cos\frac{B}{2}=1 \quad \Rightarrow \quad \cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=\frac{1}{4}$$

ដូចនេះ $\boxed{\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=\frac{1}{4}}$

ខ. បង្ហាញថា $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$

$$\begin{aligned}
\cos^2 A+\cos^2 B+\cos^2 C &= \frac{1+\cos 2A}{2}+\frac{1+\cos 2B}{2}+\cos^2 C\\
&= \frac{2+\cos 2A+\cos 2B}{2}+\cos^2 C\\
&= 1+\frac{2\cos\frac{2A+2B}{2}\cos\frac{2A-2B}{2}}{2}+\cos^2 C\\
&= 1+\cos(A+B)\cos(A-B)+\cos^2 C\\
&= 1+\cos(180^\circ-C)\cos(A-B)+\cos^2 C\\
&= 1-\cos C\cos(A-B)+\cos^2 C\\
&= 1-\cos C[\cos(A-B)-\cos C]\\
&= 1-\cos C[\cos(A-B)-\cos(180^\circ-(A+B))]\\
&= 1-\cos C[\cos(A-B)+\cos(A+B)]\\
&= 1-\cos C(\cos A\cos B+\sin A\sin B+\cos A\cos B-\sin A\sin B)\\
&= 1-2\cos C\cos A\cos B
\end{aligned}$$

ដូចនេះ $\boxed{\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C}$

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ចូរគណនារង្វាស់មុំនៃត្រីកោណ ABC បើគេដឹងថា៖

ក. $\sin 3A+\sin 3B+\sin 3C=0$

ខ. $\sin 5A+\sin 5B+\sin 5C=0$

គ. $\sin 6A+\sin 6B+\sin 6C=0$ ។

សម្រាយ.

គណនារង្វាស់មុំនៃត្រីកោណ ABC បើគេដឹងថា៖

ក. $\sin 3A + \sin 3B + \sin 3C = 0$

យើងមាន

$$\begin{aligned}
\sin 3A + \sin 3B + \sin 3C &= 2\sin\frac{3A+3B}{2}\cos\frac{3A-3B}{2} + \sin 3C \\
&= 2\sin\frac{3}{2}(A+B)\cos\frac{3}{2}(A-B) + \sin 3(180^\circ-(A+B)) \\
&= 2\sin\frac{3}{2}(A+B)\cos\frac{3}{2}(A-B) + \sin 3(A+B) \\
&= 2\sin\frac{3}{2}(A+B)\cos\frac{3}{2}(A-B) + 2\sin\frac{3}{2}(A+B)\cos\frac{3}{2}(A+B) \\
&= 2\sin\frac{3}{2}(A+B)\left[\cos\frac{3}{2}(A-B) + \cos\frac{3}{2}(A+B)\right] \\
&= 4\sin\left(\frac{3\pi}{2}-\frac{3C}{2}\right)\cos\frac{3A}{2}\cos\frac{3B}{2} \\
&= 4\sin\left(\pi+\frac{\pi}{2}-\frac{3C}{2}\right)\cos\frac{3A}{2}\cos\frac{3B}{2} \\
&= -4\sin\left(\frac{\pi}{2}-\frac{3C}{2}\right)\cos\frac{3A}{2}\cos\frac{3B}{2} \\
&= -4\cos\frac{3C}{2}\cos\frac{3A}{2}\cos\frac{3B}{2}
\end{aligned}$$

$$\sin 3A + \sin 3B + \sin 3C = 0 \Leftrightarrow -4\cos\frac{3C}{2}\cos\frac{3A}{2}\cos\frac{3B}{2} = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \cos\dfrac{3C}{2} = 0 \\ \cos\dfrac{3A}{2} = 0 \\ \cos\dfrac{3B}{2} = 0 \end{array}\right.$$

- ករណី $\cos\dfrac{3C}{2} = 0 \quad \Leftrightarrow \quad \dfrac{3C}{2} = \dfrac{\pi}{2} \quad \Rightarrow \quad C = \dfrac{\pi}{3}$

$$\Longrightarrow \quad A + B = 180^\circ - C = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \quad (*)$$

$$\Longrightarrow \quad \sin 3A + \sin 3B + \sin 3C = 0$$

$$\Leftrightarrow \quad \sin 3A + \sin 3B + \sin 3\left(\frac{\pi}{3}\right) = 0$$

$$\Leftrightarrow \quad 2\sin\frac{3A+3B}{2}\cos\frac{3A-3B}{2} = 0$$

$$\Leftrightarrow \quad \left[\begin{array}{lll} \sin\dfrac{3A+3B}{2} = 0 & \Leftrightarrow & \dfrac{3(A+B)}{2} = \pi \quad (1) \\ \cos\dfrac{3A-3B}{2} = 0 & \Leftrightarrow & \dfrac{3(A-B)}{2} = \pm\dfrac{\pi}{2} \quad (2) \end{array}\right.$$

$$(1) : A+B = \frac{2\pi}{3} \quad \text{តែ} \quad (*) : \ A+B = \frac{2\pi}{3} \quad \Rightarrow \quad \left\{\begin{array}{l} A+B = \dfrac{2\pi}{3} \\ A+B = \dfrac{2\pi}{3} \end{array}\right.$$

$$\Rightarrow \quad A, B \in \mathbb{R} \text{ ដែល } 0 < A, B < \frac{2\pi}{3}$$

$$(2) : A-B = \pm\frac{\pi}{3} \quad \text{តែ} \quad (*) : \ A+B = \frac{2\pi}{3} \quad \Rightarrow \quad \left\{\begin{array}{l} A-B = \pm\dfrac{\pi}{3} \\ A+B = \dfrac{2\pi}{3} \end{array}\right.$$

$$\Rightarrow \quad 2A = \pm\frac{\pi}{3} + \frac{2\pi}{3}$$

$$\Rightarrow \quad A = \frac{\pi}{2} \text{ ឬ } A = \frac{\pi}{6}$$

☞ $A = \dfrac{\pi}{2} \quad \Rightarrow \quad \dfrac{\pi}{2} + B = \dfrac{2\pi}{3} \quad \Rightarrow \quad B = \dfrac{\pi}{6}$

