សៀវភៅ

ត្រៀមប្រលងសិស្សពូកែ

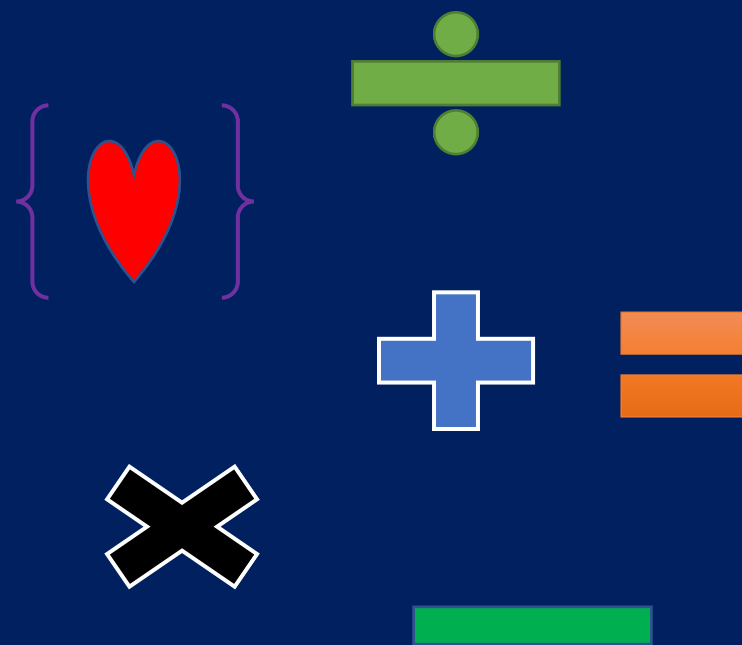
ត្រៀមប្រលងអាហារូបករណ៍

ត្រៀមប្រលងជំនាញផ្សេងៗ

រៀបរៀងដោយ: តឿន សុកាន់យ៉ា និង ផេង ភក្តី

សៀវភៅសំរាប់សិស្សពូកែ

រៀបរៀងដោយៈតឿន សុកាន់យ៉ា និង ផេង ភក្តី



អារម្ភកថា

សៀវភៅ សិស្សពូកែ ដែលអ្នកសិក្សាកំពុងកាន់នៅក្នុងដៃនេះ ខ្ញុំបាទបានរៀបរៀងឡើង ក្នុងគោលបំណងទុកជាឯកសារសម្រាប់ជាជំនួយដល់អ្នកសិក្សា។យកទៅសិក្សា ស្រាវជ្រាវ ដោយខ្លួនឯង និង ម្យ៉ាងទៀតក្នុងគោលបំណង់ចូលរួមលើកស្ទួយវិស័យគណិតវិទ្យានៅ ប្រទេសកម្ពុជាយើងឲកាន់តែរីកចម្រើនថែមទៀតដើម្បីបង្កើនធនធានមនុស្សឲមានកាន់តែ ច្រើនដើម្បីជយអភិវឌ្ឍន៍ប្រទេសជាតិរបស់យើង។

នៅក្នុងសៀវភៅនេះរួមមាន មេរៀនសង្ខេប លំហាត់គំរូ និង លំហាត់អនុវត្តន៍សម្រាប់ អ្នកសិក្សាគិតដោយខ្លួនឯងកំហុសឆ្គងដោយអចេតនាប្រាកដជាមានទាំងបច្ចេកទេស និង អក្ខរាវិរុទ្ធ។អាស្រ័យហេតុនេះ យើងខ្ញុំជាអ្នករៀបរៀងរង់ចាំដោយរីករាយជានិច្ចនូវមតិរិះគន់ បែបស្ថាបនាពីសំណាក់អ្នកសិក្សាក្នុងគ្រប់មជ្ឈដ្ឋានដើម្បីជួយកែលំអសៀវភៅនេះឲបាន កាន់តែសុក្រិត្រភាពថែមទៀត។

ជាទីបញ្ចាប់នេះយើងខ្ញុំអ្នករៀបរៀងសូមគោរពជូនដល់អ្នកសិក្សាទាំងអស់ឲមានសុខភាពល្អ និង ទទួលបានជ័យជំនះគ្រប់ភារកិច្ច។

ធ្វើនៅភ្នំពេញ ថ្ងៃទី ២៣ ខែ ៣ ឆ្នាំ ២០២០

រៀបរៀងដោយៈ តឿន សុកាន់យ៉ា និង ផេង ភក្តី

លំហាត់ និង

ដំណោះស្រាយ

លំហាត់ទី ១ កំណត់លេខខាងចុងនៃ $3^{1001}\cdot 7^{1002}\cdot 13^{1003}$ ។

ដំណោះស្រាយ

របៀបទី ១

+ចំពោះលេខចុងនៃ 3^{1001}

គេបាន $\left[3^2\right]=9$; $\left[3^3\right]=7$; $\left[3^4\right]=1$

$\left[3^8\right]=\left[3^4\cdot 3^4\right]=\left[3^4\right]\left[3^4\right]=1$; $\left[3^{4k}\right]=1$

$\left[3^{1001}\right]=\left[3^{1000}\cdot 3\right]=\left[3^{4\cdot 250}\right][3]=[1][3]=3$

នោះ $\left[3^{1001}\right]=3$ (1)

+ចំពោះលេខចុងនៃ 7^{1001}

គេបាន $\left[9^2\right]=9$; $\left[7^3\right]=3$; $\left[7^4\right]=1$; $\left[7^{4k}\right]=1$

$\left[7^{1002}\right]=\left[7^{1000}\cdot 7^2\right]=\left[7^{4\cdot 250}\right]\left[7^2\right]=[1]\left[7^2\right]=1\cdot 9=9$

នោះ $\left[7^{1002}\right]==9$ (2)

+ចំពោះលេខចុងនៃ 13^{1003}

គេបាន $\left[13^2\right]=9$; $\left[13^3\right]=7$; $\left[13^4\right]=1$; $\left[13^{4k}\right]=1$

$\left[13^{1003}\right]=\left[13^{1000}\cdot 13^3\right]=\left[13^{4\cdot 250}\right]\left[13^3\right]=7$

នោះ $\left[13^{1003}\right]=7$ (3)

តាម (1) & (2) និង (3) យើងបាន: $3^{1001}\cdot 7^{1002}\cdot 13^{1003}=[189]=9$ ។

របៀបទី ២

យើងមាន $3^2 \equiv -1(\text{mod}10)$

$3^{1000} \equiv (-1)^{500}(\text{mod}10)$

$\equiv 1(\text{mod}10)$

$3^{1001} \equiv 3(\text{mod}10)$

ហើយ $7^2 \equiv -1(\text{mod}10)$

$(7^2)^{501} \equiv -1(\text{mod}10)$

$\Rightarrow 7^{1002} \equiv -1(\text{mod}10)$

និង $13^2 \equiv -1(\text{mod}10)$

$(13^2)^{501} \equiv -1(\text{mod}10)$

$\Rightarrow 13^{1004} \equiv -13(\text{mod}10)$

$\equiv 7(\text{mod}10)$

$\Rightarrow 3^{1001} \cdot 7^{1002} \cdot 13^{1003} \equiv 3(-1)7(\text{mod}10) \equiv -21(\text{mod}10)$

