ខ្ញុំស្រឡាញ់ ♥ គណិតវិទ្យា

ពិភពគណនាលីមីតនៃអនុគមន៍

ល្អសម្រាប់ត្រៀមប្រឡងប្រជែងនានា

លំហាត់ស្រាវជ្រាវពិសេសៗ

$$\lim_{x\to 0}\frac{\sqrt{\frac{1+x}{1-x}}\cdot\sqrt[4]{\frac{1+2x}{1-2x}}\cdot\sqrt[6]{\frac{1+3x}{1-3x}}-1}{x}$$

$$\lim_{x\to 0}\left[\frac{\sin[\sin(x-\sin x)]}{x-\sin x}\times\frac{x-\sin x}{x^3}\right]$$

អ៊ិន សាគីន

$$\lim_{x\to 0^+}\frac{\left(\sqrt{x}\right)^{\sqrt{x}}-x^x}{\left(\sqrt{x}\right)^x-x^{\sqrt{x}}}$$

ថ្មី ២០២០

អារម្ភកថា

សៀវភៅ **លីមីតនៃអនុគមន៍** សម្រាប់សិស្សទូទៅ នេះជាស្នាដៃថ្មីមួយទៀតហើយ

ដែលខ្ញុំបានចងក្រងឡើង ក្នុងគោលបំណងជួយពង្រឹងស្មារតី និងជួយពង្រីកចំនេះដឹងដល់សិស្សានុសិស្សទាំងអស់គ្នា ឈានទៅដល់ការប្រឡងសញ្ញាបត្រមធ្យមសិក្សាទុតិយភូមិ ប្រឡងសិស្សពូកែ និងអាហារូបករណ៍ផ្សេងៗ។

សៀវភៅនេះផងដែរខ្ញុំបានដកស្រង់ លំហាត់ល្អៗ ចេញពីឯកសារជាតិ និងឯកសារអន្តរជាតិមានដូចជា វៀតណាម Internet និងចេញពីការស្រាវជ្រាវរបស់ខ្ញុំផ្ទាល់។

ឆ្លងតាមបទពិសោធដ៏តិចតួច និងពេលវេលាដ៏ខ្លី យើងខ្ញុំអ្នករៀបរៀងជឿជាក់ថានិងមានកំហុសដោយអចេតនាកើតឡើងជាក់ជាពុំខាន។

អាស្រ័យដូច្នេះហើយ យើងខ្ញុំអ្នករៀបរៀងរង់ចាំជានិច្ចនូវមតិរិះគន់ពីសំណាក់អ្នកអានគ្រប់មជ្ឈដ្ឋានដោយក្តីរីករាយបំផុត ដើម្បីកែលម្អសៀវភៅនេះឲ្យកាន់តែល្អប្រសើរថែមទៀត។

ជាទីបញ្ចប់ សូមជូនពរឲ្យសិស្សានុសិស្សទាំងអស់គ្នាទទួលបានជោគជ័យនៅក្នុងវិថីសិស្សានេះ។

ព្រៃវែង, ថ្ងៃទី ....ខែ....................ឆ្នាំ.......................

បច្ចេកទេសកុំព្យូទ័រ **រៀបរៀងដោយ** : **អ៊ិន សាគីន**

អ៊ិន សាគីន

សម្របសម្រួលបច្ចេកទេសដោយ

អ៊ិន សាគីន

ផ្នែកទី០១នៃលំហាត់លីមីត($Exercise\ Of\ Limit$)

$$a_1.\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}-\frac{2}{\frac{1}{S+2}+\frac{1}{S+1}-\sum_{H=S}^{\infty}\frac{3}{\sum_{A=1}^{H}\frac{2}{\sum_{B=A}^{\infty}\frac{1}{B^2+3B+2}}}}}}$$

$$a_2.\lim_{x\to 0}\frac{2019x-sin2019x}{2020x-sin2020x}$$

$$a_3.\lim_{x\to 0}\frac{(2+sinx)^{cosx}-2}{x}$$

$$a_4.\lim_{n\to+\infty}\sqrt{2}\times\sqrt[4]{2}\times\sqrt[8]{2}\times\dots\times\sqrt[n]{2}$$

$$a_5.\lim_{x\to 0}\frac{(1+x^{2019})^{2019}-1}{x^{2019}}$$

$$a_6.\lim_{n\to\infty}\frac{n+n^2+n^3+\cdots+n^n}{1^n+2^n+3^n+\cdots+n^n}$$

$$a_7.\lim_{x\to 0}\frac{(1+ax)^n-1}{x}$$

$$a_8.\lim_{n\to\infty}\frac{(n+2)!+n!}{(n+2)!+(n+1)!}$$

$$a_9.\lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots+\frac{1}{2^n}\right)$$

$$a_{10}.\lim_{n\to+\infty}\left[\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{n}{(n+1)!}\right]$$

$$a_{11}.\lim_{n\to\infty}\left[\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{n(n+1)}\right]$$

$$a_{12}.\lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)$$

$a_{13}.\lim_{n\to\infty}\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+\cdots+n}\right)$

$a_{14}.\lim_{n\to+\infty}\frac{1\cdot 1!+2\cdot 2!+3\cdot 3!+\cdots+n\cdot n!}{(n+1)!}$

$a_{15}.\lim_{x\to 0}\frac{(e^x-1)(e^{2x}-1)\ldots(e^{nx}-1)}{x^n}$

$a_{16}.\lim_{x\to 0}\frac{7^{x^2}-cos2x}{x^2}$

$a_{17}.\lim_{x\to 0}\frac{ln(1+x)+ln(1+2x)+\cdots+ln(1+nx)}{x}$

$a_{18}.\lim_{x\to 0^+}\frac{\sqrt{2+\sqrt{2+\cdots+\sqrt{2+2cosx}}}-2}{x}$

$a_{19}.\lim_{n\to\infty}\left[n^2+n-\sum_{K=1}^{n}\left(\frac{2k^3+8k^2+6k-1}{k^2+4k+3}\right)\right]$

$a_{20}.\lim_{n\to\infty}\sqrt[n]{n3^n}$

$a_{21}.\lim_{x\to 0}\frac{1-cosxcos2xcos3x\ldots cosnx}{x^2}$

$a_{22}.\lim_{x\to 1}\frac{1-x^{-\frac{1}{3}}}{1-x^{-\frac{2}{3}}}$

$a_{23}.\lim_{x\to 0}\left[\frac{1}{x^2}\left(\frac{2}{cosx}+cosx-3\right)\right]$

$a_{24}.\lim_{n\to\infty}\frac{1\cdot 2+2\cdot 3+3\cdot 4+\cdots+(n-1)n}{n^3}$

$a_{25}.\lim_{x\to\infty}x\left(e^{\frac{2}{x}}-1\right)$

$a_{26}.\lim_{x\to 0}\frac{cos^{-1}(1-x)}{\sqrt{x}}$

$a_{27}.\lim_{x\to 1}\frac{2^x-\sqrt{1+3^x}}{\sqrt{4^x-1}-3^{\frac{x}{2}}}$

$a_{28}.\lim\limits_{x\to-1}\dfrac{\left(\sqrt{\pi}-\sqrt{arccosx}\right)^2}{x+1}$

$a_{29}.\lim\limits_{x\to0}\dfrac{\sqrt[m]{1+x^n}-\sqrt[m]{1-x^n}}{x^n}$

$a_{30}.\lim\limits_{x\to1}\dfrac{x^{n+1}-x(n+1)+n}{(x-1)^2}$

$a_{31}.\lim\limits_{x\to1}\dfrac{x+x^2+\cdots+x^n-n}{x-1}$

$a_{32}.\lim\limits_{x\to1}\dfrac{x+x^2+x^3+\cdots+x^n-n}{x+x^2+x^3+\cdots+x^p-p};n,p\in IN^*$

$a_{33}.\lim\limits_{x\to1}\dfrac{nx^{n+1}-(n+1)x^n+1}{(x-1)^2}\qquad;n\in IN^*$

$a_{34}.\lim\limits_{x\to1}\left(\dfrac{1}{1-x}-\dfrac{n+1}{1-x^{n+1}}\right)$

$a_{35}.\lim\limits_{x\to1}\dfrac{x+2x^2+\cdots+n\cdot x^n-\dfrac{n(n+1)}{2}}{x-1}$

$a_{36}.\lim\limits_{x\to0}\dfrac{1-cos^nx}{xsinx}$

$a_{37}.\lim\limits_{n\to\infty}\left(\dfrac{1^3+2^3+3^3+\cdots+n^3}{n^3}-\dfrac{n}{4}\right)$

$a_{38}.\lim\limits_{x\to\infty}\dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}}$

$a_{39}.\lim\limits_{n\to\infty}\left(1+\dfrac{1}{10}\right)\left(1+\dfrac{1}{10^2}\right)\ldots\left(1+\dfrac{1}{10^{2^n}}\right)$

$a_{40}.\lim\limits_{n\to\infty}(1+x)\,(1+x^2)\ldots\left(1+x^{2^n}\right);|x|<1$

$a_{41}.\lim\limits_{n\to\infty}\left(\dfrac{1}{2}+\dfrac{3}{2^2}+\dfrac{5}{2^3}+\cdots+\dfrac{2n-1}{2^n}\right)$

$a_{42}.\lim_{n\to\infty}\left(\frac{1}{3^2}+\frac{2}{15^2}+\frac{3}{35^3}+\cdots+\frac{n}{(4n^2-1)^2}\right)$

$a_{43}.\lim_{n\to\infty}\frac{1+2+6+15+\cdots+\left(\frac{2n^3-3n^2+n+6}{6}\right)}{n^4}$

$a_{44}.\lim_{x\to 0}\frac{sin(x^2)-sin^2x}{x^2-ln(1+x^2)}$

$a_{45}.\lim_{x\to 0}\frac{sin(ax)+sin(bx)-sin[(a+b)x]}{ln[abx^3+x(a+b)+1]-ln[1+(a+b)]}$

$a_{46}.\lim_{x\to 0}\frac{e^{xtanx}-2+cos2x}{x^2}$

$a_{47}.\lim_{x\to 1}\frac{tan(lnx)}{ln(3x-2)}$

$a_{48}.\lim_{x\to 0}\frac{sin[tan\{sin(10x)\}]}{sin[sin\{sin(2x)\}]}$

$a_{49}.\lim_{x\to 1}\left(\frac{1}{lnx}-\frac{1}{x-1}\right)$

$a_{50}.\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{x}{lnx}\right)$

$a_{51}.\lim_{x\to 0}\frac{ln[1+sin(e^x-1)]}{sin(sinx)}$

$a_{52}.\lim_{x\to\frac{\pi}{2}}(sinx)^{tanx}$

$a_{53}.\lim_{x\to 0}\left(\frac{1+a^x}{1+b^x}\right)^{\frac{1}{x}}$

$a_{54}.\lim_{x\to 0}\left(\frac{3-2cosx}{2-cosx}\right)^{cot^2(2x)}$

$a_{55}.\lim_{x\to 0}\frac{1-cos2(x)^2}{x^3sinx}$

$a_{56}.\lim_{x\to 1}\frac{x^{11}+x^9+x^7-3}{x-1}$

$a_{57}.\lim_{x\to 0}2x^x$

$a_{58}.\lim_{x\to 1}\frac{(1+x)^{\frac{1}{5}}-(1-x)^{\frac{1}{5}}}{x}$

$a_{59}.\lim_{x\to 9}\frac{\sqrt{x}(x-6)^3-81}{x-9}$

$a_{60}.\lim_{x\to\infty}\left(\sqrt{x^2+2x+1}+\sqrt{x^2+4x+1}+\cdots+\sqrt{x^2+2nx+1}-\sqrt{n^2x^2+1}\right)$

$a_{61}.\lim_{n\to\infty}\sum_{k=1}^{1}\frac{2k+1}{k^2(k+1)^2}$

$a_{62}.\lim_{n\to\infty}\left(\sqrt{n+\sqrt{(n-1)+\cdots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}-\sqrt{n}\right)$

$a_{63}.\lim_{n\to\infty}\sqrt{2+\sqrt{2+\underbrace{\dots}_{n}+\sqrt{2+\sqrt{2}}}}$

$a_{64}.\lim_{n\to+\infty}\frac{1\times 2+2\times 3+3\times 4\times\ldots\times(n-1)n}{1^2+2^2+3^2+\cdots+(n-1)^2}$

$a_{65}.\lim_{x\to 1}\frac{x^2-1+\sqrt{x^3+1}-\sqrt{x^4+1}}{x-1+\sqrt{x+1}-\sqrt{x^2+1}}$

$a_{66}.\lim_{x\to\frac{\pi}{2}}\frac{sin^6x+cos^6x-1}{sin^4x+cos^4x-1}$

$a_{67}.\lim_{x\to 0^+}\frac{2-\sqrt{2+\sqrt{2+2cos2x}}}{x^2}$

$a_{68}.\lim_{n\to+\infty}[ln\{(n+1)!+n!\}-ln\{(n+1)!-n!\}]$

$a_{69}.\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}}$

$a_{70}.\lim_{n\to\infty}\left(\frac{1+2+3+\cdots+n}{n+2}-\frac{n}{2}\right)$

$a_{71}.\ \lim\limits_{n\to\infty}\left[\dfrac{1-2+3-4+\cdots-2n}{\sqrt{n^2+1}}\right]$

$a_{72}.\ \lim\limits_{n\to\infty}\left(\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\cdots+\dfrac{1}{(n-1)n}\right)$

$a_{73}.\ \lim\limits_{n\to+\infty}\left(\dfrac{7}{10}+\dfrac{29}{10^2}+\dfrac{133}{10^3}+\cdots+\dfrac{5^n+2^n}{10^n}\right)$

$a_{74}.\ \lim\limits_{n\to+\infty}\prod_{r=3}^{n}\left(\dfrac{r^3-1}{r^3+1}\right)$

វិធ្មេកទី០២នៃលំហាត់លីមីត(*Exercise Of Limit*)

$$(\mathrm{KIN})_1=\lim_{x\to 1}\cos\left[\ln x\left\{\sin\left(\frac{\pi x}{x-1}\right)\right\}\right]$$

$$(\mathrm{KIN})_2=\lim_{x\to \alpha}\frac{x^x-\alpha^\alpha}{x-\alpha}\ ;\ \alpha\in \mathrm{IN}$$

$$(\mathrm{KIN})_3=\lim_{x\to \frac{\pi}{2}}\left[\cos x\sin\left(\frac{1}{x-\frac{\pi}{2}}\right)\right]$$

$$(\mathrm{KIN})_4=\lim_{x\to 0}\frac{1+\tan^2x-\cos 2x}{x\sin x}$$

$$(\mathrm{KIN})_5=\lim_{x\to 0}\frac{1-\sqrt[7]{1+\sin x}}{\tan x}$$

$$(\mathrm{KIN})_6=\lim_{x\to 0^+}\frac{\tan 3x\cdot\sin\sqrt{2x}}{\sqrt{x^3+\sin^3 2x}}$$

$$(\mathrm{KIN})_7=\lim_{x\to \infty}\left[2^{x-1}\sin\left(\frac{a}{2^x}\right)\right]\ ;a\in \mathrm{IN}$$

$$(\mathrm{KIN})_8=\lim_{x\to 1}\left(\frac{m}{1-\sqrt[n]{x}}-\frac{n}{1-\sqrt[m]{x}}\right)\ ;(m,n)\in \mathrm{IN}$$

$$(\mathrm{KIN})_9=\lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)\ldots(1+nx)-1}{x}$$

$$(\mathrm{KIN})_{10}=\lim_{x\to 0}\frac{\ln\left(\frac{\sqrt{1-x^2}}{\cos x}\right)}{x^3}$$

$$(\mathrm{KIN})_{11}=\lim_{x\to a}\frac{x\sqrt{x}-a\sqrt{a}}{\sqrt{x}-\sqrt{a}}\ ;a\geq 0$$

$$(\mathrm{KIN})_{12}=\lim_{x\to 1}\frac{\sqrt{x+1}+\sqrt{x^2-1}-\sqrt{x^3+1}}{\sqrt{x-1}+\sqrt{x^2+1}-\sqrt{x^4+1}}$$

$$(\mathrm{KIN})_{13} = \lim_{x\to 3}\frac{\sqrt{6+\sqrt{6+\cdots+\sqrt{6+x}}}-x}{x-3}$$

$$(\mathrm{KIN})_{14} = \lim_{x\to 2}\frac{1-\sin\frac{\pi}{x}}{x^2-4x+4}$$

$$(\mathrm{KIN})_{15} = \lim_{x\to 1}\frac{x^2-1}{\cos\frac{\pi}{x+1}}$$

$$(\mathrm{KIN})_{16} = \lim_{x\to 0}\frac{\sqrt{1+\sqrt{2x^2+9}}-2}{x\sin 2x}$$

$$(\mathrm{KIN})_{17} = \lim_{x\to\frac{\pi}{2}}(\sin x)^{\frac{1}{\cos x}}$$

$$(\mathrm{KIN})_{18} = \lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin^{-1}x-\cos^{-1}x}{\sqrt{2}x-1}$$

$$(\mathrm{KIN})_{19} = \lim_{x\to\pi}\frac{1-\sin\frac{x}{2}}{x-\pi}$$

$$(\mathrm{KIN})_{20} = \lim_{x\to+\infty}\sqrt[x]{x}$$

$$(\mathrm{KIN})_{21} = \lim_{x\to 1}(2-x)^{\tan\left(\frac{\pi x}{2}\right)}$$

$$(\mathrm{KIN})_{22} = \lim_{x\to 0}\frac{1-\sqrt[n]{\cos 2x}}{x^2} = \frac{1}{3}\ ;\ \text{find } n\ ?$$

$$(\mathrm{KIN})_{23} = \lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n-1}{n^2}\right)$$

$$(\mathrm{KIN})_{24} = \lim_{x\to 2^-}\frac{x-2\cos\sqrt{2-x}}{(2-x)^2}$$

$$(\mathrm{KIN})_{25} = \lim_{x\to\infty}\left(\cos\frac{1}{x}+\sin\frac{1}{x^2}\right)^{x^2}$$

$$(\mathbf{KIN})_{26}=\lim_{x\to 0}\frac{\sin 3x+\sin 6x-\sin 9x}{ax^n}=1\ ;\ \text{find } a\ ,\ n\ ?$$

$$(\mathbf{KIN})_{27}=\lim_{x\to \infty}\left[x^2\tan^{-1}\left(\cos\frac{5}{x}-\cos\frac{2}{x}\right)\right]$$

$$(\mathbf{KIN})_{28}=\lim_{x\to 0}\left[\tan\left(\frac{\pi}{4}+x\right)^{\frac{1}{x}}\right]$$

$$(\mathbf{KIN})_{29}=\lim_{x\to 0}\frac{(2-x^2)\sin x-\sin 2x}{x^5}$$

$$(\mathbf{KIN})_{30}=\lim_{x\to 0}\frac{\sqrt{\frac{1+x}{1-x}}\cdot\sqrt[4]{\frac{1+2x}{1-2x}}\cdot\sqrt[6]{\frac{1+3x}{1-3x}}-1}{x}$$

$$(\mathbf{KIN})_{31}=\lim_{x\to 0}\frac{\sin(\tan x)-\tan(\sin x)}{\sin x-\tan x}$$

$$(\mathbf{KIN})_{32}=\lim_{x\to \frac{\pi}{2}}\frac{\cos x}{\cos 3x}$$

$$(\mathbf{KIN})_{33}=\lim_{x\to 1}\frac{\sin\pi x+\cos\frac{\pi x}{2}}{(1+3x)\cos\frac{3\pi x}{2}}$$

$$(\mathbf{KIN})_{34}=\lim_{n\to \infty}\left(\sqrt[n]{4^n+5^n}\right)$$

$$(\mathbf{KIN})_{35}=\lim_{x\to 1}\frac{(2x+1)^5-243}{(x+1)^6-32}$$

$$(\mathbf{KIN})_{36}=\lim_{n\to \infty}\left[\left(\frac{n+9}{n+10}\right)^n\right]$$

$$(\mathbf{KIN})_{37}=\lim_{x\to \infty}\left[\left(\frac{4^{\frac{1}{x}}+5^{\frac{1}{x}}+6^{\frac{1}{x}}}{3}\right)^{3x}\right]$$

$$(\mathrm{KIN})_{38}=\lim_{x\to1}\frac{x\sqrt{x+3}-2}{x^2+x-2}$$

$$(\mathrm{KIN})_{39}=\lim_{x\to e}\frac{\ln x-1}{x-e}$$

$$(\mathrm{KIN})_{40}=\lim_{x\to\frac{\pi}{3}}\frac{\sin x-\sqrt{3}\cos x}{4x-\frac{4\pi}{3}}$$

$$(\mathrm{KIN})_{41}=\lim_{x\to3}\frac{3^x-27}{2^{x+1}-16}$$

$$(\mathrm{KIN})_{42}=\lim_{x\to6}\frac{1-\sqrt{3-\sqrt{x-2}}}{x-6}$$

$$(\mathrm{KIN})_{43}=\lim_{x\to0}\frac{\cos x\cos2x\cos4x\cos8x-x\cot x}{x^2}$$

$$(\mathrm{KIN})_{44}=\lim_{x\to1}\frac{\sin(2\ln x)-2\ln x}{\ln^3(x^4)}$$

$$(\mathrm{KIN})_{45}=\lim_{x\to0}\frac{\sin2x+\sin4x-\tan6x}{1-\cos x}$$

$$(\mathrm{KIN})_{46}=\lim_{x\to0}\frac{\ln(\sin x+\cos x)^2}{\sin2x}$$

$$(\mathrm{KIN})_{47}=\lim_{x\to0}\frac{x\cos x-\sin x}{x^3}$$

$$(\mathrm{KIN})_{48}=\lim_{x\to\frac{\pi}{4}}\frac{1-\sqrt{2}\sin x}{1-\sqrt{2}\cos x}$$

$$(\mathrm{KIN})_{49}=\lim_{x\to2}\frac{x^4\sqrt{x+2}\sqrt[3]{x+6}-64}{x-2}$$

$$(\mathrm{KIN})_{50}=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-2\cos x}{\sqrt{2}-2\sin x}$$

$$(\mathrm{KIN})_{51}=\lim_{x\to\infty}\left[\left(\frac{x^2+5x+4}{x^2-3x+7}\right)^x\right]$$

$$(\mathrm{KIN})_{52}=\lim_{x\to\frac{\pi}{6}}\frac{1-\sqrt{3}\tan x}{\pi-6x}$$

$$(\mathrm{KIN})_{53}=\lim_{x\to\sqrt{\pi}}\frac{1-\cos\sqrt{\pi}x}{\pi-x^2}$$

$$(\mathrm{KIN})_{54}=\lim_{x\to 0}\frac{(x+1)^8+(x-1)^8}{(x+1)^5+(x-1)^5}$$

$$(\mathrm{KIN})_{55}=\lim_{x\to 0}\frac{\ln\left(\frac{3-\cos 2x-2x^2}{2}\right)}{x^4}$$

$$(\mathrm{KIN})_{56}=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-\cos x-\sin x}{(4x-\pi)^2}$$

$$(\mathrm{KIN})_{57}=\lim_{x\to 2}\frac{\sqrt{2x-3}\sqrt[3]{3x-5}\sqrt[4]{4x-7}\sqrt[5]{5x-9}-1}{x-2}$$

$$(\mathrm{KIN})_{58}=\lim_{m\to\infty}\left[\left(\cos\left\{\frac{x}{m}\right\}\right)^m\right]$$

$$(\mathrm{KIN})_{59}=\lim_{x\to\infty}\left(\frac{x^3+x+1}{\sqrt[3]{x^3+x-1}}-\frac{x^3-x+1}{\sqrt[3]{x^3-x+1}}\right)$$

$$(\mathrm{KIN})_{60}=\lim_{n\to\infty}\left(\frac{1}{n}\sqrt[n]{\frac{2n!}{n!}}\right)$$

$$(\mathrm{KIN})_{61}=\lim_{x\to 0}\frac{1-\cos x\cos 2x\cos 3x}{x^2}$$

$$(\mathrm{KIN})_{62}=\lim_{x\to 0}\frac{1-\sin\left(\frac{\pi}{2}\cos x+x\right)}{x^2}$$

$$(\mathrm{KIN})_{63}=\lim_{x\to 0}\frac{\ln(2x^2+\cos 2x)}{x^4}$$

$$(\mathrm{KIN})_{64}=\lim_{x\to 0}\frac{64^x-3\times 48^x+3\times 36^x-27^x}{x^3}$$

$$(\mathrm{KIN})_{65}=\lim_{x\to 0}\frac{\cos(5\tan^4x-3\sin^4x)-\cos(5\tan^4x+3\sin^4x)}{x^8}$$

$$(\mathrm{KIN})_{66}=\lim_{x\to 0}\frac{1-\cos3x}{1-\cos4x}$$

$$(\mathrm{KIN})_{67}=\lim_{x\to\frac{\pi}{6}}\frac{\sin x-\frac{1}{2}}{6x-\pi}$$

$$(\mathrm{KIN})_{68}=\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{x-\frac{\pi}{4}}$$

$$(\mathrm{KIN})_{69}=\lim_{x\to\frac{\pi}{6}}\frac{2\cos2x-1}{\cos3x}$$

$$(\mathrm{KIN})_{70}=\lim_{x\to 0}\frac{\cos x-\sqrt[3]{\cos x}}{\sin^2x}$$

$$(\mathrm{KIN})_{71}=\lim_{x\to e}\frac{x-e\ln x}{(x-e)^2}$$

$$(\mathrm{KIN})_{72}=\lim_{x\to 0}\frac{x-\tan^{-1}x}{x^3}$$

$$(\mathrm{KIN})_{73}=\lim_{x\to 0}\left(\frac{1}{x\sin x}-\frac{1}{x^2}\right)$$

$$(\mathrm{KIN})_{74}=\lim_{x\to 0}\left(\frac{2^{\sec x}-2^{\cos x}}{x^2}\right)$$

$$(\mathrm{KIN})_{75}=\lim_{x\to\frac{\pi}{2}}[(\sin x)^{\tan x}]$$

$$(\mathrm{KIN})_{76}=\lim_{x\to\frac{\pi}{4}}\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin2x}$$

$$(\mathrm{KIN})_{77}=\lim_{x\to 0}\frac{\sqrt[3]{1+\tan x}-\sqrt[3]{1+\sin x}}{x^3}$$

$$(\mathrm{KIN})_{78}=\lim_{x\to 1}\left[\left(\frac{\pi}{4}-\tan^{-1}x\right)\left(\tan\frac{\pi}{x+1}\right)\right]$$

$$(\mathrm{KIN})_{79}=\lim_{x\to 1}\frac{\sin(x^{2020}-1)}{x-1}$$

$$(\mathrm{KIN})_{80}=\lim_{x\to 0}\frac{\sqrt{1+x^4}+\sqrt[3]{8+2x^4}-\sqrt[3]{27-2x^4}}{(1-\cos 2x)\tan^2(\sin 3x)}$$

$$(\mathrm{KIN})_{81}=\lim_{x\to 0}\frac{e^{\frac{\tan^6 x}{6}}-e^{\frac{\sin^6 x}{6}}}{x^8}$$

$$(\mathrm{KIN})_{82}=\lim_{x\to 0}\frac{\ln\left(\sqrt{\frac{1-x^2}{\cos x}}\right)}{x^3}$$

$$(\mathrm{KIN})_{83}=\lim_{n\to \infty}\left(\sqrt[3]{8^n+4^n+2^n}-2^n\right)$$

$$(\mathrm{KIN})_{84}=\lim_{x\to 0}\left(\sqrt[x]{1+\sin\left[1-\left(\frac{e^x-1}{x}\right)\right]}\right)$$

$$(\mathrm{KIN})_{85}=\lim_{x\to 0}\left(\sqrt[x^2]{1+\sin\left[1-\left(\frac{\sin x}{x}\right)\right]}\right)$$

$$(\mathrm{KIN})_{86}=\lim_{x\to \frac{\pi}{4}}\frac{\cos 2x}{1-\tan x}$$

$$(\mathrm{KIN})_{87}=\lim_{x\to 0}\frac{12-6x^2-12\cos x}{x^4}$$

$$(\mathrm{KIN})_{88}=\lim_{x\to \frac{\pi}{4}}(\sin 2x)^{\tan^2 2x}$$

$$(\mathrm{KIN})_{89}=\lim_{x\to 1}\frac{3\sin\pi x-\sin 3\pi x}{(x-1)^3}$$

$$(\mathbf{KIN})_{90}=\lim_{x\to 0}\frac{\ln(e+x)-\ln(e-x)}{x}$$

$$(\mathbf{KIN})_{91}=\lim_{x\to \alpha}\left(2-\frac{\alpha}{x}\right)^{\tan\frac{\pi x}{2\alpha}}\quad \alpha\neq 0$$

$$(\mathbf{KIN})_{92}=\lim_{x\to 0}\frac{\sin^{-1}x-\sin x}{(\sin^{-1}x)^3}$$

$$(\mathbf{KIN})_{93}=\lim_{x\to \frac{\pi}{4}}\frac{\sec^2x-2\tan x}{\left(x-\frac{\pi}{4}\right)^2}$$

$$(\mathbf{KIN})_{94}=\lim_{x\to 0}\frac{\sin^2x-\sin^2 2x+3x^2}{x^4}$$

$$(\mathbf{KIN})_{95}=\lim_{x\to 0}\frac{\sin 3x+\sin 6x-\sin 9x}{x^3}$$

$$(\mathbf{KIN})_{96}=\lim_{x\to 0}\frac{\ln(1+x)^2-2x}{x^2}$$

$$(\mathbf{KIN})_{97}=\lim_{x\to 0}\frac{\frac{x}{\sin x}-2+\frac{\sin x}{x}}{x^4}$$

$$(\mathbf{KIN})_{98}=\lim_{x\to 1}\left[\sqrt[x-1-\ln x]{\sin\left(\frac{\pi x}{2}\right)}\right]$$

$$(\mathbf{KIN})_{99}=\lim_{x\to \frac{\pi}{6}}\frac{1-\sqrt{3}\tan x}{\pi-6x}$$

$$(\mathbf{KIN})_{100}=\lim_{x\to 0}\frac{e^{10x}-x-1}{e^{5x}-2x-1}$$

$$(\mathbf{KIN})_{101}=\lim_{x\to \infty}\left(\sqrt[3]{8^x+3^x}-\sqrt{4^x-2^x}\right)$$

$$(\mathbf{KIN})_{102}=\lim_{x\to 0}\left[\frac{2}{\sin^2x}+\frac{1}{\ln(\cos x)}\right]$$

$$(\mathrm{KIN})_{103}=\lim_{x\to 0}\frac{x(1-\cos x)}{x^2+x-e^x\sin x}$$

$$(\mathrm{KIN})_{104}=\lim_{x\to 0}\frac{\ln(2+x)-x}{x^2}$$

$$(\mathrm{KIN})_{105}=\lim_{x\to \frac{\pi}{3}}\frac{\tan^3 x-3\tan x}{\cos\left(x+\frac{\pi}{6}\right)}$$

$$(\mathrm{KIN})_{106}=\lim_{n\to \infty}\sqrt[n]{\frac{3n!}{2n!\,n^n}}$$

$$(\mathrm{KIN})_{107}=\lim_{x\to 1}\frac{\sin\left(\pi-\cos^4\frac{\pi x}{2}\right)}{e^{1-\sin\frac{\pi x}{2}}+e^{\sin\frac{\pi x}{2}-1}-2}$$

$$(\mathrm{KIN})_{108}=\lim_{x\to \frac{\pi}{3}}\frac{\sin\left(x-\frac{\pi}{3}\right)}{1-2\cos x}$$

$$(\mathrm{KIN})_{109}=\lim_{n\to \infty} n\tan^{-1}\left[\left\{\frac{1}{(x^2+1)n+1}\right\}\tan\left(\frac{\pi}{4}-\frac{x}{2n}\right)\right]^n$$

$$(\mathrm{KIN})_{110}=\lim_{x\to \infty}\left(\frac{x^2+1}{x^2-1}\right)^{(x^4-1)\sin^2\frac{1}{x}}$$

$$(\mathrm{KIN})_{111}=\lim_{x\to +\infty}\left(\sin\frac{\pi x+4}{2x+3}\right)^{\frac{x^2}{1+2x}}$$

$$(\mathrm{KIN})_{112}=\lim_{x\to 0}\frac{x-\sin x}{x^3}$$

$$(\mathrm{KIN})_{113}=\lim_{x\to \frac{\pi}{4}}\frac{\cos^2 x-\frac{1}{2}}{x-\frac{\pi}{4}}$$

$$(\mathrm{KIN})_{114}=\lim_{x\to 0}\frac{\tan^2 x+2\ln(\cos x)}{\sin^2 x-\ln(1+\sin^2 x)}$$

$$(\mathrm{KIN})_{115}=\lim_{x\to 1}\frac{\ln x}{\tan \pi x}$$

$$(\mathrm{KIN})_{116}=\lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)\dots(1+5x)}{\tan 5x}$$

$$(\mathrm{KIN})_{117}=\lim_{x\to \frac{\pi}{4}}\frac{1-\sqrt{2}\sin x}{1-\sqrt{2}\cos x}$$

$$(\mathrm{KIN})_{118}=\lim_{x\to 0}\frac{4(81)^x-27^x-9^x-3^x-1}{1-\sqrt[3]{1+x+x^2}}$$

$$(\mathrm{KIN})_{119}=\lim_{x\to \infty}\left[x\left\{\frac{\pi}{4}-\tan^{-1}\left(\frac{x}{x+1}\right)\right\}\right]$$

$$(\mathrm{KIN})_{120}=\lim_{x\to \frac{\pi}{2}}\frac{2\sec^2 x-3}{2\sec^2 x-\sec x+1}$$

$$(\mathrm{KIN})_{121}=\lim_{x\to 0}\frac{x(a-b)}{\sin ax-\sin bx}$$

$$(\mathrm{KIN})_{122}=\lim_{x\to 0}\frac{x-\sin x}{x-\tan x}$$

$$(\mathrm{KIN})_{123}=\lim_{x\to 1}\frac{3x^2-(7x+1)\sqrt{x}+5}{x-1}$$

$$(\mathrm{KIN})_{124}=\lim_{x\to 1}\left[\sin\frac{\pi}{2}(x^2-4x+4)\right]^{\frac{x}{(x-1)^2}}$$

$$(\mathrm{KIN})_{125}=\lim_{x\to 0}\frac{\sqrt[3]{2x+1}\sqrt[5]{3x+1}-1}{\sqrt[5]{2x+1}\sqrt[3]{3x+1}-1}$$

$$(\mathrm{KIN})_{126}=\lim_{x\to 0}\frac{\tan\left(\frac{\pi^2}{4\pi+x}\right)-1}{x}$$

$$(\mathrm{KIN})_{127}=\lim_{x\to 0}e^{\frac{\ln\cos x}{x}}$$

$$(\mathrm{KIN})_{128} = \lim_{\alpha\to 0}\frac{\sin(3x-5\alpha)-\sin 3x}{\alpha}$$

$$(\mathrm{KIN})_{129} = \lim_{n\to\infty}\left(\tan\theta+\frac{1}{2}\tan\frac{\theta}{2}+\frac{1}{2^2}\tan\frac{\theta}{2^2}+\cdots+\frac{1}{2^n}\tan\frac{\theta}{2^n}\right)$$

$$(\mathrm{KIN})_{130} = \lim_{x\to 0}\frac{\left(x-3\tan\frac{\pi}{x+4}+3\right)(e^{\cos x}-e)}{\cos(5\cos^{-1}x)-5x}$$

$$(\mathrm{KIN})_{131} = \lim_{x\to -1}(2+\cos\pi x)^{\frac{1}{x^3-3x-2}}$$

$$(\mathrm{KIN})_{132} = \lim_{x\to 1}\frac{\sin\pi x}{x^3-1}$$

$$(\mathrm{KIN})_{133} = \lim_{x\to 0}\frac{\sin^2 x+2\ln(\cos x)}{x^4}$$

$$(\mathrm{KIN})_{134} = \lim_{x\to 0}\frac{e^x-x-1}{x^2}$$

$$(\mathrm{KIN})_{135} = \lim_{x\to 0}\frac{e^{e^x}-e}{e^{3x}-\sec x}$$

$$(\mathrm{KIN})_{136} = \lim_{x\to\frac{\pi}{2}}\left(\frac{\sec^2 x+\tan^2 x-2}{2\sec^2 x+\tan^2 x-\sec x+1}\right)^{\frac{\tan^2 x}{\sec^2 x}}$$

$$(\mathrm{KIN})_{137} = \lim_{x\to 0^+}\frac{\left(\sqrt{x}\right)^{\sqrt{x}}-x^x}{\left(\sqrt{x}\right)^{x}-x^{\sqrt{x}}}$$

$$(\mathrm{KIN})_{138} = \lim_{x\to 0}\frac{\cos\left(\frac{x}{2}\right)-\sqrt{\cos x+x\sin x}}{x^2}$$

$$(\mathrm{KIN})_{139} = \lim_{x\to 0}\frac{\sin[\sin(x-\sin x)]}{x-\sin x}\times\frac{x-\sin x}{x^3}$$

$$(\mathrm{KIN})_{140} = \lim_{x\to 0}\frac{a^x-(a+1)^x}{x}$$

ផ្នែកទី០១ដំណោះលីមីត(*Solution Of Limit*)

$$a_1.\ \lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}-\frac{2}{\frac{1}{S+2}+\frac{1}{S+1}-\sum_{H=S}^{\infty}\frac{3}{\sum_{A=1}^{H}\frac{2}{\sum_{B=A}^{\infty}\frac{1}{B^2+3B+2}}}}}}$$

$$=\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}-\frac{2}{\frac{1}{S+2}+\frac{1}{S+1}-\sum_{H=S}^{\infty}\frac{3}{\sum_{A=1}^{H}\frac{2}{\frac{1}{A+1}}}}}}$$

$$=\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}-\frac{2}{\frac{1}{S+2}+\frac{1}{S+1}-\sum_{H=S}^{\infty}\frac{3}{\sum_{A=1}^{H}2(A+1)}}}}$$

$$=\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}-\frac{2}{\frac{1}{S+2}+\frac{1}{S+1}-\sum_{H=S}^{\infty}\frac{3}{H(H+3)}}}}$$

$$=\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}-\frac{2}{\frac{1}{S+2}+\frac{1}{S+1}-\left(\frac{1}{S}+\frac{1}{S+1}+\frac{1}{S+2}\right)}}}$$

$$=\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}-\frac{2}{-\frac{1}{S}}}}=\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}2S}}$$

$$=\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{I(I+1)}}=\lim_{R\to\infty}\frac{1}{\frac{(R+2)(R+3)(R+4)}{14R(R-1)(R-2)}}$$

$$=\lim_{R\to\infty}\frac{14R(R-1)(R-2)}{(R+2)(R+3)(R+4)}=\lim_{R\to\infty}\frac{14R\times R\left(1-\frac{1}{R}\right)R\left(1-\frac{2}{R}\right)}{R\left(1+\frac{2}{R}\right)R\left(1+\frac{3}{R}\right)R\left(1+\frac{4}{R}\right)}=14$$

$$a_1.\lim_{R\to\infty}\frac{1}{\prod_{I=5}^{R}\frac{(I-3)(I+4)}{\sum_{S=1}^{I}-\frac{2}{\frac{1}{S+2}+\frac{1}{S+1}-\sum_{H=S}^{\infty}\frac{3}{\sum_{A=1}^{H}\frac{2}{\sum_{B=A}^{\infty}\frac{1}{B^2+3B+2}}}}}}=14$$

$$a_2.\lim_{x\to 0}\frac{2019x-sin2019x}{2020x-sin2020x}$$

+ក្បាច់ទី០១

$$=\lim_{x\to 0}\frac{\frac{2019x-sin2019x}{x^3}}{\frac{2020x-sin2020x}{x^3}}=\lim_{x\to 0}\frac{P(x)}{Q(x)}$$