☞ $A = \dfrac{\pi}{6} \quad \Rightarrow \quad \dfrac{\pi}{6} + B = \dfrac{2\pi}{3} \quad \Rightarrow \quad B = \dfrac{\pi}{2}$

ដូចនេះ

☞ $C = \dfrac{\pi}{3};\ A = \dfrac{\pi}{2};\ B = \dfrac{\pi}{6}$

☞ $C = \dfrac{\pi}{3};\ A = \dfrac{\pi}{6};\ B = \dfrac{\pi}{2}$

☞ $C = \dfrac{\pi}{3};\ A, B \in \mathbb{R}$ ដែល $0 < A, B < \dfrac{2\pi}{3}$

• ករណី $\cos\dfrac{3A}{2} = 0$ និង $\cos\dfrac{3B}{2} = 0$ (ដោះស្រាយដូចគ្នា)

ខ. $\sin 5A + \sin 5B + \sin 5C = 0$

គ. $\sin 6A + \sin 6B + \sin 6C = 0$

សម្រាប់សំណួរ ខ. និង គ. ធ្វើសម្រាយតាមលំនាំសំណួរ ក. ។

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos x = \frac{1}{2}$ ខ. $\cos\left(3x - \frac{\pi}{6}\right) = -1$

សម្រាយ.

ដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos x = \frac{1}{2} \Leftrightarrow \boxed{x = \pm\frac{\pi}{3} + 2k\pi \quad ; k \in \mathbb{Z}}$

ខ. $\cos\left(3x - \frac{\pi}{6}\right) = -1 \Leftrightarrow 3x - \frac{\pi}{6} = \pm\pi + 2k\pi$

$\Leftrightarrow 3x = \frac{\pi}{6} \pm \pi + 2k\pi$

$\Leftrightarrow x = \frac{\pi}{18} \pm \frac{\pi}{3} + \frac{2k\pi}{3} \Leftrightarrow \boxed{\begin{cases} x = \frac{7\pi}{18} + \frac{2k\pi}{3} \\ x = -\frac{5\pi}{18} + \frac{2k\pi}{3} \end{cases} ; k \in \mathbb{Z}}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos\left(x - \frac{\pi}{4}\right) = \sin 3x$

ខ. $\sin^4 x - \cos^4 x = \frac{1}{2}$

គ. $\cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = \cos\frac{\pi}{4}$

ឃ. $1 + 3\cos x + \cos 2x = \cos 3x + 2\sin x \cdot \sin 2x$

សម្រាយ.

ដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos\left(x-\frac{\pi}{4}\right)=\sin 3x \Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\cos\left(\frac{\pi}{2}-3x\right)$

$$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{2}-3x+2k\pi \\ x-\frac{\pi}{4}=-\left(\frac{\pi}{2}-3x\right)+2k\pi \end{array}\right. ; \quad k\in\mathbb{Z}$$

$$\Leftrightarrow \boxed{\left[\begin{array}{l} x=\frac{3\pi}{16}+\frac{k\pi}{2} \\ x=\frac{\pi}{8}-k\pi \end{array}\right. ; \quad k\in\mathbb{Z}}$$

ខ. $\sin^4 x-\cos^4 x=\frac{1}{2} \Leftrightarrow \left(\sin^2 x-\cos^2 x\right)\left(\sin^2 x+\cos^2 x\right)=\frac{1}{2}$

$$\Leftrightarrow -\left(\cos^2 x-\sin^2 x\right)=\frac{1}{2}$$

$$\Leftrightarrow \cos 2x=-\frac{1}{2}$$

$$\Leftrightarrow \cos 2x=\cos\frac{2\pi}{3}$$

$$\Rightarrow \left[\begin{array}{l} 2x=\frac{2\pi}{3}+2k\pi \\ 2x=-\frac{2\pi}{3}+2k\pi \end{array}\right. ; \quad k\in\mathbb{Z}$$

$$\Rightarrow \boxed{\left[\begin{array}{l} x=\frac{\pi}{3}+k\pi \\ x=-\frac{\pi}{3}+k\pi \end{array}\right. ; \quad k\in\mathbb{Z}}$$

គ. $\cos\frac{\pi}{6}\cos x-\sin\frac{\pi}{6}\sin x=\cos\frac{\pi}{4}\Leftrightarrow\cos\left(\frac{\pi}{6}-x\right)=\cos\frac{\pi}{4}$

$$\Leftrightarrow\left[\begin{array}{l}\frac{\pi}{6}-x=\frac{\pi}{4}+2k\pi\\ \frac{\pi}{6}-x=-\frac{\pi}{4}+2k\pi\end{array}\right.;\quad k\in\mathbb{Z}$$

$$\Leftrightarrow\boxed{\left[\begin{array}{l}x=-\frac{\pi}{12}-2k\pi\\ x=\frac{5\pi}{12}-2k\pi\end{array}\right.;\quad k\in\mathbb{Z}}$$