$\Rightarrow 3^{1001} \cdot 7^{1002} \cdot 13^{1003} \equiv 9(\text{mod}10)$ ។

លំហាត់ទី២ ចូរស្រាយបញ្ជាក់ថាចំនួន $2^{n+2} + 3^{2n+1}$ ចែកដាច់នឹង 7 គ្រប់ $n \in \mathbb{N}$ ។

ដំណោះស្រាយ

យើងមាន $3^2 = 7 + 2$

គេបាន $3^{2n} = (7+2)^n = 7q + 2^n, q \in \mathbb{N}^*$

គុណអង្គទាំងពីរនឹង 3 គេបានៈ $3^{2n+1} = (7+2)^n = 3 \cdot 7q + 3 \cdot 2^n$

ហើយ $2^{n+2}=2^n\cdot 2^2=2^n\cdot 4 \quad (2)$

តាម $(1)\,\&\,(2)$ គេបាន: $2^{n+2}+3^{2n+1}=3\cdot 7q+3\cdot 2^n+2^n\cdot 4=3.7q+2^n(3+4)$

$$=3\cdot 7q+2^n(3+4)=3\cdot 7q+7\cdot 2^n=7\left(3q+2^n\right)$$

ហេតុដូចនេះ ចំនួន $2^{n+2}+3^{2n+1}$ ចែកដាចនឹង 7 គ្រប់ $n\in\mathbb{N}$ ។

លំហាត់ទី៣ : បង្ហាញថា គ្រប់ចំនួនគត់ធម្មជាតិ n

$$\frac{1}{C(2,2)}+\frac{1}{C(3,2)}+\frac{1}{C(4,2)}+\dots\dots\dots+\frac{1}{C(n,2)}=\frac{n-1}{n}$$ ។

ដំណោះស្រាយ

យើងមាន $\frac{1}{C(2,2)}+\frac{1}{C(3,2)}+\frac{1}{C(4,2)}+\dots\dots\dots+\frac{1}{C(n,2)}$

$$=\frac{1}{\frac{2!}{(2-2)!}}+\frac{1}{\frac{3!}{(3-2)!}}+\frac{1}{\frac{4!}{(4-2)!}}+\dots\dots\dots+\frac{1}{\frac{n!}{(n-2)!}}$$

$$=\frac{1}{2!}+\frac{1}{3!}+\frac{2}{4!}+\dots\dots\dots+\frac{(n-2)!}{n!}$$

$$=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\dots\dots\dots+\frac{1}{n\cdot(n-1)}$$

$$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\dots\dots\dots+\left(\frac{1}{n-1}-\frac{1}{n}\right)=1-\frac{1}{n}$$

$$=\frac{n-1}{n}$$

ដូច្នេះ គ្រប់ចំនួនគត់ធម្មជាតិ n $\frac{1}{C(2,2)}+\frac{1}{C(3,2)}+\frac{1}{C(4,2)}+\dots\dots\dots+\frac{1}{C(n,2)}=\frac{n-1}{n}$ ។

លំហាត់ទី២: គេឲស្វ៊ីត $u_1,u_2,u_3,............,u_n,....$ ផ្ទៀងផ្ទាត់៖

$$u_1=1\ ,u_2=2$$

$$u_n=2u_{n-1}+u_{n-2}$$

បង្ហាញថា $u_n=1+2C_n^2+2^2C_n^4+2^3C_n^6+..............$ ។

ដំណោះស្រាយ

បង្ហាញថា $u_n=1+2C_n^2+2^2C_n^4+2^3C_n^6+..............$

គេមាន $u_1=1\ ,u_2=2\quad u_n=2u_{n-1}+u_{n-2}$

សមីការសម្គាល់ $r^2-2r-1=0$

$$\Delta=2^2-4(-1)=8$$

$$\sqrt{\Delta}=\sqrt{8}=2\sqrt{2}$$

$$r_1=r_2=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$$

ដូច្នេះ $r_1=1+\sqrt{2}\ ,\ r_2=1-\sqrt{2}$

គេបាន $u_n=a\left(1+\sqrt{2}\right)^n+b\left(1-\sqrt{2}\right)^n\quad u_1=1\ ,u_2=2$

$$\Leftrightarrow\begin{cases}a\left(1+\sqrt{2}\right)+b\left(1-\sqrt{2}\right)=1\\a\left(3+2\sqrt{2}\right)+b\left(3-2\sqrt{2}\right)=3\end{cases}$$

គេទាញបាន $a=b=\frac{1}{2}$

នាំឲយើងបាន $u_n=\frac{1}{2}\left(1+\sqrt{2}\right)^n+\frac{1}{2}\left(1-\sqrt{2}\right)^n$

$$=\frac{1}{2}\left[\left(1+\sqrt{2}\right)^n+\left(1-\sqrt{2}\right)^n\right]$$

$$=\frac{1}{2}\left(\left(C_n^0+\sqrt{2}C_n^1+2C_n^2+2\sqrt{2}C_n^3+......\right)+\left(C_n^0-\sqrt{2}C_n^1+2C_n^2-2\sqrt{2}C_n^3+.........\right)\right)$$

$$=\frac{1}{2}\left(2\left(1+2C_n^2+2^2C_n^4+2^3C_n^6+..........\right)\right)$$

$$u_n=1+2C_n^2+2^2C_n^4+2^3C_n^6+...............$$

ដូច្នេះ $u_n=1+2C_n^2+2^2C_n^4+2^3C_n^6+...............$ ។

លំហាត់ទី៥ : បង្ហាញថា $4^{1999}+7^{1999}-2$ ចែកដាច់នឹង 9 ។

ដំណោះស្រាយ

យើងមាន $4^{1999}+7^{1999}-2$ ចែកដាច់នឹង 9 ។

ដោយ: $1999=(3\times666)+1$

$$4^{1999}=4^{3\times666+1}=\left(4^3\right)^{666}\times4=(64)^{666}\times4=(6+4)^{666}\times4\equiv4(\mathrm{mod}\,9)$$

ស្រដៀងគ្នានេះដែរ

$$7^{1999}=\left(7^3\right)^{666}\times7=(343)^{666}\times7\equiv7(\mathrm{mod}\,9)$$