+យក $P(x)=\lim\limits_{x\to 0}\frac{2019x-sin2019x}{x^3}$

តាង $u=2019x\Rightarrow x=\frac{u}{2019}$; បើ$x\to 0$; នោះ$u\to 0$

យើងបាន$\lim\limits_{u\to 0}\frac{u-sinu}{\left(\frac{u}{2019}\right)^3}=2019^3\lim\limits_{u\to 0}\frac{u-sinu}{u^3}$

តាង$u=3t$; បើ$u\to 0$; នោះ$t\to 0$

នាំឲ្យ $2019^3\lim\limits_{t\to 0}\frac{3t-sin3t}{(3t)^3}=\frac{2019^3}{27}\lim\limits_{t\to 0}\frac{3t-sin3t}{t^3}$

$$=\frac{2019^3}{27}\lim_{t\to 0}\frac{3t-3sint+4sin^3t}{t^3}=\frac{2019^3}{27}\lim_{t\to 0}\left[\frac{3(t-sint)}{t^3}+\frac{4sin^3t}{t^3}\right]$$

$$P(x)=\frac{2019^3}{27}(3P(x)+4)\equiv P(x)-3P(x)=\frac{2019^3}{27}(4)$$

$$\equiv -2P(x)=\frac{2019^3}{27}4\equiv P(x)=-2\frac{2019^3}{27}$$

+យក $Q(x)=\lim_{x\to 0}\frac{2020x-sin2020x}{x^3}$

តាង $u=2020x\Rightarrow x=\frac{u}{2020}$; បើ $x\to 0$; នោះ $u\to 0$

យើងបាន $\lim_{u\to 0}\frac{u-sinu}{\left(\frac{u}{2020}\right)^3}=2020^3\lim_{u\to 0}\frac{u-sinu}{u^3}$

តាង $u=3t$; បើ $u\to 0$; នោះ $t\to 0$

នាំឲ្យ $2020^3\lim_{t\to 0}\frac{3t-sin3t}{(3t)^3}=\frac{2030^3}{27}\lim_{t\to 0}\frac{3t-sin3t}{t^3}$

$$=\frac{2020^3}{27}\lim_{t\to 0}\frac{3t-3sint+4sin^3t}{t^3}$$

$$=\frac{2020^3}{27}\lim_{t\to 0}\left[\frac{3(t-sint)}{t^3}+\frac{4sin^3t}{t^3}\right]$$

$$Q(x)=\frac{2020^3}{27}(3Q(x)+4)\equiv Q(x)-3Q(x)=\frac{2020^3}{27}(4)$$

$$\equiv -2Q(x)=\frac{2020^3}{27}4\equiv Q(x)=-2\frac{2020^3}{27}$$

យើងបាន $\lim_{x\to 0}\frac{P(x)}{Q(x)}=\frac{-2\frac{2019^3}{27}}{-2\frac{2020^3}{27}}=\frac{2019^3}{2020^3}$

$$a_2.\lim_{x\to 0}\frac{2019x-sin2019x}{2020x-sin2020x}=\frac{2019^3}{2020^3}$$

+ក្បាច់ទី០២

$$a_2.\lim_{x\to 0}\frac{2019x-sin2019x}{2020x-sin2020x}=\lim_{x\to 0}\frac{\frac{2019x-sin2019x}{x^3}}{\frac{2020x-sin2020x}{x^3}}=\lim_{x\to 0}\frac{M(x)}{N(x)}$$

+យក $M(x)=\lim_{x\to 0}\frac{2019x-sin2019x}{x^3}$

តាង $u=2019x\Rightarrow x=\frac{u}{2019}$; បើ $x\to 0$; នោះ $u\to 0$

យើងបាន $\lim_{u\to 0}\frac{u-sinu}{\left(\frac{u}{2019}\right)^3}=2019^3\lim_{u\to 0}\frac{u-sinu}{u^3}$

តាម $Taylor\ Series: sinx=\frac{(-1)^n x^{2n+1}}{(2n+1)!}$; $n\in IN$

នោះ $Sinu=u-\frac{u^3}{3!}+\frac{u^5}{5!}-\frac{u^7}{7!}+\cdots$

$$=2019^3\lim_{u\to 0}\frac{u-u+\frac{u^3}{3!}-\frac{u^5}{5!}+\frac{u^7}{7!}-\cdots}{u^3}$$

$$=2019^3\lim_{u\to 0}\frac{\frac{u^3}{3!}-\frac{u^5}{5!}+\frac{u^7}{7!}-\cdots}{u^3}=\frac{1}{3!}2019^3$$

+យក $N(x)=\lim_{x\to 0}\frac{2020x-sin2020x}{x^3}$

តាង $u=2020x\Rightarrow x=\frac{u}{2020}$; បើ $x\to 0$; នោះ $u\to 0$

យើងបាន $\lim_{u\to 0}\frac{u-sinu}{\left(\frac{u}{2020}\right)^3}=2020^3\lim_{u\to 0}\frac{u-sinu}{u^3}$

តាម $Taylor\ Series: sinx = \frac{(-1)^n x^{2n+1}}{(2n+1)!}$; $n \in IN$

នោះ $Sinu = u - \frac{u^3}{3!} + \frac{u^5}{5!} - \frac{u^7}{7!} + \cdots$

$$= 2020^3 \lim_{u\to 0} \frac{u - u + \frac{u^3}{3!} - \frac{u^5}{5!} + \frac{u^7}{7!} - \cdots}{u^3} = 2020^3 \lim_{u\to 0} \frac{\frac{u^3}{3!} - \frac{u^5}{5!} + \frac{u^7}{7!} - \cdots}{u^3}$$

$$= \frac{1}{3!} 2020^3$$

យើងបាន $\lim_{x\to 0} \frac{M(x)}{N(x)} = \frac{-\frac{1}{3!}2019^3}{-\frac{1}{3!}2020^3} = \frac{2019^3}{2020^3}$

$a_2. \lim_{x\to 0} \frac{2019x - sin2019x}{2020x - sin2020x} = \frac{2019^3}{2020^3}$

$a_3. \lim_{x\to 0} \frac{(2 + sinx)^{cosx} - 2}{x}$

$$= \lim_{x\to 0} \frac{e^{cosxln(2+sinx)} - e^{ln2}}{x} = \lim_{x\to 0} \frac{e^{ln2}\left(e^{cosxln(2+sinx)-ln2} - 1\right)}{x}$$

$$= 2\lim_{x\to 0} \frac{\left(e^{cosxln(2+sinx)-ln2} - 1\right)}{cosxln(2+sinx) - ln2} \times \frac{cosxln\left(1 + \frac{sinx}{2}\right)}{x}$$

$$= 2\lim_{x\to 0} \frac{cosxln\left(1 + \frac{sinx}{2}\right)}{\frac{sinx}{2}} \times \frac{sinx}{2x}$$

$$= 2 \times 1 \times \frac{1}{2} = 1$$

$a_3. \lim_{x\to 0} \frac{(2 + sinx)^{cosx} - 2}{x} = 1$

$a_4. \lim_{n\to +\infty} \sqrt{2} \times \sqrt[4]{2} \times \sqrt[8]{2} \times \ldots \times \sqrt[n]{2}$

$$= \lim_{n\to+\infty} 2^{\frac{1}{2}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{8}} \times \dots \times 2^{\frac{1}{n}} = \lim_{n\to+\infty} 2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{n}}$$

$$S_n = \frac{u_1(1-q^n)}{1-q}$$

$$= \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

$$\Rightarrow \lim_{n\to+\infty} S_n = \lim_{n\to+\infty} \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

$$= 2^{1-0} = 2$$

$$a_4. \lim_{n\to+\infty} \sqrt{2} \times \sqrt[4]{2} \times \sqrt[8]{2} \times \dots \times \sqrt[n]{2} = 2$$

$$a_5. \lim_{x\to 0} \frac{(1+x^{2019})^{2019}-1}{x^{2019}}$$

ដោយ $a^x = e^{xlna}$

នោះ $(1+x^{2019})^{2019} = e^{2019ln\left(1+x^{2019}\right)}$

$$= \lim_{x\to 0} \frac{e^{2019ln\left(1+x^{2019}\right)}-1}{x^{2019}} = \lim_{x\to 0} \frac{e^{2019ln\left(1+x^{2019}\right)}-1}{2019ln(1+x^{2019})} \times \frac{2019ln(1+x^{2019})}{x^{2019}}$$

$$= 1 \times 2019 \times 1 = 2019$$

$a_5. \lim_{x\to 0} \frac{\left(1+x^{2019}\right)^{2019}-1}{x^{2019}} = 2019$

$$a_6. \lim_{n\to\infty} \frac{n+n^2+n^3+\cdots+n^n}{1^n+2^n+3^n+\cdots+n^n}$$

$$= \lim_{n\to\infty} \frac{n\left(\frac{n^n-1}{n-1}\right)}{1^n+2^n+3^n+\cdots+n^n} = \lim_{n\to\infty} \frac{\left(\frac{n}{n-1}\right)\left(\frac{n^n-1}{n^n}\right)}{\left(\frac{1}{n}\right)^n+\left(\frac{2}{n}\right)^n+\left(\frac{3}{n}\right)^n+\cdots+\left(\frac{n}{n}\right)^n}$$

$$=\lim_{n\to\infty}\frac{1}{1+\left(1-\frac{1}{n}\right)^n+\left(1-\frac{2}{n}\right)^n+\left(1-\frac{3}{n}\right)^n+\cdots}=\frac{1}{1+e^{-1}+e^{-2}+e^{-3}+\cdots}$$

$$=\frac{1}{\frac{1}{1-e^{-1}}}=\frac{e-1}{e}$$

$$a_6.\lim_{n\to\infty}\frac{n+n^2+n^3+\cdots+n^n}{1^n+2^n+3^n+\cdots+n^n}=\frac{e-1}{e}$$

$$a_7.\lim_{x\to 0}\frac{(1+ax)^n-1}{x}$$

$$=\lim_{x\to 0}\frac{[(1+ax)-1][(1+ax)^{n-1}+(1+ax)^{n-2}+\cdots+1]}{x}$$

$$=\lim_{x\to 0}\frac{ax[(1+ax)^{n-1}+(1+ax)^{n-2}+\cdots+1]}{x}=a\underbrace{((1+1+\cdots+1))}_{n}$$

$$=an$$

$$a_7.\lim_{x\to 0}\frac{(1+ax)^n-1}{x}=an$$

$$a_8.\lim_{n\to\infty}\frac{(n+2)!+n!}{(n+2)!+(n+1)!}$$

$$=\lim_{n\to\infty}\frac{n!\,(n+2)(n+1)+n!}{n!\,(n+2)(n+1)+n!\,(n+1)}=\lim_{n\to\infty}\frac{n!\,[(n+2)(n+1)+1]}{n!\,[(n+2)(n+1)+(n+1)]}$$

$$=\lim_{n\to\infty}\frac{n\left(1+\frac{2}{n}\right)n\left(1+\frac{1}{n}\right)+n^2\left(\frac{1}{n^2}\right)}{n\left(1+\frac{2}{n}\right)n\left(1+\frac{1}{n}\right)+n^2\left(\frac{1}{n}+\frac{1}{n^2}\right)}$$

$$=\lim_{n\to\infty}\frac{n^2\left[\left(1+\frac{2}{n}\right)\left(1+\frac{1}{n}\right)+\left(\frac{1}{n^2}\right)\right]}{n^2\left[\left(1+\frac{2}{n}\right)\left(1+\frac{1}{n}\right)+\left(\frac{1}{n}+\frac{1}{n^2}\right)\right]}=\frac{1\times 1+0}{1\times 1+0}=1$$

$$a_8.\lim_{n\to\infty}\frac{(n+2)!+n!}{(n+2)!+(n+1)!}=1$$

$a_9. \lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots+\frac{1}{2^n}\right)$

$$=\lim_{n\to\infty}\left[\frac{\frac{1}{2}\left\{1-\left(\frac{1}{2}\right)^n\right\}}{1-\frac{1}{2}}\right]=\lim_{n\to\infty}\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{\frac{1}{2}}=1$$

$a_9. \lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots+\frac{1}{2^n}\right)=1$

$a_{10}. \lim_{n\to+\infty}\left[\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{n}{(n+1)!}\right]$

ដោយ $\frac{n}{(n+1)!}=\frac{1}{n!}-\frac{1}{(n+1)!}$

$$=\lim_{n\to+\infty}\left[\left(\frac{1}{1!}-\frac{1}{2!}\right)+\left(\frac{1}{2!}-\frac{1}{3!}\right)+\cdots+\frac{1}{n!}-\frac{1}{(n+1)!}\right]=\lim_{n\to+\infty}\left[1-\frac{1}{(n+1)!}\right]$$
$$=1$$

$a_{10}. \lim_{n\to+\infty}\left[\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{n}{(n+1)!}\right]=1$

$a_{11}. \lim_{n\to\infty}\left[\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{n(n+1)}\right]$

ដោយ $\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$

$$=\lim_{n\to\infty}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right]=\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)=1$$

$a_{11}. \lim_{n\to\infty}\left[\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{n(n+1)}\right]=1$

$a_{12}. \lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)$

ដោយ $\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}=\frac{\sqrt{2n+1}-\sqrt{2n-1}}{2}$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(\frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\cdots+\frac{\sqrt{2n+1}-\sqrt{2n-1}}{2}\right)$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(-\frac{1}{2}+\frac{\sqrt{2n+1}}{2}\right)=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n}}\left(-\frac{1}{2\sqrt{n}}+\frac{\sqrt{2+\frac{1}{\sqrt{n}}}}{2}\right)=\frac{\sqrt{2}}{2}$$

$$a_{12}.\lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)=\frac{\sqrt{2}}{2}$$

$$a_{13}.\lim_{n\to\infty}\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+\cdots+n}\right)$$

ដោយ $\frac{1}{1+2+\cdots+n}=\frac{2}{n(n+1)}=2\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$$=\lim_{n\to\infty}2\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right]=\lim_{n\to\infty}2\left(1-\frac{1}{n+1}\right)=2$$

$$a_{13}.\lim_{n\to\infty}\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+\cdots+n}\right)=2$$

$$a_{14}.\lim_{n\to+\infty}\frac{1\cdot 1!+2\cdot 2!+3\cdot 3!+\cdots+n\cdot n!}{(n+1)!}$$

ដោយ $n!\,n=n!\,(n+1-1)=n!\,(n+1)-n!=(n+1)!-n!$

$$=\lim_{n\to+\infty}\frac{(2!-1!)+(3!-2!)+\cdots+(n+1)!-n!}{(n+1)!}=\lim_{n\to+\infty}\frac{-1+(n+1)!}{(n+1)!}$$

$$=\lim_{n\to+\infty}\left[\frac{-1}{(n+1)!}+1\right]=1$$

$$a_{14}.\lim_{n\to+\infty}\frac{1\cdot 1!+2\cdot 2!+3\cdot 3!+\cdots+n\cdot n!}{(n+1)!}=1$$

$$a_{15}.\lim_{x\to 0}\frac{(e^{x}-1)(e^{2x}-1)\dots(e^{nx}-1)}{x^{n}}$$

$$=\lim_{x\to 0}\left[\frac{(e^x-1)}{x}\times 2\frac{(e^{2x}-1)}{2x}\times\cdots\times n\frac{(e^{nx}-1)}{nx}\right]=1\times 2\times\ldots\times n=n!$$

$$a_{15}.\lim_{x\to 0}\frac{(e^x-1)(e^{2x}-1)\ldots(e^{nx}-1)}{x^n}=n!$$

$$a_{16}.\lim_{x\to 0}\frac{7^{x^2}-cos2x}{x^2}$$

$$=\lim_{x\to 0}\frac{7^{x^2}-1+1-cos2x}{x^2}=\lim_{x\to 0}\left(\frac{7^{x^2}-1}{x^2}+\frac{1-cos2x}{x^2}\right)$$

$$=\lim_{x\to 0}\left(\frac{7^{x^2}-1}{x^2}+\frac{2sin^2x}{x^2}\right)=ln7+2$$

$$a_{16}.\lim_{x\to 0}\frac{7^{x^2}-cos2x}{x^2}=ln7+2$$

$$a_{17}.\lim_{x\to 0}\frac{ln(1+x)+ln(1+2x)+\cdots+ln(1+nx)}{x}$$

$$=\lim_{x\to 0}\left[\frac{ln(1+x)}{x}+2\frac{ln(1+2x)}{2x}+\cdots+n\frac{ln(1+nx)}{nx}\right]$$

$=1+2+\cdots+n$ គណនាផលបូកស្វីតនព្វន្ត

$$=\frac{n(n+1)}{2}$$

$$a_{17}.\lim_{x\to 0}\frac{ln(1+x)+ln(1+2x)+\cdots+ln(1+nx)}{x}=\frac{n(n+1)}{2}$$

$$a_{18}.\lim_{x\to 0^+}\frac{\sqrt{2+\sqrt{2+\cdots+\sqrt{2+2cosx}}}-2}{x}$$

$*$ ដោយ$\sqrt{2+2cosx}=2cos\frac{x}{2}$

$$*\sqrt{2+\sqrt{2+cosx}}=\sqrt{2+2cos\frac{x}{2}}=2cos\frac{x}{2^2}$$

$$*\sqrt{2+\sqrt{2+\cdots+\sqrt{2+2cosx}}}=2cos\frac{x}{2^n}$$

យើងបាន $\lim\limits_{x\to0^+}\dfrac{2cos\frac{x}{2^n}-2}{x}=-2\lim\limits_{x\to0^+}\dfrac{1-cos\frac{x}{2^n}}{\frac{x}{2^n}\times2^n}=-2\times\dfrac{1}{2}\times\dfrac{1}{2^n}=-\dfrac{1}{2^n}$

$$a_{18}.\lim_{x\to0^+}\frac{\sqrt{2+\sqrt{2+\cdots+\sqrt{2+2cosx}}}-2}{x}=-\frac{1}{2^n}$$

$$a_{19}.\lim_{n\to\infty}\left[n^2+n-\sum_{K=1}^{n}\left(\frac{2k^3+8k^2+6k-1}{k^2+4k+3}\right)\right]$$

$$=\lim_{n\to\infty}\left[n^2+n-\sum_{K=1}^{n}\left(\frac{2k(k^2+4k+3)-1}{k^2+4k+3}\right)\right]$$

$$=\lim_{n\to\infty}\left[n^2+n-\sum_{K=1}^{n}\left(2k-\frac{1}{k^2+4k+3}\right)\right]$$

$$=\lim_{n\to\infty}\left[n^2+n-2\sum_{K=1}^{n}k+\sum_{K=1}^{n}\frac{1}{k^2+4k+3}\right]$$

$$=\lim_{n\to\infty}\left[n^2+n-2\frac{n(n+1)}{2}+\sum_{K=1}^{n}\frac{1}{(K+1)(k+3)}\right]$$

$$=\lim_{n\to\infty}\left[n^2+n-n^2-n+\frac{1}{2}\sum_{K=1}^{n}\frac{1}{(K+1)}-\frac{1}{(k+3)}\right]$$

$$=\frac{1}{2}\lim_{n\to\infty}\left[\sum_{K=1}^{n}\left(\frac{1}{K+1}-\frac{1}{k+2}\right)+\left(\frac{1}{K+2}-\frac{1}{k+3}\right)\right]$$

$$=\frac{1}{2}\lim_{n\to\infty}\left[\sum_{K=1}^{n}\left(\frac{1}{K+1}-\frac{1}{k+2}\right)+\sum_{k=1}^{n}\left(\frac{1}{K+2}-\frac{1}{k+3}\right)\right]$$

$$*\sum_{K=1}^{n}\left(\frac{1}{K+1}-\frac{1}{k+2}\right)=\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{n}-\frac{1}{n+1}+\frac{1}{n+1}-\frac{1}{n+2}$$

$$\Rightarrow\sum_{K=1}^{n}\left(\frac{1}{K+1}-\frac{1}{k+2}\right)=\frac{1}{2}-\frac{1}{n+2}$$

$$**\sum_{K=1}^{n}\left(\frac{1}{K+2}-\frac{1}{k+3}\right)=\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\cdots+\frac{1}{n+1}-\frac{1}{n+2}+\frac{1}{n+2}-\frac{1}{n+3}$$

$$\Rightarrow\sum_{K=1}^{n}\left(\frac{1}{K+2}-\frac{1}{k+3}\right)=\frac{1}{3}-\frac{1}{n+3}$$

តាម$(*)\&(**)$យើងបាន $\lim_{n\to\infty}\frac{1}{2}\left(\frac{1}{2}-\frac{1}{n+1}+\frac{1}{3}-\frac{1}{n+3}\right)$

$$=\frac{1}{2}\left(\frac{1}{2}-0+\frac{1}{3}-0\right)=\frac{5}{12}$$

$$a_{19}.\lim_{n\to\infty}\left[n^2+n-\sum_{K=1}^{n}\left(\frac{2k^3+8k^2+6k-1}{k^2+4k+3}\right)\right]=\frac{5}{12}$$

$$a_{20}.\lim_{n\to\infty}\sqrt[n]{n3^n}$$

$$=\lim_{n\to\infty}3n^{\frac{1}{n}}$$

តាង $\mathrm{y}=n^{\frac{1}{n}}\equiv\lim_{n\to\infty}y=\lim_{n\to\infty}n^{\frac{1}{n}}$

ដៃ $\mathrm{y}=n^{\frac{1}{n}}\equiv lny=\frac{lnn}{n}$

នោះ $\lim\limits_{n\to\infty} lny = \lim\limits_{n\to\infty} \frac{lnn}{n}$

$$\lim_{n\to\infty} lny = 0 \equiv \lim_{n\to\infty} lny = ln1 \equiv \lim_{n\to\infty} y = 1$$

យើងបាន $\lim\limits_{n\to\infty} 3n^{\frac{1}{n}} = 3 \times 1 = 3$

$a_{20}.\lim\limits_{n\to\infty} \sqrt[n]{n3^n} = 3$

$a_{21}.\lim\limits_{x\to 0} \frac{1 - cosxcos2xcos3x \dots cosnx}{x^2}$

$$= \lim_{x\to 0} \frac{1 - cosx + cosx - cosxcos2xcos3x \dots cosnx}{x^2}$$

$$= \lim_{x\to 0} \left[\frac{1 - cosx}{x^2} + \frac{cosx(1 - cos2xcos3x \dots cosnx)}{x^2}\right]$$

$$= \frac{1}{2} + \lim_{x\to 0} \left(\frac{1 - cos2x + cos2x - cos2xcos3x \dots cosnx}{x^2}\right)$$

$$= \frac{1}{2} + \lim_{x\to 0} \left(\frac{2sin^2x}{x^2} + \frac{cos2xcos3x \dots cosnx}{x^2}\right)$$

$$= \frac{1}{2} + \frac{4}{2} + \lim_{x\to 0} \left(\frac{1 - cos3x}{x^2} + \frac{cos3x \dots cosnx}{x^2}\right)$$

$$= \frac{1}{2} + \frac{4}{2} + \lim_{x\to 0} \left(\frac{9}{4} \times \frac{2sin^2\frac{3x}{2}}{\frac{9}{4}x^2} + \frac{cos3x \dots cosnx}{x^2}\right)$$

$$= \frac{1}{2} + \frac{2^2}{2} + \frac{3^2}{2} + \dots + \frac{n^2}{2} = \frac{n(n+1)(2n+1)}{12}$$

$a_{21}.\lim\limits_{x\to 0} \frac{1 - cosxcos2xcos3x \dots cosnx}{x^2} = \frac{n(n+1)(2n+1)}{12}$

$a_{22}.\lim\limits_{x\to 1} \frac{1 - x^{-\frac{1}{3}}}{1 - x^{-\frac{2}{3}}}$

$$=\lim_{x\to 1}\frac{1-\frac{1}{\sqrt[3]{x}}}{1-\sqrt[3]{x^2}}=\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[3]{x}\left(\frac{\sqrt[3]{x^2}-1}{\sqrt[3]{x^2}}\right)}=\lim_{x\to 1}\frac{\left(\sqrt[3]{x}-1\right)\sqrt[3]{x}}{\sqrt[3]{x^2}-1}=\lim_{x\to 1}\frac{\left(\sqrt[3]{x}-1\right)}{\sqrt[3]{x^2}-1}$$

$$=\lim_{x\to 1}\frac{\left(\sqrt[3]{x}-1\right)}{\left(\sqrt[3]{x}-1\right)\left(\sqrt[3]{x}+1\right)}=\lim_{x\to 1}\frac{1}{\left(\sqrt[3]{x}+1\right)}=\frac{1}{2}$$

$$a_{22}.\lim_{x\to 1}\frac{1-x^{-\frac{1}{3}}}{1-x^{-\frac{2}{3}}}=\frac{1}{2}$$

$$a_{23}.\lim_{x\to 0}\left[\frac{1}{x^2}\left(\frac{2}{cosx}+cosx-3\right)\right]$$

$$=\lim_{x\to 0}\left[\frac{1}{x^2}\left(\frac{2}{cosx}-2+cosx-1\right)\right]=\lim_{x\to 0}\left[\frac{1}{x^2}\left(\frac{2-2cosx}{cosx}+cosx-1\right)\right]$$

$$=\lim_{x\to 0}\left[\frac{1-cosx}{x^2}\left(\frac{2}{cosx}-1\right)\right]=\lim_{x\to 0}\left[\frac{2sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2\times 4}\left(\frac{2}{cosx}-1\right)\right]=\frac{1}{2}$$

$$a_{23}.\lim_{x\to 0}\left[\frac{1}{x^2}\left(\frac{2}{cosx}+cosx-3\right)\right]=\frac{1}{2}$$

$$a_{24}.\lim_{n\to\infty}\frac{1\cdot 2+2\cdot 3+3\cdot 4+\cdots+(n-1)n}{n^3}$$

ដោយ $1\cdot 2+2\cdot 3+3\cdot 4+\cdots+(n-1)n$

$$=\sum_{k=1}^{n}(k-1)k=\sum_{k=1}^{n}(k^2-k)=\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$=\frac{n(n+1)}{2}\left[\frac{2n-2}{3}\right]=\frac{n(n+1)(n-1)}{3}$$

$$\Rightarrow\sum_{k=1}^{n}(k^2-k)=\frac{n(n+1)(n-1)}{3}$$

យើងបាន $\lim_{n\to\infty}\frac{n(n+1)(n-1)}{3n^3}=\frac{1}{3}$

$a_{24}.\lim_{n\to\infty}\frac{n(n+1)(n-1)}{3n^3}=\frac{1}{3}$

$a_{25}.\lim_{x\to\infty}x\left(e^{\frac{2}{x}}-1\right)$

តាង $t=\frac{1}{x}\Rightarrow x=\frac{1}{t}$

បើ $x\to\infty$ នោះ $t\to 0$

យើងបាន $\lim_{t\to 0}\frac{1}{t}(e^{2t}-1)$

$=\lim_{t\to 0}\frac{2t}{t}\left[\frac{(e^{2t}-1)}{2t}\right]=2$

$a_{25}.\lim_{x\to\infty}x\left(e^{\frac{2}{x}}-1\right)=2$

$a_{26}.\lim_{x\to 0}\frac{cos^{-1}(1-x)}{\sqrt{x}}$

តាង $t=cos^{-1}(1-x)\equiv cost=1-x$

$1-cost=x$

បើ $x\to 0$ នោះ $t\to 0$

យើងបាន $\lim_{t\to 0}\frac{cos^{-1}(1-1+cost)}{\sqrt{1-cost}}$

$=\lim_{t\to 0}\frac{cos^{-1}(cost)}{\sqrt{2sin^2\frac{t}{2}}}=\lim_{t\to 0}\frac{1}{\sqrt{\frac{2sin^2\frac{t}{2}}{4\left(\frac{t}{2}\right)^2}}}=\sqrt{2}$

$$a_{26}.\lim_{x\to 0}\frac{cos^{-1}(1-x)}{\sqrt{x}}=\sqrt{2}$$

$$a_{27}.\lim_{x\to 1}\frac{2^x-\sqrt{1+3^x}}{\sqrt{4^x-1}-3^{\frac{x}{2}}}$$

តាង $t=x-1 \Rightarrow x=1+t$

បើ $x\to 1$ នោះ $t\to 0$

$$=\lim_{t\to 0}\frac{2^{t+1}-\sqrt{1+3^{t+1}}}{\sqrt{4^{t+1}-1}-3^{\frac{t+1}{2}}}=\lim_{t\to 0}\frac{2\times 2^t-\sqrt{1+3\times 3^t}}{\sqrt{4\times 4^t-1}-\sqrt{3\times 3^t}}$$

$$=\lim_{t\to 0}\frac{2(2^t-1)+2-\sqrt{1+3\times 3^t}}{\sqrt{4\times 4^t-1}-\sqrt{3}+\sqrt{3}-\sqrt{3\times 3^t}}$$

$$=\lim_{t\to 0}\frac{2\,\frac{(2^t-1)}{t}+\frac{2-\sqrt{1+3\times 3^t}}{t}}{\frac{\sqrt{4\times 4^t-1}-\sqrt{3}}{t}+\frac{\sqrt{3}-\sqrt{3\times 3^t}}{t}}$$

$$=\lim_{t\to 0}\frac{2ln2+\frac{4-1-3\times 3^t}{t\left(2+\sqrt{1+3\times 3^t}\right)}}{\frac{4\times 4^t-1-3}{t\left(\sqrt{4\times 4^t-1}+\sqrt{3}\right)}+\frac{3-3\times 3^t}{t\left(\sqrt{3}+\sqrt{3\times 3^t}\right)}}=\frac{\left(2ln2-\frac{3ln3}{4}\right)2\sqrt{3}}{4ln4-3ln3}$$

$$=\frac{\left(2ln2-\frac{3ln3}{4}\right)2\sqrt{3}}{8ln2-3ln3}=\frac{\frac{1}{4}(8ln2-3ln3)2\sqrt{3}}{8ln2-3ln3}=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$$

$$a_{27}.\lim_{x\to 1}\frac{2^x-\sqrt{1+3^x}}{\sqrt{4^x-1}-3^{\frac{x}{2}}}=\frac{\sqrt{3}}{2}$$

$$a_{28}.\lim_{x\to -1}\frac{\left(\sqrt{\pi}-\sqrt{arccosx}\right)^2}{x+1}$$

$$= \lim_{x\to -1}\frac{\left(\frac{\pi - arccosx}{\sqrt{\pi}+\sqrt{arccosx}}\right)^2}{x+1} = \left(\frac{1}{2\sqrt{\pi}}\right)^2 \lim_{x\to -1}\frac{(\pi - arccosx)^2}{x+1}$$