ឃ. $1+3\cos x+\cos 2x=\cos 3x+2\sin x\cdot\sin 2x$

$\Leftrightarrow\quad 3\cos x+2\cos^2 x=\cos 3x+\cos x-\cos 3x$

$\Leftrightarrow\quad 2\cos x(1+\cos x)=0$

$$\Rightarrow\left[\begin{array}{l}\cos x=0\\ 1+\cos x=0\end{array}\right.\Rightarrow\boxed{\left[\begin{array}{l}x=\pm\frac{\pi}{2}+2k\pi\\ x=\pm\pi+2k\pi\end{array}\right.;\quad k\in\mathbb{Z}}$$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin x=\frac{\sqrt{2}}{2}$

ខ. $\cos\left(3x+\frac{\pi}{3}\right)=\cos\left(x-\frac{\pi}{6}\right)$

គ. $\sin\left(2x+\frac{\pi}{4}\right)=1$

ឃ. $\sin 2x=\sin\left(\frac{\pi}{3}-x\right)$

សម្រាយ. ក. $\sin x=\frac{\sqrt{2}}{2}\Leftrightarrow\sin x=\sin\frac{\pi}{4}\Leftrightarrow\left[\begin{array}{l}x=\frac{\pi}{4}+2k\pi\\ x=\pi-\frac{\pi}{4}+2k\pi\end{array}\right.;\quad k\in\mathbb{Z}$

$$\Leftrightarrow\boxed{\left[\begin{array}{l}x=\frac{\pi}{4}+2k\pi\\ x=\frac{3\pi}{4}+2k\pi\end{array}\right.;\quad k\in\mathbb{Z}}$$

ខ. $\cos\left(3x+\frac{\pi}{3}\right)=\cos\left(x-\frac{\pi}{6}\right) \Leftrightarrow \left[\begin{array}{l} 3x+\frac{\pi}{3}=x-\frac{\pi}{6}+2k\pi \\ 3x+\frac{\pi}{3}=-\left(x-\frac{\pi}{6}\right)+2k\pi \end{array}\right. ; \quad k\in\mathbb{Z}$

$\Leftrightarrow \boxed{\left[\begin{array}{l} x=-\frac{\pi}{4}+k\pi \\ x=-\frac{\pi}{24}+\frac{k\pi}{2} \end{array}\right. ; \quad k\in\mathbb{Z}}$

គ. $\sin\left(2x+\frac{\pi}{4}\right)=1 \Leftrightarrow \sin\left(2x+\frac{\pi}{4}\right)=\sin\frac{\pi}{2}$

$\Leftrightarrow \left[\begin{array}{l} 2x+\frac{\pi}{4}=\frac{\pi}{2}+2k\pi \\ 2x+\frac{\pi}{4}=\pi-\frac{\pi}{2}+2k\pi \end{array}\right. ; \quad k\in\mathbb{Z}$

$\Leftrightarrow \boxed{\left[\begin{array}{l} x=\frac{\pi}{8}+k\pi \\ x=-\frac{\pi}{8}+k\pi \end{array}\right. ; \quad k\in\mathbb{Z}}$

ឃ. $\sin 2x=\sin\left(\frac{\pi}{3}-x\right) \Leftrightarrow \left[\begin{array}{l} 2x=\frac{\pi}{3}-x+2k\pi \\ 2x=\pi-\left(\frac{\pi}{3}-x\right)+2k\pi \end{array}\right. ; \quad k\in\mathbb{Z}$

$\Leftrightarrow \boxed{\left[\begin{array}{l} x=\frac{\pi}{9}+\frac{2k\pi}{3} \\ x=\frac{2\pi}{3}+2k\pi \end{array}\right. ; \quad k\in\mathbb{Z}}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin\frac{x}{2}\cos\frac{\pi}{3}-\cos\frac{x}{2}\sin\frac{\pi}{3}=\frac{1}{2}$

ខ. $2\sin x\cos x-3\sin 2x=0$

គ. $2\sin x\cos x+\sqrt{3}-2\cos x-\sqrt{3}\sin x=0$

សម្រាយ.

ដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin\frac{x}{2}\cos\frac{\pi}{3}-\cos\frac{x}{2}\sin\frac{\pi}{3}=\frac{1}{2} \Leftrightarrow \sin\left(\frac{x}{2}-\frac{\pi}{3}\right)=\sin\frac{\pi}{6}$

$$\Leftrightarrow \begin{cases} \frac{x}{2}-\frac{\pi}{3}=\frac{\pi}{6}+2k\pi \\ \frac{x}{2}-\frac{\pi}{3}=\pi-\frac{\pi}{6}+2k\pi \end{cases} ; k\mathbb{Z}$$

$$\Leftrightarrow \boxed{\begin{cases} x=\pi+4k\pi \\ x=\frac{7\pi}{3}+4k\pi \end{cases} ; k\mathbb{Z}}$$

ខ. $2\sin x\cos x-3\sin 2x=0 \Leftrightarrow \sin 2x-3\sin 2x=0$

$$\Leftrightarrow -2\sin 2x=0$$

$$\Leftrightarrow \sin 2x=0$$

$$\Leftrightarrow \sin 2x=\sin\pi$$

$$\Leftrightarrow \begin{cases} 2x=\pi+2k\pi \\ 2x=\pi-\pi+2k\pi \end{cases} ; \quad k\in\mathbb{Z}$$

$$\Leftrightarrow \boxed{\begin{cases} x=\frac{\pi}{2}+k\pi \\ x=k\pi \end{cases} ; \quad k\in\mathbb{Z}}$$