គេបាន $4^{1999}+7^{1999}-2\equiv4+7-2(\mathrm{mod}\,9)\equiv0(\mathrm{mod}\,9)$

ដូច្នេះ $4^{1999}+7^{1999}-2$ ចែកដាច់នឹង 9 ។

លំហាត់ទី៦:បើ $f(x)=\frac{2}{1-x^2}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\frac{8}{1+x^8}+\frac{16}{1+x^{16}}-\frac{32}{1-x^{32}}$ គណនា $f(x)$ កាលណា $x\in\{4,3,\}$ ។

ដំណោះស្រាយ

យើងមាន $f(x)=\frac{2}{1-x^2}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\frac{8}{1+x^8}+\frac{16}{1+x^{16}}-\frac{32}{1-x^{32}}$

$=\frac{4}{1-x^4}+\frac{4}{1+x^4}+\frac{8}{1+x^8}+\frac{16}{1+x^{16}}-\frac{32}{1-x^{32}}$

$=\frac{8}{1-x^4}++\frac{8}{1+x^8}+\frac{16}{1+x^{16}}-\frac{32}{1-x^{32}}$

$=\frac{16}{1-x^{16}}+\frac{16}{1+x^{16}}-\frac{32}{1-x^{32}}$

$=\frac{32}{1-x^{32}}-\frac{32}{1-x^{32}}=0$

+រក $f(4), f(3)$

យើងមាន $f(x)=0 \Rightarrow f(4)=0$ & $f(3)=0$

ដូច្នេះ $f(4)=0$ & $f(3)=0$ ។

លំហាត់ទី៣: រកតម្លៃនៃ $\sqrt{3\sqrt{3\sqrt{3\sqrt{3.........}}}}-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+.......}}}}$ ។

ដំណោះស្រាយ

យើងមាន $\sqrt{3\sqrt{3\sqrt{3\sqrt{3.........}}}}-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+.......}}}}$

តាង $x=\sqrt{3\sqrt{3\sqrt{3\sqrt{3.........}}}}, y=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+.......}}}}$

ដោយ $x=\sqrt{3}x$ លុះត្រា $x^2-3x=0$ នោះ $x=3$ ព្រោះ $x>0$

ដោយ $y=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+.......}}}}$

គេបាន $y=\sqrt{2+y}$

$\Leftrightarrow y^2=2+y$ នឹង $y>0$

$\Rightarrow y = 2$

គេបាន $\sqrt{3\sqrt{3\sqrt{3\sqrt{3.........}}}} - \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+.......}}}} = 3-2 = 1$

ដូច្នេះ $\sqrt{3\sqrt{3\sqrt{3\sqrt{3.........}}}} - \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+.......}}}} = 1$ ។

លំហាត់ទី៨:គណនាតម្លៃ

$$\frac{20092008^2}{20092007^2 + 20092009^2 - 2}$$

ដំណោះស្រាយ

យើងមាន $\frac{20092008^2}{20092007^2 + 20092009^2 - 2}$

គេបាន $\frac{20092008^2}{20092007^2 + 20092009^2 - 2}$

$$= \frac{20092008^2}{20092007^2 - 1 + 20092009^2 - 1}$$

$$= \frac{20092008^2}{(20092007-1)(2009200+1)+(20092009-1)(20092009+1)}$$

$$= \frac{20092008^2}{(20092006)(20092008)+(20092008)(20092010)}$$

$$= \frac{20092008^2}{20092008\left[(20092006)+(20092010)\right]} = \frac{20092008^2}{2\left(20092008^2\right)}$$

$$= \frac{1}{2}$$

ដូច្នេះ $\frac{20092008^2}{20092007^2 + 20092009^2 - 2} = \frac{1}{2}$ ។

លំហាត់ទី៩:គ្រប់ចំនួនគត់វិជ្ជមាន n គេ $f(n)=\dfrac{1}{\sqrt[3]{n^2-2n+1}+\sqrt[3]{n^2-1}+\sqrt[3]{n^2+2n+1}}$

កំណត់តម្លៃនៃ $f(1)+f(2)+f(3)+f(4)+........+f(999997)+f(999999)$

ដំណោះស្រាយ

យើងដឹងថា $n^2-2n+1=(n-1)^2$ និង $n^2-1=(n-1)(n+1)$

ហើយ $n^2+2n+1=(n+1)^2$

យើងស្មានថា $x=\sqrt[3]{n+1}$, $y=\sqrt[3]{n-1}$

គេបាន $f(x)=\dfrac{1}{x^2+xy+y^2}$

$$f(n)=\frac{x-y}{x^3-y^3}$$

$$=\frac{1}{2}(x-y)$$

$$=\frac{1}{2}\left(\sqrt[3]{n+1}-\sqrt[3]{n-1}\right)$$

គេបាន $f(1)+f(2)+f(3)+f(4)+........+f(999997)+f(999999)$

$$=\frac{1}{2}\left[\left(\sqrt[3]{2}-\sqrt[3]{0}\right)+\left(\sqrt[3]{4}-\sqrt[3]{2}\right)+\left(\sqrt[3]{6}-\sqrt[3]{4}\right)+........+\left(\sqrt[3]{999998}-\sqrt[3]{999996}\right)+\left(\sqrt[3]{100000}-\sqrt[3]{999998}\right)\right]$$

$$=\frac{1}{2}\left[\sqrt[3]{1000000}-\sqrt[3]{0}\right]=50$$

ដូច្នេះ $f(1)+f(2)+f(3)+f(4)+........+f(999997)+f(999999)=50$ ។

លំហាត់ទី១០:បើ a,b,c,x,y,z គ្រប់ចំនួនគត់ទាំងអស់បង្ហាញថា៖

$(ab+xy)(ax+by)\geq 4abxy$ ។

ដំណោះស្រាយ

តាមវិសមភាព $AM-GM$ យើងមាន

$$\frac{ab+xy}{2}\geq\sqrt{abxy}\Rightarrow(ab+xy)\geq2\sqrt{abxy}\quad(1)$$

និង $\frac{ax+by}{2}\geq\sqrt{abxy}\Rightarrow(ax+by)\geq2\sqrt{abxy}\quad(2)$

តាម$(1)\&(2)$ គេបាន $(ab+xy)(ax+by)\geq4\sqrt{abxy}\sqrt{abxy}\backslash$

$$\Rightarrow(ab+xy)(ax+by)\geq4abxy$$

ហេតុដូចនេះ $(ab+xy)(ax+by)\geq4abxy$ ។

លំហាត់ទី១១:រកតម្លៃនៃកន្សោមដូចខាងក្រោម៖

$$A=6+\log_{\frac{3}{2}}\left[\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots\ldots\ldots}}}\right]$$ ។

ដំណោះស្រាយ

យើងមាន $A=6+\log_{\frac{3}{2}}\left[\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots\ldots\ldots}}}\right]$