តាង $t = \pi - arccosx$

$cost = cos(\pi - arccosx)$

$cost = cos\pi cos(arccosx) - sin\pi sin(arcsinx)$

$cost = -x \ \Rightarrow x = -cost$

បើ $x \to -1$ នោះ $t \to 0$

$$= \left(\frac{1}{2\sqrt{\pi}}\right)^2 \lim_{x\to -1}\frac{1}{\frac{1-cost}{t^2}} = \frac{1}{2\pi}$$

$$a_{28}.\lim_{x\to -1}\frac{\left(\sqrt{\pi}-\sqrt{arccosx}\right)^2}{x+1} = \frac{1}{2\pi}$$

$$a_{29}.\lim_{x\to 0}\frac{\sqrt[m]{1+x^n}-\sqrt[m]{1-x^n}}{x^n}$$

$$= \lim_{x\to 0}\frac{\left(\sqrt[m]{1+x^n}-1\right)-\left(\sqrt[m]{1+x^n}-1\right)}{x^n}$$

$$= \lim_{x\to 0}\frac{\frac{x^n}{\sqrt[m]{(1+x^n)^{m-1}+\cdots+1}}+\frac{x^n}{\sqrt[m]{(1+x^n)^{m-1}+\cdots+1}}}{x^n}$$

$$= \lim_{x\to 0}\frac{x^n}{x^n}\left[\frac{1}{\sqrt[m]{(1+x^n)^{m-1}+\cdots+1}}+\frac{1}{\sqrt[m]{(1+x^n)^{m-1}+\cdots+1}}\right]$$

$$= \frac{1}{\underbrace{1+1+\cdots+1}_{m}}+\frac{1}{\underbrace{1+1+\cdots+1}_{m}} = \frac{1}{m}+\frac{1}{m} = \frac{2}{m}$$

$$a_{29}.\lim_{x\to 0}\frac{\sqrt[m]{1+x^n}-\sqrt[m]{1-x^n}}{x^n} = \frac{2}{m}$$

$$a_{30}.\lim_{x\to 1}\frac{x^{n+1}-x(n+1)+n}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{x^{n+1}-nx-x+n}{(x-1)^2}=\lim_{x\to 1}\frac{x(x^n-1)-n(x-1)}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{x(x-1)(x^{n-1}+x^{n-2}+\cdots+1)-n(x-1)}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{(x-1)[x(x^{n-1}+x^{n-2}+\cdots+1)-n]}{(x-1)(x-1)}=\lim_{x\to 1}\frac{x^n+x^{n-1}+\cdots+x-n}{(x-1)}$$

$$=\lim_{x\to 1}\frac{(x^n-1)+(x^{n-1}-1)+\cdots+(x-1)}{(x-1)}$$

$$=\lim_{x\to 1}\frac{(x-1)[(x^{n-1}+\cdots+1)+(x^{n-2}+\cdots+1)+\cdots+1]}{(x-1)}$$

$$=n+(n-1)+\cdots+1=\frac{n(n+1)}{2}$$

$$a_{30}.\lim_{x\to 1}\frac{x^{n+1}-x(n+1)+n}{(x-1)^2}=\frac{n(n+1)}{2}$$

$$a_{31}.\lim_{x\to 1}\frac{x+x^2+\cdots+x^n-n}{x-1}$$

$$=\lim_{x\to 1}\frac{x+x^2+\cdots+x^n-(1+1+\cdots+1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)+(x^2-1)+\cdots+(x^n-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)[1+(x+1)+\cdots+(x^{n-1}+\cdots+1)]}{x-1}$$

$$=1+2+\cdots+\underbrace{1+1+\cdots+1}_{n}$$

$$=1+2+\cdots+n=\frac{n(n+1)}{2}$$

$$a_{31}.\lim_{x\to1}\frac{x+x^2+\cdots+x^n-n}{x-1}=\frac{n(n+1)}{2}$$

$$a_{32}.\lim_{x\to1}\frac{x+x^2+x^3+\cdots+x^n-n}{x+x^2+x^3+\cdots+x^p-p}$$

$$=\lim_{x\to1}\frac{x+x^2+x^3+\cdots+x^n-(1+1+\cdots+1)}{x+x^2+x^3+\cdots+x^p-(1+1+\cdots+1)}$$

$$=\lim_{x\to1}\frac{(x-1)+(x^2-1)+(x^3-1)+\cdots+(x^n-1)}{(x-1)+(x^2-1)+(x^3-1)+\cdots+(x^p-1)}$$

$$=\lim_{x\to1}\frac{(x-1)[1+(x+1)+(x^2+x+1)+\cdots+(x^{n-1}+\cdots+1)]}{(x-1)[1+(x+1)+(x^2+x+1)+\cdots+(x^{p-1}+\cdots+1)]}$$

$$=\frac{1+2+3+\cdots+\underbrace{(1+\cdots+1)}_{n}}{1+2+3+\cdots+\underbrace{(1+\cdots+1)}_{p}}=\frac{1+2+3+\cdots+n}{1+2+3+\cdots+p}=\frac{n\frac{(n+1)}{2}}{p\frac{(p+1)}{2}}$$

$$=\frac{n(n+1)}{p(p+1)}$$

$$a_{32}.\lim_{x\to1}\frac{x+x^2+x^3+\cdots+x^n-n}{x+x^2+x^3+\cdots+x^p-p}=\frac{n(n+1)}{p(p+1)}$$

$$a_{33}.\lim_{x\to1}\frac{nx^{n+1}-(n+1)x^n+1}{(x-1)^2}$$

$$=\lim_{x\to1}\frac{nx^{n+1}-nx^n-x^n+1}{(x-1)^2}=\lim_{x\to1}\frac{nx^n(x-1)-(x^n-1)}{(x-1)^2}$$

$$=\lim_{x\to1}\frac{nx^n(x-1)-(x-1)(x^{n-1}+\cdots+1)}{(x-1)^2}$$

$$=\lim_{x\to1}\frac{(x-1)[nx^n-(x^{n-1}+x^{n-2}+\cdots+1)]}{(x-1)^2}$$

$$=\lim_{x\to1}\frac{(x^n+x^n+\cdots+x^n)-x^{n-1}-x^{n-2}-\cdots-1}{x-1}$$

$$=\lim_{x\to 1}\frac{(x^n-x^{n-1})+(x^n-x^{n-2})+\cdots+(x^n-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{x^{n-1}(x-1)+x^{n-2}(x^2-1)+\cdots+(x^n-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)[x^{n-1}+x^{n-2}(x+1)+\cdots+x(x^{n-2}+\cdots+1)+n]}{x-1}$$

$$=1^{n-1}+1^{n-2}(2+1)+\cdots+1(1^{n-2}+\cdots+1)+n$$

$$=1+2+\cdots+(n-1)+n$$

$$=\frac{n(n+1)}{2}$$

$$a_{33}.\lim_{x\to 1}\frac{nx^{n+1}-(n+1)x^n+1}{(x-1)^2}=\frac{n(n+1)}{2}$$

$$a_{34}.\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{n+1}{1-x^{n+1}}\right)$$

$$=\lim_{x\to 1}\left[\frac{1}{1-x}-\frac{n+1}{(1-x)(1+x+\cdots+x^n)}\right]=\lim_{x\to 1}\frac{1}{1-x}\left[1-\frac{n+1}{(1+x+\cdots+x^n)}\right]$$

$$=\lim_{x\to 1}\frac{1}{1-x}\left[\frac{1+x+\cdots+x^n-n-1}{(1+x+\cdots+x^n)}\right]=\lim_{x\to 1}\frac{1}{1-x}\left[\frac{(x-1)+(x^2-1)+\cdots+(x^n-1)}{1+x+\cdots+x^n}\right]$$

$$=\lim_{x\to 1}\frac{(x-1)}{1-x}\left[\frac{1+(x+1)+\cdots+n}{1+n}\right]=-\frac{n\frac{n+1}{2}}{1+n}=-\frac{n}{2}$$

$$a_{34}.\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{n+1}{1-x^{n+1}}\right)=-\frac{n}{2}$$

$$a_{35}.\lim_{x\to 1}\frac{x+2x^2+\cdots+n\cdot x^n-\frac{n(n+1)}{2}}{x-1}$$

$$=\lim_{x\to 1}\frac{x+2x^2+\cdots+nx^n-1-1-\cdots-n}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)+(2x^2-2)+\cdots+(nx^n-n)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)[1+2(x+1)+\cdots+n\times n]}{x-1}=1^2+2^2+\cdots+n^2$$

$$=\frac{n(n+1)(2n+1)}{6}$$

$$a_{35}.\lim_{x\to 1}\frac{x+2x^2+\cdots+nx^n-\dfrac{n(n+1)}{2}}{x-1}=\frac{n(n+1)(2n+1)}{6}$$

$$a_{36}.\lim_{x\to 0}\frac{1-cos^n x}{xsinx}$$

$$=\lim_{x\to 0}\frac{(1-cosx)(1+cosx+\cdots+cos^{n-1}x)}{xsinx}$$

$$=\lim_{x\to 0}\frac{\dfrac{(1-cosx)}{x^2}(1+cosx+\cdots+cos^{n-1}x)}{\dfrac{xsinx}{x^2}}$$

$$=\frac{1}{2}\left(\underbrace{1+1+\cdots+1}_{n}\right)=\frac{n}{2}$$

$$a_{36}.\lim_{x\to 0}\frac{1-cos^n x}{xsinx}=\frac{n}{2}$$

$$a_{37}.\lim_{n\to\infty}\left(\frac{1^3+2^3+3^3+\cdots+n^3}{n^3}-\frac{n}{4}\right)$$

$$=\lim_{n\to\infty}\left[\left(n\frac{n+1}{2}\right)^2\times\frac{1}{n^3}-\frac{n}{4}\right]=\lim_{n\to\infty}\left[n^2\frac{(n+1)^2}{4n^3}-\frac{n}{4}\right]=\lim_{n\to\infty}\left[\frac{(n+1)^2-n^2}{4n}\right]$$

$$=\lim_{n\to\infty}\left[\frac{2n+1}{4n}\right]=\frac{1}{2}$$

$$a_{37}.\lim_{n\to\infty}\left(\frac{1^3+2^3+3^3+\cdots+n^3}{n^3}-\frac{n}{4}\right)=\frac{1}{2}$$

$$a_{38}.\lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}}$$

$$= \lim_{x\to\infty}\frac{\sqrt{x\left(1+\frac{\sqrt{x+\sqrt{x}}}{x}\right)}+\sqrt{x}}{\sqrt{x\left(1+\frac{\sqrt{x}}{x}\right)}+\sqrt{x}} = \lim_{x\to\infty}\frac{\sqrt{x}\left[\sqrt{\left(1+\sqrt{\frac{1}{x}+\frac{1}{\sqrt[3]{x}}}\right)}+1\right]}{\sqrt{x}\left[\sqrt{\left(1+\frac{1}{\sqrt{x}}\right)}+1\right]} = 1$$

$$a_{38}.\lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}} = 1$$

$$a_{39}.\lim_{n\to\infty}\left(1+\frac{1}{10}\right)\left(1+\frac{1}{10^2}\right)\dots\left(1+\frac{1}{10^{2^n}}\right)$$

ដោយ $1+\frac{1}{10^{2^n}} = \frac{1-\left(\frac{1}{10^{2^n}}\right)^2}{1-\frac{1}{10^{2^n}}}$

$$= \lim_{n\to\infty}\left[\frac{1-\left(\frac{1}{10}\right)^2}{1-\frac{1}{10}}\right]\left[\frac{1-\left(\frac{1}{10^2}\right)^2}{1-\frac{1}{10^2}}\right]\dots\left[\frac{1-\left(\frac{1}{10^{2^n}}\right)^2}{1-\frac{1}{10^{2^n}}}\right] = \lim_{n\to\infty}\frac{1}{1-\frac{1}{10}}\left[1-\left(\frac{1}{10^{2^n}}\right)^2\right]$$

$$= \frac{1}{1-\frac{1}{10}} = \frac{10}{9}$$

$$a_{39}.\lim_{n\to\infty}\left(1+\frac{1}{10}\right)\left(1+\frac{1}{10^2}\right)\dots\left(1+\frac{1}{10^{2^n}}\right) = \frac{10}{9}$$

$$a_{40}.\lim_{n\to\infty}(1+x)\,(1+x^2)\dots\left(1+x^{2^n}\right)$$

ដោយ $1+x^{2^n} = \frac{1-\left(x^{2^n}\right)^2}{1+x^{2^n}}$

$$= \lim_{n\to\infty}\left(\frac{1-x^2}{1-x}\right)\left(\frac{1-x^{2^2}}{1-x^2}\right)\dots\left[\frac{1-\left(x^{2^n}\right)^2}{1+x^{2^n}}\right] = \lim_{n\to\infty}\left(\frac{1}{1-x}\right)\left[1-\left(x^{2^n}\right)^2\right]$$

$$= \frac{1}{1-x}$$

$$a_{40}. \lim_{n\to\infty}(1+x)\,(1+x^2)\dots\left(1+x^{2^n}\right) = \frac{1}{1-x}$$

$$a_{41}. \lim_{n\to\infty}\left(\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n}\right)$$

តាង $M' = \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n}$

$$M' - \frac{M'}{2} = \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n} - \frac{M'}{2}$$

$$= \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n} - \frac{1}{2}\left(\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n}\right)$$

$$= \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n} - \frac{3}{2^2} - \frac{5}{2^3} - \cdots - \frac{2n-1}{2^{n+1}}$$

$$= \frac{1}{2}+\frac{2}{2^2}+\frac{2}{2^3}+\cdots+\frac{2}{2^n} - \frac{2n-1}{2^{n+1}}$$

$$= \frac{1}{2}\left(\frac{1}{2}+\frac{2}{2^2}+\cdots+\frac{2}{2^{n-1}}+\frac{1}{2^n}\right) - \frac{1}{2^n} - \frac{2n-1}{2^{n+1}}$$

$$M' - \frac{M'}{2} = \frac{1}{2} + \left[\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}\right] - \frac{1}{2^n} - \frac{2n-1}{2^{n+1}}$$

$$= \frac{1}{2} + \left(1-\left(\frac{1}{2}\right)^n\right) - \frac{1}{2^n} - \frac{2n-1}{2^{n+1}} = \frac{3}{2} - \frac{2}{2^n} - \frac{2n-1}{2^{n+1}}$$

$$\Rightarrow \frac{M'}{2} = \frac{3}{2} - \frac{2}{2^n} - \frac{2n-1}{2^{n+1}}$$

$$M' = 3 - \frac{4}{2^n} - \frac{4n-2}{2^{n+1}}$$

$$\Rightarrow \lim_{n\to\infty}\left(3 - \frac{4}{2^n} - \frac{4n-2}{2^{n+1}}\right) = 3 - 0 - 0 = 3$$

$$a_{41}.\lim_{n\to\infty}\left(\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n}\right)=3$$

$$a_{42}.\lim_{n\to\infty}\left(\frac{1}{3^2}+\frac{2}{15^2}+\frac{3}{35^3}+\cdots+\frac{n}{(4n^2-1)^2}\right)$$

ដោយ $\frac{n}{(4n^2-1)^2}=\frac{1}{[(2n-1)(2n+1)]^2}=\frac{1}{8}\left[\frac{1}{(2n-1)^2}-\frac{1}{(2n+1)^2}\right]$

$$=\lim_{n\to\infty}\frac{1}{8}\left[\left(\frac{1}{1}-\frac{1}{3^2}\right)+\left(\frac{1}{3^2}-\frac{1}{5^2}\right)+\cdots+\left(\frac{1}{(2n-1)^2}-\frac{1}{(2n+1)^2}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{8}\left[1--\frac{1}{(2n+1)^2}\right]=\frac{1}{8}$$

$$a_{42}.\lim_{n\to\infty}\left(\frac{1}{3^2}+\frac{2}{15^2}+\frac{3}{35^3}+\cdots+\frac{n}{(4n^2-1)^2}\right)=\frac{1}{8}$$

$$a_{43}.\lim_{n\to\infty}\frac{1+2+6+15+\cdots+\left(\frac{2n^3-3n^2+n+6}{6}\right)}{n^4}$$

ដោយ $\frac{2n^3-3n^2+n+6}{6}=\sum_{k=1}^{n}\frac{2n^3-3n^2+n+6}{6}$

$$\sum_{k=1}^{n}k=\frac{n(n+1)}{2}\quad ;\quad \sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n}k^3=\left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{k=1}^{n}\frac{2n^3-3n^2+n+6}{6}=\frac{1}{3}\left[\frac{n(n+1)}{2}\right]^2-\frac{1}{2}\frac{n(n+1)(2n+1)}{6}+\frac{1}{6}\frac{n(n+1)}{2}+n$$

$$=\lim_{n\to\infty}\frac{\frac{1}{3}\left[\frac{n(n+1)}{2}\right]^2-\frac{1}{2}\frac{n(n+1)(2n+1)}{6}+\frac{1}{6}\frac{n(n+1)}{2}+n}{n^4}=12$$

$$a_{43}.\lim_{n\to\infty}\frac{1+2+6+15+\cdots+\left(\frac{2n^3-3n^2+n+6}{6}\right)}{n^4}=12$$

$$a_{44}.\lim_{x\to 0}\frac{sin(x^2)-sin^2x}{x^2-ln(1+x^2)}$$

$$=\lim_{x\to 0}\frac{sin(x^2)-x^2-sin^2x+x^2}{lne^{x^2}-ln(1+x^2)}=\lim_{x\to 0}\frac{sin(x^2)-x^2-(sin^2x-x^2)}{ln\left(\frac{e^{x^2}}{1+x^2}\right)}$$

$$=\lim_{x\to 0}\frac{[(sinx^2-x^2)-(sinx-x)(sinx+1)]\left(\frac{e^{x^2}}{1+x^2}-1\right)}{\left(\frac{e^{x^2}}{1+x^2}-1\right)ln\left(\frac{e^{x^2}}{1+x^2}\right)}$$

$$=\lim_{x\to 0}\frac{[(sinx^2-x^2)-(sinx-x)(sinx+1)](x^2+1)}{e^{x^2}-x^2-1}$$

$$=\lim_{x\to 0}\left[\frac{\{(sinx^2-x^2)-(sinx-x)(sinx+1)\}}{x^4}\right]\cdot\frac{x^4}{e^{x^2}-x^2-1}$$

$$=2\lim_{x\to 0}\left[\frac{sinx^2-x^2}{x^4}-\frac{(sinx-x)(sinx+1)}{x^3x}\right]$$

$$=2\lim_{x\to 0}\left[\frac{sinx^2-x^2}{(x^3)^2}\cdot x^2-\frac{(sinx-x)\left(\frac{sinx}{x}+1\right)}{x^3}\right]$$

$$=2\left[-\frac{1}{6}\times 0+2\left(\frac{1}{6}\right)2\right]=\frac{2}{3}$$

កំណត់សម្គាល់ $\lim_{x\to 0}\frac{sinx-x}{x^3}=-\frac{1}{6}\lim_{x\to 0}\frac{e^x-x-1}{x^2}=\frac{1}{2}$

$$a_{44}.\lim_{x\to 0}\frac{sin(x^2)-sin^2x}{x^2-ln(1+x^2)}=\frac{2}{3}$$

$$a_{45}.\lim_{x\to 0}\frac{sin(ax)+sin(bx)-sin[(a+b)x]}{ln[abx^3+x(a+b)+1]-ln[1+(a+b)x]}$$

$$= \lim_{x\to 0}\frac{sin(ax) - ax + sin(bx) - bx - sin[(a+b)x] + (a+b)x}{ln[abx^3 + x(a+b) + 1] - ln[1 + (a+b)x]}\cdot\frac{x^3}{x^3}$$

យក $M = \lim_{x\to 0}\frac{sin(ax) - ax + sin(bx) - bx - [sin(a+b)x - (a+b)x]}{x^3}$

$$= \lim_{x\to 0}\left[\frac{a^3(sinax - ax)}{(ax)^3} + \frac{b^3(sinbx - bx)}{(bx)^3} - \frac{(a+b)^3[sin(a+b)x - (a+b)x]}{[(a+b)x]^3}\right]$$

$$M = -\frac{1}{6}[a^3 + b^3 - (a+b)^3] = \frac{1}{2}ab(a+b)$$

យក $N = \lim_{x\to 0}\frac{ln[abx^3 + x(a+b) + 1] - ln[1 + (a+b)x]}{x^3}$

$$N = \lim_{x\to 0}\frac{ln\left[\frac{abx^3 + x(a+b) + 1}{1 + (a+b)x}\right]}{x^3}$$

$$= \lim_{x\to 0}\frac{ln\left[\frac{abx^3}{1 + (a+b)x} + 1\right]}{\frac{abx^3}{1 + (a+b)x}}\times\frac{\frac{abx^3}{1 + (a+b)x}}{x^3} = ab$$

យើងបាន $\frac{\frac{1}{2}ab(a+b)}{ab} = \frac{1}{2}(a+b)$

$$a_{45}.\lim_{x\to 0}\frac{sin(ax) + sin(bx) - sin[(a+b)x]}{ln[abx^3 + x(a+b) + 1] - ln[1 + (a+b)x]} = \frac{a+b}{2}$$

$$a_{46}.\lim_{x\to 0}\frac{e^{xtanx} - 2 + cos2x}{x^2}$$

$$= \lim_{x\to 0}\frac{e^{xtanx} - 1 + cos2x - 1}{x^2} = \lim_{x\to 0}\frac{e^{xtanx} - 1 - (1 - cos2x)}{x^2}$$

$$= \lim_{x\to 0}\frac{e^{xtanx} - 1}{x^2} - \lim_{x\to 0}\frac{1 - cos2x}{x^2} = \lim_{x\to 0}\frac{e^{xtanx} - 1}{xtanx}\times\frac{xtanx}{x^2} - \lim_{x\to 0}\frac{2sin^2x}{x^2}$$

$$= 1\times 1 - 2 = -1$$

$$a_{46}.\lim_{x\to 0}\frac{e^{xtanx}-2+cos2x}{x^2}=-1$$

$$a_{47}.\lim_{x\to 1}\frac{tan(lnx)}{ln(3x-2)}$$

$$=\lim_{x\to 1}\frac{tan(lnx)}{lnx}\cdot\frac{lnx}{x-1}\cdot\frac{3x-3}{ln(3x-2)}\cdot\frac{x-1}{3x-3}=1\times 1\times\frac{1}{3}\times 1=\frac{1}{3}$$

$$a_{47}.\lim_{x\to 1}\frac{tan(lnx)}{ln(3x-2)}=\frac{1}{3}$$

$$a_{48}.\lim_{x\to 0}\frac{sin[tan\{sin(10x)\}]}{sin[sin\{sin(2x)\}]}$$

$$=\lim_{x\to 0}\frac{\frac{sin[tan\{sin(10x)\}]}{x}}{\frac{sin[sin\{sin(2x)\}]}{x}}$$

$$M=\lim_{x\to 0}\frac{sin[tan\{sin(10x)\}]}{x}\cdot\frac{tan\{sin(10x)\}}{sin(10x)}\cdot\frac{sin(10x)}{10x}\cdot\frac{10x}{x}$$

$$=1\times 1\times 1\times 10$$

$$N=\lim_{x\to 0}\frac{sin[sin\{sin(2x)\}]}{sin\{sin(2x)\}}\cdot\frac{sin\{sin(2x)\}}{sin(2x)}\cdot\frac{sin(2x)}{2x}\cdot\frac{2x}{x}$$

$$=1\times 1\times 1\times 2$$

$$k_5=\frac{10}{5}=5$$

$$a_{48}.\lim_{x\to 0}\frac{sin[tan\{sin(10x)\}]}{sin[sin\{sin(2x)\}]}=5$$

$$a_{49}.\lim_{x\to 1}\left(\frac{1}{lnx}-\frac{1}{x-1}\right)$$

$$=\lim_{x\to 1}\frac{x-1-lnx}{(x-1)lnx}$$

$\boldsymbol{Let}: x-1=y; if\, x\to 1\,; y\to 0$

$$=\lim_{y\to 0}\frac{y-ln(y+1)}{yln(y+1)}=\lim_{y\to 0}\frac{y-ln(y+1)}{y^2}\cdot\frac{y}{ln(y+1)}$$

$\boldsymbol{Let}: ln(y+1)=t; if\, y\to 0\,; t\to 0$

$$=\lim_{t\to 0}\frac{e^t-1-t}{(e^t-1)^2}=\lim_{t\to 0}\frac{e^t-1-t}{t^2}\cdot\frac{t^2}{(e^t-1)^2}=\lim_{t\to 0}\frac{e^t-1-t}{t}\cdot\left(\frac{t}{(e^t-1)}\right)^2$$

$$=\frac{1}{2}\times 1=\frac{1}{2}$$

$$a_{49}.\lim_{x\to 1}\left(\frac{1}{lnx}-\frac{1}{x-1}\right)=\frac{1}{2}$$

$$a_{50}.\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{x}{lnx}\right)$$

$$=\lim_{x\to 1}\left(\frac{xlnx-x(x-1)}{lnx(x-1)}\right)$$

តាម $\boldsymbol{Taylor\ Series}$: $lnx=(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\cdots$

$$=\lim_{x\to 1}\frac{x\left[(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\cdots\right]-x(x-1)}{(x-1)\left[(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\cdots\right]}$$

$$=\lim_{x\to 1}\frac{x\left[1-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\cdots\right]-x}{(x-1)\left[(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\cdots\right]}$$

$$=\lim_{x\to 1}\frac{x(x-1)^2\left[-\frac{1}{2}+\frac{1}{3}(x-1)-\cdots\right]}{(x-1)^2\left[1-\frac{1}{2}(x-1)+\frac{1}{3}(x-1)^2-\cdots\right]}=\frac{1\left[-\frac{1}{2}+0\right]}{1-0}=-\frac{1}{2}$$

$$a_{50}.\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{x}{lnx}\right)=-\frac{1}{2}$$

$$a_{51}.\lim_{x\to 0}\frac{ln[1+sin(e^x-1)]}{sin(sinx)}$$

$$= \lim_{x\to 0}\frac{\frac{ln[1+sin(e^x-1)]}{x}}{\frac{sin(sinx)}{x}} = \lim_{x\to 0}\frac{\frac{ln[1+sin(e^x-1)]}{sin(e^x-1)}\times\frac{sin(e^x-1)}{(e^x-1)}\times\frac{(e^x-1)}{x}}{\frac{sin(sinx)}{sinx}\times\frac{sinx}{x}}$$

$$= \frac{1\times 1\times 1}{1\times 1} = 1$$

$$a_{51}.\lim_{x\to 0}\frac{ln[1+sin(e^x-1)]}{sin(sinx)} = 1$$

$$a_{52}.\lim_{x\to\frac{\pi}{2}}(sinx)^{tanx}$$

$$= \lim_{x\to\frac{\pi}{2}}\left[(1+sinx-1)^{tanx(sinx-1)}\right]^{\frac{1}{sinx-1}} = e^{\lim\limits_{x\to\frac{\pi}{2}}tanx(sinx-1)} = e^{\lim\limits_{x\to\frac{\pi}{2}}-\frac{sinx}{cosx}\cdot\frac{(1-sinx)(1+sinx)}{(1+sinx)}}$$

$$= e^{\lim\limits_{x\to\frac{\pi}{2}}-\frac{1}{2}\frac{sinx(1-sin^2x)}{cosx}} = e^{\lim\limits_{x\to\frac{\pi}{2}}-\frac{1}{2}\frac{sinxcos^2x}{cosx}} = e^{\lim\limits_{x\to\frac{\pi}{2}}-\frac{1}{2}sinxcosx} = e^0 = 1$$

$$a_{52}.\lim_{x\to\frac{\pi}{2}}(sinx)^{tanx} = 1$$

$$a_{53}.\lim_{x\to 0}\left(\frac{1+a^x}{1+b^x}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to 0}\left(1-\frac{1+a^x}{1+b^x}-1\right)^{\frac{1}{x}} = \lim_{x\to 0}\left[\left(1-\frac{a^x-b^x}{1+b^x}\right)^{\frac{1-b^x}{a^x+b^x}}\right]^{\frac{a^x-b^x}{1+b^x}\times\frac{1}{x}}$$

$$= e^{\lim\limits_{x\to 0}\frac{a^x-b^x}{1+b^x}\times\frac{1}{x}} = e^{\lim\limits_{x\to 0}\frac{1}{2}\frac{a^x-1-(b^x-1)}{x}} = e^{\frac{1}{2}ln\frac{a}{b}} = e^{ln\sqrt{\frac{a}{b}}} = \sqrt{\frac{a}{b}}$$

$$a_{53}.\lim_{x\to 0}\left(\frac{1+a^x}{1+b^x}\right)^{\frac{1}{x}} = \sqrt{\frac{a}{b}}$$

$$a_{54}.\lim_{x\to 0}\left(\frac{3-2cosx}{2-cosx}\right)^{cot^2(2x)}$$

$$= \lim_{x\to 0}\left(1-\frac{3-2cosx}{2-cosx}+1\right)^{cot^2(2x)} = \lim_{x\to 0}\left[\left(1-\frac{1-cosx}{2-cosx}\right)^{\frac{2-cosx}{1-cosx}}\right]^{\frac{(1-cosx)}{2-cosx}cot^2(2x)}$$

$$= e^{\lim_{x\to 0}\frac{(1-cosx)}{2-cosx}cot^2(2x)} = e^{\lim_{x\to 0}\frac{(1-cosx)}{2-cosx}\times\frac{cos^2 2x}{sin^2 2x}} = e^{\lim_{x\to 0}\frac{(1-cosx)}{(1+cos2x)(1-cos2x)(2-cosx)}}$$

$$= e^{\lim_{x\to 0}\frac{(1-cosx)}{2(1-cos2x)}} = e^{\lim_{x\to 0}\frac{sin^2\left(\frac{x}{2}\right)}{2sin^2 x}} = e^{\frac{1}{8}} = \sqrt[8]{e}$$

$$a_{54}.\lim_{x\to 0}\left(\frac{3-2cosx}{2-cosx}\right)^{cot^2(2x)} = \sqrt[8]{e}$$

$$a_{55}.\lim_{x\to 0}\frac{1-cos2(x)^2}{x^3 sinx}$$

$$= \lim_{x\to 0}\frac{1-cos2x^2}{x^3 sinx} = \lim_{x\to 0}\frac{2sin^2x^2}{x^3 sinx} = 2\lim_{x\to 0}\frac{sin^2x^2}{x^4}\times\frac{x}{sinx} = 2\times 1\times 1 = 2$$

$$a_{55}.\lim_{x\to 0}\frac{1-cos2(x)^2}{x^3 sinx} = 2$$

$$a_{56}.\lim_{x\to 1}\frac{x^{11}+x^9+x^7-3}{x-1}$$

$$= \lim_{x\to 1}\frac{x^{11}-1+x^9-1+x^7-1}{x-1}$$

$$= \lim_{x\to 1}\frac{(x-1)(x^{10}+\cdots+1)+(x-1)(x^8+\cdots+1)+(x-1)(x^6+\cdots+1)}{x-1}$$

$$= 11+9+7 = 27$$

$$a_{56}.\lim_{x\to 1}\frac{x^{11}+x^9+x^7-3}{x-1} = 27$$

$$a_{57}.\lim_{x\to 0}2x^x$$

$$= 2\lim_{x\to 0}(1+x-1)^{x} = 2\lim_{x\to 0}\left[(1+x-1)^{\frac{1}{x-1}}\right]^{x(x-1)} = 2e^{\lim_{x\to 0}x(x-1)} = 2e^{0}$$
$$= 2$$

$a_{57}.\lim_{x\to 0} 2x^{x} = 2$

$$a_{58}.\lim_{x\to 1}\frac{(1+x)^{\frac{1}{5}}-(1-x)^{\frac{1}{5}}}{x}$$

$$= \lim_{x\to 1}\frac{(1+x)^{\frac{1}{5}}-1+1-(1-x)^{\frac{1}{5}}}{1+x-1} = \frac{1}{5}+\frac{1}{5} = \frac{2}{5}$$

$$a_{58}.\lim_{x\to 1}\frac{(1+x)^{\frac{1}{5}}-(1-x)^{\frac{1}{5}}}{x} = \frac{2}{5}$$

$$a_{59}.\lim_{x\to 9}\frac{\sqrt{x}(x-6)^{3}-81}{x-9}$$

$Let: x-9 = y; if x \to 0\ ; y \to 0$

$$= \lim_{y\to 0}\frac{\sqrt{y+9}(y+3)^{3}-81}{y} = \lim_{y\to 0}\frac{\sqrt{y+9}(y+3)^{3}+3(y+3)^{3}-3(y+3)^{3}-81}{y}$$

$$= \lim_{y\to 0}\frac{(y+3)^{3}\left(\sqrt{y+9}-3\right)+3(y+3)^{3}-81}{y}$$

$$= \lim_{y\to 0}\left[\frac{(y+3)^{3}\left(\sqrt{y+9}-3\right)}{y+9-9}+\frac{3(y+3)^{3}-81}{y+3-3}\right]$$

$$= \lim_{y\to 0}\left[\frac{(y+3)^{3}\left(\sqrt{y+9}-3\right)}{\left(\sqrt{y+9}-3\right)\left(\sqrt{y+9}+3\right)}+\frac{3(y+3)^{3}-81}{y+3-3}\right]$$

$$= \lim_{y\to 0}\left[\frac{(y+3)^{3}}{\left(\sqrt{y+9}+3\right)}+\frac{3(y+3)^{3}-81}{y+3-3}\right] = \frac{(0+3)^{3}}{\sqrt{0+9}+3}+3\times 3\times 3^{2} = \frac{171}{2}$$

$$a_{60}.\lim_{x\to\infty}\left(\sqrt{x^{2}+2x+1}+\sqrt{x^{2}+4x+1}+\cdots+\sqrt{x^{2}+2nx+1}-\sqrt{n^{2}x^{2}+1}\right)$$

$$=\lim_{x\to\infty}\left[\sqrt{x^2+2x+1}+\sqrt{x^2+4x+1}+\cdots+\sqrt{x^2+2nx+1}-n\sqrt{x^2+\frac{1}{n^2}}\right]$$

$$=\lim_{x\to\infty}\left[\sqrt{x^2+2x+1}+\sqrt{x^2+4x+1}+\cdots+\sqrt{x^2+2nx+1}-\left(\sqrt{x^2+\frac{1}{n^2}}+\sqrt{x^2+\frac{1}{n^2}}+\sqrt{x^2+\frac{1}{n^2}}\right)\right]$$

$$=\lim_{x\to\infty}\left[\left(\sqrt{x^2+2x+1}-\sqrt{x^2+\frac{1}{n^2}}\right)+\left(\sqrt{x^2+4x+1}-\sqrt{x^2+\frac{1}{n^2}}\right)+\cdots+\left(\sqrt{x^2+2nx+1}-\sqrt{x^2+\frac{1}{n^2}}\right)\right]$$

$$=\lim_{x\to\infty}\left[\left(\frac{2x+1-\frac{1}{n^2}}{\sqrt{x^2+2x+1}+\sqrt{x^2+\frac{1}{n^2}}}\right)+\left(\frac{4x+1-\frac{1}{n^2}}{\sqrt{x^2+4x+1}+\sqrt{x^2+\frac{1}{n^2}}}\right)+\cdots+\left(\frac{2nx+1-\frac{1}{n^2}}{\sqrt{x^2+2nx+1}+\sqrt{x^2+\frac{1}{n^2}}}\right)\right]$$

$$=\frac{2}{2}+\frac{4}{2}+\cdots+\frac{2n}{2}=\frac{1}{2}(2+4+\cdots+2n)=\frac{1}{2}\left(n\frac{(2+2n)}{2}\right)=\frac{n(n+1)}{2}$$

$$a_{61}.\lim_{n\to\infty}\sum_{k=1}^{1}\frac{2k+1}{k^2(k+1)^2}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{1}\left(\frac{1}{k^2}-\frac{1}{(k+1)^2}\right)=\lim_{n\to\infty}\left[\left(\frac{1}{1}-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\cdots+\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right)\right]$$

$$=\lim_{n\to\infty}\left(1-\frac{1}{(n+1)^2}\right)=1-0=1$$

$$a_{61}.\lim_{n\to\infty}\sum_{k=1}^{1}\frac{2k+1}{k^2(k+1)^2}=1$$

$$a_{62}.\lim_{n\to\infty}\left(\sqrt{n+\sqrt{(n-1)+\cdots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}-\sqrt{n}\right)$$

$$=\lim_{n\to\infty}\frac{\left(\sqrt{n+\sqrt{(n-1)+\cdots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}-\sqrt{n}\right)\left(\sqrt{n+\sqrt{(n-1)+\cdots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}+\sqrt{n}\right)}{\sqrt{n+\sqrt{(n-1)+\cdots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}+\sqrt{n}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{(n-1)+(n-2)\ldots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}{\sqrt{n+\sqrt{(n-1)+\cdots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}+\sqrt{n}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}\sqrt{\left(1-\frac{1}{\sqrt{n}}\right)+\frac{(n-2)}{\sqrt{n}}\ldots+\frac{\sqrt{3+\sqrt{2+\sqrt{1}}}}{\sqrt{n}}}}{\sqrt{n}\left(\sqrt{1\frac{+\sqrt{(n-1)+\cdots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}{\sqrt{n}}}+1\right)}=\frac{1}{2}$$

$$a_{62}.\lim_{n\to\infty}\left(\sqrt{n+\sqrt{(n-1)+\cdots+\sqrt{3+\sqrt{2+\sqrt{1}}}}}-\sqrt{n}\right)=1$$

$$a_{63}.\lim_{n\to\infty}\sqrt{2+\sqrt{2+\underbrace{\ldots}_{n}+\sqrt{2+\sqrt{2}}}}$$

$$Let: L_0=\sqrt{2}=2\frac{\sqrt{2}}{2}=2cos\frac{\pi}{4}=2cos\frac{\pi}{2^2}=2cos\frac{\pi}{2^{0+2}}$$

$$L_1=\sqrt{2+\sqrt{2}}=\sqrt{2+2cos\frac{\pi}{4}}=\sqrt{4cos^2\frac{\pi}{8}}=2cos\frac{\pi}{8}$$

$$=2cos\frac{\pi}{2^3}=2cos\frac{\pi}{2^{1+2}}$$

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$$L_n=2cos\frac{\pi}{2^{n+2}}$$

$$=\lim_{n\to\infty}L_n=\lim_{n\to\infty}2cos\frac{\pi}{2^{n+2}}$$

$$=2$$

$$a_{63}.\lim_{n\to\infty}\sqrt{2+\sqrt{2+\underbrace{\dots}_{n}+\sqrt{2+\sqrt{2}}}}=2$$

$$a_{64}.\lim_{n\to+\infty}\frac{1\times2+2\times3+3\times4\times\dots\times(n-1)n}{1^2+2^2+3^2+\cdots+(n-1)^2}$$

យើងមាន $1^2+2^2+3^2+\cdots+(n-1)n=\sum_{k=2}^{n}(k-1)=\sum_{k=2}^{n}[(k-1)k]$
$=\sum_{k=2}^{n}k^2-\sum_{k=1}^{n}k$

$$=\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}=\frac{n(n+1)(2n+1-3)}{6}=\frac{2n(n+1)(n-1)}{6}$$