គ. $2\sin x\cos x+\sqrt{3}-2\cos x-\sqrt{3}\sin x=0$

$$\Leftrightarrow 2\sin x\cos x-\sqrt{3}\sin x+\sqrt{3}-2\cos x=0$$

$$\Leftrightarrow \sin x\left(2\cos x-\sqrt{3}\right)-\left(2\cos x-\sqrt{3}\right)=0$$

$$\Leftrightarrow \left(2\cos x-\sqrt{3}\right)(\sin x-1)=0$$

$$\Rightarrow \begin{cases} 2\cos x-\sqrt{3}=0 & (1) \\ \sin x-1=0 & (2) \end{cases}$$

$$(1):\ 2\cos x-\sqrt{3}=0 \Leftrightarrow \cos x=\frac{\sqrt{3}}{2}$$
$$\Leftrightarrow \cos x=\cos\frac{\pi}{6}$$
$$\Rightarrow \begin{cases} x=\dfrac{\pi}{6}+2k\pi \\ x=-\dfrac{\pi}{6}+2k\pi \end{cases} \quad ; \quad k\in\mathbb{Z}$$

$$(2):\ \sin x-1=0 \Leftrightarrow \sin x=1$$
$$\Leftrightarrow \sin x=\sin\frac{\pi}{2}$$
$$\Rightarrow \begin{cases} x=\dfrac{\pi}{2}+2k\pi \\ x=\pi-\dfrac{\pi}{2}+2k\pi \end{cases} \quad ; \quad k\in\mathbb{Z}$$
$$\Rightarrow x=\frac{\pi}{2}+2k\pi \quad ; \quad k\in\mathbb{Z}$$

ដូចនេះ $\begin{cases} x=\dfrac{\pi}{6}+2k\pi \\ x=-\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{\pi}{2}+2k\pi \end{cases} ; \quad k\in\mathbb{Z}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\tan 3x = \tan\left(\frac{\pi}{3} - 2x\right)$

ខ. $\tan x = \frac{1}{\sqrt{3}}$

គ. $\tan 3x = \sqrt{3}$

ឃ. $\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan x \tan\frac{\pi}{4}} = \sqrt{3}$

សម្រាយ.

ដោះស្រាយសមីការខាងក្រោម៖

ក. $\tan 3x = \tan\left(\frac{\pi}{3} - 2x\right) \Leftrightarrow 3x = \frac{\pi}{3} - 2x + k\pi$

$\Leftrightarrow \boxed{x = \frac{\pi}{15} + \frac{k\pi}{5} \quad ; \quad k \in \mathbb{Z}}$

ខ. $\tan x = \frac{1}{\sqrt{3}} \Leftrightarrow \tan x = \frac{\sqrt{3}}{3}$

$\Leftrightarrow \tan x = \tan\frac{\pi}{6}$

$\Leftrightarrow \boxed{x = \frac{\pi}{6} + k\pi \quad ; \quad k \in \mathbb{Z}}$

គ. $\tan 3x = \sqrt{3} \Leftrightarrow \tan 3x = \tan\frac{\pi}{3}$

$\Leftrightarrow 3x = \frac{\pi}{3} + k\pi$

$\Leftrightarrow \boxed{x = \frac{\pi}{9} + \frac{k\pi}{3} \quad ; \quad k \in \mathbb{Z}}$

ឃ. $\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan x \tan\frac{\pi}{4}} = \sqrt{3} \Leftrightarrow \tan\left(\frac{\pi}{4} - x\right) = \tan\frac{\pi}{3}$

$\Leftrightarrow \frac{\pi}{4} - x = \frac{\pi}{3} + k\pi$

$\Leftrightarrow \boxed{x = -\frac{\pi}{12} - k\pi \quad ; \quad k \in \mathbb{Z}}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\dfrac{2\tan x}{1-\tan^2 x}=\sqrt{3}$ ខ. $2\tan x\cos x+1=2\cos x+\tan x$

សម្រាយ.

ដោះស្រាយសមីការខាងក្រោម៖

ក. $\dfrac{2\tan x}{1-\tan^2 x}=\sqrt{3} \Leftrightarrow \tan 2x=\tan\dfrac{\pi}{3}$

$\Leftrightarrow 2x=\dfrac{\pi}{3}+k\pi$

$\Leftrightarrow \boxed{x=\dfrac{\pi}{6}+\dfrac{k\pi}{2} \quad ; \quad k\in\mathbb{Z}}$

ខ. $2\tan x\cos x+1=2\cos x+\tan x \Leftrightarrow 2\tan x\cos x-\tan x-2\cos x+1=0$

$\Leftrightarrow \tan x(2\cos x-1)-(\cos 2x-1)=0$

$\Leftrightarrow (2\cos x-1)(\tan x-1)=0$

$\Rightarrow \left[\begin{array}{ll} 2\cos x-1=0 & (1)\\ \tan x-1=0 & (2)\end{array}\right.$

$(1): 2\cos x-1=0 \Leftrightarrow \cos x=\dfrac{1}{2}$

$\Leftrightarrow \cos x=\cos\dfrac{\pi}{3}$

$\Leftrightarrow \left[\begin{array}{l} x=\dfrac{\pi}{3}+2k\pi\\ x=-\dfrac{\pi}{3}+2k\pi\end{array}\right. \quad ; \quad k\in\mathbb{Z}$

$(2): \tan x-1=0 \Leftrightarrow \tan x=1$

$\Leftrightarrow \tan x=\tan\dfrac{3\pi}{4} \Rightarrow x=\dfrac{3\pi}{4}+k\pi \quad ; \quad k\in\mathbb{Z}$

ដូចនេះ $\left[\begin{array}{l} x=\dfrac{\pi}{3}+2k\pi \\ x=-\dfrac{\pi}{3}+2k\pi \\ x=\dfrac{3\pi}{4}+k\pi \end{array}\right. ; \quad k\in\mathbb{Z}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cot\left(2x+\dfrac{\pi}{3}\right)=\sqrt{3}$

ខ. $\cot x=-\dfrac{1}{\sqrt{3}}$

គ. $\cot 3x=\sqrt{3}$

ឃ. $\cot\left(\dfrac{x}{2}-3\right)=-1$

សម្រាយ.