តាង $B=\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots\ldots\ldots}}}\Rightarrow B=\sqrt{4-\frac{1}{3\sqrt{2}}B}\Rightarrow B^2=4-\frac{1}{3\sqrt{2}}B$

$$\Rightarrow B=\frac{-\frac{1}{3\sqrt{2}}\pm\sqrt{\frac{1}{18}+16}}{2}$$

$$\Rightarrow B=\frac{-\frac{1}{3\sqrt{2}}\pm\frac{17}{3\sqrt{2}}}{2} \quad ,B>0$$

$$\Rightarrow B=\frac{1}{2}\left(\frac{-1}{3\sqrt{2}}+\frac{17}{3\sqrt{2}}\right)=\frac{16}{3\cdot 2\sqrt{2}}=\frac{8}{3\sqrt{2}}$$

គេបាន $A=6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\times\frac{8}{3\sqrt{2}}\right)$

$$\Rightarrow A=6+\log_{\frac{3}{2}}\left(\frac{4}{9}\right)=6+\log_{\frac{3}{2}}\left(\frac{3}{2}\right)^{-2}=6-2=4$$

ដូចនេះ $A=4$ ។

លំហាត់ទី១២: ក.បង្ហាញថា $\forall k\in\mathbb{N}^*$ គេបាន $kk!=(k+1)!-k!$

ខ.ទាញបង្ហាញថា $1\cdot 1!+2\cdot 2!+3\cdot 3!+........+n\cdot n!<(n+1)!$

ដំណោះស្រាយ

ក.យើងមាន $\forall k\in\mathbb{N}^*$ គេបាន $kk!=(k+1)!-k!$

$$\Leftrightarrow (k+1)!=(k+1)k!$$

$$(k+1)!=kk!+k!$$

$$kk!=(k+1)!-k!$$

ដូចច្នេះ $\forall k\in\mathbb{N}^*$ គេបាន $kk!=(k+1)!-k!$ ។

ខ.ទាញបង្ហាញថា $1\cdot 1!+2\cdot 2!+3\cdot 3!+........+n\cdot n!<(n+1)!$

យើងមាន $\forall k\in\mathbb{N}^*$ គេបាន $kk!=(k+1)!-k!$

ចំពោះ $k=1,2,3,4,........$

គេបាន $1\cdot 1!+2\cdot 2!+3\cdot 3!+........+n\cdot n!=2!-1!+3!-2!+.......+(n+1)!-n!$

$$1\cdot 1!+2\cdot 2!+3\cdot 3!+........+n\cdot n!=(n+1)!-1!$$

ដោយ $(n+1)!-1!<(n+1)!$

យើងបាន $1\cdot 1!+2\cdot 2!+3\cdot 3!+........+n\cdot n!<(n+1)!$

ដូចនេះ គេទាញបាន $1\cdot 1!+2\cdot 2!+3\cdot 3!+........+n\cdot n!<(n+1)!$ ។

លំហាត់ទី១៣: បើ $\cos x=\dfrac{2\cos y-1}{2-\cos y}, x\in(0,\pi)$ នោះបង្ហាញថា $\tan\dfrac{x}{2}\cot\dfrac{y}{2}=\sqrt{3}$ ។

ដំណោះស្រាយ

ដោយ $\cos x=\dfrac{2\cos y-1}{2-\cos y}$

$$\Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\frac{2\left[\dfrac{1-\tan^2\frac{y}{2}}{1+\tan^2\frac{y}{2}}\right]-1}{2-\left[\dfrac{1-\tan^2\frac{y}{2}}{1+\tan^2\frac{y}{2}}\right]}$$

$$\Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\frac{2\left(1-\tan^2\frac{y}{2}\right)-\left(1+\tan^2\frac{y}{2}\right)}{2\left(1+\tan^2\frac{y}{2}\right)-\left(1-\tan^2\frac{y}{2}\right)}$$

$$\Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\frac{1-3\tan^2\frac{y}{2}}{1+3\tan^2\frac{y}{2}}$$

$$\Rightarrow 1+3\tan^2\frac{y}{2}-\tan^2\frac{x}{2}-3\tan^2\frac{x}{2}\tan^2\frac{y}{2}=1-3\tan^2\frac{y}{2}+\tan^2\frac{x}{2}-3\tan^2\frac{x}{2}\tan^2\frac{y}{2}$$

$$\Rightarrow 6\tan^2\frac{y}{2}=2\tan^2\frac{x}{2}$$

$$\Rightarrow \tan^2\frac{x}{2}\cdot\frac{1}{\tan^2\frac{y}{2}}=3$$

$$\Rightarrow \tan\frac{x}{2}\cot\frac{y}{2}=\sqrt{3}$$

ដូចនេះ $\tan\frac{x}{2}\cot\frac{y}{2}=\sqrt{3}$ ។

លំហាត់ទី១៤: គណនាតម្លៃ $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$ ។

ដំណោះស្រាយ

យើងមាន $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 4x)}}}}$$ ព្រោះតែ $1+\cos 2x=2\cos^2 x$

$$=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4\cos^2 2x}}}}$$

$$=\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 2x)}}}$$

$$=\sqrt{2+\sqrt{2+\sqrt{4\cos^2 x}}}$$

$$=\sqrt{2+\sqrt{2+2\cos x}}$$

$$=\sqrt{2+\sqrt{4\cos^2\frac{x}{2}}}=\sqrt{2+2\cos\frac{x}{2}}=\sqrt{2\left(1+\cos\frac{x}{2}\right)}=2\cos\frac{x}{4}$$

ដូច្នេះ $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}=2\cos\frac{x}{4}$ ។

លំហាត់ទី១៥ :បើ $x=9+4\sqrt{5}$ និង $xy=1$ នោះ $\frac{1}{x^2}+\frac{1}{y^2}$ ស្មើប៉ុន្មាន? ។

ដំណោះស្រាយ

បើ $x=9+4\sqrt{5}$ និង $xy=1$

$$y=\frac{1}{x}=\frac{1}{9+4\sqrt{5}}\times\frac{9-4\sqrt{5}}{9-4\sqrt{5}} \qquad \Rightarrow y=\frac{9-4\sqrt{5}}{81-80}=4\sqrt{5}$$

គេបាន $\frac{1}{x^2}+\frac{1}{y^2}=x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=(x+y)^2-2 \ =18^2-2=322$