និង $1^2+2^2+3^2+\cdots+(n-1)^2$

$$=\frac{(n-1)[(n-1)+1][2(n-1)+1]}{6}=\frac{n(n-1)(2n-1)}{6}$$

គេបាន $\lim\limits_{n\to+\infty}\dfrac{2n(n+1)(n-1)}{n(n-1)(2n-1)}$

$$= \lim_{n\to+\infty} \frac{2(n+1)}{2n-1} = \lim_{n\to+\infty} \frac{2n\left(1+\frac{1}{n}\right)}{2n\left(1-\frac{1}{n}\right)} = 1$$

$$a_{64}.\lim_{n\to+\infty} \frac{1\times 2+2\times 3+3\times 4\times \ldots \times (n-1)n}{1^2+2^2+3^2+\cdots+(n-1)^2} = 1$$

$$a_{65}.\lim_{x\to1} \frac{x^2-1+\sqrt{x^3+1}-\sqrt{x^4+1}}{x-1+\sqrt{x+1}-\sqrt{x^2+1}}$$

$$= \lim_{x\to1} \frac{(x-1)(x+1)+\frac{x^3+1-x^4-1}{\sqrt{x^3+1}+\sqrt{x^4+1}}}{(x-1)+\frac{x+1-x^2-1}{\sqrt{x+1}+\sqrt{x^2+1}}}$$

$$= \lim_{x\to1} \frac{(x-1)(x+1)-\frac{x^3(x-1)}{\sqrt{x^3+1}+\sqrt{x^4+1}}}{(x-1)-\frac{x(x-1)}{\sqrt{x+1}+\sqrt{x^2+1}}}$$

$$= \lim_{x\to1} \frac{(x-1)\left[(x+1)-\frac{x^3}{\sqrt{x^3+1}+\sqrt{x^4+1}}\right]}{(x-1)\left(1-\frac{x(x-1)}{\sqrt{x+1}+\sqrt{x^2+1}}\right)} = \lim_{x\to1} \frac{x+1-\frac{x^3}{\sqrt{x^3+1}+\sqrt{x^4+1}}}{\left(1-\frac{x}{\sqrt{x+1}+\sqrt{x^2+1}}\right)}$$

$$= \frac{2-\frac{1}{2\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} = \frac{7+3\sqrt{2}}{2}$$

$$a_{65}.\lim_{x\to1} \frac{x^2-1+\sqrt{x^3+1}-\sqrt{x^4+1}}{x-1+\sqrt{x+1}-\sqrt{x^2+1}} = \frac{7+3\sqrt{2}}{2}$$

$$a_{66}.\lim_{x\to\frac{\pi}{2}} \frac{sin^6x+cos^6x-1}{sin^4x+cos^4x-1}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{(sin^2x+cos^2x)^3-3sin^2xcos^2x(sin^2x+cos^2x)-1}{(sin^2x+cos^2x)^2-2sin^2xcos^2x-1}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1^3 - 3sin^2xcos^2x(1) - 1}{1^2 - 2sin^2xcos^2x - 1}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{-3sin^2xcos^2x}{-2sin^2xcos^2x} = \frac{3}{2}$$

$$a_{66}. \lim_{x\to\frac{\pi}{2}} \frac{sin^6x + cos^6x - 1}{sin^4x + cos^4x - 1} = \frac{3}{2}$$

$$a_{67}. \lim_{x\to 0^+} \frac{2 - \sqrt{2 + \sqrt{2 + 2cos2x}}}{x^2}$$

$$= \lim_{x\to 0^+} \frac{2 - \sqrt{2 + \sqrt{2(1 + cos2x)}}}{x^2} = \lim_{x\to 0^+} \frac{2 - \sqrt{2 + \sqrt{2 \times 2cos^2x}}}{x^2}$$

$$= \lim_{x\to 0^+} \frac{2 - \sqrt{2 + 2cosx}}{x^2} = \lim_{x\to 0^+} \frac{2 - \sqrt{2(1 + cosx)}}{x^2} = \lim_{x\to 0^+} \frac{2 - \sqrt{2 \times 2cos^2\frac{x}{2}}}{x^2}$$

$$= \lim_{x\to 0^+} \frac{2 - 2cos\frac{x}{2}}{x^2} = \lim_{x\to 0^+} \frac{2\left(1 - cos\frac{x}{2}\right)}{x^2} = \lim_{x\to 0^+} \frac{2 \times 2sin^2\frac{x}{4}}{x^2}$$

$$= \lim_{x\to 0^+} \frac{2 \times 2sin^2\frac{x}{4}}{\left(\frac{x}{4}\right)^2 \times 16} = \frac{4}{16} = \frac{1}{4}$$

$$a_{67}. \lim_{x\to 0^+} \frac{2 - \sqrt{2 + \sqrt{2 + 2cos2x}}}{x^2} = \frac{1}{4}$$

$$a_{68}. \lim_{n\to+\infty} [ln\{(n+1)! + n!\} - ln\{(n+1)! - n!\}]$$

$$= \lim_{n\to+\infty} ln\left[\frac{(n-1)! + n!}{(n+1)! - n!}\right] = \lim_{n\to+\infty} ln\left[\frac{(n-1)n! + n!}{(n+1)n! - n!}\right] = \lim_{n\to+\infty} ln\left[\frac{(n+1+1)n!}{(n-1)n!}\right]$$

$$= \lim_{n\to+\infty} ln\left[\frac{(n+2)}{n}\right] = \lim_{n\to+\infty} ln\left(\frac{(n+2)}{n}\right) = \lim_{n\to+\infty} ln\left(1 + \frac{2}{n}\right) = ln1 = 0$$

$$a_{68}. \lim_{n\to+\infty} [ln\{(n+1)! + n!\} - ln\{(n+1)! - n!\}] = 0$$

$$a_{69}.\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}}$$

$$=\lim_{n\to\infty}\frac{\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}}{\frac{1-\left(\frac{1}{3}\right)^{n+1}}{1-\frac{1}{3}}}=\lim_{n\to\infty}\frac{\frac{1-\left(\frac{1}{2}\right)^{n+1}}{\frac{1}{2}}}{\frac{1-\left(\frac{1}{3}\right)^{n+1}}{\frac{2}{3}}}=\frac{\frac{1-0}{\frac{1}{2}}}{\frac{1-0}{\frac{2}{3}}}=\frac{2}{\frac{3}{2}}=\frac{4}{3}$$

$$a_{69}.\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}}=\frac{4}{3}$$

$$a_{70}.\lim_{n\to\infty}\left(\frac{1+2+3+\cdots+n}{n+2}-\frac{n}{2}\right)$$

$$=\lim_{n\to\infty}\left[\frac{\frac{n(n+1)}{2}}{n+2}-\frac{n}{2}\right]=\lim_{n\to\infty}\left[\frac{n(n+1)}{2(n+2)}-\frac{n}{2}\right]=\lim_{n\to\infty}\left[\frac{n(n+1)-n(n+2)}{2(n+2)}\right]$$

$$=\lim_{n\to\infty}\left[\frac{n^2-n^2-2n+n}{2(n+2)}\right]=\lim_{n\to\infty}\frac{-n}{2(n+2)}=\lim_{n\to\infty}\frac{-n}{2n\left(1+\frac{2}{n}\right)}=-\frac{1}{2}$$

$$a_{70}.\lim_{n\to\infty}\left(\frac{1+2+3+\cdots+n}{n+2}-\frac{n}{2}\right)=-\frac{1}{2}$$

$$a_{71}.\lim_{n\to\infty}\left(\frac{1-2+3-4+\cdots-2n}{\sqrt{n^2+1}}\right)$$

$$=\lim_{n\to\infty}\left[\frac{1+3+\cdots+(2n-1)-(2+4+\cdots+2n)}{\sqrt{n^2+1}}\right]$$

$$=\lim_{n\to\infty}\frac{\frac{n(1+2n-1)}{2}-\frac{n(2+2n)}{2}}{\sqrt{n^2+1}}$$

$$= \lim_{n\to\infty} \frac{\frac{2n^2}{2} - \frac{2n+2n^2}{2}}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{-n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{-n}{n\sqrt{1+\frac{1}{n^2}}} = -1$$

$a_{71}.\ \lim_{n\to\infty}\left(\frac{1-2+3-4+\cdots-2n}{\sqrt{n^2+1}}\right) = -1$

$a_{72}.\ \lim_{n\to\infty}\left[\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)n}\right]$

ដោយ $\frac{1}{(n-1)n} = \frac{1}{n-1} - \frac{1}{n}$

$$= \lim_{n\to\infty}\left(\frac{1}{n-1}-\frac{1}{n}\right) = \lim_{n\to\infty}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n-1}-\frac{1}{n}\right)\right]$$

$$= \lim_{n\to\infty}\left(1-\frac{1}{n}\right) = 1$$

$a_{72}.\ \lim_{n\to\infty}\left[\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)n}\right] = 1$

$a_{73}.\ \lim_{n\to+\infty}\left(\frac{7}{10}+\frac{29}{10^2}+\frac{133}{10^3}+\cdots+\frac{5^n+2^n}{10^n}\right)$

$$= \lim_{n\to+\infty}\sum_{k=0}^{n}\frac{5^k+2^k}{10^k} = \lim_{n\to\infty}\sum_{k=0}^{n}\left[\left(\frac{1}{2}\right)^k+\left(\frac{1}{5}\right)^k\right] = \lim_{n\to+\infty}\left[\sum_{k=0}^{n}\left(\frac{1}{2}\right)^k+\sum_{k=0}^{n}\left(\frac{1}{5}\right)^k\right]$$

$$= \lim_{n\to+\infty}\left[\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}+\frac{\frac{1}{5}\left(1-\frac{1}{5^n}\right)}{1-\frac{1}{5}}\right] = \lim_{n\to+\infty}\left[\left(1-\frac{1}{2^n}\right)+\frac{\left(1-\frac{1}{5^n}\right)}{4}\right] = \frac{5}{4}$$

$a_{73}.\ \lim_{n\to+\infty}\left(\frac{7}{10}+\frac{29}{10^2}+\frac{133}{10^3}+\cdots+\frac{5^n+2^n}{10^n}\right) = \frac{5}{4}$

$a_{74}.\ \lim_{n\to+\infty}\prod_{r=3}^{n}\left(\frac{r^3-1}{r^3+1}\right)$

$$*\frac{r^3-1}{r^3+1}=\frac{r-1}{r+1}\times\frac{r^2+r+1}{(r-1)^2+(r-1)+1}$$

$$*\prod_{r=3}^{n}\left(\frac{r^3-1}{r^3+1}\right)=\prod_{r=3}^{n}\left(\frac{r-1}{r+1}\times\frac{r^2+r+1}{(r-1)^2+(r-1)+1}\right)$$

$$=\prod_{r=3}^{n}\left(\frac{r-1}{r+1}\right)\times\prod_{r=3}^{n}\frac{r^2+r+1}{(r-1)^2+(r-1)+1}$$

$$\prod_{r=3}^{n}\left(\frac{r-1}{r+1}\right)=\frac{2}{4}\times\frac{3}{5}\times\frac{4}{6}\times\ldots\times\frac{n-3}{n-1}\times\frac{n-2}{n}\times\frac{n-1}{n+1}$$

$$=\frac{6}{n^2+n}$$

$$\prod_{r=3}^{n}\frac{r^2+r+1}{(r-1)^2+(r-1)+1}$$

$$=\frac{3^2+3+1}{2^2+2+1}\times\frac{4^2+4+1}{3^2+3++1}\times\ldots\times\frac{n^2+n+1}{(n-1)^2+(n-1)+1}$$

$$=\frac{n^2+n+1}{2^2+2+1}=\frac{n^2+n+1}{7}$$

$$=\lim_{n\to+\infty}\prod_{r=3}^{n}\left(\frac{r^3-1}{r^3+1}\right)=\lim_{n\to+\infty}\left[\left(\frac{6}{n^2+n}\right)\left(\frac{n^2+n+1}{7}\right)\right]$$

$$=\frac{6}{7}\lim_{n\to+\infty}\frac{n^2+n+1}{n^2+n}=\frac{6}{7}$$

$$a_{74}.\lim_{n\to+\infty}\prod_{r=3}^{n}\left(\frac{r^3-1}{r^3+1}\right)=\frac{6}{7}$$

ផ្នែកទី០២ដំណោះលីមីត(*Solution Of Limit*)

$$(\mathrm{KIN})_1=\lim_{x\to1}\cos\left[\ln x\left\{\sin\left(\frac{\pi x}{x-1}\right)\right\}\right]$$

$$=\cos\lim_{x\to1}\left[\ln x\left\{\sin\left(\frac{\pi x}{x-1}\right)\right\}\right]$$

$$\therefore\ \lim_{x\to1}\ln x=0$$

$$\therefore\ \lim_{x\to1}\sin\left(\frac{\pi x}{x-1}\right)=\sin(\infty)\ \Rightarrow\left|\sin\left(\frac{\pi x}{x-1}\right)\right|\le1$$

$$\boxed{\text{So: }(\mathrm{KIN})_1=\lim_{x\to1}\cos\left[\ln x\left\{\sin\left(\frac{\pi x}{x-1}\right)\right\}\right]=1}$$

$$(\mathrm{KIN})_2=\lim_{x\to\alpha}\frac{x^x-\alpha^\alpha}{x-\alpha}\ ;\ \alpha\in\mathrm{IN}$$

$$=\lim_{x\to\alpha}\frac{x^x-x^\alpha}{x-\alpha}+\lim_{x\to\alpha}\frac{x^\alpha-\alpha^\alpha}{x-\alpha}$$

$$=\lim_{x\to\alpha}x^\alpha\frac{x^{x-\alpha}-1}{x-\alpha}+\lim_{x\to\alpha}\frac{(x-\alpha)(x^{\alpha-1}+x^{\alpha-2}+\cdots+1)}{x-\alpha}$$

$$=\lim_{x\to\alpha}x^\alpha\ln x+\left(\underbrace{x^{\alpha-1}+x^{\alpha-2}+\cdots+\alpha^{\alpha-1}}_{n}\right)$$

$$=\alpha^\alpha\ln\alpha+\alpha^\alpha$$

$$\boxed{\text{So: }(\mathrm{KIN})_2=\lim_{x\to\alpha}\frac{x^x-\alpha^\alpha}{x-\alpha}=\alpha^\alpha\ln\alpha+\alpha^\alpha}$$

$$(\mathrm{KIN})_3=\lim_{x\to\frac{\pi}{2}}\left[\cos x\sin\left(\frac{1}{x-\frac{\pi}{2}}\right)\right]$$

$$\therefore\ \lim_{x\to\frac{\pi}{2}}\cos x\lim_{x\to\frac{\pi}{2}}\sin\left(\frac{1}{x-\frac{\pi}{2}}\right)$$

$$\therefore \lim_{x\to\frac{\pi}{2}} \cos x = 0$$

$$\therefore \lim_{x\to\frac{\pi}{2}} \sin\left(\frac{1}{x-\frac{\pi}{2}}\right) = \sin(\infty) \Rightarrow \left|\sin\left(\frac{\pi x}{x-1}\right)\right| \le 1$$

$$\text{So}: (\text{KIN})_3 = \lim_{x\to\frac{\pi}{2}} \left[\cos x \sin\left(\frac{1}{x-\frac{\pi}{2}}\right)\right] = 0$$

$$(\text{KIN})_4 = \lim_{x\to 0} \frac{1+\tan^2 x - \cos 2x}{x\sin x}$$

$$= \lim_{x\to 0} \frac{\frac{\cos^2 x + \sin^2 x - \cos^2 x\cos 2x}{\cos^2 x}}{x\sin x}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x\cos 2x}{x\sin x\cos^2 x} = \lim_{x\to 0} \frac{1-\cos^2 x(\cos^2 x - \sin^2 x)}{x\sin x\cos^2 x}$$

$$= \lim_{x\to 0} \frac{1-\cos^4 x + \cos^2 x\sin^2 x}{x\sin x\cos^2 x}$$

$$= \lim_{x\to 0} \frac{1-\cos^4 x}{x\sin x\cos^2 x} + \lim_{x\to 0} \frac{\cos^2 x\sin^2 x}{x\sin x\cos^2 x}$$

$$= \lim_{x\to 0} \frac{(1-\cos^2 x)(1+\cos^2 x)}{x\sin x\cos^2 x} + \lim_{x\to 0} \frac{\sin x}{x}$$

$$= \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)(1+\cos^2 x)}{x\sin x\cos^2 x} + 1$$

$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}(1+\cos x)(1+\cos^2 x)}{x\sin x\cos^2 x} + 1$$

$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}(1+\cos x)(1+\cos^2 x)}{2x\sin\frac{x}{2}\cos\frac{x}{2}\cos^2 x} + 1$$

$$= 4\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{x\sin\frac{x}{2}\cos\frac{x}{2}\cos^2x}+1$$

$$= 4\lim_{x\to0}\frac{\sin\frac{x}{2}}{x\cos\frac{x}{2}\cos^2x}+1 = 4\lim_{x\to0}\frac{\sin\frac{x}{2}}{\frac{x}{2}\times\frac{2}{x}x\cos\frac{x}{2}\cos^2x}+1$$

$$= 4\lim_{x\to0}\frac{\sin\frac{x}{2}}{\frac{x}{2}\times2\cos\frac{x}{2}\cos^2x}+1 = 4\times1\times\frac{1}{2}+1 = 3$$

$$\boxed{\text{So: }(\text{KIN})_4 = \lim_{x\to0}\frac{1+\tan^2x-\cos2x}{x\sin x} = 3}$$

$$(\text{KIN})_5 = \lim_{x\to0}\frac{1-\sqrt[7]{1+\sin x}}{\tan x}$$

$$= \lim_{x\to0}\cos x\lim_{x\to0}\frac{1-\sqrt[7]{1+\sin x}}{\sin x}$$

Let: $y^7 = 1+\sin x \Rightarrow \sin x = y^7-1$

when: $x \to o$ than $y \to 1$

$$= \lim_{y\to1}\frac{1-y}{y^7-1} = -\lim_{y\to1}\frac{y-1}{y^7-1}$$

$$= -\lim_{y\to1}\frac{y-1}{(y-1)(y^6+y^5+y^4+y^3+y^2+y+1)}$$

$$= -\lim_{y\to1}\frac{1}{(y^6+y^5+y^4+y^3+y^2+y+1)}$$

$$= -\frac{1}{(1^6+1^5+1^4+1^3+1^2+1+1)} = -\frac{1}{7}$$

$$\boxed{\text{So: }(\text{KIN})_5 = \lim_{x\to0}\frac{1-\sqrt[7]{1+\sin x}}{\tan x} = -\frac{1}{7}}$$

$$(\text{KIN})_6 = \lim_{x\to0^+}\frac{\tan3x\cdot\sin\sqrt{2x}}{\sqrt{x^3+\sin^3 2x}}$$

$$= \lim_{x\to 0^+} \tan 3x \cdot \lim_{x\to 0^+} \frac{\sin\sqrt{2x}}{\sqrt{x^3+\sin^3 2x}}$$

$$= \lim_{x\to 0^+} \tan 3x \cdot \lim_{x\to 0^+} \frac{\sin\sqrt{2x}}{\sqrt{x^3\left(1+\frac{\sin^3 2x}{x^3}\right)}}$$

$$= \lim_{x\to 0^+} \frac{\tan 3x}{x} \cdot \lim_{x\to 0^+} \frac{\sin\sqrt{2x}}{\sqrt{x}\sqrt{\left(1+\frac{\sin^3 2x}{x^3}\right)}}$$

$$= \lim_{x\to 0^+} 3\frac{\tan 3x}{3x} \cdot \lim_{x\to 0^+} \frac{\sin\sqrt{2}\cdot\sqrt{x}}{\sqrt{2}\cdot\sqrt{x}\sqrt{\left(1+\frac{\sin^3 2x}{x^3}\right)}}\cdot\sqrt{2}$$

$$= \sqrt{2}\lim_{x\to 0^+} 3\frac{\tan 3x}{3x} \cdot \lim_{x\to 0^+} \frac{\sin\left(\sqrt{2}\cdot\sqrt{x}\right)}{\left(\sqrt{2}\cdot\sqrt{x}\right)\sqrt{\left(1+\frac{\sin^3 2x}{x^3}\right)}}$$

$$= 3\sqrt{2}\lim_{x\to 0^+} \frac{1}{\sqrt{\left(1+\frac{\sin^3 2x}{x^3}\right)}} = 3\sqrt{2}\lim_{x\to 0^+} \frac{1}{\sqrt{\left(1+\frac{\sin^3 2x}{(2x)^3}8\right)}}$$

$$= 3\sqrt{2}\lim_{x\to 0^+} \frac{1}{\sqrt{\left(1+\left(\frac{\sin 2x}{2x}\right)^3 8\right)}} = 3\sqrt{2}\times\frac{1}{\sqrt{9}} = \sqrt{2}$$

$$\boxed{\textbf{So}: (\text{KIN})_6 = \lim_{x\to 0^+} \frac{\tan 3x\cdot\sin\sqrt{2x}}{\sqrt{x^3+\sin^3 2x}} = \sqrt{2}}$$

$$(\text{KIN})_7 = \lim_{x\to\infty}\left[2^{x-1}\sin\left(\frac{a}{2^x}\right)\right] ; a \in \text{IN}$$

$$= \frac{1}{2}\lim_{x\to\infty}\left[2^x\sin\left(\frac{a}{2^x}\right)\right] = \frac{1}{2}\lim_{x\to\infty}\left[\frac{\sin\left(\frac{a}{2^x}\right)}{2^x}\right]$$

$$=\frac{1}{2}\lim_{x\to\infty}\left[\frac{\sin\left(\frac{a}{2^x}\right)}{\frac{a}{2^x}\frac{1}{a}}\right]=\frac{a}{2}\lim_{x\to\infty}\left[\frac{\sin\left(\frac{a}{2^x}\right)}{\frac{a}{2^x}}\right]=\frac{a}{2}$$

$$\text{So: }(\text{KIN})_7=\lim_{x\to\infty}\left[2^{x-1}\sin\left(\frac{a}{2^x}\right)\right]=\frac{a}{2}\quad ;a\in\text{IN}$$

$$(\text{KIN})_8=\lim_{x\to 1}\left(\frac{m}{1-\sqrt[n]{x}}-\frac{n}{1-\sqrt[m]{x}}\right)\ ;(m,n)\in\text{IN}$$

Let: $x=t^{mn}$ when $x\to 1\ ;t\to 1$

$$=\lim_{t\to 1}\left(\frac{m}{1-t^m}-\frac{n}{1-t^n}\right)$$

$$=\lim_{t\to 1}\left[\left(\frac{m}{1-t^m}-\frac{1}{1-t}\right)-\left(\frac{n}{1-t^n}-\frac{1}{1-t}\right)\right]$$

$$*\lim_{t\to 1}\left(\frac{m}{1-t^m}-\frac{1}{1-t}\right)=\lim_{t\to 1}\frac{m-(1+t+\cdots+t^{m-1})}{1-t^m}$$

$$=\lim_{t\to 1}\frac{(1-1)(1-t)(1-t^2)\ldots(1-t^{m-1})}{(1-t)(1+t+\cdots+t^{m-1})}$$

$$=\frac{1+2+3+\cdots+(m-1)}{m}=]\frac{(m-1)(1+m-1)}{2m}=\frac{m-1}{2}$$

$$**\lim_{t\to 1}\left(\frac{n}{1-t^n}-\frac{1}{1-t}\right)=\frac{n-1}{2}$$

$$=\lim_{t\to 1}\left[\left(\frac{m}{1-t^m}-\frac{1}{1-t}\right)-\left(\frac{n}{1-t^n}-\frac{1}{1-t}\right)\right]$$

$$=\frac{m-1}{2}-\frac{n-1}{2}=\frac{m-n}{2}$$

$$\text{So: }(\text{KIN})_8=\lim_{x\to 1}\left(\frac{m}{1-\sqrt[n]{x}}-\frac{n}{1-\sqrt[m]{x}}\right)=\frac{m-n}{2}\ ;(m,n)\in\text{IN}$$

$$(\text{KIN})_9=\lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)\ldots(1+nx)-1}{x}$$

$$= \lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)\dots(1+nx)-(1+nx)+(1+nx)-1}{x}$$

$$= \lim_{x\to 0}\frac{(1+nx)[(1+x)(1+2x)(1+3x)\dots(1+(n-1)x)]+nx}{x}$$

$$u_n = \lim_{x\to 0}\frac{[(1+x)(1+2x)(1+3x)\dots(1+(n-1)x)]-1}{x}+\lim_{x\to 0}\frac{nx}{x}$$

$$u_n = u_{n-1}+n \equiv n = u_n - u_{n-1} \equiv \sum_{k=2}^{n} u_k - u_{k-1} = \sum_{k=2}^{n} k$$

$$= u_2 - u_1 + u_3 - u_2 + \cdots + u_n - u_{n-1} = 2+3+4+\cdots+n$$

$$\Rightarrow u_n = u_1 + 2+3+4+\cdots+n\ ;\ u_1 = \lim_{x\to 0}\frac{(1+x)-1}{x} = 1$$

$$\Rightarrow u_n = 1+2+3+4+\cdots+n = \frac{n(n+1)}{2}$$

$$\text{So: } (\mathrm{KIN})_9 = \lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)\dots(1+nx)-1}{x} = \frac{n(n+1)}{2}$$

$$(\mathrm{KIN})_{10} = \lim_{x\to 0}\frac{\ln\left(\frac{\sqrt{1-x^2}}{\cos x}\right)}{x^3}$$

Let: $x = -u$; when $x \to 0$; $u \to 0$

$$= \lim_{u\to 0}\frac{\ln\left(\frac{\sqrt{1-(-u)^2}}{\cos(-u)}\right)}{(-u)^3} = \lim_{u\to 0}\frac{\ln\left(\frac{\sqrt{1-u^2}}{\cos u}\right)}{-u^3}$$

$$= -\lim_{u\to 0}\frac{\ln\left(\frac{\sqrt{1-u^2}}{\cos u}\right)}{u^3} \equiv (\mathrm{KIN})_{10} = -(\mathrm{KIN})_{10}$$

$$\equiv (\mathrm{KIN})_{10} + (\mathrm{KIN})_{10} = 0 \equiv 2(\mathrm{KIN})_{10} = 0 \Rightarrow (\mathrm{KIN})_{10} = 0$$

$$\text{So: } (\mathrm{KIN})_{10} = \lim_{x\to 0}\frac{\ln\left(\frac{\sqrt{1-x^2}}{\cos x}\right)}{x^3} = 0$$

$$(\mathrm{KIN})_{11}=\lim_{x\to a}\frac{x\sqrt{x}-a\sqrt{a}}{\sqrt{x}-\sqrt{a}}\ ;a\geq 0$$

$$=\lim_{x\to a}\frac{x\sqrt{x}-a\sqrt{x}+a\sqrt{x}-a\sqrt{a}}{\sqrt{x}-\sqrt{a}}$$
$$=\lim_{x\to a}\frac{\sqrt{x}(x-a)+a(\sqrt{x}-\sqrt{a})}{\sqrt{x}-\sqrt{a}}$$
$$=\lim_{x\to a}\frac{\sqrt{x}(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})+a(\sqrt{x}-\sqrt{a})}{\sqrt{x}-\sqrt{a}}$$
$$=\lim_{x\to a}\frac{(\sqrt{x}-\sqrt{a})[\sqrt{x}(\sqrt{x}+\sqrt{a})+a]}{\sqrt{x}-\sqrt{a}}$$
$$=\sqrt{a}(\sqrt{a}+\sqrt{a})+a=3a$$

So: $(\mathrm{KIN})_{11}=\lim_{x\to a}\frac{x\sqrt{x}-a\sqrt{a}}{\sqrt{x}-\sqrt{a}}=3a\ ;a\geq 0$

$$(\mathrm{KIN})_{12}=\lim_{x\to 1}\frac{\sqrt{x+1}+\sqrt{x^2-1}-\sqrt{x^3+1}}{\sqrt{x-1}+\sqrt{x^2+1}-\sqrt{x^4+1}}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2-1}+\left(\sqrt{x+1}-\sqrt{x^3+1}\right)}{\sqrt{x-1}+\left(\sqrt{x^2+1}-\sqrt{x^4+1}\right)}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2-1}+\dfrac{x+1-x^3-1}{\sqrt{x+1}-\sqrt{x^3+1}}}{\sqrt{x-1}+\dfrac{x^2+1-x^3-1}{\sqrt{x^2+1}-\sqrt{x^4+1}}}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2-1}-\dfrac{x(x-1)(x+1)}{\sqrt{x+1}-\sqrt{x^3+1}}}{\sqrt{x-1}-\dfrac{x^2(x-1)(x+1)}{\sqrt{x^2+1}-\sqrt{x^4+1}}}$$

$$=\lim_{x\to 1}\frac{\sqrt{x-1}\left[\sqrt{x+1}-\dfrac{x(x+1)\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x^3+1}}\right]}{\sqrt{x-1}-\left[1-\dfrac{x^2(x+1)\sqrt{x-1}}{\sqrt{x^2+1}-\sqrt{x^4+1}}\right]}$$

$$=\lim_{x\to1}\frac{\sqrt{x+1}-\dfrac{x(x+1)\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x^3+1}}}{\sqrt{x-1}-\dfrac{x^2(x+1)\sqrt{x-1}}{\sqrt{x^2+1}-\sqrt{x^4+1}}}=\frac{\sqrt{2}-0}{1-0}=\sqrt{2}$$

$$\text{So}: (KIN)_{12}=\lim_{x\to1}\frac{\sqrt{x+1}+\sqrt{x^2-1}-\sqrt{x^3+1}}{\sqrt{x-1}+\sqrt{x^2+1}-\sqrt{x^4+1}}=\sqrt{2}$$

$$(KIN)_{13}=\lim_{x\to3}\frac{\sqrt{6+\sqrt{6+\cdots+\sqrt{6+x}}}-x}{x-3}$$

$$\text{let } a_n=\lim_{x\to3}\frac{\sqrt{6+\sqrt{6+\cdots+\sqrt{6+x}}}-x}{x-3}$$

$$=\lim_{x\to3}\frac{6+\overbrace{\sqrt{6+\sqrt{6+\cdots+\sqrt{6+x}}}}^{(n-1)}-x^2}{(x-3)\left(\sqrt{6+\sqrt{6+\cdots+\sqrt{6+x}}}-x\right)}$$

$$=\lim_{x\to3}\left(\frac{1}{\sqrt{6+\sqrt{6+\cdots+\sqrt{6+x}}}-x}\times\frac{\sqrt{6+\sqrt{6+\cdots+\sqrt{6+x}}}-x+x+6-x^2}{x-3}\right)$$

$$=\frac{1}{6}\left[\lim_{x\to3}\frac{\sqrt{6+\sqrt{6+\cdots+\sqrt{6+x}}}-x}{x-3}+\lim_{x\to3}\frac{-x^2+x+6}{x-3}\right]$$

$$=\frac{1}{6}\left[a_n+\lim_{x\to3}\frac{-x(x-3)-2(x-3)}{x-3}\right]$$

$$=\frac{1}{6}\left[a_n+\lim_{x\to3}\frac{(x+3)(-x-2)}{x-3}\right]$$

$$a_n = \frac{1}{6}(a_{n-1} - 5) \text{ or } a_{n-1} = \frac{1}{6}a_n - \frac{5}{6}$$

$$\text{If } n = 1 \text{ than } a_1 = \lim_{x\to 3}\frac{\sqrt{6+x} - x}{x - 3} \text{ form } \frac{0}{0}$$

$$= \lim_{x\to 3}\frac{6 + x - x^2}{(x-3)\left(\sqrt{6+x} + x\right)} = \frac{1}{6}(-5) = -\frac{5}{6}$$

$$* \, r = \frac{1}{6}r - \frac{5}{6} \equiv \frac{5}{6}r - \frac{5}{6} \Rightarrow r = 1$$

$$\text{let } b_n = a_{n+1} + 1 = \frac{1}{6}a_n - \frac{5}{6} + 1 = \frac{1}{6}a_n + \frac{1}{6}$$

$$b_{n+1} = \frac{1}{6}(a_n + 1) = \frac{1}{6}b_n$$

$$\text{we have } (b_n) \text{is sequence } \frac{00}{00}$$

$$\text{have } q = \frac{1}{6} \text{ , } b_1 = a_1 + 1 = -\frac{5}{6} + 1 = \frac{1}{6}$$

$$* \, b_n = b_1 q^{n-1} = \frac{1}{6}\left(\frac{1}{6}\right)^{n-1} = \left(\frac{1}{6}\right)^n$$

$$\Rightarrow a_n = \left(\frac{1}{6}\right)^n - 1$$

$$\text{So: } (\text{KIN})_{13} = \lim_{x\to 3}\frac{\sqrt{6 + \sqrt{6 + \cdots + \sqrt{6+x}}} - x}{x - 3} = \left(\frac{1}{6}\right)^n - 1$$

$$(\text{KIN})_{14} = \lim_{x\to 2}\frac{1 - \sin\frac{\pi}{x}}{x^2 - 4x + 4}$$

$$= \lim_{x\to 2}\frac{1-\cos\left(\frac{\pi}{2}-\frac{\pi}{x}\right)}{(x-2)^2} = \lim_{x\to 2}\frac{1-\cos\left(\frac{\pi x-2\pi}{2x}\right)}{(x-2)^2}$$

$$= \lim_{x\to 2}\frac{1-\cos\pi\left(\frac{x-2}{2x}\right)}{(x-2)^2} = \lim_{x\to 2}\frac{2\sin^2\pi\left(\frac{x-2}{4x}\right)}{(x-2)^2}$$

$$= \lim_{x\to 2}\frac{2\sin^2\pi\left(\frac{x-2}{4x}\right)}{\left(\pi\frac{x-2}{4x}\right)^2}\times\frac{\pi^2}{(4x)^2} = \frac{\pi^2}{32}$$

$$\boxed{\text{So}: (\text{KIN})_{14} = \lim_{x\to 2}\frac{1-\sin\frac{\pi}{x}}{x^2-4x+4} = \frac{\pi^2}{32}}$$

$$(\text{KIN})_{15} = \lim_{x\to 1}\frac{x^2-1}{\cos\frac{\pi}{x+1}}$$

$$= \lim_{x\to 1}\frac{x^2-1}{\sin\left(\frac{\pi}{2}-\frac{\pi}{x+1}\right)} = \lim_{x\to 1}\frac{(x-1)(x+1)}{\sin\left[\frac{\pi x-\pi}{2(x+1)}\right]}$$

$$= \lim_{x\to 1}\frac{\frac{\pi x-\pi}{2(x+1)}}{\sin\left[\frac{\pi x-\pi}{2(x+1)}\right]}\times\frac{(x-1)(x+1)}{\frac{\pi x-\pi}{2(x+1)}} = \lim_{x\to 1}\frac{(x+1)^2}{\frac{\pi}{2}} = \frac{8}{\pi}$$

$$\boxed{\text{So}: (\text{KIN})_{15} = \lim_{x\to 1}\frac{x^2-1}{\cos\frac{\pi}{x+1}} = \frac{8}{\pi}}$$

$$(\text{KIN})_{16} = \lim_{x\to 0}\frac{\sqrt{1+\sqrt{2x^2+9}}-2}{x\sin 2x}$$

$$= \lim_{x\to 0}\frac{1+\sqrt{2x^2+9}-4}{x\sin 2x\left(\sqrt{1+\sqrt{2x^2+9}}+2\right)} = \lim_{x\to 0}\frac{\sqrt{2x^2+9}-3}{4x\sin 2x}$$

$$= \lim_{x\to 0}\frac{2x^2+9-9}{4x\sin 2x(\sqrt{2x^2+9}+3)} = \lim_{x\to 0}\frac{2x^2}{4\times 6x\sin 2x} = \frac{1}{24}$$

$$\boxed{\text{So}: (\text{KIN})_{16} = \lim_{x\to 0}\frac{\sqrt{1+\sqrt{2x^2+9}}-2}{x\sin 2x} = \frac{1}{24}}$$

$$(\mathrm{KIN})_{17}=\lim_{x\to\frac{\pi}{2}}(\sin x)^{\frac{1}{\cos x}}$$

$$=e^{\lim_{x\to\frac{\pi}{2}}\frac{\ln\sin x}{\cos x}}=e^{\lim_{x\to\frac{\pi}{2}}\frac{\ln\sin x}{(\sin x-1)}\times\frac{\sin x-1}{\cos x}}$$

$$=e^{\lim_{x\to\frac{\pi}{2}}\frac{\ln\sin x}{(\sin x-1)}\times\frac{\sin x-1}{\cos x}\cdot\frac{1+\sin x}{1+\sin x}}=e^{\lim_{x\to\frac{\pi}{2}}\frac{\ln\sin x}{(\sin x-1)}\times\frac{\cos^2 x}{\cos x(1+\sin x)}}$$

$$=e^{\lim_{x\to\frac{\pi}{2}}\frac{\ln\sin x}{(\sin x-1)}\times\frac{\cos x}{(1+\sin x)}}=e^{1\times\frac{0}{2}}=1$$