ដោះស្រាយសមីការខាងក្រោម៖

ក. $\cot\left(2x+\dfrac{\pi}{3}\right)=\sqrt{3} \Leftrightarrow \cot\left(2x+\dfrac{\pi}{3}\right)=\cot\dfrac{\pi}{6}$

$\Leftrightarrow 2x+\dfrac{\pi}{3}=\dfrac{\pi}{6}+k\pi$

$\Leftrightarrow \boxed{x=-\dfrac{\pi}{12}+\dfrac{k\pi}{2} \quad ; \quad k\in\mathbb{Z}}$

ខ. $\cot x=-\dfrac{1}{\sqrt{3}} \Leftrightarrow \cot x=-\dfrac{\sqrt{3}}{3}$

$\Leftrightarrow \cot x=\cot\dfrac{2\pi}{3}$

$\Leftrightarrow \boxed{x=\dfrac{2\pi}{3}+k\pi \quad ; \quad k\in\mathbb{Z}}$

គ. $\cot 3x=\sqrt{3} \Leftrightarrow \cot 3x=\cot\dfrac{\pi}{6}$

$\Leftrightarrow 3x=\dfrac{\pi}{6}+k\pi$

$\Leftrightarrow \boxed{x=\dfrac{\pi}{18}+\dfrac{k\pi}{3} \quad ; \quad k\in\mathbb{Z}}$

ឃ. $\cot\left(\frac{x}{2}-3\right)=-1 \Leftrightarrow \cot\left(\frac{x}{2}-3\right)=\cot\frac{3\pi}{4}$

$\Leftrightarrow \frac{x}{2}-3=\frac{3\pi}{4}+k\pi$

$\Leftrightarrow \boxed{x=\frac{3\pi+12}{2}+2k\pi \quad ; \quad k\in\mathbb{Z}}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $3\cot x-\sqrt{3}=0$

ខ. $2(\cot 2x-\cot 3x)=\tan 2x+\cot 3x$

សម្រាយ.

ដោះស្រាយសមីការខាងក្រោម៖

ក. $3\cot x-\sqrt{3}=0 \Leftrightarrow \cot x=\frac{\sqrt{3}}{3}$

$\Leftrightarrow \cot x=\cot\frac{\pi}{3}$

$\Rightarrow \boxed{x=\frac{\pi}{3}+k\pi \quad ; \quad k\in\mathbb{Z}}$

ខ. $2(\cot 2x-\cot 3x)=\tan 2x+\cot 3x \Leftrightarrow 2\left(\frac{\cos 2x}{\sin 2x}-\frac{\cos 3x}{\sin 3x}\right)=\frac{\sin 2x}{\cos 2x}+\frac{\cos 3x}{\sin 3x}$ $(*)$

លក្ខខណ្ឌនៃសមីការ $\sin 2x\neq 0 \quad ; \quad \sin 3x\neq 0 \quad ; \quad \cos 2x\neq 0$

$(*)\Rightarrow 2\left(\frac{\cos 2x\sin 3x-\cos 3x\sin 2x}{\sin 2x\sin 3x}\right)=\frac{\sin 2x\sin 3x+\cos 3x\cos 2x}{\cos 2x\sin 3x}$

$\Leftrightarrow \frac{2\sin(3x-2x)}{\sin 2x\sin 3x}=\frac{\cos(3x-2x)}{\cos 2x\sin 3x}$

$\Leftrightarrow \frac{2\sin x}{\sin 2x\sin 3x}-\frac{\cos x}{\cos 2x\sin 3x}$

$\Leftrightarrow \frac{2\sin x\cos 2x-\cos x\sin 2x}{\sin 2x\sin 3x\cos 2x}=0$

$\Leftrightarrow \frac{2\sin x\cos 2x-2\sin x\cos^2 x}{\sin 2x\sin 3x\cos 2x}=0$

$$\Leftrightarrow \quad \frac{2\sin x\left(\cos 2x-\cos^2 x\right)}{\sin 2x\sin 3x\cos 2x}=0$$

$$\Leftrightarrow \quad \frac{2\sin x\left(2\cos^2 x-1-\cos^2 x\right)}{\sin 2x\sin 3x\cos 2x}=0$$

$$\Leftrightarrow \quad \frac{-2\sin^3 x}{\sin 2x\sin 3x\cos 2x}=0$$

$$\Leftrightarrow \quad \sin^3 x=0 \quad \Leftrightarrow \quad \sin x=0$$

តាមលក្ខខណ្ឌ $\sin 2x\neq 0 \quad \Rightarrow \quad \sin x\neq 0$ យើងបាន សមីការគ្មានឫសៗ

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ចូរដោះស្រាយវិសមីការខាងក្រោម៖

ក. $2\cos x+1<0$

ខ. $\tan x\geq -1$

គ. $2\cos x\geq -\sqrt{2}$

ឃ. $\cos 2x>\cos\frac{2\pi}{3}$

ង. $\sqrt{2}\cos x-1<0$

ច. $\cot x<\frac{\sqrt{3}}{3}$

ឆ. $2\sin^2 x+3\sin x-2\geq 0$

សម្រាយ.