ហេតុដូចនេះ $\frac{1}{x^2}+\frac{1}{y^2}$ ស្មើ 322 ។

លំហាត់ទី១៥:បើ $a+b+c=0$ នោះរកតម្លៃ $\frac{a^2+b^2+c^2}{a^2b^2+c^2b^2+a^2c^2}$ ។

ដំណោះស្រាយ

យើងមាន

$$a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2=0$$

គេបាន $a^2+b^2+c^2=-(2ab+2bc+2ca)=-2(ab+bc+ca)$

នោះ $$\frac{a^2+b^2+c^2}{a^2b^2+b^2c^2+c^2a^2}=\frac{[-2(ab+bc+ca)]^2}{\left[a^2b^2+b^2c^2+c^2a^2\right]}$$

$$=\frac{\left[4\left((ab+bc+ca)^2\right]\right.}{\left[a^2b^2+b^2c^2+c^2a^2\right]}$$

$$=\frac{\left[4\left(a^2b^2+b^2c^2+c^2a^2+2abc(a+b+c)\right)\right]}{\left[a^2b^2+b^2c^2+c^2a^2\right]}$$

$$=\frac{\left[4\left(a^2b^2+b^2c^2+c^2a^2\right)\right]}{\left[a^2b^2+b^2c^2+c^2a^2\right]}$$ ព្រោះ $a+b+c=0$

$= 4$

ដូចនេះ $\frac{a^2+b^2+c^2}{a^2b^2+c^2b^2+a^2c^2} = 4$ ។

លំហាត់ទី១៦:ក. បង្ហាញថាគ្រប់ចំនួនពិតមិនអវិជ្ជមាន a, b, c

គេបាន $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \geq \frac{3}{2}$ ។

ដំណោះស្រាយ

ក.ពិនិត្យកន្សោមខាងក្រោម

$$S = \frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$$
$$M = \frac{b}{b+c}+\frac{c}{c+a}+\frac{a}{a+b}$$
$$N = \frac{c}{b+c}+\frac{a}{c+a}+\frac{b}{a+b}$$

យើងមាន $M+N=3$ ។តាមវិសមភាព $AM-GM$ យើងបាន

$$M+S = \frac{a+b}{b+c}+\frac{b+c}{c+a}+\frac{c+a}{a+b} \geq 3$$
$$N+S = \frac{a+c}{b+c}+\frac{a+b}{c+a}+\frac{b+c}{a+b} \geq 3$$

នាំឲ $M+N+2S \geq \sim 3$ និង $2S \geq 3$ រឺ $S \geq \frac{3}{2}$ ។

ដូចនេះ គេបាន $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \geq \frac{3}{2}$ ។

លំហាត់ទី១៦: បង្ហាញថា $\sqrt[3]{ax^2+by^2+cz^2}=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$

បើ $ax^3=by^3=cz^3$ និង $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$ ។

ដំណោះស្រាយ

យើងមាន $ax^3=by^3=cz^3$ និង $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

គេបាន:

$$\sqrt[3]{ax^2+by^2+cz^2}=\sqrt[3]{\frac{ax^3}{x}+\frac{by^3}{y}+\frac{cz^2}{z}}=\sqrt[3]{ax^3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}$$

$=x\sqrt[3]{a}\quad(1)$

$=y\sqrt[3]{b}\quad(2)$

$=z\sqrt[3]{c}\quad(3)$

តាមសមីការ (1), (2) , (3) គេបាន:

គេបាន $$\frac{\sqrt[3]{ax^2+by^2+cz^2}}{x}+\frac{\sqrt[3]{ax^2+by^2+cz^2}}{y}+\frac{\sqrt[3]{ax^2+by^2+cz^2}}{z}=\sqrt[3]{a}+\sqrt[2]{b}+\sqrt[3]{c}$$

$$\sqrt[3]{ax^2+by^2+cz^2}\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$$

នាំឲ $\sqrt[3]{ax^2+by^2+cz^2}=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$

ដូចច្នេះ $\sqrt[3]{ax^2+by^2+cz^2}=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$ ។

លំហាត់ទី១៧:រកតម្លៃ maximun និង minimum នៃអនុគមន៍ $f(x)=x\left(1993-\sqrt{1995-x^2}\right)$ លើដែនកំណត់របស់វា។

ដំណោះស្រាយ

+ដែនកំណត់វា $D=\left[-\sqrt{1995},\sqrt{1995}\right]$

ដឹងថា $f(x)$ ជាអនុគមន៍សេស និង $f(x)\geq 0$, $\forall x\in\left[0,\sqrt{1995}\right]$

នោះ $\max_{x\in D} f(x)=\max_{x\in[0,\sqrt{1995}]} f(x);\min_{x\in D}=-\text{mix}_{x\in[0,\sqrt{1995}]}$

, $\forall x\in\left[0,\sqrt{1995}\right]$ យើងមាន $f(x)=x\left(1993-\sqrt{1995-x^2}\right)$

$=x\left(\sqrt{1993}\cdot\sqrt{1993}+1-\sqrt{1995-x^2}\right)$

$\leq x\left(\sqrt{1993+1}-\sqrt{1993+\left(1995-x^2\right)}\right)$

តាមវិសមភាព $Cauchy-Schwarz$

$$=x\sqrt{1994}\cdot\sqrt{1993+1995-x^2}\leq\sqrt{1994}\cdot\frac{x^2+1993+1995-x^2}{2}=\sqrt{1994}\cdot 1994$$

បើសមភាពកើតឡើង និង លក្ខខ័ណ្ឌ:

$$\begin{cases}1=\sqrt{1995-x^2}\\ x=\sqrt{1993+1995-x^2}\end{cases}\Leftrightarrow x=\sqrt{1994}\in[0,\sqrt{1995}]$$

ដូចនេះ $\max_{x\in D} f(x)=\max_{x\in[0,\sqrt{1995}]} f(x)=1994\sqrt{1994}$ បាន $x=\sqrt{1994}$

$\min_{x\in D} f(x)=-\max_{x\in[0,\sqrt{1995}]} f(x)=-1994\sqrt{1994}$ បាន $x=-\sqrt{1994}$ ។

លំហាត់ទី១៨: ដោះស្រាយសមីការ $e^{x+\pi}=\pi^{x+e}$ ។

ដំណោះស្រាយ

គេមាន $e^{x+\pi}=\pi^{x+e}$

$\Leftrightarrow \ln e^{x+\pi}=\ln\pi^{x+e}$

$$(x+\pi)\ln e=(x+e)\ln\pi$$
$$x+\pi=x\ln\pi+e\ln\pi$$
$$x-x\ln\pi=e\ln\pi-\pi$$
$$\Rightarrow x=\frac{e\ln\pi-\pi}{1-\ln\pi}$$

ដូចនេះ រឹសសមីការគឺៈ $x=\dfrac{e\ln\pi-\pi}{1-\ln\pi}$ ។

លំហាត់ទី១៩:

គណនា $A=\left[\sqrt[\sqrt[\sqrt{3}]{\sqrt{3}}]{(3\sqrt{3})^{\sqrt[\sqrt{3}]{\sqrt{3}^{\sqrt{3}+1}}}}\right]^{\sqrt{3}}$ ។

ដំណោះស្រាយ

គេមាន $A=\left[\sqrt[\sqrt[\sqrt{3}]{\sqrt{3}}]{(3\sqrt{3})^{\sqrt[\sqrt{3}]{\sqrt{3}^{\sqrt{3}+1}}}}\right]^{\sqrt{3}}$