So: $(\mathrm{KIN})_{17}=\lim_{x\to\frac{\pi}{2}}(\sin x)^{\frac{1}{\cos x}}=1$

$$(\mathrm{KIN})_{18}=\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin^{-1}x-\cos^{-1}x}{\sqrt{2}x-1}$$

Let: $\sin^{-1}x=y \Rightarrow x=\sin y\ ;\ \cos^{-1}x=\frac{\pi}{2}-y$

if: $x\to\frac{1}{\sqrt{2}}\ ;\ y\to\frac{\pi}{4}$

we have: $(\mathrm{KIN})_{18}=\lim_{y\to\frac{\pi}{4}}\frac{y-\left(\frac{\pi}{2}-y\right)}{\sqrt{2}\left(\sin y-\frac{1}{\sqrt{2}}\right)}$

$$=\lim_{y\to\frac{\pi}{4}}\frac{2\left(y-\frac{\pi}{4}\right)}{\sqrt{2}\left(\sin y-\sin\frac{\pi}{4}\right)}=\lim_{y\to\frac{\pi}{4}}\frac{4\left(\frac{y-\frac{\pi}{4}}{2}\right)}{\sqrt{2}\cdot 2\sin\left(\frac{y-\frac{\pi}{4}}{2}\right)\cos\left(\frac{y+\frac{\pi}{4}}{2}\right)}$$

$$=\frac{2}{\sqrt{2}}\lim_{y\to\frac{\pi}{4}}\frac{\left(\frac{y-\frac{\pi}{4}}{2}\right)}{\sin\left(\frac{y-\frac{\pi}{4}}{2}\right)}\times\frac{1}{\cos\left(\frac{y+\frac{\pi}{4}}{2}\right)}$$

$$= \frac{2}{\sqrt{2}\times\frac{\sqrt{2}}{2}} = 2$$

$$\text{So: } (\text{KIN})_{18} = \lim_{x\to\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x - \cos^{-1}x}{\sqrt{2}x - 1} = 1$$

$$(\text{KIN})_{19} = \lim_{x\to\pi} \frac{1 - \sin\frac{x}{2}}{x - \pi}$$

$$\therefore\ 1 + \sin x = 2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\therefore\ 1 - \sin x = 2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\text{we have: } (\text{KIN})_{19} = \lim_{x\to\pi} \frac{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{4\left(\frac{x}{4} - \frac{\pi}{4}\right)}$$

$$= \lim_{x\to\pi} \frac{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)}{-4\left(\frac{\pi}{4} - \frac{x}{2}\right)} \times \sin\left(\frac{\pi}{4} - \frac{x}{4}\right) = -\frac{1}{2}\times 0 = 0$$

$$\text{So: } (\text{KIN})_{19} = \lim_{x\to\pi} \frac{1 - \sin\frac{x}{2}}{x - \pi} = 0$$

$$(\text{KIN})_{20} = \lim_{x\to+\infty} \sqrt[x]{x}$$

$$\therefore\ \lim_{x\to+\infty} \frac{\ln x}{x} = 0\ ;\ \lim_{x\to0^+} x\ln x = 0$$

$$\text{we have: } (\text{KIN})_{20} = \lim_{x\to+\infty} \sqrt[x]{x} = e^{\lim_{x\to+\infty}\frac{1}{x}\ln x} = e^0 = 1$$

$$\text{So: } (\text{KIN})_{20} = \lim_{x\to+\infty} \sqrt[x]{x} = 1$$

$$(\text{KIN})_{21} = \lim_{x\to1} (2 - x)^{\tan\left(\frac{\pi x}{2}\right)}$$

$$\therefore\ \lim_{x\to1} \frac{\ln x}{x - 1} = 1$$

we have: $\lim_{x\to1}(2-x)^{\tan\left(\frac{\pi x}{2}\right)} = e^{\lim_{x\to1}\tan\left(\frac{\pi x}{2}\right)\ln(2-x)}$

$$\therefore \lim_{x\to1}\tan\left(\frac{\pi x}{2}\right)\ln(2-x) = \lim_{x\to1}\cot\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)\frac{\ln(2-x)}{1-x}$$

$$= \lim_{x\to1}\frac{1-x}{\tan\frac{\pi}{2}(1-x)}\times\frac{\ln(2-x)}{1-x}\times(1-x) = \frac{2}{\pi}$$

$$\boxed{\text{So: } (KIN)_{21} = \lim_{x\to1}(2-x)^{\tan\left(\frac{\pi x}{2}\right)} = \frac{2}{\pi}}$$

$$(KIN)_{22} = \lim_{x\to0}\frac{1-\sqrt[n]{\cos2x}}{x^2} = \frac{1}{3}\text{ ; find n ?}$$

$$\therefore \lim_{x\to1}\frac{x^m-a^m}{x^n-a^n} = \frac{m}{n}a^{m-n}$$

we have: $\lim_{x\to0}\frac{1-(\cos2x)^{\frac{1}{n}}}{1-\cos2x}\times\frac{1-\cos2x}{x^2} = \frac{1}{3}$

$$\equiv \frac{1}{n}(1)^{\frac{1}{n}-1}\times\lim_{x\to0}\frac{2\sin^2x}{x^2} = \frac{1}{3} \equiv \frac{2}{n} == \frac{1}{3} \Rightarrow n = 6$$

$$\boxed{\text{So: } (KIN)_{22} = \lim_{x\to0}\frac{1-\sqrt[n]{\cos2x}}{x^2} = \frac{1}{3}\text{ ; } n = 6}$$

$$(KIN)_{23} = \lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n-1}{n^2}\right)$$

$$\therefore 1+2+2+\cdots+(n-1) = \frac{n(n-1)}{2}$$

we have: $\lim_{n\to\infty}\frac{1}{n^2}\left[\frac{n(n-1)}{2}\right] = \lim_{n\to\infty}\frac{n^2}{n^2}\left[\frac{\left(1-\frac{1}{n^2}\right)}{2}\right] = \frac{1}{2}$

$$\boxed{\text{So: } (KIN)_{23} = \lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n-1}{n^2}\right) = \frac{1}{2}}$$

$$(\mathrm{KIN})_{24}=\lim_{x\to 2^-}\frac{x-2\cos\sqrt{2-x}}{(2-x)^2}$$

$$\therefore \lim_{x\to 0}\frac{\sin x-x}{x^3}=-\frac{1}{6}$$

Let: $2-x=y\ \Rightarrow x=2-y$, if: $x\to 2^-$; $y\to 0$

we have: $\lim_{x\to 0^+}\frac{2-y-2\cos\sqrt{y}}{y^2}=\lim_{x\to 0^+}\frac{2\left(1-\cos\sqrt{y}\right)-y}{y^2}$

$$=\lim_{x\to 0^+}\frac{2\times 2\sin^2\left(\frac{\sqrt{y}}{2}\right)-y}{y^2}=\frac{4}{16}\lim_{x\to 0^+}\frac{\left[\sin^2\left(\frac{\sqrt{y}}{2}\right)-\left(\frac{\sqrt{y}}{2}\right)^2\right]}{\left(\frac{\sqrt{y}}{2}\right)^4}$$

$$=\frac{1}{4}\lim_{x\to 0^+}\frac{\sin\left(\frac{\sqrt{y}}{2}\right)+\left(\frac{\sqrt{y}}{2}\right)}{\left(\frac{\sqrt{y}}{2}\right)}\times\frac{\sin\left(\frac{\sqrt{y}}{2}\right)-\left(\frac{\sqrt{y}}{2}\right)}{\left(\frac{\sqrt{y}}{2}\right)^3}=-\frac{1}{12}$$

So: $(\mathrm{KIN})_{24}=\lim_{x\to 2^-}\frac{x-2\cos\sqrt{2-x}}{(2-x)^2}=-\frac{1}{12}$

$$(\mathrm{KIN})_{25}=\lim_{x\to\infty}\left(\cos\frac{1}{x}+\sin\frac{1}{x^2}\right)^{x^2}$$

$$\therefore \lim_{x\to 0}\frac{\ln(1+x)}{x}=1$$

Let: $x=\frac{1}{y}$, if: $x\to\infty$; $y\to 0$

we have: $\lim_{x\to\infty}\left(\cos\frac{1}{x}+\sin\frac{1}{x^2}\right)^{x^2}=\lim_{y\to 0}(\cos y+\sin y^2)^{\frac{1}{y^2}}$

$$=e^{\lim_{y\to 0}\frac{\ln(\cos y+\sin y^2)}{y^2}}=e^{\lim_{y\to 0}\frac{\ln(\cos y+\sin y^2)}{\cos y+\sin y^2-1}\times\frac{\cos y+\sin y^2-1}{y^2}}$$

$$= e^{\lim\limits_{y\to 0} 1\times\frac{(\cos y-1)+\sin y^2}{y^2}} = e^{\lim\limits_{y\to 0}\frac{-2\sin^2\left(\frac{y}{2}\right)+\sin y^2}{y^2}}$$

$$= e^{\lim\limits_{y\to 0}\left(\frac{-2\sin^2\left(\frac{y}{2}\right)}{y^2}+\frac{\sin y^2}{y^2}\right)} = e^{\lim\limits_{y\to 0}\frac{-2\sin^2\left(\frac{y}{2}\right)}{4\left(\frac{y}{2}\right)^2}+\frac{\sin y^2}{y^2}}$$

$$= e^{-\frac{1}{2}+1} = e^{\frac{1}{2}} = \sqrt{e}$$

$$\boxed{\text{So}: (\text{KIN})_{25} = \lim_{x\to\infty}\left(\cos\frac{1}{x}+\sin\frac{1}{x^2}\right)^{x^2} = \sqrt{e}}$$

$$(\text{KIN})_{26} = \lim_{x\to 0}\frac{\sin 3x+\sin 6x-\sin 9x}{ax^n} = 1\text{ ; find } a , n\ ?$$

$$\therefore \lim_{x\to 0}\frac{\sin x - x}{x^3} = -\frac{1}{6}$$

$$\Rightarrow \lim_{x\to 0}\frac{(3x)^3\frac{\sin 3x-3x}{(3x)^3}+(6x)^3\frac{\sin 6x-6x}{(6x)^3}-(9x)^3\frac{\sin 9x-9x}{(9x)^3}}{ax^n} = 1$$

$$\equiv \lim_{x\to 0}\frac{-\frac{1}{6}\left[(3x)^3+(6x)^3-(9x)^3\right]}{ax^n} = 1$$

$$\equiv \lim_{x\to 0}\frac{-\frac{1}{6}\left[27x^3+216x^3-729x^3\right]}{ax^n} = 1\ \ ; x^n = x^3 \Rightarrow n = 3$$

$$\equiv -\frac{1}{6}(-486) = a \Rightarrow a = 81$$

$$\text{So}:(\text{KIN})_{26}=\lim_{x\to 0}\frac{\sin 3x+\sin 6x-\sin 9x}{ax^n}=1\,;\begin{cases}n=3\\a=81\end{cases}$$

$$(\text{KIN})_{27}=\lim_{x\to\infty}\left[x^2\tan^{-1}\left(\cos\frac{5}{x}-\cos\frac{2}{x}\right)\right]$$

$$\text{Let}: y=\frac{1}{x}\ ,\text{if}: x\to\infty\,;y\to 0$$

$$\text{we have}:\lim_{y\to 0}\left[\frac{1}{y^2}\tan^{-1}(\cos 5y-\cos 2y)\right]$$

$$=\lim_{y\to 0}\left[\frac{\tan^{-1}\left[-2\sin\left(\frac{7y}{2}\right)\sin\left(\frac{3y}{2}\right)\right]}{y^2}\right]$$

$$=\lim_{y\to 0}\left[\frac{\tan^{-1}\left[-2\sin\left(\frac{7y}{2}\right)\sin\left(\frac{3y}{2}\right)\right]}{-2\sin\left(\frac{7y}{2}\right)\sin\left(\frac{3y}{2}\right)}\right]\cdot\frac{-2\sin\left(\frac{7y}{2}\right)\sin\left(\frac{3y}{2}\right)}{y^2}$$

$$=-2\lim_{y\to 0}\frac{\sin\left(\frac{7y}{2}\right)}{y}\times\frac{\sin\left(\frac{3y}{2}\right)}{y}$$

$$=-2\lim_{y\to 0}\frac{\sin\left(\frac{7y}{2}\right)}{\frac{7y}{2}\times\frac{2}{7}}\times\frac{\sin\left(\frac{3y}{2}\right)}{\frac{3y}{2}\times\frac{2}{3}}=-2\times\frac{7}{2}\times\frac{3}{2}=-\frac{21}{2}$$

$$\text{So}:(\text{KIN})_{27}=\lim_{x\to\infty}\left[x^2\tan^{-1}\left(\cos\frac{5}{x}-\cos\frac{2}{x}\right)\right]=-\frac{21}{2}$$

$$(\text{KIN})_{28}=\lim_{x\to 0}\left[\tan\left(\frac{\pi}{4}+x\right)^{\frac{1}{x}}\right]$$

$$\text{we have}:\lim_{x\to 0}\left[1+\tan\left(\frac{\pi}{4}+x\right)-1\right]^{\frac{1}{x}}$$

$$=e^{\lim\limits_{x\to 0}\frac{\tan\left(\frac{\pi}{4}+x\right)-1}{x}}=e^{\lim\limits_{x\to 0}\frac{\tan\left(\frac{\pi}{4}+x\right)-\tan\frac{\pi}{4}}{x}}$$

$$= e^{\lim\limits_{x\to 0}\frac{\tan\left(\frac{\pi}{4}+x-\frac{\pi}{4}\right)\left[1+\left(\frac{\pi}{4}+x\right)\tan\frac{\pi}{4}\right]}{x}} = e^{\lim\limits_{x\to 0}\frac{\tan x\left[1+\tan\left(\frac{\pi}{4}+x\right)\right]}{x}} = e^2$$

$$\boxed{\text{So: } (\text{KIN})_{28} = \lim_{x\to 0}\left[\tan\left(\frac{\pi}{4}+x\right)^{\frac{1}{x}}\right] = e^2}$$

$$(\text{KIN})_{29} = \lim_{x\to 0}\frac{(2-x^2)\sin x - \sin 2x}{x^5}$$

$$\therefore \lim_{x\to 0}\frac{\sin x - x}{x^3} = -\frac{1}{6}$$

$$\Rightarrow \lim_{x\to 0}\frac{2\sin x - x^2\sin x - 2\sin x\cos x}{x^5} = \lim_{x\to 0}\frac{2\sin x(1-\cos x) - x^2\sin x}{x^5}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{4\sin^2\left(\frac{x}{2}\right)-x^2}{x^4} = \lim_{x\to 0}\frac{2\sin\left(\frac{x}{2}\right)+x}{x}\cdot\frac{2\sin\left(\frac{x}{2}\right)-x}{x^3}$$

$$= \lim_{x\to 0}\left[\frac{2\sin\left(\frac{x}{2}\right)}{x}+1\right]\cdot\left[\frac{2\sin\left(\frac{x}{2}\right)}{x^3}-\frac{2\frac{x}{2}}{x^3}\right]$$

$$= \left(2\times\frac{1}{2}+1\right)2\lim_{x\to 0}\left[\frac{\sin\left(\frac{x}{2}\right)-\frac{x}{2}}{\left(\frac{x}{2}\right)^3}\times\frac{\left(\frac{x}{2}\right)^3}{x^3}\right] = 2\times 2\times\frac{-1}{6}\times\frac{1}{8} = -\frac{1}{12}$$

$$\boxed{\text{So: } (\text{KIN})_{29} = \lim_{x\to 0}\frac{(2-x^2)\sin x - \sin 2x}{x^5} = -\frac{1}{12}}$$

$$(\text{KIN})_{30} = \lim_{x\to 0}\frac{\sqrt{\frac{1+x}{1-x}}\cdot\sqrt[4]{\frac{1+2x}{1-2x}}\cdot\sqrt[6]{\frac{1+3x}{1-3x}}-1}{x}$$

$$= \lim_{x\to 0}\frac{\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\cdot\left(\frac{1+2x}{1-2x}\right)^{\frac{1}{4}}\cdot\left(\frac{1+3x}{1-3x}\right)^{\frac{`1}{6}}-1}{\left[\left(\frac{1+3x}{1-3x}\right)-1\right]}\cdot 6$$

$$+\lim_{x\to0}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\frac{\left(\frac{1+2x}{1-2x}\right)^{\frac{1}{4}}\cdot\left[\left(\frac{1+3x}{1-3x}\right)^{\frac{1}{6}}-1\right]}{\left[\left(\frac{1+3x}{1-3x}\right)-1\right]}\cdot4+\lim_{x\to0}\frac{\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1}}{\left(\frac{1+x}{1-x}\right)-1}\cdot2$$

$$=\frac{1}{6}(1)^{-\frac{5}{6}}\times6+\frac{1}{4}(1)^{-\frac{3}{4}}\times4+\frac{1}{2}(1)^{-\frac{1}{2}}\times2=3$$

$$\text{So: }(KIN)_{30}=\lim_{x\to0}\frac{\sqrt{\frac{1+x}{1-x}}\cdot\sqrt[4]{\frac{1+2x}{1-2x}}\cdot\sqrt[6]{\frac{1+3x}{1-3x}}-1}{x}=3$$

$$(KIN)_{31}=\lim_{x\to0}\frac{\sin(\tan x)-\tan(\sin x)}{\sin x-\tan x}$$

$$\therefore\lim_{x\to0}\frac{\sin x-x}{x^3}=-\frac{1}{6}$$

$$\therefore\lim_{x\to0}\frac{\sin(\tan x)-\tan(\sin x)}{\sin x-\tan x}$$

$$\therefore\lim_{x\to0}\frac{\sin[(\tan x)-\tan][\tan x(\sin x)-\sin x]-(\sin x-\tan x)}{\sin x-\tan x}$$

$$=\lim_{x\to0}\frac{\sin(\tan x)-\tan x-[\tan x(\sin x)-\sin x]}{\sin x-\tan x}-1$$

$$=\lim_{x\to0}\frac{\left[\frac{\sin(\tan x)-\tan x}{\tan^3x}\right]\cdot\frac{\tan^3x}{x^3}-\left[\frac{\tan x(\sin x)-\sin x}{\sin^3x}\right]\cdot\frac{\tan^3x}{x^3}}{\sin x-\tan x}-1$$

$$=\frac{-\frac{1}{6}(1)^3-\frac{1}{3}(1)^3}{-\frac{1}{6}-\frac{1}{3}}-1=1-1=0$$

$$\text{So: }(KIN)_{31}=\lim_{x\to0}\frac{\sin(\tan x)-\tan(\sin x)}{\sin x-\tan x}=0$$

$$(\mathrm{KIN})_{32}=\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\cos 3x}$$

$\therefore \cos 3x = 4\cos^3 x - 3\cos x$

we have: $\displaystyle\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{4\cos^3 x-3\cos x}=\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\cos x(4\cos^2 x-3)}$

$$=\lim_{x\to\frac{\pi}{2}}\frac{1}{(4\cos^2 x-3)}=-\frac{1}{3}$$

So: $\displaystyle(\mathrm{KIN})_{32}=\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\cos 3x}=-\frac{1}{3}$

$$(\mathrm{KIN})_{33}=\lim_{x\to 1}\frac{\sin\pi x+\cos\frac{\pi x}{2}}{(1+3x)\cos\frac{3\pi x}{2}}$$

$$=\lim_{x\to 1}\frac{\sin\pi x+\sin\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}{(1+3x)\sin\left(\frac{\pi}{2}-\frac{3\pi x}{2}\right)}$$

$$=\lim_{x\to 1}\left[\frac{\sin\pi x}{(1+3x)\sin\frac{\pi}{2}(1-3x)}+\frac{\sin\frac{\pi}{2}(1-x)}{(1+3x)\sin\frac{\pi}{2}(1-3x)}\right]$$

$$=\lim_{x\to 1}\left[\frac{\pi x}{(1+3x)\frac{\pi}{2}(1-3x)}+\frac{(1-x)}{(1+3x)(1-3x)}\right]=-\frac{1}{4}$$

So: $\displaystyle(\mathrm{KIN})_{33}=\lim_{x\to 1}\frac{\sin\pi x+\cos\frac{\pi x}{2}}{(1+3x)\cos\frac{3\pi x}{2}}=-\frac{1}{4}$

$$(\mathrm{KIN})_{34}=\lim_{n\to\infty}\left(\sqrt[n]{4^n+5^n}\right)$$

$$=\lim_{n\to\infty}(4^n+5^n)^{\frac{1}{n}}=\lim_{n\to\infty}(5^n)^{\frac{1}{n}}\left[\left(\frac{4}{5}\right)^n+1\right]^{\frac{1}{n}}$$

$$=5\lim_{n\to\infty}\left[1+\left(\frac{4}{5}\right)^n\right]^{\frac{1}{n}}=5e^{\lim_{n\to\infty}\ln\left(\frac{4}{5}\right)^n\times\frac{1}{n}}=5e^0=5$$

$$\text{So}: (\text{KIN})_{34} = \lim_{n\to\infty}\left(\sqrt[n]{4^n + 5^n}\right) = 5$$

$$(\text{KIN})_{35} = \lim_{x\to 1}\frac{(2x+1)^5 - 243}{(x+1)^6 - 32}$$

$$\therefore (x-1) = (x+1) - 2 \equiv [(2x+1) - 3]\frac{1}{2}$$

$$\text{we have}: \lim_{x\to 1}\frac{(2x+1)^5 - 243}{(x+1)^6 - 32} = \lim_{x\to 1}\frac{2\,\dfrac{(2x+1)^5 - 3^5}{(2x+1) - 3}}{\dfrac{(x+1)^6 - 2^6}{(x+1) - 2}}$$

$$= \frac{2 \times \frac{5}{1} \times 3^4}{\frac{6}{1} \times 2^5} = \frac{135}{32}$$

$$\text{So}: (\text{KIN})_{35} = \lim_{x\to 1}\frac{(2x+1)^5 - 243}{(x+1)^6 - 32} = \frac{135}{32}$$

$$(\text{KIN})_{36} = \lim_{n\to\infty}\left[\left(\frac{n+9}{n+10}\right)^n\right]$$

$$= e^{\lim\limits_{n\to\infty} n\ln\frac{n+9}{n+10}} = e^{\lim\limits_{n\to\infty} n\frac{\ln\frac{(n+9)}{n+10}}{\left[\frac{(n+9)}{n+10} - 1\right]}\times\left[\frac{(n+9)}{n+10} - 1\right]} = e^{\lim\limits_{n\to\infty} n\times\left[\frac{-1}{n+10}\right]} = \frac{1}{e}$$

$$\text{So}: (\text{KIN})_{36} = \lim_{n\to\infty}\left[\left(\frac{n+9}{n+10}\right)^n\right] = \frac{1}{e}$$

$$(\text{KIN})_{37} = \lim_{x\to\infty}\left[\left(\frac{4^{\frac{1}{x}} + 5^{\frac{1}{x}} + 6^{\frac{1}{x}}}{3}\right)^{3x}\right]$$

$$\text{Let}: y = \frac{1}{x}\ ,\text{if}: x \to \infty\ ; y \to 0$$

we have: $\lim_{y\to 0}\left[\left(\frac{4^y+5^y+6^y}{3}\right)^{\frac{3}{y}}\right]=e^{\lim_{y\to 0}\frac{3}{y}\ln\left(\frac{4^y+5^y+6^y}{3}\right)}$

$$=e^{\lim_{y\to 0}\frac{3\ln\left(\frac{4^y+5^y+6^y}{3}\right)}{y\left(\frac{4^y+5^y+6^y}{3}-1\right)}\times\left(\frac{4^y+5^y+6^y}{3}-1\right)}=e^{\lim_{y\to 0}\left(\frac{4^y+5^y+6^y-3}{y}\right)}$$

$$=e^{\lim_{y\to 0}\left(\frac{4^y-1+5^y-1+6^y-1}{y}\right)}=e^{\ln 4+\ln 5+\ln 6}=e^{\ln 120}=120$$

So: $(\mathrm{KIN})_{37}=\lim_{x\to\infty}\left[\left(\frac{4^{\frac{1}{x}}+5^{\frac{1}{x}}+6^{\frac{1}{x}}}{3}\right)^{3x}\right]=120$

$$(\mathrm{KIN})_{38}=\lim_{x\to 1}\frac{x\sqrt{x+3}-2}{x^2+x-2}$$

$$=\lim_{x\to 1}\frac{x\left(\sqrt{x+3}-\sqrt{4}+2(x-1)\right)}{x^2+x-2}$$

$$=\lim_{x\to 1}\frac{x\left(\sqrt{x+3}-\sqrt{4}\right)}{x^2+x-2}+\lim_{x\to 1}\frac{2(x-1)}{x^2+x-2}$$

$$=\frac{1}{3}\times\frac{1}{2}(4)^{-\frac{1}{2}}+\frac{2}{3}=\frac{3}{4}$$

So: $(\mathrm{KIN})_{38}=\lim_{x\to 1}\frac{x\sqrt{x+3}-2}{x^2+x-2}=\frac{3}{4}$

$$(\mathrm{KIN})_{39}=\lim_{x\to e}\frac{\ln x-1}{x-e}$$

$$\therefore \lim_{x\to 1}\frac{\ln x}{x-1}=1\ ,\ \therefore \lim_{x\to 0}\frac{\ln(x+1)}{x}=1$$

$$=\lim_{x\to e}\frac{\ln x-\ln e}{e\left(\frac{x}{e}-1\right)}=\lim_{x\to e}\frac{\ln\left(\frac{x}{e}\right)}{e\left(\frac{x}{e}-1\right)}=\frac{1}{e}\times 1=\frac{1}{e}$$

$$\text{So}: (\text{KIN})_{39} = \lim_{x\to e}\frac{\ln x - 1}{x - e} = \frac{1}{e}$$

$$(\text{KIN})_{40} = \lim_{x\to\frac{\pi}{3}}\frac{\sin x - \sqrt{3}\cos x}{4x - \frac{4\pi}{3}}$$

$$= \lim_{x\to\frac{\pi}{3}}\left(\cos x\cdot\frac{\tan x - \sqrt{3}}{4x - \frac{4\pi}{3}}\right) = \frac{1}{2}\lim_{x\to\frac{\pi}{3}}\frac{\tan x - \tan\frac{\pi}{3}}{4\left(x - \frac{\pi}{3}\right)}$$

$$= \frac{1}{2}\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(x - \frac{\pi}{3}\right)}{4\left(x - \frac{\pi}{3}\right)} \times \frac{1}{\cos x\cdot\cos\frac{\pi}{3}} = \frac{1}{2}\times\frac{1}{4}\times\frac{1}{\frac{1}{2}\times\frac{1}{2}} = \frac{1}{2}$$

$$\text{So}: (\text{KIN})_{40} = \lim_{x\to\frac{\pi}{3}}\frac{\sin x - \sqrt{3}\cos x}{4x - \frac{4\pi}{3}} = \frac{1}{2}$$

$$(\text{KIN})_{41} = \lim_{x\to 3}\frac{3^x - 27}{2^{x+1} - 16}$$

$$\therefore a^x = (b^x)^{\log_b a} \therefore \lim_{x\to 0}\frac{x^m - a^m}{x^n - a^n} = \frac{m}{n}\cdot a^{m-n}$$

$$= \frac{1}{2}\lim_{x\to 3}\frac{(2^x)^{\log_2 3} - (2^3)^{\log_2 3}}{2^x - 2^3}$$

$$= \frac{1}{2}\times\frac{\log_2 3}{1}\times(2^3)^{\log_2 3} = \frac{1}{2}\times\log_2 3\times 2^{\log_2 27}\times\frac{1}{8}$$

$$= \frac{27}{16}\log_2 3$$

$$\text{So}: (\text{KIN})_{41} = \lim_{x\to 3}\frac{3^x - 27}{2^{x+1} - 16} = \frac{27}{16}\log_2 3$$

$$(\text{KIN})_{42} = \lim_{x\to 6}\frac{1 - \sqrt{3 - \sqrt{x - 2}}}{x - 6}$$

$$=\lim_{x\to6}\frac{\left(1-\sqrt{3-\sqrt{x-2}}\right)\left(1+\sqrt{3-\sqrt{x-2}}\right)}{(x-6)\left(1+\sqrt{3-\sqrt{x-2}}\right)}$$

$$=\lim_{x\to6}\frac{(x-2)^{\frac{1}{2}}-4^{\frac{1}{2}}}{(x-6)\left(1+\sqrt{3-\sqrt{x-2}}\right)}=\frac{1}{2}\times4^{\frac{1}{2}}\times\frac{1}{2}=\frac{1}{8}$$

$$\text{So}:(\text{KIN})_{42}=\lim_{x\to6}\frac{1-\sqrt{3-\sqrt{x-2}}}{x-6}=\frac{1}{8}$$

$$(\text{KIN})_{43}=\lim_{x\to1}\frac{\sin(2\ln x)-2\ln x}{\ln^3(x^4)}$$

$$\therefore\lim_{x\to0}\frac{\sin x-x}{x^3}=-\frac{1}{6}$$

$$=\lim_{x\to1}\frac{\sin(2\ln x)-2\ln x}{\ln x^4\times\ln x^4\times\ln x^4}\times\frac{8\ln^3x}{64\ln^3x}$$

$$=-\frac{1}{6}\times\frac{1}{8}=-\frac{1}{48}$$

$$\text{So}:(\text{KIN})_{43}=\lim_{x\to1}\frac{\sin(2\ln x)-2\ln x}{\ln^3(x^4)}=-\frac{1}{48}$$

$$(\text{KIN})_{44}=\lim_{x\to0}\frac{\cos x\cos2x\cos4x\cos8x-x\cot x}{x^2}$$

$$\therefore\lim_{x\to0}\frac{\sin x-x}{x^3}=-\frac{1}{6}$$

$$\therefore\cos x\cos2x\cos4x\cos8x$$

$$=\frac{\sin2x\cos2x\cos4x\cos8x}{2\sin x}=\frac{\sin4x\cos4x\cos8x}{4\sin x}$$

$$=\frac{\sin8x\cos8x}{8\sin x}=\frac{\sin16x}{16\sin x}$$

$$\therefore\lim_{x\to0}\frac{\frac{\sin16x}{16\sin x}-x\cdot\cot x}{x^2}$$

$$=\lim_{x\to0}\frac{\sin16x-16x\cos x}{16x^2\sin x}$$

$$=\lim_{x\to0}\frac{\sin16x-16x+16x\cos x}{16x^2\sin x}$$

$$=\lim_{x\to0}\frac{\sin16x-16x}{16x^2\sin x}\times\frac{(16x)^3}{16x^3\sin x}+\lim_{x\to0}\frac{16x2\sin^2\left(\frac{x}{2}\right)}{16x^2\sin x}$$

$$=-\frac{1}{6}\times256+\frac{1}{2}=-\frac{42}{6}=-7$$

$$\text{So}: (\mathrm{KIN})_{44}=\lim_{x\to0}\frac{\cos x\cos2x\cos4x\cos8x-x\cot x}{x^2}=-7$$

$$(\mathrm{KIN})_{45}=\lim_{x\to0}\frac{\sin2x+\sin4x-\tan6x}{1-\cos x}$$

$$=\lim_{x\to0}\left(\frac{2x-\sin2x}{1-\cos x}-\frac{4x-\sin4x}{2\sin^2\left(\frac{x}{2}\right)}+\frac{6x-\tan6x}{2\sin^2\left(\frac{x}{2}\right)}\right)$$

$$=-\lim_{x\to0}\left(\frac{\frac{2x-\sin2x}{(2x)^2}(2x)}{\frac{2\sin^2\left(\frac{x}{2}\right)}{(2x)^2}}-\frac{\frac{4x-\sin4x}{(4x)^2}(4x)}{\frac{2\sin^2\left(\frac{x}{2}\right)}{(4x)^2}}+\frac{\frac{6x-\sin6x}{(4x)^2}(6x)}{\frac{2\sin^2\left(\frac{x}{2}\right)}{(6x)^2}}\right)$$

$$=-\frac{\frac{1}{6}}{2\left(\frac{1}{4}\right)^2}\times0\,\frac{\frac{1}{6}}{2\left(\frac{1}{8}\right)^2}\times0+\frac{\frac{1}{6}}{2\left(\frac{1}{12}\right)^2}\times0=0$$

$$\text{So}: (\mathrm{KIN})_{45}=\lim_{x\to0}\frac{\sin2x+\sin4x-\tan6x}{1-\cos x}=0$$

$$(\mathrm{KIN})_{46}=\lim_{x\to0}\frac{\ln(\sin x+\cos x)^2}{\sin2x}$$

$$=\lim_{x\to0}\frac{\ln(\sin x+\cos x)^2}{(\sin x+\cos x)^2-1}\times\frac{(\sin x+\cos x)^2-1}{2\sin x}$$

$$= \lim_{x\to 0}\frac{\sin^2 x + \cos^2 x + 2sinxcosx - 1}{2\sin x} = \lim_{x\to 0}\frac{1 + sin2x - 1}{sin2x} = 1$$

$$\text{So}: (\text{KIN})_{46} = \lim_{x\to 0}\frac{\ln(\sin x + \cos x)^2}{\sin 2x} = 1$$

$$(\text{KIN})_{47} = \lim_{x\to 0}\frac{x\cos x - \sin x}{x^3}$$

$$= \lim_{x\to 0}\frac{x\cos x - x + x - \sin x}{x^3} = \lim_{x\to 0}\frac{x(\cos x - 1)}{x^3} + \lim_{x\to 0}\frac{(x - \sin x)}{x^3}$$

$$= \frac{1}{6}\lim_{x\to 0}\frac{x\left(-2\sin^2\left(\frac{x}{2}\right)\right)}{x^3} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$$

$$\text{So}: (\text{KIN})_{47} = \lim_{x\to 0}\frac{x\cos x - \sin x}{x^3} = -\frac{1}{3}$$

$$(\text{KIN})_{48} = \lim_{x\to\frac{\pi}{4}}\frac{1 - \sqrt{2}\sin x}{1 - \sqrt{2}\cos x}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{1 - \sqrt{2}\sin x}{1 - \sqrt{2}\cos x} \times \frac{1 + \sqrt{2}\cos x}{1 + \sqrt{2}\cos x} \times \frac{1 + \sqrt{2}\sin x}{1 + \sqrt{2}\sin x}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{(1 - \sqrt{2}sin^2 x)(1 + \sqrt{2}\cos x)}{(1 - \sqrt{2}cos^2 x)(1 + \sqrt{2}\sin x)}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\cos 2x(1 + \sqrt{2}\cos x)}{\left(1 - 2\sin^2\left(\frac{\pi}{2} - x\right)\right)(1 + \sqrt{2}\sin x)}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\cos 2x(1 + 1)}{\cos(\pi - 2x)(1 + 1)} = -1$$

$$\text{So}: (\text{KIN})_{48} = \lim_{x\to\frac{\pi}{4}}\frac{1 - \sqrt{2}\sin x}{1 - \sqrt{2}\cos x} = -1$$

$$(\text{KIN})_{49} = \lim_{x\to 2}\frac{x^4\sqrt{x + 2}\sqrt[3]{x + 6} - 64}{x - 2}$$

$$= \lim_{x\to 2}\frac{\left(x^4\sqrt{x+2}\left(\sqrt[3]{x+6}-\sqrt[3]{8}\right)\right)+2x^4\left(\sqrt{x+2}-\sqrt{4}\right)-4(x^4-2^4)}{x-2}$$

$$= \lim_{x\to 2}\left[x^4\sqrt{x+2}\,\frac{(x+6)^{\frac{1}{3}}-8^{\frac{1}{3}}}{x-2}+2x^4\left[\frac{(x+2)^{\frac{1}{2}}-4^{\frac{1}{2}}}{x-2}\right]+4\left(\frac{x^4-2^4}{x-2}\right)\right]$$

$$= \frac{416}{3}$$

$$\text{So: }(\mathrm{KIN})_{49} = \lim_{x\to 2}\frac{x^4\sqrt{x+2}\sqrt[3]{x+6}-64}{x-2} = \frac{416}{3}$$

$$(\mathrm{KIN})_{50} = \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-2\cos x}{\sqrt{2}-2\sin x}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-2\cos x}{\sqrt{2}-2\sin x}\times\frac{\sqrt{2}+2\sin x}{\sqrt{2}+2\cos x}\times\frac{\sqrt{2}+2\sin x}{\sqrt{2}+2\sin x}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{2(1-2\cos^2 x)}{2(1-2\sin^2 x)}\times\frac{\sqrt{2}+2\sin x}{\sqrt{2}+2\cos x}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{-\cos 2x}{\cos 2x}\times\frac{\sqrt{2}+2\sin x}{\sqrt{2}+2\cos x} = \lim_{x\to\frac{\pi}{4}}\frac{-\sqrt{2}+2\sin x}{\sqrt{2}+2\cos x} = -1$$