ដោះស្រាយវិសមីការ៖

ក. $2\cos x+1<0 \quad \Leftrightarrow \quad \cos x<-\frac{1}{2}$

$$\Leftrightarrow \quad \cos x<\cos\frac{2\pi}{3}$$

$$\Leftrightarrow \quad \boxed{\frac{2\pi}{3}+2k\pi<x<-\frac{2\pi}{3}+2k\pi \quad ; \quad k\in\mathbb{Z}}$$

ខ. $\tan x\geq -1 \quad \Leftrightarrow \quad \tan x\geq \tan\frac{3\pi}{4}$

$$\Leftrightarrow \quad \boxed{\frac{3\pi}{4}+k\pi\leq x\leq\frac{\pi}{2}+k\pi \quad ; \quad k\in\mathbb{Z}}$$

គ. $2\cos x \geq -\sqrt{2} \Leftrightarrow \cos x \geq -\dfrac{\sqrt{2}}{2}$

$\Leftrightarrow \cos x \geq \cos\dfrac{3\pi}{4}$

$\Leftrightarrow \boxed{-\dfrac{3\pi}{4}+2k\pi \leq x \leq \dfrac{3\pi}{4}+2k\pi \quad ; \quad k\in\mathbb{Z}}$

ឃ. $\cos 2x > \cos\dfrac{2\pi}{3} \Leftrightarrow -\dfrac{2\pi}{3}+2k\pi < 2x < \dfrac{2\pi}{3}+2k\pi$

$\Leftrightarrow \boxed{-\dfrac{\pi}{3}+k\pi < x < \dfrac{\pi}{3}+k\pi \quad ; \quad k\in\mathbb{Z}}$

ង. $\sqrt{2}\cos x - 1 < 0 \Leftrightarrow \cos x < \dfrac{1}{\sqrt{2}}$

$\Leftrightarrow \cos x < \dfrac{\sqrt{2}}{2}$

$\Leftrightarrow \cos x < \cos\dfrac{\pi}{4}$

$\Leftrightarrow \boxed{\dfrac{\pi}{4}+2k\pi < x < -\dfrac{\pi}{4}+2k\pi \quad ; \quad k\in\mathbb{Z}}$

ច. $\cot x < \dfrac{\sqrt{3}}{3} \Leftrightarrow \cot x < \cot\dfrac{\pi}{3}$

$\Leftrightarrow \boxed{\dfrac{\pi}{3}+k\pi < x < \pi+k\pi \quad ; \quad k\in\mathbb{Z}}$

ឆ. $2\sin^2 x + 3\sin x - 2 \geq 0$

តាង $t = \sin x$ ដែល $-1 \leq t \leq 1 \Rightarrow 2t^2+3t-2 \geq 0$

សិក្សាសញ្ញានៃ $2t^2+3t-2$

បើ $2t^2+3t-2=0 \Leftrightarrow (2t-1)(t+2)=0 \Rightarrow \left[\begin{array}{l} t=\dfrac{1}{2} \\ t=-2 \end{array}\right.$

t	$-\infty$		-2		$\frac{1}{2}$		$+\infty$
$2t^2+3t-2$		$+$	0	$-$	0	$+$	

$2t^2+3t-2 \geq 0$ ពេល $t \leq -2$ ឬ $t \geq \dfrac{1}{2}$ $\quad t \leq -2$ មិនយក

☞ $t \geq \frac{1}{2}$; $t = \sin x \Rightarrow \sin x \geq \frac{1}{2} \Rightarrow$ $\boxed{\frac{\pi}{6} + 2k\pi \leq x \leq \frac{5\pi}{6} + 2k\pi \quad ; k \in \mathbb{Z}}$

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ចូរដោះស្រាយវិសមីការខាងក្រោម៖

ក. $\sin\left(\frac{\pi}{2} - x\right) < \cos\left(\frac{\pi}{2} + \frac{x}{2}\right)$ ខ. $\sin\left(x - \frac{\pi}{3}\right) > \sin x$

សម្រាយ.

ដោះស្រាយវិសមីការ៖

ក. $\sin\left(\frac{\pi}{2} - x\right) < \cos\left(\frac{\pi}{2} + \frac{x}{2}\right) \Leftrightarrow \cos x < \cos\left(\frac{\pi}{2} + \frac{x}{2}\right)$

$\Leftrightarrow \frac{\pi}{2} + \frac{x}{2} + 2k\pi < x < -\left(\frac{\pi}{2} + \frac{x}{2}\right) + 2k\pi$

$\Leftrightarrow \begin{cases} x > \frac{\pi}{2} + \frac{x}{2} + 2k\pi \\ x < -\left(\frac{\pi}{2} + \frac{x}{2}\right) + 2k\pi \end{cases}$

$\Leftrightarrow \boxed{\begin{cases} x > \pi + 4k\pi \\ x < -\frac{\pi}{3} + \frac{4k\pi}{3} \end{cases} \quad ; \quad k \in \mathbb{Z}}$

ខ. $\sin\left(x - \frac{\pi}{3}\right) > \sin x \Leftrightarrow \sin\left(x - \frac{\pi}{3}\right) + \sin(-x) > 0$