គេបាន $A=\left(\sqrt[\sqrt[\sqrt{3}]{\sqrt{3}}]{\sqrt{3}^{3^{\sqrt[\sqrt{3}]{\sqrt{3}^{\sqrt{3}+1}}}}}\right)^{\sqrt{3}}$

$$=\left(\sqrt{3}^{3\frac{\sqrt[\sqrt{3}]{\sqrt{3}^{\sqrt{3}+1}}}{\sqrt[\sqrt{3}]{\sqrt{3}}}}\right)^{\sqrt{3}}$$

$$=\left(\sqrt{3}^{3^{\sqrt[\sqrt{3}]{\sqrt{3}^{\sqrt{3}+1-1}}}}\right)^{\sqrt{3}}$$

$$=\left(\sqrt{3}^{3^{\sqrt[\sqrt{3}]{\sqrt{3}^{\sqrt{3}}}}}\right)^{\sqrt{3}}$$

$$= \left(\sqrt{3}^{3\sqrt{3}}\right)^{\sqrt{3}}$$

$$= \sqrt{3}^{3\cdot 3} = 81\sqrt{3}$$

ដូច្នេះ $A = 81\sqrt{3}$ ។

លំហាត់ទី២០: រកតម្លៃ $E = 3 + 7x^{-x} + 11x^{-2x} + 15x^{-3x} + ...$ ។

បើ $(x+5)^x = 23x + 1 + \sqrt[x]{x\sqrt[x]{x\sqrt[x]{x\sqrt[x]{x\sqrt[x]{x}}}}} + ...$ ។

ដំណោះស្រាយ

គេមាន $E = 3 + 7x^{-x} + 11x^{-2x} + 15x^{-3x} + ...$ (1)

$(x+5)^x = 23x + 1 + \sqrt[x]{x\sqrt[x]{x\sqrt[x]{x\sqrt[x]{x\sqrt[x]{x}}}}} + ...$ (2)

តាង $B = \sqrt[x]{x\sqrt[x]{x\sqrt[x]{x\sqrt[x]{x\sqrt[x]{x}}}}}$

គេបាន $B^x = xB$ នាំឲ $B = \sqrt[x-1]{x}$

យក B ជំនួសក្នុងសមីការ (2) គេបាន:

$$(x+5)^x = 23x + 1 + \sqrt[x-1]{x}$$

នាំឲ $x = 2$

គេបាន $x^{-x}E = 3x^{-x} + 7x^{-2x} + 11x^{-3x} + 15x^{-4x} + ...(3)$

យក $(1)-(3)$ យើងបាន $\left(1 - x^{-x}\right)E = 3 + 4x^{-x}\left(1 + x^{-x} + x^{-2x} + ...\right)$ (4)

តាង $A = 1 + x^{-x} + x^{-2x} + ...$ នាំឲ $A = \dfrac{1}{1 - x^{-x}} = \dfrac{4}{3}$

យក A ជំនួសក្នុងសមីការ(4) គេបាន : $\frac{4}{3}E=3+4\left(2^{-2}\right)\left(\frac{4}{3}\right)\Rightarrow E=\frac{52}{9}$

ដូច្នេះ តម្លៃ $E=\frac{52}{9}$ ។

លំហាត់ទី២១: បង្ហាញថា

$$\tan^{-1}\left(\frac{x}{1+1.2x^2}\right)+\tan^{-1}\left(\frac{x}{1+2.3x^2}\right)+\cdots+\tan^{-1}\left(\frac{x}{1+n(n+1)x^2}\right)==\tan^{-1}(n+1)x-\tan^{-1}x$$

$\forall n\in\mathbb{N}$

ដំណោះស្រាយ

តាមវិចារដោយកំណើន:

-ចំពោះ $n=1$ គេបាន: $\tan^{-1}\left(\frac{x}{1+1.2x^2}\right)=\tan^{-1}2x-\tan^{-1}x$ ពិត

-ឧបមាថាពិតដល់ $n=k$ គេបាន:

$$\tan^{-1}\left(\frac{x}{1+k.(k+1)x^2}\right)=\tan^{-1}(k+1)x-\tan^{-1}x \quad (*)$$

-បង្ហាញថាវាពិតដល់ $n=k+1$ គឺបង្ហាថា:

$$\tan^{-1}\left(\frac{x}{1+1.2x^2}\right)+\tan^{-1}\left(\frac{x}{1+2.3x^2}\right)+\cdots+\tan^{-1}\left(\frac{x}{1+(k+1)(k+2)x^2}\right)=\tan^{-1}(k+2)x-\tan^{-1}x?$$

ថែមអង្គទាំងពីរនៃ $(*)$ នឹង $\tan^{-1}\left(\frac{x}{1+(k+1)(k+2)x^2}\right)$ យើងបាន:

$$\tan^{-1}\left(\frac{x}{1+1.2x^2}\right)+\tan^{-1}\left(\frac{x}{1+2.3x^2}\right)+\cdots+\tan^{-1}\left(\frac{x}{1+(k+1)(k+2)x^2}\right)$$

$$=\tan^{-1}(k+1)x-\tan^{-1}x+\tan^{-1}\left(\frac{x}{1+(k+1)(k+2)x^2}\right)$$

$$= \tan^{-1}(k+1)x + \tan^{-1}\left(\frac{x}{1+(k+1)(k+2)x^2}\right) - \tan^{-1}x$$

$$= \tan^{-1}\left(\frac{(k+1)x+(k+1)^2(k+2)x^3+x}{1+(k+1)^2(k+2)x^2}\right) - \tan^{-1}x$$

$$= \tan^{-1}\left(\frac{(k+2)x+(k+1)^2(k+2)x^2}{1+(k+1)(k+2)x^2}\right) - \tan^{-1}x$$

$$= \tan^{-1}\left(\frac{(k+2)x+(k+1)^2(k+2)x^2}{1+(k+1)^2(k+2)x^2}\right) - \tan^{-1}x$$

$$= \tan^{-1}\left(\frac{(k+2)x\left[1+(k+1)^2x^2\right]}{1+(k+1)^2(k+2)x^2}\right) - \tan^{-1}x$$

$\tan^{-1}(k+2)x - \tan^{-1}x$ ពិត

ដូច្នេះ $\tan^{-1}\left(\frac{x}{1+1.2x^2}\right)+\tan^{-1}\left(\frac{x}{1+2.3x^2}\right)+\cdots+\tan^{-1}\left(\frac{x}{1+n(n+1)x^2}\right) == \tan^{-1}(n+1)x - \tan^{-1}x$