$$\text{So: }(\mathrm{KIN})_{50} = \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-2\cos x}{\sqrt{2}-2\sin x} = -1$$

$$(\mathrm{KIN})_{51} = \lim_{x\to\infty}\left[\left(\frac{x^2+5x+4}{x^2-3x+7}\right)^x\right]$$

$$= \lim_{x\to\infty}\left[\left(1+\frac{8x-3}{x^2-3x+7}\right)^x\right] = e^{\lim\limits_{x\to\infty}\left(\frac{8x-3}{x^2-3x+7}\right)^x} = e^8$$

$$\text{So: }(\mathrm{KIN})_{51} = \lim_{x\to\infty}\left[\left(\frac{x^2+5x+4}{x^2-3x+7}\right)^x\right] = e^8$$

$$(\mathrm{KIN})_{52}=\lim_{x\to\frac{\pi}{6}}\frac{1-\sqrt{3}\tan x}{\pi-6x}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\sqrt{3}}{6}\left(\frac{\frac{1}{\sqrt{3}}-\tan x}{\frac{\pi}{6}-x}\right)=\frac{\sqrt{3}}{6}\lim_{x\to\frac{\pi}{6}}\left(\frac{\tan\frac{\pi}{6}-\tan x}{\left(\frac{\pi}{6}-x\right)}\right)$$

$$=\frac{\sqrt{3}}{6}\lim_{x\to\frac{\pi}{6}}\left(\frac{\frac{\sin\left(\frac{\pi}{6}-x\right)}{\cos\frac{\pi}{6}\cos x}}{\left(\frac{\pi}{6}-x\right)}\right)=\frac{\sqrt{3}}{6}\times\left(\frac{2}{\sqrt{3}}\right)^2=\frac{2\sqrt{3}}{9}$$

$$\boxed{\text{So: }(\mathrm{KIN})_{52}=\lim_{x\to\frac{\pi}{6}}\frac{1-\sqrt{3}\tan x}{\pi-6x}=\frac{2\sqrt{3}}{9}}$$

$$(\mathrm{KIN})_{53}=\lim_{x\to\sqrt{\pi}}\frac{1-\cos\sqrt{\pi}x}{\pi-x^2}$$

$$=\lim_{x\to\sqrt{\pi}}\left(\frac{2\cos^2-\frac{\sqrt{\pi}x}{2}}{\pi-x^2}\right)=\lim_{x\to\sqrt{\pi}}\left(\frac{2\sin^2\left(\frac{\pi}{2}-\frac{\sqrt{\pi}x}{2}\right)}{\pi-x^2}\right)$$

$$=\lim_{x\to\sqrt{\pi}}2\left(\frac{\sin\frac{\sqrt{\pi}(\sqrt{\pi}-x)}{2}}{\sqrt{\pi}-x}\times\frac{\sin\frac{\sqrt{\pi}(\sqrt{\pi}-x)}{2}}{\sqrt{\pi}-x}\right)$$

$$=2\times\frac{\sqrt{\pi}}{2}\times0=0$$

$$\boxed{\text{So: }(\mathrm{KIN})_{53}=\lim_{x\to\sqrt{\pi}}\frac{1-\cos\sqrt{\pi}x}{\pi-x^2}=0}$$

$$(\mathrm{KIN})_{54}=\lim_{x\to0}\frac{(x+1)^8+(x-1)^8}{(x+1)^5+(x-1)^5}$$

$$= \lim_{x\to 0}\frac{[(x+1)^8-(1)^8+(x-1)^8-(1)^8]}{[(x+1)^5-(1)^5-(1)^5+(x-1)^5-(1)^5]}$$

$$= \lim_{x\to 0}\frac{\left[\frac{(x+1)^8-(1)^8}{(x+1)-1}-\frac{(x-1)^8-(-1)^8}{(x-1)-(-1)}\right]}{\left[\frac{(x+1)^5-(1)^5-(1)^5}{(x+1)-1}+\frac{(x-1)^5-(1)^5}{(x-1)-(-1)}\right]}$$

$$= \frac{8(1)^7-8(-1)^7}{4(1)^4+5(-1)^4}=\frac{8}{5}$$

$$\text{So: } (KIN)_{54} = \lim_{x\to 0}\frac{(x+1)^8+(x-1)^8}{(x+1)^5+(x-1)^5}=\frac{8}{5}$$

$$(KIN)_{55} = \lim_{x\to 0}\frac{\ln\left(\frac{3-\cos 2x-2x^2}{2}\right)}{x^4}$$

$$= \lim_{x\to 0}\ln\left[\frac{2}{2}+\left(\frac{1-\cos 2x-2x^2}{2}\right)\right]^{\frac{1}{x^4}}$$

$$= \lim_{x\to 0}\ln(1+\sin^2 x-x^2)^{\frac{1}{x^4}}$$

$$= \ln\lim_{x\to 0} e^{\frac{\sin x-x}{x^3}\times\frac{\sin x+x}{x}}$$

$$= \ln e^{-\frac{1}{6}\times 2}$$

$$= \ln e^{-\frac{1}{3}}$$

$$= -\frac{1}{3}$$

$$\text{So: } (KIN)_{55} = \lim_{x\to 0}\frac{\ln\left(\frac{3-\cos 2x-2x^2}{2}\right)}{x^4}=-\frac{1}{3}$$

$$(\text{KIN})_{56}=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-\cos x-\sin x}{(4x-\pi)^2}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}+(\cos x+\sin x)}{\sqrt{2}+(\cos x+\sin x)}\times\frac{\sqrt{2}-(\cos x+\sin x)}{16\left(x-\frac{\pi}{4}\right)^2}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2-(\cos^2x+\sin^2x+2\sin x\cos x)}{16\left(x-\frac{\pi}{4}\right)^2}\times\frac{1}{2\sqrt{2}}$$

$$=\frac{1}{32\sqrt{2}}\lim_{x\to\frac{\pi}{4}}\frac{2-1-\sin 2x}{\left(x-\frac{\pi}{4}\right)^2}$$

$$=\frac{1}{32\sqrt{2}}\lim_{x\to\frac{\pi}{4}}\frac{1-\cos\left(\frac{\pi}{2}-2x\right)}{\left(x-\frac{\pi}{4}\right)^2}$$

$$=\frac{1}{32\sqrt{2}}\lim_{x\to\frac{\pi}{4}}\frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{\left(x-\frac{\pi}{4}\right)^2}$$

$$=\frac{2}{32\sqrt{2}}=\frac{\sqrt{2}}{32}$$

$$\boxed{\text{So: }(\text{KIN})_{56}=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-\cos x-\sin x}{(4x-\pi)^2}=\frac{\sqrt{2}}{32}}$$

$$(\text{KIN})_{57}=\lim_{x\to 2}\frac{\sqrt{2x-3\sqrt[3]{3x-5\sqrt[4]{4x-7\sqrt[5]{5x-9}}}}-1}{x-2}$$

$\therefore$ **Tayler series**

$$(1+x)^n=1+\frac{n}{1!}x+\frac{n(n-1)}{2!}x^2+\cdots$$

$$=\lim_{x\to 2}\frac{\left(1+(2x-4)\right)^{\frac{1}{2}}\left(1+(3x-6)\right)^{\frac{1}{3}}\left(1+(4x-8)\right)^{4}\left(1+(5x-10)\right)^{\frac{1}{5}}-1}{x-2}$$

$$=\lim_{x\to 2}\frac{\left(1+\frac{2x-4}{2}+\cdots\right)\left(1+\frac{3x-6}{3}+\cdots\right)\left(1+\frac{4x-8}{4}+\cdots\right)\left(1+\frac{5x-10}{5}+\cdots\right)-1}{x-2}$$

$$=\lim_{x\to 2}\frac{(1+x-2)(1+x-2)(1+x-2)(1+x-2)-1}{x-2}$$

$$=\lim_{x\to 2}\frac{(1+x-2)^4-1^4}{(x-1)-1}=4(1)^3=4$$

$$\text{So: }(KIN)_{57}=\lim_{x\to 2}\frac{\sqrt{2x-3}\sqrt[3]{3x-5}\sqrt[4]{4x-7}\sqrt[5]{5x-9}-1}{x-2}=4$$

$$(KIN)_{58}=\lim_{m\to\infty}\left[\left(\cos\left\{\frac{x}{m}\right\}\right)^m\right]$$

$$\text{let } m=\frac{1}{n}\ ,\ \lim_{m\to\infty}\left[(\cos\{nx\})^{\frac{1}{n}}\right]$$

$$=e^{\lim\limits_{n\to 0}\frac{\ln\cos(nx)}{n}}=e^{\lim\limits_{n\to 0}\frac{\ln\cos(nx)}{\cos nx-1}\times\frac{\cos(nx)-1}{n}}=e^{1\times\lim\limits_{n\to 0}\frac{-2\sin^2(nx)}{\cos nx-1}}$$

$$=e^{\lim\limits_{n\to 0}\frac{\sin(nx)}{n}\times(-2\sin(nx))=e^{1\times(-2)\times 0}=e^0=1}$$

$$\text{So: }(KIN)_{58}=\lim_{m\to\infty}\left[\left(\cos\left\{\frac{x}{m}\right\}\right)^m\right]=1$$

$$(KIN)_{59}=\lim_{x\to\infty}\left(\frac{x^3+x+1}{\sqrt[3]{x^3+x-1}}-\frac{x^3-x+1}{\sqrt[3]{x^3-x+2}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{x^3+x-1}{\sqrt[3]{x^3+x-1}}+\frac{2}{\sqrt[3]{x^3-x-1}}\right)-\lim_{x\to\infty}\left(\frac{x^3+x+2}{\sqrt[3]{x^3+x+2}}-\frac{1}{\sqrt[3]{x^3-x+2}}\right)$$

$$=\lim_{x\to\infty}(x^3+x-1)^{\frac{2}{3}}+0+(x^3-x+2)^{\frac{2}{3}}-0$$

$$=\lim_{x\to\infty}(x^3+x-1)^{\frac{1}{3}}-(x^3-x+2)^{\frac{1}{3}}(x^3+x-1)^{\frac{1}{3}}+(x^3-x+2)^{\frac{1}{3}}$$

$$= \lim_{x\to\infty}\frac{(x^3+x-1-x^3+x-2)(x^3+x-1)^{\frac{1}{3}}+(x^3-x+2)^{\frac{1}{3}}}{(x^3+x-1)^{\frac{2}{3}}+(x^3+x-1)^{\frac{1}{3}}(x^3-x+2)^{\frac{1}{3}}+(x^3-x+2)^{\frac{2}{3}}}$$

$$= \lim_{x\to\infty}\frac{(2x-3)(x^3+x-1)^{\frac{1}{3}}+(x^3-x+2)^{\frac{1}{3}}}{(x^3+x-1)^{\frac{2}{3}}+(x^3+x-1)^{\frac{1}{3}}(x^3-x+2)^{\frac{1}{3}}+(x^3-x+2)^{\frac{2}{3}}}$$

$$= \frac{2(1+1)}{1+1+1} = \frac{4}{3}$$

So: $(KIN)_{59} = \lim_{x\to\infty}\left(\frac{x^3+x+1}{\sqrt[3]{x^3+x-1}} - \frac{x^3-x+1}{\sqrt[3]{x^3-x+2}}\right) = \frac{4}{3}$

$$(KIN)_{60} = \lim_{n\to\infty}\left(\frac{1}{n}\sqrt[n]{\frac{2n!}{n!}}\right)$$

$$= \lim_{n\to\infty}\frac{1}{n}\left(\frac{2n!}{n!}\right)^{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{n}\left(\frac{\sqrt{2\pi\times 2n}}{\sqrt{2\pi n}}\times\left(\frac{e}{n}\right)^{n}\right)^{\frac{1}{n}}$$

$$= \lim_{n\to\infty}\frac{1}{n}(\sqrt{2})^{\frac{1}{n}}\left(\left(\frac{4n^2}{e^2}\times\frac{e}{n}\right)^{n}\right)^{\frac{1}{n}} = \lim_{n\to\infty}\left(\sqrt{2}^{\frac{1}{n}}\times\frac{1}{n}\times\frac{4n}{e}\right)$$

$$= 1\times\lim_{n\to\infty}\frac{4n}{ne} = \frac{4}{e}$$

So: $(KIN)_{60} = \lim_{n\to\infty}\left(\frac{1}{n}\sqrt[n]{\frac{2n!}{n!}}\right) = \frac{4}{e}$

$$(KIN)_{61} = \lim_{x\to 0}\frac{\ln(2x^2+\cos 2x)}{x^4}$$

$$= \lim_{x\to 0}\ln(1+2x^2+\cos 2x-2)^{\frac{1}{x^4}}$$

$$= \ln e^{\lim_{x\to 0}(2x^2-2\sin^2 x)}$$

$$= \ln e^{\lim_{x\to 0}\frac{2(x-\sin x)}{x^3}\times\frac{(x+\sin x)}{x}} = \ln e^{2\times\frac{1}{6}\times 2} = \frac{2}{3}$$

$$\text{So}: (\text{KIN})_{61} = \lim_{x\to 0}\frac{\ln(2x^2+\cos 2x)}{x^4} = \frac{2}{3}$$

$$(\text{KIN})_{62} = \lim_{x\to 0}\frac{1-\sin\left(\frac{\pi}{2}\cos x + x\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{\sin\frac{\pi}{2} - \sin\left(\frac{\pi}{2}\cos x + x\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{2\sin\left(\frac{\frac{\pi}{2}-\frac{\pi}{2}\cos x - x}{2}\right)\sin\left(\frac{\pi}{2}-\frac{\frac{\pi}{2}+\frac{\pi}{2}\cos x + x}{2}\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{2\sin^2\left(\frac{\frac{\pi}{2}-\frac{\pi}{2}\cos x - x}{2}\right)}{x^2}$$

$$= \lim_{x\to 0}\left[\frac{2\sin^2\left(\frac{\frac{\pi}{2}(1-\cos x)-x}{2}\right)}{x^2}\times\frac{2\sin^2\left(\frac{\frac{\pi}{2}2\sin^2\left(\frac{x}{2}\right)-x}{2}\right)}{\left(\frac{\pi\sin^2\left(\frac{x}{2}\right)-x}{2}\right)^2}\times\frac{\left(\frac{\pi\sin^2\left(\frac{x}{2}\right)-x}{2}\right)^2}{x^2}\right]$$

$$= 1\times 2\times\left(0-\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\text{So}: (\text{KIN})_{62} = \lim_{x\to 0}\frac{1-\sin\left(\frac{\pi}{2}\cos x + x\right)}{x^2} = \frac{1}{2}$$

$$= \lim_{x\to 0}\frac{1-\frac{(2\sin x\cos x)\cos 2x\cos 3x}{2\sin x}}{x^2}$$

$$= \lim_{x\to 0}\frac{1-\frac{(2\sin 2x\cos 2x)\cos 3x}{4\sin x}}{x^2} = \lim_{x\to 0}\frac{1-\frac{\sin 4x\cos 3x}{4\sin x}}{x^2}$$

$$= \lim_{x\to 0}\frac{1-\frac{\frac{\sin(4x+3x)+\sin(4x-3x)}{2}}{4\sin x}}{x^2}$$

$$= \lim_{x\to 0} \frac{8\sin x - \sin 7x - \sin x}{8x^2 \sin x}$$

$$= \lim_{x\to 0} \frac{7\sin x - (7x) - \sin 7x + (7x)}{8x^2 \sin x}$$

$$= \lim_{x\to 0} \left[\frac{7\sin x - 7x}{x^3} \times \frac{x^3}{8x^2 \sin x} - \frac{\sin 7x - (7x)}{(7x)^3} \times \frac{(7x)^3}{8x^2 \sin x}\right]$$

$$= 7 \times \left(-\frac{1}{6}\right)\left(\frac{1}{8}\right) - \left(-\frac{1}{6}\right)\left(\frac{7^3}{8}\right) = 7$$

$$\text{So: } (\text{KIN})_{63} = \lim_{x\to 0} \frac{1 - \cos x \cos 2x \cos 3x}{x^2} = 7$$

$$(\text{KIN})_{64} = \lim_{x\to 0} \frac{64^x - 3 \times 48^x + 3 \times 36^x - 27^x}{x^3}$$

$$= \lim_{x\to 0} \frac{(2^{2x} - 3^x)^3}{x^3}$$

$$= \lim_{x\to 0} \frac{\left(e^{2x\ln 2} - 3^{x\ln 3}\right)^3}{x}$$

$$= \lim_{x\to 0} \left(\frac{e^{2x\ln 2} - 1}{x} - \frac{3^{x\ln 3} - 1}{x}\right)^3$$

$$= \lim_{x\to 0} \left(\frac{e^{2x\ln 2} - 1}{2x\ln 2} \times \frac{2x\ln 2}{x} - \frac{3^{x\ln 3} - 1}{3x\ln 3} \times \frac{3x\ln 3}{x}\right)^3$$

$$= (1 \times 2\ln 2 - 1 \times \ln 3)^3$$

$$= \left(\ln\frac{4}{3}\right)^3$$

$$\text{So: } (\text{KIN})_{64} = \lim_{x\to 0} \frac{64^x - 3 \times 48^x + 3 \times 36^x - 27^x}{x^3} = \left(\ln\frac{4}{3}\right)^3$$

$$(\mathrm{KIN})_{65} = \lim_{x\to 0}\frac{\cos(5\tan^4x - 3\sin^4x) - \cos(5\tan^4x + 3\sin^4x)}{x^8}$$

$$= \lim_{x\to 0}\frac{\cos(4\tan^4x - 3\sin^4x) - 1 - \cos(5\tan^4x + 3\sin^4x) - 1}{x^8}$$

$$= \lim_{x\to 0}\frac{\frac{-2\sin^2(5\tan^4x - 3\sin^4x)}{2} + \frac{2\sin^2(5\tan^4x + 3\sin^4x)}{2}}{x^8}$$

$$= -2\lim_{x\to 0}\frac{\left[\sin\frac{(5\tan^4x - 3\sin^4x)}{2} - \sin\frac{(5\tan^4x + 3\sin^4x)}{2}\right]}{x^4}$$

$$\times \lim_{x\to 0}\frac{\left[\sin\frac{(4\tan^4x - 3\sin^4x)}{2} - \sin\frac{(4\tan^4x + 3\sin^4x)}{2}\right]}{x^4}$$

$$= -2 \times \left(\frac{5-3}{2} - \frac{5+3}{2}\right)\left(\frac{5-3}{2} + \frac{5+3}{2}\right)$$

$$= -2 \times (-3) \times 5 = 30$$

So: $(\mathrm{KIN})_{65} = \lim_{x\to 0}\frac{\cos(5\tan^4x - 3\sin^4x) - \cos(5\tan^4x + 3\sin^4x)}{x^8} = 30$

$$(\mathrm{KIN})_{66} = \lim_{x\to 0}\frac{1-\cos3x}{1-\cos4x}$$

$$= \lim_{x\to 0}\frac{2\sin^2\left(\frac{3x}{2}\right)}{2\sin^2\left(\frac{4x}{2}\right)} = \frac{9}{16}$$

So: $(\mathrm{KIN})_{66} = \lim_{x\to 0}\frac{1-\cos3x}{1-\cos4x} = \frac{9}{16}$

$$(\mathrm{KIN})_{67} = \lim_{x\to\frac{\pi}{6}}\frac{\sin x - \frac{1}{2}}{6x - \pi}$$

$$= \lim_{x\to\frac{\pi}{6}}\frac{\sin x - \sin\frac{\pi}{6}}{6\left(x - \frac{\pi}{6}\right)} = \lim_{x\to\frac{\pi}{6}}\frac{2\sin\left(\frac{x-\frac{\pi}{6}}{2}\right)\cos\left(\frac{x+\frac{\pi}{6}}{2}\right)}{6\left(x-\frac{\pi}{6}\right)}$$

$$= \lim_{x\to\frac{\pi}{6}}\left[\frac{2\sin\left(\frac{x-\frac{\pi}{6}}{2}\right)}{6\left(x-\frac{\pi}{6}\right)} \times \frac{\cos\left(\frac{x+\frac{\pi}{6}}{2}\right)}{6}\right] = 2 \times \frac{1}{2} \times \frac{\frac{\sqrt{3}}{2}}{6} = \frac{\sqrt{3}}{2}$$

$$\boxed{\text{So: } (KIN)_{67} = \lim_{x\to\frac{\pi}{6}}\frac{\sin x - \frac{1}{2}}{6x-\pi} = \frac{\sqrt{3}}{2}}$$

$$(KIN)_{68} = \lim_{x\to\frac{\pi}{4}}\frac{\tan x - 1}{x-\frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\tan x - \tan\frac{\pi}{4}}{x-\frac{\pi}{4}} = \lim_{x\to\frac{\pi}{4}}\frac{\sin\left(x-\frac{\pi}{4}\right)}{\left(x-\frac{\pi}{4}\right)\cos x\cos\frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}}\left[\frac{\sin\left(x-\frac{\pi}{4}\right)}{\left(x-\frac{\pi}{4}\right)} \times \frac{1}{\cos x\cos\frac{\pi}{4}}\right] = 1 \times \frac{1}{\frac{\sqrt{2}}{2}} \times \frac{1}{\frac{\sqrt{2}}{2}} = 2$$

$$\boxed{\text{So: } (KIN)_{68} = \lim_{x\to\frac{\pi}{4}}\frac{\tan x - 1}{x-\frac{\pi}{4}} = 2}$$

$$(KIN)_{69} = \lim_{x\to\frac{\pi}{6}}\frac{2\cos 2x - 1}{\cos 3x}$$

$$= \lim_{x\to\frac{\pi}{6}}\frac{2\left(\cos 2x - \frac{1}{2}\right)}{\sin\left(\frac{\pi}{2}-3x\right)}$$

$$= \lim_{x\to\frac{\pi}{6}}\frac{2\left(\cos 2x - \cos\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{2}-3x\right)}$$

$$= \lim_{x\to\frac{\pi}{6}}\frac{2\times(-2)\sin\left(\frac{2x+\frac{\pi}{3}}{2}\right)\sin\left(\frac{2x-\frac{\pi}{3}}{2}\right)}{\sin\left(\frac{\pi}{2}-3x\right)}$$

$$= \lim_{x \to \frac{\pi}{6}} \left[\frac{\sin \frac{1}{6}(6x - \pi)s}{\sin \frac{1}{2}(\pi - 6x)} \times 2(-2)\sin\left(\frac{\pi}{6} + x\right) \right]$$

$$= -\frac{1}{3} \times (-4)\left(\frac{\sqrt{3}}{2}\right) = \frac{2\sqrt{3}}{3}$$

$$\text{So: } (KIN)_{69} = \lim_{x \to \frac{\pi}{6}} \frac{2\cos 2x - 1}{\cos 3x} = \frac{2\sqrt{3}}{3}$$

$$(KIN)_{70} = \lim_{x \to 0} \frac{\cos x - \sqrt[3]{\cos x}}{\sin^2 x}$$

$$= \lim_{x \to 0} \frac{(\cos x - 1) - \left(\sqrt[3]{\cos x} - 1\right)}{\sin^2 x}$$

$$= -\lim_{x \to 0} \frac{(\cos x - 1) - \left(\sqrt[3]{\cos x} - 1\right)}{\cos^2 x - 1}$$

$$= -\lim_{x \to 0} \left[\frac{(\cos x - 1)}{\cos^2 x - 1} + \frac{\cos^{\frac{1}{3}} z - 1}{\cos^2 x - 1} \right] = -\frac{1}{2} + \frac{\frac{1}{3}}{2} = -\frac{1}{3}$$

$$\text{So: } (KIN)_{70} = \lim_{x \to 0} \frac{\cos x - \sqrt[3]{\cos x}}{\sin^2 x} = -\frac{1}{3}$$

$$(KIN)_{71} = \lim_{x \to e} \frac{x - e\ln x}{(x - e)^2}$$

let $x = ey$, if $x \Rightarrow e \ \equiv y \Rightarrow 1$

$$= \lim_{x \to e} \frac{e}{e^2} \times \frac{(y - \ln y - \ln e)}{(y - 1)^2}$$

$$= \frac{1}{e} \lim_{x \to e} \frac{(y - \ln y - \ln e)}{(y - 1)^2} = \frac{1}{e} \times \frac{1}{2} = \frac{1}{2e}$$

$$\text{So: } (KIN)_{71} = \lim_{x \to e} \frac{x - e\ln x}{(x - e)^2} = \frac{1}{2e}$$

$$(\mathrm{KIN})_{72} = \lim_{x\to 0}\frac{x-\tan^{-1}x}{x^3}$$

let $y = \tan^{-1}x$

$$= \lim_{x\to 0}\frac{x-y}{x^3} = \lim_{x\to 0}\frac{\tan y - y}{\tan^3 y} = \lim_{x\to 0}\frac{\sin y - y\cos y}{y^3\cos y}$$

$$= \lim_{x\to 0}\frac{(\sin y - y) + (y - y\cos y)}{y^3}\times\frac{1}{\cos y}$$

$$= \lim_{x\to 0}\frac{(\sin y - y)}{y^3}\times\frac{1}{\cos y} + \lim_{x\to 0}\frac{y2\sin^2\frac{y}{2}}{y^3}\times\frac{1}{\cos y}$$

$$= -\frac{1}{6}\times 1 + 2\times\frac{1}{4} = \frac{1}{3}$$

$$\boxed{\text{So: } (\mathrm{KIN})_{72} = \lim_{x\to 0}\frac{x-\tan^{-1}x}{x^3} = \frac{1}{3}}$$

$$(\mathrm{KIN})_{73} = \lim_{x\to 0}\left(\frac{1}{x\sin x} - \frac{1}{x^2}\right)$$

$$= \lim_{x\to 0}\left(\frac{x-\sin x}{x^2\sin x}\right) = \lim_{x\to 0}\frac{x-\sin x}{x^3}\times\frac{x^3}{x^2\sin x}$$

$$= \lim_{x\to 0}\left(\frac{x-\sin x}{x^3}\times\frac{x}{\sin x}\right) = \frac{1}{6}\times 1 = \frac{1}{6}$$

$$\boxed{\text{So: } (\mathrm{KIN})_{73} = \lim_{x\to 0}\left(\frac{1}{x\sin x} - \frac{1}{x^2}\right) = \frac{1}{6}}$$

$$(\mathrm{KIN})_{74} = \lim_{x\to 0}\left(\frac{2^{\sec x} - 2^{\cos x}}{x^2}\right)$$

$$= \lim_{x\to 0}\left(\frac{2^{\ln 2\sec x} - 2^{\ln 2\cos x}}{x^2}\right)$$

$$= \lim_{x\to 0}\frac{e^{\ln 2\sec x} - e^{\ln 2\cos x}}{x^2}$$

$$= \lim_{x\to 0} \frac{e^{\ln 2\cos x}\left(e^{\ln 2\frac{1}{\cos x}\ln 2\cos x} - 1\right)}{x^2}$$

$$= \lim_{x\to 0} \frac{e^{\ln 2\cos x}\left(e^{\ln 2\left(\frac{1\cos^2 x}{\cos x}\right)} - \ln 2\left(\frac{\sin^2 x}{\cos x}\right) - 1\right)}{\left(\ln 2\frac{\sin^2 x}{\cos x}\right)^2} \times \frac{\left(\ln 2\frac{\sin^2 x}{\cos x}\right)^2}{x^2}$$

$$= 2 \times \frac{1}{2} \times 0 + 2 \times \ln 2 \times 1 = 2\ln 2$$

So: $(\mathrm{KIN})_{74} = \lim_{x\to 0}\left(\frac{2^{\sec x} - 2^{\cos x}}{x^2}\right) = 2\ln 2$

$$(\mathrm{KIN})_{75} = \lim_{x\to\frac{\pi}{2}}\left[(\sin x)^{\tan x}\right]$$

$$\text{put } y = \frac{\pi}{2} - x$$

$$= \lim_{y\to 0}\left[\left(\sin x\left(\frac{\pi}{2} - y\right)\right)^{\tan\left(\frac{\pi}{2}-y\right)}\right]$$

$$= \lim_{y\to 0}(\cos y)^{\cot y}$$

$$= \lim_{y\to 0}\big(1 + (\cos y - 1)\big)^{\cot y}$$

$$= \lim_{y\to 0}\left(1 - 2\sin^2\left(\frac{y}{2}\right)\right)^{\cot y}$$

$$= e^{\lim_{y\to 0}\left(-2\sin^2\left(\frac{y}{2}\right)\right)\times\frac{\cos y}{\sin y}} = e^{\lim_{y\to 0}(-2)\times\frac{\sin\left(\frac{y}{2}\right)}{\sin y}\times\sin\left(\frac{y}{2}\right)}$$

$$= e^{(-2)\times\frac{1}{2}\times 0} = 1$$

So: $(KIN)_{75} = \lim_{x \to \frac{\pi}{2}} [(\sin x)^{\tan x}] = 1$

$$(KIN)_{76} = \lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)^5}{1 - \cos\left(\frac{\pi}{2} - 2x\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2}\left(1 - \cos^5\left(\frac{\pi}{4} - x\right)\right)}{1 - 2\cos^5\left(\frac{\frac{\pi}{2} - 2x}{2}\right) - 1}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2}\left(1 - \cos^5\left(\frac{\pi}{4} - x\right)\right)}{2\left(1 - \cos^5\left(\frac{\pi}{4} - x\right)\right)}$$

$$= \frac{4\sqrt{2}}{2} \times \frac{5}{2}(1)^{5-2} = 5\sqrt{2}$$

So: $(KIN)_{76} = \lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x} = 5\sqrt{2}$

$$(KIN)_{77} = \lim_{x \to 0} \frac{\sqrt[3]{1 + \tan x} - \sqrt[3]{1 + \sin x}}{x^3}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt[3]{1 + \tan x} - \sqrt[3]{1 + \sin x}\right)\left(\sqrt[3]{(1 + \tan x)^2} + \sqrt[3]{1 + \tan x} + \sqrt[3]{1 + \sin x} + \sqrt[3]{(1 + \sin x)^2}\right)}{x^3\left(\sqrt[3]{(1 + \tan x)^2} + \sqrt[3]{1 + \tan x} + \sqrt[3]{1 + \sin x} + \sqrt[3]{(1 + \sin x)^2}\right)}$$

$$= \lim_{x \to 0} \left[\frac{(1 + \tan x) - (1 + \sin x)}{x^3} \times \frac{1}{\left(\sqrt[3]{(1 + \tan x)^2} + \sqrt[3]{1 + \tan x} + \sqrt[3]{1 + \sin x} + \sqrt[3]{(1 + \sin x)^2}\right)}\right]$$

$$= \lim_{x \to 0} \frac{\tan x - \sin x}{x^3} \times \frac{1}{3} = \frac{1}{3} \lim_{x \to 0} \frac{(\tan x - x) - (x - \sin x)}{x^3}$$

$$=\frac{1}{3}\lim_{x\to 0}\frac{(\tan x-x)}{x^3}+\frac{1}{3}\lim_{x\to 0}\frac{x-\sin x}{x^3}$$

$$=\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{6}=\frac{1}{6}$$

$$\text{So: }(KIN)_{77}=\lim_{x\to 0}\frac{\sqrt[3]{1+\tan x}-\sqrt[3]{1+\sin x}}{x^3}=\frac{1}{6}$$

$$(KIN)_{78}=\lim_{x\to 1}\left[\left(\frac{\pi}{4}-\tan^{-1}x\right)\left(\tan\frac{\pi}{x+1}\right)\right]$$

let $y=\tan^{-1}x\equiv x=\tan y$

$$=\lim_{x\to 1}\frac{\left(\frac{\pi}{4}-\tan^{-1}x\right)}{\cot\left(\frac{\pi}{x+1}\right)}=\lim_{x\to 1}\frac{\left(\frac{\pi}{4}-\tan^{-1}x\right)}{\tan\left(\frac{\pi}{2}-\frac{\pi}{x+1}\right)}$$

$$=\lim_{x\to 1}\frac{\left(\frac{\pi}{4}-\tan^{-1}x\right)}{\tan\frac{\pi}{2}\left(\frac{\pi}{x+1}\right)}=\lim_{x\to 1}\frac{\left(\frac{x-1}{x+1}\right)}{\tan\frac{\pi}{2}\left(\frac{\pi}{x+1}\right)}\times\frac{\left(\frac{\pi}{4}-\tan^{-1}x\right)}{\left(\frac{x-1}{x+1}\right)}$$

$$=\frac{2}{\pi}\lim_{y\to\frac{\pi}{4}}\frac{\left(\frac{\pi}{4}-y\right)}{\left(\frac{\tan y-1}{\tan y+1}\right)}=\frac{2}{\pi}\lim_{y\to\frac{\pi}{4}}\frac{\left(\frac{\pi}{4}-y\right)}{\tan\left(y-\frac{\pi}{4}\right)}=\frac{2}{\pi}(-1)=-\frac{2}{\pi}$$

$$\text{So: }(KIN)_{78}=\lim_{x\to 1}\left[\left(\frac{\pi}{4}-\tan^{-1}x\right)\left(\tan\frac{\pi}{x+1}\right)\right]=-\frac{2}{\pi}$$

$$(KIN)_{79}=\lim_{x\to 1}\frac{\sin(x^{2020}-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{\sin(x^{2020}-1)}{(x^{2019}-1)}\times\frac{(x^{2019}-1)}{x-1}$$

$$=1\times\frac{2019}{1}\times 1^{2019-1}=2019$$

$$\text{So: }(KIN)_{79}=\lim_{x\to 1}\frac{\sin(x^{2020}-1)}{x-1}=2019$$

$$(KIN)_{80} = \lim_{x\to 0}\frac{\sqrt{1+x^4}+\sqrt[3]{8+2x^4}-\sqrt[3]{27-2x^4}}{(1-\cos 2x)\tan^2(\sin 3x)}$$

$$\therefore \lim_{x\to 0}\frac{(1-\cos 2x)\tan^2(\sin 3x)}{x^4}$$

$$= \lim_{x\to 0}\left[\frac{2\sin^2 x}{x^2}\times\frac{\tan^2(\sin 3x)}{\sin^2(3x)}\times\frac{\sin^2 3x}{x^2}\right] = 21\times\times 9 = 18$$

$$\therefore \lim_{x\to 0}\frac{\sqrt{1+x^4}+\sqrt[3]{8+2x^4}-\sqrt[3]{27-2x^4}}{x^4}$$

$$= \lim_{x\to 0}\left[\frac{\sqrt{1+x^4}-1}{(1+x^4)-1}+\frac{\sqrt[3]{8+2x^4}-\sqrt[3]{8}}{(8+2x^4)-8}\times(2)-\frac{\sqrt[3]{27-2x^4}-\sqrt[3]{27}}{(27-2x^4)-27}\times(-2)\right]$$

$$= \frac{1}{2}(1)+\frac{1}{3}(8)^{-\frac{2}{3}}(2)-\frac{1}{3}(27)^{-\frac{2}{3}}(-2)$$

$$= \frac{1}{2}+\frac{1}{6}+\frac{2}{27} = \frac{20}{27}$$

$$\therefore \lim_{x\to 0}\frac{\sqrt{1+x^4}+\sqrt[3]{8+2x^4}-\sqrt[3]{27-2x^4}}{(1-\cos 2x)\tan^2(\sin 3x)}$$

$$= \frac{\frac{20}{27}}{18} = \frac{10}{243}$$

So: $(KIN)_{80} = \lim_{x\to 0}\frac{\sqrt{1+x^4}+\sqrt[3]{8+2x^4}-\sqrt[3]{27-2x^4}}{(1-\cos 2x)\tan^2(\sin 3x)} = \frac{10}{243}$

$$(KIN)_{81} = \lim_{x\to 0}\frac{e^{\frac{\tan^6 x}{6}}-e^{\frac{\sin^6 x}{6}}}{x^8}$$

$$= \lim_{x\to 0}\frac{e^{\frac{\tan^6 x}{6}}\left(1-e^{\frac{\sin^6 x}{6}-\frac{\tan^6 x}{6}}\right)}{x^8} = \lim_{x\to 0}\frac{e^{\frac{\tan^6 x}{6}}\left(1-e^{\frac{\tan^6 x}{6}(\cos^6 x-1)}\right)}{x^8}$$

$$= \lim_{x\to 0}\left[e^{\frac{\tan^6 x}{6}} \times \frac{\left(1 - e^{\frac{\tan^6 x}{6}(\cos^6 x - 1)}\right)}{x^8}\right]$$