$\Leftrightarrow 2\sin\frac{x - \frac{\pi}{3} - x}{2}\cos\frac{x - \frac{\pi}{3} + x}{2} > 0$

$\Leftrightarrow -2\sin\frac{\pi}{6}\cos\frac{6x - \pi}{6} > 0$

$\Leftrightarrow -\cos\frac{6x - \pi}{6} > 0$

$\Leftrightarrow \cos\frac{6x - \pi}{6} < 0$

$\Leftrightarrow \frac{\pi}{2} + 2k\pi < \frac{6x - \pi}{6} < -\frac{\pi}{2} + 2k\pi$

$\Leftrightarrow \boxed{\frac{2\pi}{3} + 2k\pi < x < -\frac{\pi}{3} + 2k\pi}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin\theta = \dfrac{\sqrt{3}}{2}$

ខ. $\cos\theta = -\dfrac{1}{2}$

គ. $\cos^2 x = 1$

ឃ. $\sin\sqrt{x} = -1$

ង. $\cot x = 1$

សម្រាយ.

ដោះស្រាយសមីការ៖

ក. $\sin\theta = \dfrac{\sqrt{3}}{2} \Leftrightarrow \left[\begin{array}{l} \theta = \dfrac{\pi}{3} + 2k\pi \\ \theta = \dfrac{2\pi}{3} + 2k\pi \end{array}\right. ; \quad k \in \mathbb{Z}$

ខ. $\cos\theta = -\dfrac{1}{2} \Leftrightarrow \left[\begin{array}{l} \theta = \dfrac{2\pi}{3} + 2k\pi \\ \theta = -\dfrac{2\pi}{3} + 2k\pi \end{array}\right. ; \quad k \in \mathbb{Z}$

គ. $\cos^2 x = 1 \Rightarrow \cos x = \pm 1$

☞ $\cos x = 1 \Rightarrow x = 2k\pi \; ; \quad k \in \mathbb{Z}$

☞ $\cos x = -1 \Rightarrow x = k\pi \; ; \quad k \in \mathbb{Z}$

ដូចនេះ $x = 2k\pi$ ឬ $x = k\pi \; ; \quad k \in \mathbb{Z}$

ឃ. $\sin\sqrt{x} = -1 \Leftrightarrow \sqrt{x} = \dfrac{3\pi}{2} + 2k\pi \Leftrightarrow x = \left(\dfrac{3\pi}{2} + 2k\pi\right)^2 \; ; k \in \mathbb{Z}$

ង. $\cot x = 1 \Leftrightarrow x = \dfrac{\pi}{4} + k\pi \; ; \quad k \in \mathbb{Z}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\frac{1}{\cos 2x} = \sqrt{2}$ ខ. $2\sin\left(2x - \frac{\pi}{3}\right) = 1$

សម្រាយ.

ដោះស្រាយសមីការ៖

ក. $\frac{1}{\cos 2x} = \sqrt{2} \Leftrightarrow \cos 2x = \frac{1}{\sqrt{2}}$

$\Leftrightarrow \cos 2x = \frac{\sqrt{2}}{2}$

$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{4} + 2k\pi \\ 2x = -\frac{\pi}{4} + 2k\pi \end{cases}$

$\Leftrightarrow \boxed{\begin{cases} x = \frac{\pi}{8} + k\pi \\ x = -\frac{\pi}{8} + k\pi \end{cases} ; \quad k \in \mathbb{Z}}$

ខ. $2\sin\left(2x - \frac{\pi}{3}\right) = 1 \Leftrightarrow \sin\left(2x - \frac{\pi}{3}\right) = \frac{1}{2}$

$\Leftrightarrow \begin{cases} 2x - \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi \\ 2x - \frac{\pi}{3} = \frac{5\pi}{6} + 2k\pi \end{cases}$

$\Leftrightarrow \boxed{\begin{cases} x = \frac{\pi}{4} + k\pi \\ x = \frac{7\pi}{12} + k\pi \end{cases} ; \quad k \in \mathbb{Z}}$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos\left(2x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

ខ. $\tan^3\left(x-\frac{\pi}{4}\right)=\tan x-1$

គ. $\cot\left(2x+\frac{\pi}{3}\right)=2$

ឃ. $\frac{2\tan x}{1-\tan^2 x}=5$

សម្រាយ.

ដោះស្រាយសមីការ៖

ក. $\cos\left(2x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2} \Leftrightarrow \begin{cases} 2x-\frac{\pi}{3}=\frac{\pi}{6}+2k\pi \\ 2x-\frac{\pi}{3}=-\frac{\pi}{6}+2k\pi \end{cases}$

$\Leftrightarrow \boxed{\begin{cases} x=\frac{\pi}{4}+k\pi \\ x=-\frac{\pi}{12}+k\pi \end{cases} ; \quad k\in\mathbb{Z}}$

ខ. $\tan^3\left(x-\frac{\pi}{4}\right)=\tan x-1 \Leftrightarrow \left(\frac{\tan x-\tan\frac{\pi}{4}}{1+\tan x\tan\frac{\pi}{4}}\right)^3=\tan x-1$

$\Leftrightarrow \left(\frac{\tan x-1}{1+\tan x}\right)^3-(\tan x-1)=0$

$\Leftrightarrow (\tan x-1)\left(\frac{(\tan x-1)^2}{(1+\tan x)^3}\right)=0$

$\Leftrightarrow \tan x-1=0$

$\Leftrightarrow \tan x=1$

$\Rightarrow \boxed{x=\frac{\pi}{4}+k\pi \quad ; \quad k\in\mathbb{Z}}$

គ. $\cot\left(2x+\frac{\pi}{3}\right)=2 \Leftrightarrow 2x+\frac{\pi}{3}=\cot^{-1}(2)+k\pi$