$\forall n \in \mathbb{N}$ ។

លំហាត់ទី២២: បើ a,b,c និង d ជាចំនួនពិតវិជ្ជមានហើយចំពោះ $abcd=1$ នោះ

បង្ហាញថា $(1+a)(1+b)(1+c)(1+d) \geq 16$ ។

ដំណោះស្រាយ

យើងដឹងថា $(1+a)(1+b)(1+c)(1+d)$

$=1+(a+b+c+d)+(ab+ac+ad+bc+bd+cd)$

$+(abc+acd+abd+bcd)+abcd$

$$=1+abcd+(a+bcd)+(b+acd)+(c+abd)+(d+abc)+(ab+cd)+(ac+bd)+(ad+bc)$$

$$=1+1+\left(a+\frac{1}{a}\right)+\left(b+\frac{1}{b}\right)+\left(c+\frac{1}{c}\right)+\left(d+\frac{1}{d}\right)+\left(ab+\frac{1}{ab}\right)+\left(ac+\frac{1}{ac}\right)+\left(ad+\frac{1}{ad}\right)$$

ប៉ុន្តែគ្រប់ចំនួនពិត $k>0$, $k+\frac{1}{k}\geq 2$ គេបានៈ

$$(1+a)(1+b)(1+c)(1+d)$$

$$=2+\left(a+\frac{1}{a}\right)+\left(b+\frac{1}{b}\right)+\left(c+\frac{1}{c}\right)+\left(d+\frac{1}{d}\right)+\left(ab+\frac{1}{ab}\right)+\left(ac+\frac{1}{ac}\right)+\left(ad+\frac{1}{ad}\right)$$

$$\geq 2+2\cdot 7=16$$

ដូចនេះ $(1+a)(1+b)(1+c)(1+d)\geq 16$ ។

លំហាត់ទី២៣ៈ រកតម្លៃ $m\in\mathbb{N}^*-\{1\}$ ដើម្បីឲសំណល់នៃវិធីចែកទាំងបី $69,90,125$ នឹង m មានតម្លៃស្មើនឹងគ្នា។

ដំណោះស្រាយ

គេមាន $\begin{cases}69\equiv r\pmod m\\90\equiv r\pmod m\\125\equiv r\pmod m\end{cases}$ ដែល $0\leq r\leq m$

តែ

$$\begin{cases}69=mk+r & (1)\\90\equiv mp+r & (2)\\125\equiv mq+r & (3)\end{cases}$$

តាម $(2)-(1)$ គេបានៈ $m(p-k)=21$

ចំពោះ $m \in \mathbb{N}^* - \{1\} \quad \Rightarrow m\{3,7,21\}$

$+$ ករណី $m = 3$

$$\begin{cases} 69 \equiv 0(\mod 3) \\ 90 \equiv 0(\mod 3) \\ 125 \equiv 2(\mod 3) \end{cases} \Rightarrow m \neq 3$$

$+$ ករណី $m = 7$

$$\begin{cases} 69 \equiv 6(\mod 7) \\ 90 \equiv 6(\mod 7) \\ 125 \equiv 6(\mod 7) \end{cases} \Rightarrow m = 7 \text{ ពិត}$$

$+$ ករណី $m = 21$

$$\begin{cases} 69 \equiv 6(\mod 21) \\ 90 \equiv 6(\mod 21) \\ 125 \equiv 20(\mod 21) \end{cases} \Rightarrow m \neq 21$$

ដូចនេះ $m = 7$ ។

លំហាត់ទី២៤: បង្ហាញថា ចំពោះ$\forall n \in \mathbb{N}$ គេបាន:$3^{3^{2n+1}} + 3 = 0(\mod 30)$ ។

ដំណោះស្រាយ

តាមសម្មតិកម្មគេបាន:

.គេមាន $3^{3^{2n+1}} = 3^{3 \cdot 3^{2n}} = 27^{3^{2n}} = 27^{9^n}$

លុះត្រាតែ$3^{3^{2n+1}} = 27^{9^n} = \left(\ldots \left(\ldots \left(27^9 \right)^9 \ldots \right)^9 \ldots . \right)^9$

.គេមាន $27^9 \equiv 27\pmod{30}$

$$\Rightarrow 27^{9n} = \left(\ldots\left(\ldots\left(27^9\right)^9\ldots\right)^9\ldots.\right)^9 \equiv 27 \pmod{30} \quad (1)$$

.គេមាន $3 \equiv 3 \pmod{30} \quad (2)$

តាមស (1) & (2) គេបាន $3^{3^{2n+1}} + 3 \equiv 27 + 3 \pmod{30} \equiv 30 \pmod{30}$

ព្រោះ $30 \equiv 0 \pmod{30}$

ដូចនេះ គេបាន: ចំពោះ $\forall n \in \mathbb{N}$ គេបាន: $3^{3^{2n+1}} + 3 = 0 \pmod{30}$

៧(របៀបទី២ ស្រាយតាមវិចារកំណើន)

លំហាត់ទី២៥: រកដេរីវេ y' នៃអនុគមន៍ $y = Argth\left(\frac{1}{\sqrt{Argsh\sqrt{Argthx}}}\right)^{\frac{3}{4}}$

ដំណោះស្រាយ

$y = Argthu$ ដែល $u = \left(\frac{1}{\sqrt{Argsh\sqrt{Argthx}}}\right)^{\frac{3}{4}} = u = (v)^{\frac{3}{4}}$

$$v = \frac{1}{\sqrt{Argsh\sqrt{Argthx}}} = \frac{1}{w}$$

$$w = Argsh\sqrt{Argthx} = Argshk$$

$$k = \sqrt{Argthx} = \sqrt{R}$$

$$R = Argthx \Rightarrow R' = \frac{1}{1-x^2}$$

$$\Rightarrow k'=\frac{R'}{2R}=\frac{1}{2\left(1-x^2\right)\sqrt{Argthx}}$$

$$\Rightarrow w=\frac{k'}{\sqrt{k^2+1}}=\frac{1}{2\left(1-x^2\right)\sqrt{Argthx\left(Argthx+1\right)}}$$

$$\Rightarrow v'=\frac{-w}{2\sqrt{w^3}}=\frac{-1}{4(1-x^2)\sqrt{Argthx\left(Argthx+1\right)Argsh\sqrt{Argthx}^3}}$$

$$\Rightarrow u'=\frac{3}{4}v'v^{\frac{-1}{4}}=\frac{-3\sqrt[8]{Argshx\sqrt{Argthx}}}{16(1-x^2)\sqrt{Argthx(Argthx+1)Argsh\sqrt{Argthx}^3}}$$

ដូច្នេះ

$$y'=\frac{u'}{1-u^2}=\frac{3\sqrt[8]{\left(Argsh\sqrt{Argthx}\right)^7}}{16\left(1-x^2\right)\left[1-\left(Argsh\sqrt{Argthx}\right)^{\frac{3}{4}}\right]\sqrt{Argthx\left(Argthx+1\right)Argsh\sqrt{Argthx}^3}}$$ ។