$$= 1 \times \lim_{x\to 0}\left[\frac{\left(1 - e^{\frac{\tan^6 x}{6}(\cos^6 x - 1)}\right)}{\frac{\tan^6 x}{6}(\cos^6 x - 1)} \times \frac{\frac{\tan^6 x}{6}(\cos^6 x - 1)}{x^8}\right]$$

$$= 1 \times (-1)\lim_{x\to 0}\left[\frac{\frac{\tan^6 x}{6}(\cos^6 x - 1)(\cos^4 x + \cos^2 x - 1)}{x^8}\right]$$

$$= -\lim_{x\to 0}\left[\frac{\frac{\tan^6 x}{6}}{x^6} \times \frac{-\sin^2 x}{x^2} \times (\cos^4 x + \cos^2 x - 1)\right]$$

$$= -1 \times \frac{1}{6} \times (-1) \times 3 = \frac{1}{2}$$

So: $(\text{KIN})_{81} = \lim_{x\to 0}\frac{e^{\frac{\tan^6 x}{6}} - e^{\frac{\sin^6 x}{6}}}{x^8} = \frac{1}{2}$

$$(\text{KIN})_{82} = \lim_{x\to 0}\frac{\ln\left(\sqrt{\frac{1 - x^2}{\cos x}}\right)}{x^3}$$

$$= \lim_{x\to 0}\frac{\sqrt{1 - x^2} - \cos x}{x^3 \cos x} \times \frac{\sqrt{1 - x^2} + \cos x}{\sqrt{1 - x^2} + \cos x}$$

$$= \lim_{x\to 0}\frac{1 - x^2 - \cos^2 x}{x^3} \times \frac{1}{\cos x\sqrt{2 - x^2} + \cos x}$$

$$= \lim_{x\to 0}\frac{\sin^2 x - x^2}{x^3} \times 1$$

$$=\lim_{x\to 0}\frac{(\sin x-x)}{x^3}\times(\sin x+x)=-\frac{1}{6}\times 0=0$$

$$\text{So: }(\text{KIN})_{82}=\lim_{x\to 0}\frac{\ln\left(\sqrt{\frac{1-x^2}{\cos x}}\right)}{x^3}=0$$

$$(\text{KIN})_{83}=\lim_{n\to\infty}\left(\sqrt[3]{8^n+4^n+2^n}-2^n\right)$$

$$=\lim_{n\to\infty}\frac{\left(\sqrt[3]{8^n+4^n+2^n}-2^n\right)\left(\sqrt[3]{8^n+4^n+2^n}+2^n\right)\left(\sqrt[3]{8^n+4^n+2^n}+2^{2n}\right)}{\left(\sqrt[3]{8^n+4^n+2^n}\right)+2^n\left(\sqrt[3]{8^n+4^n+2^n}+2^{2n}\right)}$$

$$=\lim_{n\to\infty}\frac{(8^n+4^n+2^n-2^{3n})}{\left(\sqrt[3]{8^n+4^n+2^n}\right)+2^n\left(\sqrt[3]{8^n+4^n+2^n}+2^{2n}\right)}$$

$$=\lim_{n\to\infty}\frac{(4^n+2^n)}{\left(\sqrt[3]{8^n+4^n+2^n}\right)+2^n\left(\sqrt[3]{8^n+4^n+2^n}+2^{2n}\right)}$$

$$=\frac{1}{1+1+1}=\frac{1}{3}$$

$$\text{So: }(\text{KIN})_{83}=\lim_{n\to\infty}\left(\sqrt[3]{8^n+4^n+2^n}-2^n\right)=\frac{1}{3}$$

$$(\text{KIN})_{84}=\lim_{x\to 0}\left(\sqrt[x]{1+\sin\left[1-\left(\frac{e^x-1}{x}\right)\right]}\right)$$

$$=\lim_{x\to 0}1+\sin\left[1-\left(\frac{e^x-1}{x}\right)\right]^{\frac{1}{x}}$$

$$=e^{\lim_{x\to 0}\left(\frac{\sin\left(1-\left(\frac{e^x-1}{x}\right)\right)}{x}\right)}$$

$$\therefore \lim_{x\to 0}\frac{\sin\left(1-\left(\frac{e^x-1}{x}\right)\right)}{x}$$

$$=\lim_{x\to 0}\frac{\sin\left(1-\left(\frac{e^x-1}{x}\right)\right)}{1-\left(\frac{e^x-1}{x}\right)}\times\frac{1-\left(\frac{e^x-1}{x}\right)}{x}$$

$$=1\times\lim_{x\to 0}\frac{-(e^x-x-1)}{1-\left(\frac{e^x-1}{x}\right)}=\frac{-1}{2}$$

$$=\lim_{x\to 0}\left(\sqrt[x]{1+\sin\left[1-\left(\frac{e^x-1}{x}\right)\right]}\right)=e^{\frac{-1}{2}}$$

$$\text{So: } (KIN)_{84}=\lim_{x\to 0}\left(\sqrt[x]{1+\sin\left[1-\left(\frac{e^x-1}{x}\right)\right]}\right)=e^{\frac{-1}{2}}$$

$$(KIN)_{85}=\lim_{x\to 0}\left(\sqrt[x^2]{1+\sin\left[1-\left(\frac{\sin x}{x}\right)\right]}\right)$$

$$=\lim_{x\to 0}1+\sin\left[1-\left(\frac{\sin x}{x}\right)\right]^{\frac{1}{x^2}}$$

$$=e^{\lim_{x\to 0}\left(\frac{\sin\left(1-\left(\frac{\sin x}{x}\right)\right)}{x^2}\right)}$$

$$\therefore \lim_{x\to 0}\frac{\sin\left(1-\left(\frac{\sin x}{x}\right)\right)}{x}$$

$$=\lim_{x\to 0}\frac{\sin\left(1-\left(\frac{\sin x}{x}\right)\right)}{1-\left(\frac{\sin x}{x}\right)}\times\frac{1-\left(\frac{\sin x}{x}\right)}{x^2}$$

$$= 1 \times \lim_{x \to 0} \frac{(x - \sin x)}{x^3} = \frac{1}{6} = \lim_{x \to 0} \left(\sqrt[x^2]{1 + \sin\left[1 - \left(\frac{\sin x}{x}\right)\right]} \right) = e^{\frac{1}{6}}$$

$$\text{So: } (\text{KIN})_{85} = \lim_{x \to 0} \left(\sqrt[x^2]{1 + \sin\left[1 - \left(\frac{\sin x}{x}\right)\right]} \right) = e^{\frac{1}{6}}$$

$$(\text{KIN})_{86} = \lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{1 - \tan x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos\left(\frac{\pi}{4} - 2x\right)}{\tan\frac{\pi}{4} - \tan x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos\left(\frac{\pi}{4}\right)\cos x}{\sin\left(\frac{\pi}{4} - x\right)} \times \sin 2\left(\frac{\pi}{4} - x\right)$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sin 2\left(\frac{\pi}{4} - x\right)}{\sin\left(\frac{\pi}{4} - x\right)} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2 \times \frac{1}{2} = 1$$

$$\text{So: } (\text{KIN})_{86} = \lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{1 - \tan x} = 1$$

$$(\text{KIN})_{87} = \lim_{x \to 0} \frac{12 - 6x^2 - 12\cos x}{x^4}$$

$$= \lim_{x \to 0} \frac{12(1 - \cos x) - 6x^2}{x^4}$$

$$= \lim_{x \to 0} \frac{12 \times 2\sin^2\left(\frac{x}{2}\right) - 6x^2}{x^4}$$

$$= \lim_{x \to 0} \frac{24\sin^2\left(\frac{x}{2}\right) - 24\left(\frac{x}{2}\right)^2}{x^4}$$

$$= 24\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)+\frac{x}{2}}{\frac{x}{2}}\times\frac{1}{2}\times\left[\frac{\sin\left(\frac{x}{2}\right)-\frac{x}{2}}{\left(\frac{x}{2}\right)^3}\times\frac{1}{8}\right]$$

$$= 24\times\frac{1}{2}\times 2\times\frac{1}{8}\times\left(-\frac{1}{6}\right)=-\frac{1}{2}$$

$$\text{So: } (\text{KIN})_{87}=\lim_{x\to 0}\frac{12-6x^2-12\cos x}{x^4}=-\frac{1}{2}$$

$$(\text{KIN})_{88}=\lim_{x\to\frac{\pi}{4}}(\sin 2x)^{\tan^2 2x}$$

$$= e^{\lim\limits_{x\to\frac{\pi}{4}}(\sin 2x-1)\tan^2 2x}$$

$$\therefore \lim_{x\to\frac{\pi}{4}}(\sin 2x-1)\tan^2 2x$$

$$= \lim_{x\to\frac{\pi}{4}}\left(\sin 2x-\sin\frac{\pi}{2}\right)\cot^2\left(\frac{\pi}{2}-2x\right)$$

$$= \lim_{x\to\frac{\pi}{4}}\left(2\sin\frac{2x-\frac{\pi}{2}}{2}\right)\cos\left(\frac{2x+\frac{\pi}{2}}{2}\right)\cot^2 2\left(\frac{\pi}{4}-x\right)$$

$$= \lim_{x\to\frac{\pi}{4}}\left(\frac{2\sin\left(x-\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}-x\right)}{\tan^2\left(\frac{\pi}{4}-x\right)}\right)$$

$$= -\frac{2}{4}=-\frac{1}{2}$$

$$\text{So: } (\text{KIN})_{88}=\lim_{x\to\frac{\pi}{4}}(\sin 2x)^{\tan^2 2x}=-\frac{1}{2}$$

$$(\mathrm{KIN})_{89}=\lim_{x\to 1}\frac{3\sin\pi x-\sin 3\pi x}{(x-1)^3}$$

$$=\lim_{x\to 1}\frac{3\sin\pi x-(3\sin\pi x-4\sin^3\pi x)}{(x-1)^3}$$

$$=\lim_{x\to 1}\frac{4\sin^3\pi x}{(x-1)^3}=\lim_{x\to 1}\frac{4\sin^3(\pi-\pi x)}{(x-1)^3}=\lim_{x\to 1}\frac{4\sin^3\pi(1-x)}{(x-1)^3}=-4\pi^3$$

So: $(\mathrm{KIN})_{89}=\lim\limits_{x\to 1}\frac{3\sin\pi x-\sin 3\pi x}{(x-1)^3}=-4\pi^3$

$$(\mathrm{KIN})_{90}=\lim_{x\to 0}\frac{\ln(e+x)-\ln(e-x)}{x}$$

$$=\lim_{x\to 0}\frac{\ln\left(\frac{e+x}{e-x}\right)}{x}$$

$$=\lim_{x\to 0}\frac{\ln\left(\frac{e+x}{e-x}\right)}{\left(\frac{e+x}{e-x}-1\right)}\times\left(\frac{\frac{e+x-e+x}{e-x}}{x}\right)$$

$$=1\times\lim_{x\to 0}\frac{\frac{2x}{e-x}}{x}=\lim_{x\to 0}\frac{2x}{e-x}=\frac{2}{e}$$

So: $(\mathrm{KIN})_{90}=\lim\limits_{x\to 0}\frac{\ln(e+x)-\ln(e-x)}{x}=\frac{2}{e}$

$$(\mathrm{KIN})_{91}=\lim_{x\to \alpha}\left(2-\frac{\alpha}{x}\right)^{\tan\frac{\pi x}{2\alpha}}\quad \alpha\neq 0$$

$$=\lim_{x\to \alpha}\left(1+\left(1-\frac{\alpha}{x}\right)\right)^{\tan\frac{\pi x}{2\alpha}}=e^{\lim\limits_{x\to\alpha}\left(1-\frac{\alpha}{x}\right)\tan\frac{\pi x}{2\alpha}}$$

$$\therefore \lim_{x\to \alpha}\left(1-\frac{\alpha}{x}\right)\tan\left(\frac{\pi x}{2\alpha}\right)$$

$$=\lim_{x\to \alpha}\left(\frac{x-\alpha}{x}\right)\cot\left(\frac{\pi}{2}-\frac{\pi x}{2\alpha}\right)=\lim_{x\to \alpha}\left(\frac{\frac{x-\alpha}{x}}{\tan\frac{\pi}{2}\left(\frac{a-x}{a}\right)}\right)$$

$$= \lim_{x\to\alpha}\left(\frac{\frac{a-x}{a}}{\tan\frac{\pi}{2}\left(\frac{a-x}{a}\right)}\times\frac{\frac{x-\alpha}{x}}{\frac{a-x}{a}}\right) = \frac{2}{\pi}\times\left(-\frac{1}{1}\right) = -\frac{2}{\pi}$$

$$\boxed{\text{So: } (KIN)_{91} = \lim_{x\to\alpha}\left(2-\frac{\alpha}{x}\right)^{\tan\frac{\pi x}{2\alpha}} = -\frac{2}{\pi}}$$

$$(KIN)_{92} = \lim_{x\to 0}\frac{\sin^{-1}x - \sin x}{(\sin^{-1}x)^3}$$

$$\text{let } y = \sin^{-1}x \equiv x = \sin y$$

$$\therefore \lim_{x\to 0}\frac{\sin^{-1}x - x + x - \sin x}{(\sin^{-1}x)^3}$$

$$= \lim_{x\to 0}\frac{\sin^{-1}x - x}{(\sin^{-1}x)^3} + = \lim_{x\to 0}\frac{x - \sin x}{(\sin^{-1}x)^3}$$

$$= \lim_{y\to 0}\frac{y - \sin y}{y^3} + \lim_{x\to 0}\frac{\sin y - \sin(\sin y)}{y^3}$$

$$= \frac{1}{6} + \lim_{y\to 0}\frac{\frac{\sin y - \sin(\sin y)}{\sin^3 y}}{\frac{y^3}{\sin^3 y}} = \frac{1}{6}+\frac{1}{6} = \frac{1}{3}$$

$$\boxed{\text{So: } (KIN)_{92} = \lim_{x\to 0}\frac{\sin^{-1}x - \sin x}{(\sin^{-1}x)^3} = \frac{1}{3}}$$

$$(KIN)_{93} = \lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x - 2\tan x}{\left(x-\frac{\pi}{4}\right)^2}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x + 1 - 2\tan x}{\left(x-\frac{\pi}{4}\right)^2}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{(1-\tan x)^2}{\left(x-\frac{\pi}{4}\right)^2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\left(\tan\frac{\pi}{4} - \tan x\right)^2}{\left(x - \frac{\pi}{4}\right)^2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\tan^2\left(\frac{\pi}{4} - x\right)\left(1 + \tan x \tan\frac{\pi}{4}\right)^2}{\left(x - \frac{\pi}{4}\right)^2}$$

$$= 1 \times (1+1)^2 = 4$$

So: $(KIN)_{93} = \lim_{x\to\frac{\pi}{4}} \frac{\sec^2 x - 2\tan x}{\left(x - \frac{\pi}{4}\right)^2} = 4$

$$(KIN)_{94} = \lim_{x\to 0} \frac{\sin^2 x - \sin^2 2x + 3x^2}{x^4}$$

$$= \lim_{x\to 0} \frac{3x^2 - \sin x \sin 3x}{x^4}$$

$$= \lim_{x\to 0} \frac{3x^2 - \sin x(3\sin 3 - 4\sin^3 x)}{x^4}$$

$$= \lim_{x\to 0} \frac{3x^2 - 3\sin^2 x + 4\sin^4 x}{x^4}$$

$$= \lim_{x\to 0} \frac{3(x^2 - \sin^2 x)}{x^4} + \lim_{x\to 0} \frac{4\sin^4 x}{x^4}$$

$$= 3\lim_{x\to 0} \frac{(x - \sin x)}{x^3} \times \lim_{x\to 0} \frac{(x + \sin x)}{x^4} + 4$$

$$= 3 \times \frac{1}{6} \times 2 + 4 = 5$$

So: $(KIN)_{94} = \lim_{x\to 0} \frac{\sin^2 x - \sin^2 2x + 3x^2}{x^4} = 5$

$$(KIN)_{95} = \lim_{x\to 0} \frac{\sin 3x + \sin 6x - \sin 9x}{x^3}$$

$$= \lim_{x \to 0} \frac{\sin 3x + 2\sin 3x\cos 3x - 3\sin 3x - 4\sin^3 3x}{x^3}$$

$$= \lim_{x \to 0} \frac{-2\sin 3x[-2\sin^2 3x - \cos 3x + 1]}{x^3}$$

$$= \lim_{x \to 0} \frac{-2\sin 3x[-2(1 - \cos^2 3x - \cos 3x + 1)]}{x^3}$$

$$= \lim_{x \to 0} \frac{-2\sin 3x(2\cos^2 3x) - (\cos 3x - 1)}{x^3}$$

$$= \lim_{x \to 0} \frac{-2\sin 3x(2\cos 3x + 1)(\cos 3x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{-2\sin 3x(2\cos 3x + 1)}{x} \times \frac{\sin^2 \frac{3x}{2}}{x^2}$$

$$= -2 \times 3 \times 3 \times (-2) \times \frac{9}{4} = 81$$

So: $(KIN)_{95} = \lim_{x \to 0} \frac{\sin 3x + \sin 6x - \sin 9x}{x^3} = 81$

$$(KIN)_{96} = \lim_{x \to 0} \frac{\ln(1+x)^2 - 2x}{x^2}$$

let $\ln(1+x) = y \equiv x = e^y - 1$

$$= \lim_{x \to 0} \frac{2y - 2(e^y - 1)}{(e^y - 1)^2} = \lim_{x \to 0} \frac{\frac{-2(e^y - y - 1)}{y^2}}{\left(\frac{e^y - 1}{y}\right)^2} = -\frac{2 \times \frac{1}{2}}{1^2} = -1$$

So: $(KIN)_{96} = \lim_{x \to 0} \frac{\ln(1+x)^2 - 2x}{x^2} = -1$

$$(KIN)_{97} = \lim_{x \to 0} \frac{\frac{x}{\sin x} - 2 + \frac{\sin x}{x}}{x^4}$$

$$= \lim_{x \to 0} \frac{x^2 - 2x\sin x + \sin^2 x}{x^4 x\sin x}$$

$$= \lim_{x\to 0}\left(\frac{\sin x - x}{x^3}\right)^2 \times \frac{x}{\sin x}$$

$$= \left(-\frac{1}{6}\right)^2 \times 1 = \frac{1}{36}$$

$$\text{So: } (\text{KIN})_{97} = \lim_{x\to 0}\frac{\frac{x}{\sin x} - 2 + \frac{\sin x}{x}}{x^4} = \frac{1}{36}$$

$$(\text{KIN})_{98} = \lim_{x\to 1}\left[\sqrt[x-1-\ln x]{\sin\left(\frac{\pi x}{2}\right)}\right]$$

$$= \lim_{x\to 1}\left[1 + \left(\sin\frac{\pi x}{2} - 1\right)\right]^{\frac{1}{x-1-\ln x}} = e^{\lim\limits_{x\to 1}\left(\frac{\sin\frac{\pi x}{2} - 1}{x-1-\ln x}\right)}$$

$$\therefore \lim_{x\to 1}\frac{\sin\frac{\pi x}{2} - \sin\frac{\pi}{2}}{x-1-\ln x} = \lim_{x\to 1}\frac{2\sin\frac{\pi}{4}(x-1)\cos\frac{\pi}{4}(x+1)}{x-1-\ln x}$$

$$= \lim_{x\to 1}\frac{2\sin\frac{\pi}{4}(x-1) - \sin\frac{\pi}{4}(x-1)}{x-1-\ln x}$$

$$\text{let } x = e^y \equiv x \to 1 \equiv y \to 0$$

$$\therefore \lim_{x\to 1}\frac{-2\sin^2\frac{\pi}{4}(e^y - 1)}{e^y - 1 - \ln x}$$

$$= \lim_{x\to 1}\left(-2\frac{\sin^2\frac{\pi}{4}(e^y-1)}{(e^y-1)^2} \times \frac{y^2}{e^y - y - 1} \times \frac{(e^y-1)^2}{y^2}\right)$$

$$= -2 \times \left(\frac{\pi}{4}\right)^2 \times \frac{1}{\frac{1}{2}} \times 1 = -\frac{\pi^2}{4}$$

$$\text{So: } (\text{KIN})_{98} = \lim_{x\to 1}\left[\sqrt[x-1-\ln x]{\sin\left(\frac{\pi x}{2}\right)}\right] = e^{-\frac{\pi^2}{4}}$$

$$(\text{KIN})_{99} = \lim_{x\to\frac{\pi}{6}}\frac{1 - \sqrt{3}\tan x}{\pi - 6x}$$

$$= \lim_{x\to\frac{\pi}{6}} \frac{\sqrt{3}}{6} \times \frac{\tan\frac{\pi}{6} - \tan x}{\left(\frac{\pi}{6} - x\right)}$$

$$= \lim_{x\to\frac{\pi}{6}} \frac{\sqrt{3}}{6} \times \frac{\sin\left(\frac{\pi}{6} - x\right)}{\left(\frac{\pi}{6} - x\right)} \times \frac{1}{\cos\frac{\pi}{6}\cos x} = \frac{\sqrt{3}}{6} \times \frac{1}{\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{9}$$

So: $(KIN)_{99} = \lim_{x\to\frac{\pi}{6}} \frac{1 - \sqrt{3}\tan x}{\pi - 6x} = \frac{2\sqrt{3}}{9}$

$$(KIN)_{100} = \lim_{x\to 0} \frac{e^{10x} - x - 1}{e^{5x} - 2x - 1}$$

$$= \lim_{x\to 0} \frac{\frac{(e^x)^{10} - 1^{10}}{e^x - 1} - \frac{x}{e^x - 1}}{\frac{(e^x)^5 - 1^5}{e^x - 1} - \frac{2x}{e^x - 1}} = \frac{\frac{10}{1} - 1}{\frac{5}{1} - 2} = \frac{9}{3} = 3$$

So: $(KIN)_{100} = \lim_{x\to 0} \frac{e^{10x} - x - 1}{e^{5x} - 2x - 1} = 3$

$$(KIN)_{101} = \lim_{x\to\infty} \left(\sqrt[3]{8^x + 3^x} - \sqrt{4^x - 2^x}\right)$$

$$= \lim_{x\to\infty} \left(\sqrt[3]{8^x + 3^x} - \sqrt[3]{8^x}\right) - \left(\sqrt{4^x - 2^x} - \sqrt{4^x}\right)$$

$$= \lim_{x\to\infty} \left(\sqrt[3]{8^x + 3^x} - \sqrt[3]{8^x}\right) \times \frac{\sqrt[3]{(8^x + 3^x)^2} + \sqrt[3]{8^x(8^x + 3^x)} + \sqrt[3]{(8^x)^2}}{\sqrt[3]{(8^x + 3^x)^2} + \sqrt[3]{8^x(8^x + 3^x)} + \sqrt[3]{(8^x)^2}}$$

$$-\left(\sqrt{4^x - 2^x} - \sqrt{4^x}\right) \times \frac{\left(\sqrt{4^x - 2^x} + \sqrt{4^x}\right)}{\left(\sqrt{4^x - 2^x} + \sqrt{4^x}\right)}$$

$$= \lim_{x\to\infty} \frac{8^x + 3^x - 8^x}{4^x\left(\sqrt[3]{\left(1 + \left(\frac{3}{4}\right)^x\right)^2} + \sqrt[3]{\left(1 + \left(\frac{3}{4}\right)^x\right)} + \sqrt[3]{1}\right)} - \frac{4^x - 2^x - 4^x}{\left(\sqrt{4^x - 2^x} + \sqrt{4^x}\right)}$$

$$= 0 - \lim_{x\to\infty}\frac{4^x - 2^x - 4^x}{2^x\left(\sqrt{1-\left(\frac{1}{2}\right)^x} + \sqrt{1}\right)} = 0 + \frac{1}{2} = \frac{1}{2}$$

$$\text{So: } (\text{KIN})_{101} = \lim_{x\to\infty}\left(\sqrt[3]{8^x + 3^x} - \sqrt{4^x - 2^x}\right) = \frac{1}{2}$$

$$(\text{KIN})_{102} = \lim_{x\to 0}\left[\frac{2}{\sin^2 x} + \frac{1}{\ln(\cos x)}\right]$$

$$= \lim_{x\to 0}\frac{2\ln\cos x + \sin^2 x}{\sin^2 x\ln(\cos x)} = 2\lim_{x\to 0}\frac{\ln(1-\sin^2 x) + \sin^2 x}{\sin^2 x\ln(1-\sin^2 x)}$$

put $1 - \sin^2 x = e^y$

$$= 2\lim_{x\to 0}\frac{y + 1 - e^y}{(1-e^y)y} = 2\times\lim_{x\to 0}\frac{\frac{e^y - y - 1}{y^2}}{-\frac{e^y - 1}{y}} = 2\times\left(\frac{-\frac{1}{2}}{-1}\right) = 1$$

$$\text{So: } (\text{KIN})_{102} = \lim_{x\to 0}\left[\frac{2}{\sin^2 x} + \frac{1}{\ln(\cos x)}\right] = 1$$

$$(\text{KIN})_{103} = \lim_{x\to 0}\frac{x(1-\cos x)}{x^2 + x - e^x\sin x}$$

$$= \lim_{x\to 0}\frac{x\left(2\sin^2\frac{x}{2}\right)}{x^2 + x - x\sin x - \sin x - \sin x(e^x - x - 1)}$$

$$= \lim_{x\to 0}\frac{2x\sin^2\frac{x}{2}}{(x+1)(x-\sin x) - \sin x(e^x - x - 1)}$$

$$= \lim_{x\to 0}\frac{\frac{2x\sin^2\frac{x}{2}}{x^3}}{\frac{(x+1)(x-\sin x)}{x^3} - \frac{\sin x}{x}\times\frac{(e^x - x - 1)}{x^2}} = \frac{2\times\frac{1}{4}}{\frac{1}{6} - \frac{1}{2}} = -\frac{3}{2}$$

$$\text{So: } (\text{KIN})_{103} = \lim_{x\to 0}\frac{x(1-\cos x)}{x^2 + x - e^x\sin x} = -\frac{3}{2}$$

$$(KIN)_{104} = \lim_{x\to 0}\frac{\ln(2+x)-x}{x^2}$$

$$= \lim_{x\to 0}\frac{\ln(1+x)-\ln e^x}{x^2} = \lim_{x\to 0}\frac{\ln\frac{1+x}{e^x}-\ln e^x}{x^2}$$

$$= \lim_{x\to 0}\frac{\ln\left(\frac{1+x}{e^x}\right)}{\left(\frac{1+x}{e^x}\right)-1}\times\frac{\left(\frac{1+x}{e^x}\right)-1}{x^2}$$

$$= 1\times\lim_{x\to 0}\frac{-(e^x-x-1)}{x^2}\times\frac{1}{e^x} = 1\times\left(-\frac{1}{2}\right)\times 1 = -\frac{1}{2}$$

So: $(KIN)_{104} = \lim_{x\to 0}\frac{\ln(2+x)-x}{x^2} = -\frac{1}{2}$

$$(KIN)_{105} = \lim_{x\to\frac{\pi}{3}}\frac{\tan^3x-3\tan x}{\cos\left(x+\frac{\pi}{6}\right)}$$

$$= \lim_{x\to\frac{\pi}{3}}\frac{\tan\left(\frac{\pi}{3}-y\right)\left(\tan\left(\frac{\pi}{3}-y\right)-\sqrt{3}\right)\left(\tan\left(\frac{\pi}{3}-y\right)+\sqrt{3}\right)}{\cos\left(\frac{\pi}{3}-y+\frac{\pi}{6}\right)}$$

$$= \lim_{x\to\frac{\pi}{3}}\frac{\sqrt{3}\left[\left(\tan\left(\frac{\pi}{3}-y\right)\right)-\tan\frac{\pi}{3}\right]\times 2\sqrt{3}}{\sin y}$$

$$= \lim_{x\to\frac{\pi}{3}}\frac{6\left[\left(\tan\frac{\pi}{3}-y-\frac{\pi}{3}\right)\left(1+\tan\frac{\pi}{3}\times\tan\left(\frac{\pi}{3}-y\right)\right)\right]}{\sin y}$$

$$= 6\times\lim_{x\to\frac{\pi}{3}}\frac{\left[\left(\tan(-y)\times 1+\sqrt{3}\times\sqrt{3}\right)\right]}{\sin y} = 24$$

So: $(KIN)_{105} = \lim_{x\to\frac{\pi}{3}}\frac{\tan^3x-3\tan x}{\cos\left(x+\frac{\pi}{6}\right)} = 24$

$$(\mathrm{KIN})_{106}=\lim_{n\to\infty}\sqrt[n]{\frac{3n!}{2n!\,n^n}}$$

$$=\lim_{n\to\infty}\left(\frac{3n!}{2n!\,n^n}\right)^{\frac{1}{n}}=\lim_{n\to\infty}\left(\frac{\sqrt{2\pi 3n}}{\sqrt{2\pi 2n}}\times\frac{\left(\frac{3n}{e}\right)^{3n}}{\left(\frac{2n}{e}\right)^{2n}n^n}\right)^{\frac{1}{n}}$$

$$=\lim_{n\to\infty}\left(\frac{3}{2}\right)^{\frac{1}{n}}\times\frac{\left(\frac{3n}{e}\right)^{3}}{\left(\frac{2n}{e}\right)^{n}n}=\lim_{n\to\infty}1\times\frac{27n^3e^2}{4n^2e^3n}=\frac{27}{4e}$$

So: $(\mathrm{KIN})_{106}=\lim_{n\to\infty}\sqrt[n]{\frac{3n!}{2n!\,n^n}}=\frac{27}{4e}$

$$(\mathrm{KIN})_{107}=\lim_{x\to 1}\frac{\sin\left(\pi-\cos^4\frac{\pi x}{2}\right)}{e^{1-\sin\frac{\pi x}{2}}+e^{\sin\frac{\pi x}{2}-1}-2}$$

$$=\lim_{x\to 1}\frac{\dfrac{\sin^2\frac{\pi}{2}(1-x)\sin^2\frac{\pi}{2}(1-x)}{\left(2\sin\frac{\pi}{4}(1-x)\cos\frac{\pi}{4}(2+x)\right)^2}}{\dfrac{\left(e^{2\sin\frac{\pi}{4}(1-x)\cos\frac{\pi}{4}(1+x)}-1\right)^2}{\left(2\sin\frac{\pi}{4}(1-x)\cos\frac{\pi}{4}(2+x)\right)^2}}=\lim_{x\to 1}\frac{\sin^2\frac{\pi}{2}(1-x)}{\sin^2\frac{\pi}{2}-\frac{\pi}{4}(1+x)}$$

$$=\lim_{x\to 1}\frac{\sin^2\frac{\pi}{2}(1-x)}{\sin^2\frac{\pi}{4}(1-x)}=\frac{\pi^2}{4}\times\frac{16}{\pi^2}=4$$

So: $(\mathrm{KIN})_{107}=\lim_{x\to 1}\frac{\sin\left(\pi-\cos^4\frac{\pi x}{2}\right)}{e^{1-\sin\frac{\pi x}{2}}+e^{\sin\frac{\pi x}{2}-1}-2}=4$

$$(\mathrm{KIN})_{108}=\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(x-\frac{\pi}{3}\right)}{1-2\cos x}$$

$$=\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(x-\frac{\pi}{3}\right)}{2\left(\frac{1}{2}-\cos x\right)}=\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(x-\frac{\pi}{3}\right)}{2\left(cos\frac{\pi}{4}-\cos x\right)}$$

$$=\lim_{x\to\frac{\pi}{3}}\frac{\frac{\sin\left(x-\frac{\pi}{3}\right)}{x-\frac{\pi}{3}}}{2\times(-2)\left(\frac{sin\frac{\frac{\pi}{3}-x}{2}}{x-\frac{\pi}{3}}\times\frac{\sin\left(x+\frac{\pi}{3}\right)}{2}\right)}=\frac{1}{4\times\frac{1}{2}\times\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

So: $(\mathrm{KIN})_{108}=\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(x-\frac{\pi}{3}\right)}{1-2\cos x}=\frac{\sqrt{3}}{3}$

$$(\mathrm{KIN})_{109}=\lim_{n\to\infty}n\tan^{-1}\left[\left\{\frac{1}{(x^2+1)n+1}\right\}\tan\left(\frac{\pi}{4}-\frac{x}{2n}\right)\right]^n$$

$$\text{let } y=\tan^{-1}\left(\frac{1}{(x^2+1)n+1}\right)\equiv(x^2+1)n+1=\cot y$$

$$\therefore n=\frac{\cot y-\cot\frac{\pi}{4}}{x^2+1}=\frac{\tan\left(\frac{\pi}{4}-y\right)(1+\tan y)}{(x^2+1)\tan y}$$

$$\therefore n\to\infty\equiv y\to 0$$

$$\therefore\lim_{y\to 0}\frac{y\tan\left(\frac{\pi}{4}-y\right)(1+\tan y)}{(x^2+1)\tan y}=\frac{1}{(x^2+1)}\quad(1)$$

$$\therefore\lim_{n\to\infty}\left(1+\tan\left(\frac{\pi}{4}+\frac{x}{2n}\right)-\tan\frac{\pi}{4}\right)^n=\lim_{n\to\infty}\left(1+\tan\left(\frac{\pi}{4}+\frac{x}{2n}-\frac{\pi}{4}\right)-\tan\frac{\pi}{4}\right)^n$$

$$=\lim_{n\to\infty}\left(\left(1+\tan\left(\frac{x}{2n}\right)\right)\left(1+\tan\left(\frac{\pi}{4}+\frac{x}{2n}\right)\right)\right)^n$$

$$= e^{\lim\limits_{n\to\infty} n\tan\left(\frac{x}{2n}\right)\tan 1 + \tan\left(\frac{\pi}{4}+\frac{x}{2n}\right)} = e^{\lim\limits_{n\to\infty}\frac{\tan\left(\frac{x}{2n}\right)}{\frac{1}{n}}\left(1+\tan\left(\frac{\pi}{4}+\frac{x}{2n}\right)\right)}$$

$$= e^{\frac{x}{2}(1+1)} = e^{x}\ (2)$$

$$\therefore \lim_{n\to\infty} f(n) = \frac{e^{x}}{x^{2}+1}$$

$$\text{So: } (KIN)_{109} = \lim_{n\to\infty} n\tan^{-1}\left[\left\{\frac{1}{(x^{2}+1)n+1}\right\}\tan\left(\frac{\pi}{4}-\frac{x}{2n}\right)\right]^{n} = \frac{e^{x}}{x^{2}+1}$$

$$(KIN)_{110} = \lim_{x\to\infty}\left(\frac{x^{2}+1}{x^{2}-1}\right)^{(x^{4}-1)\sin^{2}\frac{1}{x}}$$

$$\text{let } y = \frac{1}{x};\ \therefore \lim_{x\to\infty}\left(\frac{1+y^{2}}{1-y^{2}}\right)^{\left(\frac{1-y^{2}}{y^{4}}\right)\sin^{2}y}$$

$$= e^{\lim\limits_{y\to 0}\left(\frac{(1-y^{2})(1+y^{2})\sin^{2}y}{y^{4}}\right)\ln\frac{1+y^{2}}{1-y^{2}}} = e^{\lim\limits_{y\to 0}(1-y^{2})(1+y^{2})\left(\frac{\sin^{2}y}{y^{4}}\right)\left(\ln\frac{1+y^{2}}{y^{2}}\right)+\left(\ln\frac{1-y^{2}}{y^{2}}\right)}$$

$$= e^{1\times 1\times 1\times(1+1)} = e^{2}$$

$$\text{So: } (KIN)_{110} = \lim_{x\to\infty}\left(\frac{x^{2}+1}{x^{2}-1}\right)^{(x^{4}-1)\sin^{2}\frac{1}{x}} = e^{2}$$

$$(KIN)_{111} = \lim_{x\to+\infty}\left(\sin\frac{\pi x+4}{2x+3}\right)^{\frac{x^{2}}{1+2x}}$$

$$\text{let } y = \frac{1}{x} \equiv x\to+\infty\ ,y\to 0$$

$$= \lim_{y\to 0}\left(\sin\frac{4y+\pi}{3y+2}\right)^{\frac{1}{y(y+2)}} = \lim_{y\to 0}\left(1+\sin\frac{4y+\pi}{3y+2}-\sin\frac{\pi}{2}\right)^{\frac{1}{y(y+2)}}$$

$$\therefore \sin\frac{4y+\pi}{3y+2}-\sin\frac{\pi}{2} = \left[2\sin\frac{\frac{y(8-3\pi)}{2(3y+2)}}{2}\cos\frac{\frac{4\pi+y(8-3\pi)}{2(3y+2)}}{2}\right]$$

$$= \lim_{y \to 0}\left(1 + 2\sin\frac{y(8-3\pi)}{4(3y+2)}\cos\frac{4\pi + y(8-3\pi)}{4(3y+2)}\right)^{\frac{1}{y(y+2)}}$$

$$= e^{\lim\limits_{y\to 0} 2\sin\frac{y(8-3\pi)}{4(3y+2)}\cos\frac{4\pi+y(8-3\pi)}{4(3y+2)}\times\frac{1}{y(y+2)}}$$

$$= e^{\lim\limits_{y\to 0} 2\frac{\sin\frac{y(8-3\pi)}{4(3y+2)}}{\frac{y(8-3\pi)}{4(3y+2)}}\cos\frac{4\pi+y(8-3\pi)}{4(3y+2)}\times\frac{y(8-3\pi)}{4(3y+2)}\times\frac{1}{y(y+2)}} = e^{2\times 1\times\cos\frac{4\pi}{8}\times\frac{(8-3\pi)}{4\times 2\times 2}} = 1$$

$$\text{So: } (KIN)_{111} = \lim_{x\to+\infty}\left(\sin\frac{\pi x + 4}{2x+3}\right)^{\frac{x^2}{1+2x}} = 1$$

$$(KIN)_{112} = \lim_{x\to 0}\frac{x - \sin x}{x^3}$$

$$\text{let } x = 2y \text{ and } p = \lim_{x\to 0}\frac{x-\sin x}{x^3}$$

$$p = \lim_{x\to 0}\frac{x-\sin x}{x^3} = \lim_{y\to 0}\frac{2y - \sin 2y}{8y^3}$$

$$p = \lim_{y\to 0}\frac{2y - 2\sin y + 2\sin y - 2\sin y\cos y}{8y^3}$$

$$p = \lim_{y\to 0}\frac{2(y-\sin y)}{8y^3} + \lim_{x\to 0}\frac{2\sin y(1-\cos y)}{8y^3}$$

$$p = p\frac{1}{4} + \lim_{y\to 0}\frac{2\sin y 2\sin^2\frac{x}{2}}{8y^3}$$

$$p\frac{1}{4} = \frac{1}{8} \equiv p = \frac{1}{6}$$

$$\text{So: } (KIN)_{112} = \lim_{x\to 0}\frac{x-\sin x}{x^3} = \frac{1}{6}$$

$$(\mathrm{KIN})_{113}=\lim_{x\to\frac{\pi}{4}}\frac{\cos^2x-\frac{1}{2}}{x-\frac{\pi}{4}}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\cos^2x-\cos^2\frac{\pi}{4}}{x-\frac{\pi}{4}}=\lim_{x\to\frac{\pi}{4}}\frac{\left(\cos x-\cos\frac{\pi}{4}\right)\left(\cos x+\cos\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$$

$$=\sqrt{2}\lim_{x\to\frac{\pi}{4}}\frac{\left(-2\sin\frac{x-\frac{\pi}{4}}{2}\right)}{x-\frac{\pi}{4}}\times\sin\left(\frac{x+\frac{\pi}{4}}{2}\right)=\sqrt{2}\times(-2)\times\frac{1}{2}\times\frac{1}{\sqrt{2}}=-1$$