$\Leftrightarrow \boxed{x=-\frac{\pi}{6}+\frac{1}{2}\cot^{-1}(2)+\frac{1}{2}k\pi \quad ; \quad k\in\mathbb{Z}}$

ឃ. $\dfrac{2\tan x}{1-\tan^2 x}=5 \quad\Leftrightarrow\quad 2x=\tan^{-1}5+k\pi$

$$\Leftrightarrow \quad \boxed{x=\frac{1}{2}\tan^{-1}5+\frac{1}{2}k\pi \quad ; \quad k\in\mathbb{Z}}$$

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ចូរដោះស្រាយសមីការខាងក្រោម៖

ក. $\tan^3 3x-2\sin^3 3x=0$

ខ. $\cos\left(\dfrac{3\pi}{2}+x\right)=\sqrt{2}\sin x\cos x$

គ. $\sin 2x=(\cos x-\sin x)^2$

ឃ. $\sqrt{3}\sin x+\sin x=\dfrac{1}{\cos x}$

សម្រាយ.

ដោះស្រាយសមីការ

ក. $\tan^3 3x-2\sin^3 3x=0 \quad\Leftrightarrow\quad \sin^3 3x\left(\dfrac{1}{\cos^3 3x}-2\right)=0$

$$\Leftrightarrow \quad \left[\begin{array}{l}\sin^3 3x=0\\ \dfrac{1}{\cos^3 3x}-2=0\end{array}\right.$$

$$\Leftrightarrow \quad \left[\begin{array}{l}\sin 3x=0\\ \cos 3x=\dfrac{1}{\sqrt[3]{2}}\end{array}\right.$$

$$\Leftrightarrow \quad \left[\begin{array}{l}3x=k\pi\\ 3x=\cos^{-1}\dfrac{1}{\sqrt[3]{2}}+2k\pi\end{array}\right.$$

$$\Leftrightarrow \quad \boxed{\left[\begin{array}{l}x=\dfrac{k\pi}{3}\\ x=\dfrac{1}{3}\left(\cos^{-1}\dfrac{1}{\sqrt[3]{2}}+2k\pi\right)\end{array}\right. \quad ; \quad k\in\mathbb{Z}}$$

ខ. $\cos\left(\frac{3\pi}{2}+x\right)=\sqrt{2}\sin x\cos x \Leftrightarrow \cos\frac{3\pi}{2}\cos x-\sin\frac{3\pi}{2}\sin x=\sqrt{2}\sin x\cos x$

$\Leftrightarrow 0+\sin x=\sqrt{2}\sin x\cos x$

$\Leftrightarrow \sin x\left(\sqrt{2}\cos x-1\right)=0$

$\Leftrightarrow \begin{cases}\sin x=0\\ \sqrt{2}\cos x-1=0\end{cases}$

$\Leftrightarrow \begin{cases}x=k\pi\\ \cos x=\frac{\sqrt{2}}{2}\end{cases}$

$\Leftrightarrow \boxed{\begin{cases}x=k\pi\\ x=\pm\frac{\pi}{4}+2k\pi\end{cases};\quad k\in\mathbb{Z}}$

គ. $\sin 2x=(\cos x-\sin x)^2 \Leftrightarrow \sin 2x=\cos^2 x-2\cos x\sin x+\sin^2 x$

$\Leftrightarrow 2\sin x\cos x=1-2\cos x\sin x$

$\Leftrightarrow 4\sin x\cos x=1$

$\Leftrightarrow 2\sin 2x=1$

$\Leftrightarrow \sin 2x=\frac{1}{2}$

$\Leftrightarrow \begin{cases}2x=\frac{\pi}{6}+2k\pi\\ 2x=\frac{5\pi}{6}+2k\pi\end{cases}$

$\Leftrightarrow \boxed{\begin{cases}x=\frac{\pi}{12}+k\pi\\ x=\frac{5\pi}{12}+k\pi\end{cases};\quad k\in\mathbb{Z}}$

ឃ. $\sqrt{3}\sin x+\sin x=\dfrac{1}{\cos x}$

$$\Leftrightarrow \quad \sin x\left(\sqrt{3}+1\right)=\frac{1}{\cos x}$$
$$\Leftrightarrow \quad \sin x\cos x=\frac{1}{\sqrt{3}+1}$$
$$\Leftrightarrow \quad \frac{1}{2}\sin 2x=\frac{1}{\sqrt{3}+1}$$
$$\Leftrightarrow \quad \sin 2x=\frac{2}{\sqrt{3}+1}$$
$$\Leftrightarrow \quad 2x=\arcsin\left(\frac{2}{\sqrt{3}+1}\right)+2k\pi$$
$$\Leftrightarrow \quad \boxed{x=\frac{1}{2}\arcsin\left(\frac{2}{\sqrt{3}+1}\right)+k\pi \quad ; \quad k\in\mathbb{Z}}$$

សម្រាប់លំហាត់ទី៥០ ដល់ទៅ១០3 មានចម្លើយក្នុងស្បៃវែនៅតាគ២

សូមអរគុណ

រួមគិត រួមធ្វើ រួមទទួលខុសត្រូវ
ដើម្បីអនាគតប្រទេសជាតិ និងកូនចៅជំនាន់ក្រោយ