So: $(\mathrm{KIN})_{113}=\lim_{x\to\frac{\pi}{4}}\frac{\cos^2x-\frac{1}{2}}{x-\frac{\pi}{4}}=-1$

$$(\mathrm{KIN})_{114}=\lim_{x\to0}\frac{\tan^2x+2\ln(\cos x)}{\sin^2x-\ln(1+\sin^2x)}$$

let $\cos x=e^{\frac{1}{2}y}$

$$\therefore\lim_{x\to0}\frac{\frac{1-e^y}{e^y}+y}{(1-e^y)^2}=\lim_{x\to0}\frac{1-e^y+y}{e^y(1-e^y)^2}+\frac{ye^y-y}{e^y(1-e^y)^2}$$

$$=-\frac{1}{2}+\lim_{x\to0}\frac{(e^y-1)y}{e^y(1-e^y)(e^y-1)}=-\frac{1}{2}+1=\frac{1}{2}$$

So: $(\mathrm{KIN})_{114}=\lim_{x\to0}\frac{\tan^2x+2\ln(\cos x)}{\sin^2x-\ln(1+\sin^2x)}=\frac{1}{2}$

$$(\mathrm{KIN})_{115}=\lim_{x\to1}\frac{\ln x}{\tan\pi x}$$

$$=\lim_{x\to1}\frac{\frac{\ln(1+(x-1))}{\pi(1-x)}}{-\frac{\tan\pi(x-1)}{\pi(x-1)}}=\lim_{x\to1}\frac{(x-1)\frac{1-(x-1)}{2+(x-1)^2}\frac{}{3-\cdots}}{-\pi(1-x)}=\frac{1}{\pi}$$

$$\text{So}: (\text{KIN})_{115} = \lim_{x\to 1}\frac{\ln x}{\tan \pi x} = \frac{1}{\pi}$$

$$(\text{KIN})_{116} = \lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)\dots(1+5x)}{\tan 5x}$$

$$= \lim_{x\to 0}\frac{1+(x+2x+3x+4x+5x)\dots - 1}{\tan 5x}$$

$$= \lim_{x\to 0}\frac{15x+\cdots \text{hight degree of x}}{\tan 5x}$$

$$= \lim_{x\to 0}\frac{15x+\cdots f(x)\text{hight degree of x}}{\frac{\tan 5x}{x}} = \frac{15}{5}+0 = 3$$

$$\text{So}: (\text{KIN})_{116} = \lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)\dots(1+5x)}{\tan 5x} = 3$$

$$(\text{KIN})_{117} = \lim_{x\to \frac{\pi}{4}}\frac{1-\sqrt{2}\sin x}{1-\sqrt{2}\cos x}$$

$$= \lim_{x\to \frac{\pi}{4}}\frac{\frac{1}{\sqrt{2}}-\sin x}{\frac{1}{\sqrt{2}}-\cos x} = \lim_{x\to \frac{\pi}{4}}\frac{\sin\frac{\pi}{4}-\sin x}{\cos\frac{\pi}{4}-\cos x}$$

$$= \lim_{x\to \frac{\pi}{4}}\frac{2\sin\left(\frac{\frac{\pi}{4}-x}{2}\right)\cos\left(\frac{\frac{\pi}{4}+x}{2}\right)}{-2\sin\left(\frac{\frac{\pi}{4}-x}{2}\right)\sin\left(\frac{\frac{\pi}{4}+x}{2}\right)} = -1$$

$$\text{So}: (\text{KIN})_{117} = \lim_{x\to \frac{\pi}{4}}\frac{1-\sqrt{2}\sin x}{1-\sqrt{2}\cos x} = -1$$

$$(\text{KIN})_{118} = \lim_{x\to 0}\frac{4(81)^x-27^x-9^x-3^x-1}{1-\sqrt[3]{1+x+x^2}}$$

$$\therefore \lim_{x\to 0}\frac{4(3)^{4x}-3^{3x}-3^{2x}-3^{x}-1}{x}$$

$$=\lim_{x\to 0}\frac{4e^{4x\ln 3}-e^{3x\ln 3}-e^{2x\ln 3}-e^{x\ln 3}-1}{x}$$

$$=\lim_{x\to 0}\frac{\left(e^{x\ln 3}-1\right)\left(4e^{3x\ln 3}+3e^{2x\ln 3}+2e^{x\ln 3}+1\right)}{x}$$

$$-\lim_{x\to 0}\frac{\left(e^{x\ln 3}-1\right)}{x}\times\left(4e^{3x\ln 3}+3e^{2x\ln 3}+2e^{x\ln 3}+1\right)$$

$$=\ln 3(4+3+2+1)=10\ln 3$$

$$\therefore \lim_{x\to 0}\frac{1-\sqrt[3]{1+x+x^2}}{x}$$

$$=\lim_{x\to 0}\frac{1^{\frac{1}{3}}-(1+x+x^2)^{\frac{1}{3}}}{1-(1+x+x^2)}\times[-(x+1)]=-\frac{1}{3}(1+0)==-\frac{1}{3}\equiv\frac{10\ln 3}{-\frac{1}{3}}$$

$$=-30\ln 3$$

$$\boxed{\text{So: }(KIN)_{118}=\lim_{x\to 0}\frac{4(81)^{x}-27^{x}-9^{x}-3^{x}-1}{1-\sqrt[3]{1+x+x^2}}=-30\ln 3}$$

$$(KIN)_{119}=\lim_{x\to\infty}\left[x\left\{\frac{\pi}{4}-\tan^{-1}\left(\frac{x}{x+1}\right)\right\}\right]$$

$$\text{let } y=\tan^{-1}\left(\frac{x}{x+1}\right)\equiv x=\frac{\tan y}{1-\tan y}$$

$$x\to\infty\equiv y\to\frac{\pi}{4}$$

$$=\lim_{y\to\frac{\pi}{4}}\left[\frac{\tan y}{\tan\left(\frac{\pi}{4}-y\right)(1+\tan y)}\times\left(\frac{\pi}{4}-y\right)\right]=\lim_{y\to\frac{\pi}{4}}\left[\frac{\left(\frac{\pi}{4}-y\right)}{\tan\left(\frac{\pi}{4}-y\right)}\times\frac{\tan y}{1+\tan y}\right]$$

$$=1\times\frac{1}{1+1}=\frac{1}{2}$$

$$\text{So: } (\text{KIN})_{119} = \lim_{x\to\infty}\left[x\left\{\frac{\pi}{4} - \tan^{-1}\left(\frac{x}{x+1}\right)\right\}\right] = \frac{1}{2}$$

$$(\text{KIN})_{120} = \lim_{x\to\frac{\pi}{2}} \frac{2\sec^2 x - 3}{2\sec^2 x - \sec x + 1}$$

$$= \lim_{x\to\frac{\pi}{2}}\left(1 + \frac{2\sec^2 x - 3}{2\sec^2 x - \sec x + 1} - 1\right) = \lim_{x\to\frac{\pi}{2}}\left(1 + \frac{\sec x - 4}{2\sec^2 x - \sec x + 1}\right)$$

$$\therefore\ e^{\lim_{x\to\frac{\pi}{2}}\left(1+\frac{\sec x-4}{2\sec^2 x-\sec x+1}\right)\times\frac{\sin^2 x}{\cos x}}$$

$$= e^{\lim_{x\to\frac{\pi}{2}}\left(\frac{\sec x-4}{(2\sec x+1)(\sec x-1)}\right)\times\frac{\sin^2 x}{\cos x}} = e^{\lim_{x\to\frac{\pi}{2}}\left(\frac{\sec x-4}{(2\sec x+1)(\sec x-1)}\right)\times\frac{(1-\cos x)(1+\cos x)}{\cos x}}$$

$$= e^{\lim_{x\to\frac{\pi}{2}}\left(\frac{\sec x-4}{(2+\cos x)(1-\cos x)}\right)\times\frac{\cos x(1-\cos x)(1+\cos x)}{1}} = e^{\lim_{x\to\frac{\pi}{2}}\left(\frac{1-4\cos x}{(2+\cos x)}\right)\times\frac{(1+\cos x)}{1}}$$

$$= e^{\frac{1-4\times 1+0}{2+0}} = e^{\frac{1}{2}}$$

$$\text{So: } (\text{KIN})_{120} = \lim_{x\to\frac{\pi}{2}} \frac{2\sec^2 x - 3}{2\sec^2 x - \sec x + 1} = e^{\frac{1}{2}}$$

$$(\text{KIN})_{121} = \lim_{x\to 0} \frac{x(a-b)}{\sin ax - \sin bx}$$

$$= \lim_{x\to 0} \frac{x(a-b)}{2\cos\frac{x(a+b)}{2} - \sin\frac{x(a-b)}{2}}$$

$$= \lim_{x\to 0} \frac{1}{2\cos\frac{x(a+b)}{2}} \times \frac{\frac{x(a-b)}{2}}{\sin\frac{x(a-b)}{2}} = 1 \times 1 = 1$$

$$\text{So: } (\text{KIN})_{121} = \lim_{x\to 0} \frac{x(a-b)}{\sin ax - \sin bx} = 1$$

$$(\text{KIN})_{122} = \lim_{x\to 0} \frac{x - \sin x}{x - \tan x}$$

$$\therefore \lim_{x\to 0}\frac{x-\sin x}{x^3}\ \text{founded by lopital or Tayler series}$$

let $x = 3y$ and the lim it $= p$

$$p = \lim_{x\to 0}\frac{3y - 3\sin y - 4\sin^2 y}{27y^3}$$

$$p = \lim_{x\to 0}\frac{3y - 3\sin y - 4\sin^2 y}{27y^3} + \lim_{x\to 0}\frac{4\sin^2 y}{27y^3}$$

$$p = \frac{3p}{27} + \frac{4}{27} \Rightarrow p = \frac{1}{6}$$

So: $(\mathrm{KIN})_{122} = \lim\limits_{x\to 0}\frac{x-\sin x}{x-\tan x} = \frac{1}{6}$

$$(\mathrm{KIN})_{123} = \lim_{x\to 1}\frac{3x^2-(7x+1)\sqrt{x}+5}{x-1}$$

$$= \lim_{x\to 1}\frac{3x^2 - 7\sqrt{x^3} - \sqrt{x} + 5}{x-1}$$

$$= \lim_{x\to 1}\frac{3x^2 - 3\sqrt{x^3} - 4\sqrt{x^3} + 4x - 4x + 4\sqrt{x} - 5\sqrt{x} + 5}{x-1}$$

$$= \lim_{x\to 1}\frac{3\sqrt{x^3}(\sqrt{x}-1) - 4x(\sqrt{x}-1) - 4\sqrt{x}(\sqrt{x}-1) - 5(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x^3}-4x-4\sqrt{x}-5)}{(\sqrt{x}-1)(\sqrt{x}+1)} = -5$$

So: $(\mathrm{KIN})_{123} = \lim\limits_{x\to 1}\frac{3x^2-(7x+1)\sqrt{x}+5}{x-1} = -5$

$$(\mathrm{KIN})_{124} = \lim_{x\to 1}\left[\sin\frac{\pi}{2}(x^2-4x+4)\right]^{\frac{x}{(x-1)^2}}$$

$$= \lim_{x\to 1}\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{2}(x^2-4x+4)\right)\right]^{\frac{x}{(x-1)^2}}$$

$$= \lim_{x\to 1}\left[\cos\left(\frac{\pi}{2}(x^2-4x+3)\right)\right]^{\frac{x}{(x-1)^2}}$$

$$= \lim_{x\to 1}\left[1+\left(\cos\frac{\pi}{2}(x-3)(x-1)-1\right)\right]^{\frac{x}{(x-1)^2}}$$

$$= \lim_{x\to 1}\left[1+\left(-2\sin^2\frac{\pi}{4}(x-3)(x-1)\right)\right]^{\frac{x}{(x-1)^2}} = e^{\lim\limits_{x\to 1}\frac{x}{(x-1)^2}\times\left(-2\sin^2\frac{\pi}{4}(x-3)(x-1)\right)}$$

$$= e^{\lim\limits_{x\to 1}\frac{-2\sin^2\frac{\pi}{4}(x-3)(x-1)}{\left(\frac{\pi}{4}\right)^2(x-3)^2(x-1)^2}\times\left(\frac{\pi}{4}\right)^2\times(x-3)^2x} = e^{\left(-\frac{\pi}{2}\right)^2}$$

So: $(KIN)_{124} = \lim\limits_{x\to 1}\left[\sin\frac{\pi}{2}(x^2-4x+4)\right]^{\frac{x}{(x-1)^2}} = e^{\left(-\frac{\pi}{2}\right)^2}$

$$(KIN)_{125} = \lim_{x\to 0}\frac{\sqrt[3]{2x+1}\sqrt[5]{3x+1}-1}{\sqrt[5]{2x+1}\sqrt[3]{3x+1}-1}$$

$$= \lim_{x\to 0}\frac{\sqrt[3]{2x+1}\sqrt[5]{3x+1}-\sqrt[3]{2x+1}+\sqrt[3]{2x+1}-1}{x}$$

$$\div \lim_{x\to 0}\frac{\sqrt[3]{3x+1}\sqrt[5]{2x+1}-\sqrt[3]{3x+1}\sqrt[3]{3x+1}-1}{x} = \frac{L_1}{L_2}$$

$$L_1 = 3\lim_{x\to 0}\frac{\sqrt[5]{3x+1}-1}{(3x+1)-1}+2\lim_{x\to 0}\frac{\sqrt[3]{2x+1}-1}{(2x+1)-1} = \frac{3}{5}+\frac{2}{3} = \frac{19}{15} \quad (1)$$

$$L_2 = 2\lim_{x\to 0}\frac{\sqrt[5]{2x+1}-1}{(2x+1)-1}+3\lim_{x\to 0}\frac{\sqrt[3]{3x+1}-1}{(3x+1)-1} = \frac{3}{5}+1 = \frac{7}{5} \quad (2)$$

$$\Rightarrow \frac{L_1}{L_2} = \frac{\frac{19}{15}}{\frac{7}{5}} = \frac{19}{21}$$

$$\text{So}: (\text{KIN})_{125} = \lim_{x\to 0}\frac{\sqrt[3]{2x+1}\sqrt[5]{3x+1}-1}{\sqrt[5]{2x+1}\sqrt[3]{3x+1}-1} = \frac{19}{21}$$

$$(\text{KIN})_{126} = \lim_{x\to 0}\frac{\tan\left(\frac{\pi^2}{4\pi+x}\right)-1}{x}$$

$$= \lim_{x\to 0}\frac{\tan\left(\frac{\pi^2}{4\pi+x}\right)-\tan\frac{\pi}{4}}{x}$$

$$= \lim_{x\to 0}\frac{\tan\left(\frac{\pi^2}{4\pi+x}-\frac{\pi}{4}\right)\left(1+\tan\frac{\pi}{4}\tan\left(\frac{\pi^2}{4\pi+x}\right)\right)}{x}$$

$$= \lim_{x\to 0}\frac{\tan\left(\frac{-\pi x}{4(4\pi+x)}\right)\left(1+\tan\frac{\pi}{4}\tan\left(\frac{\pi^2}{4\pi+x}\right)\right)}{x}$$

$$= \lim_{x\to 0}\frac{\tan\left(\frac{-\pi x}{4(4\pi+x)}\right)\left(1+\tan\frac{\pi}{4}\tan\left(\frac{\pi^2}{4\pi+x}\right)\right)}{x\left(\frac{-\pi}{4(4\pi+x)}\right)}\times\left(\frac{-\pi}{4(4\pi+x)}\right)$$

$$= \frac{-\pi}{4(4\pi+0)}\times(1+1) = -\frac{1}{8}$$

$$\text{So}: (\text{KIN})_{126} = \lim_{x\to 0}\frac{\tan\left(\frac{\pi^2}{4\pi+x}\right)-1}{x} = -\frac{1}{8}$$

$$(\text{KIN})_{127} = \lim_{x\to 0} e^{\frac{\ln\cos x}{x}}$$

$$= \lim_{x\to 0} e^{\frac{\ln\cos x}{x}} = \lim_{x\to 0}\ln(\cos x)^{\frac{1}{x}}$$

$$= \lim_{x\to 0}\ln\left(1-2\sin^2\frac{x}{2}\right)^{\frac{1}{x}} = \lim_{x\to 0}\ln e^{\frac{-2\sin^2\frac{x}{2}}{x}} = \ln e^0 = \ln 1 = 0$$

$$\lim_{x\to 0} e^{\frac{\ln\cos x}{x}} = e^{\ln 1} = e^0 = 1$$

So: $(\mathrm{KIN})_{127}=\lim_{x\to 0} e^{\frac{\ln\cos x}{x}}=1$

$$(\mathrm{KIN})_{128}=\lim_{\alpha\to 0}\frac{\sin(3x-5\alpha)-\sin 3x}{\alpha}$$

$$=\lim_{\alpha\to 0}\frac{2\sin\frac{(3x-5\alpha-3x)}{2}\cos\frac{(3x-5\alpha+3x)}{2}}{\alpha}$$

$$=\lim_{\alpha\to 0}\frac{2\sin\frac{(-5\alpha)}{2}\cos\frac{(6x)}{2}}{\alpha}=\lim_{\alpha\to 0}\frac{2\sin\frac{(-5\alpha)}{2}\cos(3x)}{\alpha}$$

$$=2\times\left(-\frac{5}{2}\right)\cos 3x=-5\cos 3x$$

So: $(\mathrm{KIN})_{128}=\lim_{\alpha\to 0}\frac{\sin(3x-5\alpha)-\sin 3x}{\alpha}=-5\cos 3x$

$$(\mathrm{KIN})_{129}=\lim_{n\to\infty}\left(\tan\theta+\frac{1}{2}\tan\frac{\theta}{2}+\frac{1}{2^2}\tan\frac{\theta}{2^2}+\cdots+\frac{1}{2^n}\tan\frac{\theta}{2^n}\right)$$

$$=\lim_{n\to\infty}\left((\cot\theta-2\cot\theta)+\left(\frac{1}{2}\cot\frac{\theta}{2}-\cot\theta\right)+\left(\frac{1}{4}\cot\frac{\theta}{4}-\frac{1}{2}\cot\frac{\theta}{2}\right)+\cdots\right.$$
$$\left.+\left(\frac{1}{2^n}\cot\frac{\theta}{2^n}-\frac{1}{2^{n-1}}\cot\frac{\theta}{2^{n-1}}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{2^n}\cot\frac{\theta}{2^n}-2\cot 2\theta\right)=\lim_{n\to\infty}\frac{1}{2^n}\left(\frac{\cos\frac{\theta}{2^n}}{\frac{\theta}{2^n}}\times\frac{\frac{\theta}{2^n}}{\sin\frac{\theta}{2^n}}-2\cot 2\theta\right)$$

$$=\frac{1}{\theta}-2\cot 2\theta$$

So: $(\mathrm{KIN})_{129}=\lim_{n\to\infty}\left(\tan\theta+\frac{1}{2}\tan\frac{\theta}{2}+\frac{1}{2^2}\tan\frac{\theta}{2^2}+\cdots+\frac{1}{2^n}\tan\frac{\theta}{2^n}\right)=\frac{1}{\theta}-2\cot 2\theta$

$$(\mathrm{KIN})_{130}=\lim_{x\to 1}\frac{\sin\pi x}{x^3-1}$$

$$=\lim_{x\to 1}\frac{\sin(\pi-\pi x)}{(x-1)(x^2+x+1)}=\lim_{x\to 1}\frac{\sin\pi(1-x)}{(x-1)(x^2+x+1)}$$

$$= \lim_{x\to 1}\left[\frac{\sin\pi(1-x)}{\pi(1-x)}\times\frac{\pi(1-x)}{(x-1)(x^2+x+1)}\right] = \lim_{x\to 1}\frac{-\pi}{(x^2+x+1)} = -\frac{\pi}{3}$$

$$\text{So: } (KIN)_{130} = \lim_{x\to 1}\frac{\sin\pi x}{x^3-1} = -\frac{\pi}{3}$$

$$(KIN)_{131} = \lim_{x\to -1}(2+\cos\pi x)^{\frac{1}{x^3-3x-2}}$$

$$= \lim_{x\to -1}\left(1+(1+\cos\pi x)\right)^{\frac{1+\cos\pi x}{x^3-3x-2}\times\frac{1}{1+\cos\pi x}} = e^{\lim\limits_{x\to -1}\frac{2\cos^2\frac{\pi x}{2}}{(x+1)^2(x-2)}}$$

$$= e^{\lim\limits_{x\to -1}\frac{2\cos^2\left(\frac{\pi x}{2}+\frac{\pi}{2}\right)}{(x+1)^2(x-2)}} = e^{\lim\limits_{x\to -1}\frac{2\sin^2\frac{\pi}{2}(1+x)}{(x+1)^2(x-2)}} = e^{\lim\limits_{x\to -1}\frac{2\sin^2\frac{\pi}{2}(1+x)}{(x+1)^2}\times\frac{1}{x-2}} = e^{2\times\frac{\pi^2}{4}\times\left(-\frac{1}{3}\right)} = e^{\frac{\pi^2}{6}}$$

$$\text{So: } (KIN)_{131} = \lim_{x\to -1}(2+\cos\pi x)^{\frac{1}{x^3-3x-2}} = e^{\frac{\pi^2}{6}}$$

$$(KIN)_{132} = \lim_{x\to 0}\frac{\sin^2x+2\ln(\cos x)}{x^4}$$

$$= \lim_{x\to 0}\frac{y+\ln(1-y)}{y^2}$$

$$\text{let } 1-y = e^t \Rightarrow L = \lim_{x\to 0}\frac{1-e^t+t}{(1-e^t)^2} = \lim_{x\to 0}\frac{e^t-t+t}{t^2} = -\frac{1}{2}$$

$$\text{So: } (KIN)_{132} = \lim_{x\to 0}\frac{\sin^2x+2\ln(\cos x)}{x^4} = -\frac{1}{2}$$

$$(KIN)_{133} = \lim_{x\to 0}\frac{\left(x-3\tan\frac{\pi}{x+4}+3\right)(e^{\cos x}-e)}{\cos(5\cos^{-1}x)-5x}$$

$$\text{sol : put } y = \cos^{-1}x \Rightarrow \cos y = x$$

$$\therefore \cos(5\cos^{-1}x)-5x = \cos5y-5\cos y = \cos5y-\cos y-4\cos y$$

$$= -2\sin3y\sin2y-4\cos y = -4\cos y[\sin y\sin3y+1]$$

$$= -4\cos y[\sin y(3\sin y-4\sin^2y)+1]$$

$$= 4\cos y[(4\sin^2 y - 3\sin^2 y) - 1]$$

$$= 4\cos y(\sin^2 - 1)(4\sin^2 y + 1) = -4\cos^2 y(5 - 4\cos^2 y)$$

$$= -4x^3(5 - 4x^2)$$

$$\therefore L = -\frac{1}{4}\lim_{x\to 0}\left[\frac{x + 3\left(1 - \tan\frac{\pi}{x+4}\right)}{x} \times \frac{1 - \cos x}{x^2} \times \left(\frac{1 - e^{1-\cos x}}{1 - \cos x} \times \frac{e^{\cos x}}{5 - 4x^2}\right)\right]$$

$$= -\frac{1}{4} \times A \times B \times C \times D \quad (1)$$

$$A = \lim_{x\to 0}\left[1 + \frac{3}{x+4} \times \frac{\tan\frac{\pi}{4}\left(\frac{\pi}{x+4}\right)}{x} \times \frac{1 - \cos x}{x^2} \times \left(\frac{1 - e^{1-\cos x}}{1 - \cos x} \times \frac{e^{\cos x}}{5 - 4x^2}\right)\right]$$

$$= \left(1 + \frac{3}{4} \times \frac{\pi}{4} \times 2\right) = \frac{8 + 3\pi}{8} \quad (2)$$

$$B = \lim_{x\to 0}\frac{1 - \cos x}{x^2} = \lim_{x\to 0}\left(\frac{\sin^2 x}{x^2} \times \frac{1}{1 + \cos x}\right) = \frac{1}{2} \quad (3)$$

$$C = \lim_{x\to 0}\frac{1 - e^{1-\cos x}}{1 - \cos x} = -1 \quad (4) \ ; D = \frac{e^{\cos x}}{5 - 4x^2} = \frac{e}{5} \quad (5)$$

$$\Rightarrow L = -\frac{1}{4}\left(\frac{8+3\pi}{8} \times \frac{1}{2} \times (-1) \times \frac{e}{5}\right) = \frac{(8+3\pi)e}{320}$$

So: $(KIN)_{133} = \lim_{x\to 0}\frac{\left(x - 3\tan\frac{\pi}{x+4} + 3\right)(e^{\cos x} - e)}{\cos(5\cos^{-1}x) - 5x} = \frac{(8 + 3\pi)e}{320}$

$$(KIN)_{134} = \lim_{x\to 0}\frac{e^x - x - 1}{x^2}$$

let the lim it p ; and $x = 2y$

$$p = \lim_{y\to 0}\frac{e^{2y} - 2y - 1}{4y^2}$$

$$p = \lim_{y\to 0}\frac{e^{2y} - 2y - 1 - y^2 + y^2}{4y^2}$$

$$p=\lim_{y\to 0}\frac{(e^y-y-1)}{y^2}\times\frac{2}{4}+\frac{1}{4}$$

$$p=p\frac{2}{4}+\frac{1}{4}\equiv p=\frac{1}{2}$$

$$\text{So: }(\text{KIN})_{134}=\lim_{x\to 0}\frac{e^x-x-1}{x^2}=\frac{1}{2}$$

$$(\text{KIN})_{135}=\lim_{x\to 0}\frac{e^{e^x}-e}{e^{3x}-\sec x}$$

$$=\lim_{x\to 0}\frac{e+xe^{-x}e^{e^x}-x^2\left(e^{-x}e^{e^x}+e^{-2x}e^{e^x}\right)+\cdots-e}{\left(1+3x+\frac{(3x)^2}{2!}+\cdots\right)-\left(1+\frac{x^2}{2}+\frac{5x^4}{34}+\cdots\right)}$$

$$=\lim_{x\to 0}\frac{x\left(e^{-x}e^{e^x}-x\left(e^{-x}e^{e^x}+e^{-2x}e^{e^x}\right)\right)+\cdots}{x\left(3+\frac{9x}{2!}+\cdots-\frac{1}{2}x-\frac{5x^3}{34}\right)-\cdots}=\lim_{x\to 0}\frac{e^{-x}e^{e^x}}{3}=\frac{e}{3}$$

$$\text{So: }(\text{KIN})_{135}=\lim_{x\to 0}\frac{e^{e^x}-e}{e^{3x}-\sec x}=\frac{e}{3}$$

$$(\text{KIN})_{136}=\lim_{x\to\frac{\pi}{2}}\left(\frac{\sec^2x+\tan^2x-2}{2\sec^2x+\tan^2x-\sec x+1}\right)^{\frac{\tan^2x}{\sec^2x}}$$

$$=\lim_{x\to\frac{\pi}{2}}\left(\frac{2\sec^2x-3}{2\sec^2x-\sec x+1}\right)^{\frac{\tan^2x}{\sec^2x}}=\lim_{x\to\frac{\pi}{2}}\left(\frac{2-3\sin^2y}{\sin^2y-\sin y+2}\right)^{\cot^2y\sin y}$$

$$=\lim_{x\to\frac{\pi}{2}}\left(1+\frac{\sin y-4\sin^2y}{\sin^2y-\sin y+2}\right)^{\cot^2y\sin y}$$

$$=e^{\lim_{x\to\frac{\pi}{2}}\frac{\cot^2y\sin y\sin y(1-4\sin y)}{\sin^2y-\sin y+2}}=e^{\frac{1(1-0)}{0-0+2}}=e^{\frac{1}{2}}=\sqrt{e}$$

$$\text{So: }(\text{KIN})_{136}=\lim_{x\to\frac{\pi}{2}}\left(\frac{\sec^2x+\tan^2x-2}{2\sec^2x+\tan^2x-\sec x+1}\right)^{\frac{\tan^2x}{\sec^2x}}=\sqrt{e}$$

$$(\mathbf{KIN})_{137}=\lim_{x\to 0^+}\frac{\left(\sqrt{x}\right)^{\sqrt{x}}-x^x}{\left(\sqrt{x}\right)^x-x^{\sqrt{x}}}$$

$$=\lim_{x\to 0^+}\frac{1+\sqrt{x}\ln\sqrt{x}+\left(\frac{\sqrt{x}\ln\sqrt{x}}{2!}\right)^2+\cdots-1-x\ln x-\left(\frac{x\ln x}{2!}\right)^2-\cdots}{1+\sqrt{x}\ln\sqrt{x}+\left(\frac{x\ln\sqrt{x}}{2!}\right)^2+\cdots-1-\sqrt{x}\ln x-\left(\frac{\sqrt{x}\ln x}{2!}\right)^2-\cdots}$$

$$=\lim_{x\to 0^+}\frac{\sqrt{x}\ln\sqrt{x}+\left(\frac{1}{2}-\sqrt{x}\right)^2}{\sqrt{x}\ln x\left(\frac{1}{2}\sqrt{x}-1\right)}=-\frac{1}{2}$$

$$\text{So: }(\mathbf{KIN})_{137}=\lim_{x\to 0^+}\frac{\left(\sqrt{x}\right)^{\sqrt{x}}-x^x}{\left(\sqrt{x}\right)^x-x^{\sqrt{x}}}=-\frac{1}{2}$$

$$(\mathbf{KIN})_{138}=\lim_{x\to 0}\frac{\cos\left(\frac{x}{2}\right)-\sqrt{\cos x+x\sin x}}{x^2}$$

$$=\lim_{x\to 0}\frac{\cos^2\left(\frac{x}{2}\right)-\cos x-x\sin x}{x^2\left(\cos\left(\frac{x}{2}\right)+\sqrt{\cos x+x\sin x}\right)}$$

$$=\lim_{x\to 0}\frac{\cos^2\left(\frac{x}{2}\right)-2\cos^2\left(\frac{x}{2}\right)+1-x\sin x}{x^2\left(\cos\left(\frac{x}{2}\right)+\sqrt{\cos x+x\sin x}\right)}$$

$$=\lim_{x\to 0}\frac{1-\cos^2\left(\frac{x}{2}\right)-x\sin x}{x^2\left(\cos\left(\frac{x}{2}\right)+\sqrt{\cos x+x\sin x}\right)}=\lim_{x\to 0}\frac{\sin^2\left(\frac{x}{2}\right)-x\sin x}{x^2\left(\cos\left(\frac{x}{2}\right)+\sqrt{\cos x+x\sin x}\right)}$$

$$=\lim_{x\to 0}\frac{\dfrac{\frac{\sin^2\left(\frac{x}{2}\right)}{x^2-x\sin x}}{x^2}}{\dfrac{x^2\left(\cos\left(\frac{x}{2}\right)+\sqrt{\cos x+x\sin x}\right)}{x^2}}=\frac{\frac{1}{4}-1}{1+1}=-\frac{3}{8}$$

$$\text{So: } (\text{KIN})_{138} = \lim_{x\to 0}\frac{\cos\left(\frac{x}{2}\right) - \sqrt{\cos x + x\sin x}}{x^2} = -\frac{3}{8}$$

$$(\text{KIN})_{139} = \lim_{x\to 0}\frac{\sin[\sin(x-\sin x)]}{x-\sin x}\times\frac{x-\sin x}{x^3}$$

$$\therefore \frac{x-\sin x}{x^3}\ \text{let } x = 2y\ ; \frac{x-\sin x}{x^3} = k$$

$$k = \lim_{x\to 0}\frac{2y-\sin 2y}{(2y)^3}$$

$$k = \lim_{y\to 0}\frac{2y - 2\sin y + 2\sin y - 2\sin y\cos y}{8y^3}$$

$$k = \lim_{y\to 0}\frac{y-\sin y}{y^3} + \lim_{y\to 0}\frac{2\sin y(1-\cos y)}{8y^3}$$

$$k = \frac{1}{4}k + \lim_{y\to 0}\frac{2\sin y}{8y}\times\frac{2\sin^2\left(\frac{y}{2}\right)}{y^2}$$

$$\frac{3}{4}k = \frac{2\times 2}{8\times 4} \Rightarrow k = \frac{1}{6}$$

$$\text{So: } (\text{KIN})_{139} = \lim_{x\to 0}\frac{\sin[\sin(x-\sin x)]}{x-\sin x}\times\frac{x-\sin x}{x^3} = \frac{1}{6}$$

$$(\text{KIN})_{140} = \lim_{x\to 0}\frac{a^x-(a+1)^x}{x}$$

$$\therefore \lim_{x\to 0}\frac{e^{x\ln a} - e^{x\ln(a+1)}}{x}$$

$$= \lim_{x\to 0}\frac{1 + x\ln a + \frac{(x\ln a)^2}{2!} + \cdots - 1 + x\ln(a+1)\frac{x\ln(a+1)^2}{2!} + \cdots}{x}$$

$$= \lim_{x\to 0}\frac{x\ln a\frac{1+x\ln a}{2!} + \frac{(x\ln a)^2}{3!} + \cdots - \ln(a+1)\frac{1+x\ln(a+1)}{2!} + \frac{x\ln(a+1)^2}{3!} + \cdots}{x}$$

$$= \lim_{x\to 0}\frac{\ln a(1+0+0+\cdots) - \ln(a+1)(1+0+0+\cdots)}{1} = \frac{\ln a}{\ln(a+1)}$$

$$\text{So}: (\text{KIN})_{140} = \lim_{x\to 0}\frac{a^x-(a+1)^x}{x} = \frac{\ln a}{\ln(a+1)}$$

សូមអធ្យាស្រ័យរាល់កំហុសឆ្គងពីខ្ញុំបាទអ៊ិន សាគីន។ សូមអរគុណ!