លំហាត់ទី២៦: គណនាលីមីតដូចខាងក្រោមៈ

$$A=\lim_{n\to\infty}\frac{\sqrt[n]{\sqrt{2!}\cdot\sqrt[3]{3!}\ldots\sqrt[n]{n!}}}{\sqrt[n+1]{(2n+1)!!}}$$ ។

ដំណោះស្រាយ

$$A=\lim_{n\to\infty}\frac{\sqrt[n]{\sqrt{2!}\cdot\sqrt[3]{3!}\ldots\sqrt[n]{n!}}}{\sqrt[n+1]{(2n+1)!!}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[n]{\dfrac{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n]{n!}}{n^n}}}{\sqrt[n]{\dfrac{(2n+1)!!}{n^n}}}\times\lim_{n\to\infty}\frac{\sqrt[n]{(2n-1)!!}}{\sqrt[n+1]{(2n+1)!!}}$$

$$=\lim_{n\to\infty}\frac{\frac{\sqrt[n+1]{(n+1)!}}{(n+1)^{n+1}}n^n}{\frac{2n+1}{(n+1)^{n+1}}n^n}=\lim_{n\to\infty}\frac{n+1}{2n+1}\times\lim_{n\to\infty}\sqrt[n+1]{\frac{(n+1)!}{(n+1)^{n+1}}}$$

$$=\frac{1}{2}\lim_{n\to\infty}\frac{n+2}{(n+2)^{n+2}}(n+1)^{n+1}$$

$$=\frac{1}{2}\lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n+1}\right)^{n+1}}=\frac{1}{2e}$$

ដូចនេះ $A=\frac{1}{2e}$ ។

លំហាត់ទី២៦:គេឲ $f:\mathbb{Z}\to\mathbb{Z}$ ជាអនុគមន៍ផ្ទៀងផ្ទាត់ $f(x)\neq 0, f(1)=0$ និង

$(i), f(xy)+f(x)f(y)=f(x)+f(y)$

$(ii), f(x-y)-f(0))f(x)f(y)=0 \quad ,\forall x,y\in\mathbb{Z}$

(a),រកសំណុំដែលគ្រប់តម្លៃនៃអនុគមន៍ f ។

(b)បើ $f(10)\neq 0$ និង $f(2)=0$ ។រកគ្រប់សំណុំចំនួនគត់ n ចំពោះ $f(n)\neq 0$ ។

ដំណោះស្រាយ

(a)

បើ $x=y=0$ ជំនួសក្នុង (i)

$f(0)f(0)=f(0)\Rightarrow f(0)=1$

តាម (ii) នាំឲ $f(x-y)-1)f(x)f(y)=0$

បើ $y=0$

$\left(f(x)-1\right)f(x)=0 \Rightarrow f(x)=1$ ឬ 0

ដូចនេះ រកសំណុំដែលគ្រប់តម្លៃនៃអនុគមន៍ f គឺ 1 , 0 ។

(b), $f(2)=0$ ពិនិត្យ $y=2$

ដែល $f(2x)=f(x)\Rightarrow f(2)=f(4)=f(8)=0$

$x=5\Rightarrow f(5)\neq 0$

បើ $y=5$ ជំនួសក្នុង (ii)

$(f(x-5)f(x)=0$

បើ $x=9\Rightarrow\left(f(4)-1\right)f(9)=0$

$f(2)=f(4)=0\Rightarrow f(9)=0$

$x=7\Rightarrow\left(f(2)-1\right)f(7)=0\Rightarrow f(7)=0$

$x=6\Rightarrow\left(f(6)-1\right)f(6)=0\Rightarrow f\left(6\right)=0\Rightarrow f(3)=0$

បើ $x=0$ $f(5)=1\Rightarrow f(5)\neq 0$

ដូច្នេះ $f(2x)=f(x)\Rightarrow n\left\{\pm 5,\pm 10,\pm 20,\pm 40,........\right\}$ ។

លំហាត់ទី២៧:គេឲចំនួនគត់វិជ្ជមាន $n\geqslant 2$ ។ឧបមាថាចំនួនគត់វិជ្ជមាន $a_i (i=1,2,\cdots,n)$ ផ្ទៀងផ្ទាត់ថា $a_1<a_1<\cdots<a_n$ និង $\sum_{i=1}^{n}\frac{1}{a_i}\leqslant 1$ ។បង្ហាញថាគ្រប់ចំនួនពិត x ដូចវិសមភាពខាងក្រោមៈ

$$\left(\sum_{i=1}^{n}\frac{1}{a_i^2+x^2}\right)^2\leqslant\frac{1}{2}\cdot\frac{1}{a_1\left(a_1-1\right)+x^2}$$ ។

ដំណោះស្រាយ

ដោយ $x^2 \geqslant a_1(a_1-1)$, $\sum_{i=1}^{n}\frac{1}{a_i} \leqslant 1$

យើងមាន $\left(\sum_{i=1}^{n}\frac{1}{a_i^2+x^2}\right)^2 \leqslant \left(\sum_{i=1}^{n}\frac{1}{2a_i|x|}\right)^2 = \frac{1}{4x^2}\left(\sum_{i=1}^{n}\frac{1}{a_i}\right)^2$

$$\leqslant \frac{1}{4x^2} \leqslant \frac{1}{2}\cdot\frac{1}{a_1(a_1-1)+x^2}$$

ដោយ $x^2 \geqslant a_1(a_1-1)$ ដោយប្រើវិសមភាព Cauchy យើងមាន

$$\left(\sum_{i=1}^{n}\frac{1}{a_i^2+x^2}\right)^2 \leqslant \left(\sum_{i=1}^{n}\frac{1}{a_i}\right)\left(\sum_{i=1}^{n}\frac{a_i}{\left(a_i^2+x^2\right)^2}\right) \leqslant \sum_{i=1}^{n}\frac{a_i}{\left(a_i^2+x^2\right)^2} \text{ ។}$$

ចំពោះចំនួនគត់វិជ្ជមាន $a_1 < a_1 < \ldots < a_n$ ដែលយើងមាន $a_{i+1} \geqslant a_i + 1$ និង

$$\frac{2a_i}{\left(a_i^2+x^2\right)^2} \leqslant \frac{2a_i}{\left(a_i^2+x^2+\frac{1}{4}\right)^2 - a_i^2}$$

$$= \frac{2a_i}{\left(\left(a_i-\frac{1}{2}\right)^2+x^2\right)\left(\left(a_i+\frac{1}{2}\right)^2+x^2\right)}$$

$$= \frac{1}{\left(a_i-\frac{1}{2}\right)^2+x^2} - \frac{1}{\left(a_i+\frac{1}{2}\right)^2+x^2}$$

$$\leqslant \frac{1}{\left(a_i-\frac{1}{2}\right)^2+x^2} - \frac{1}{\left(a_{i+1}-\frac{1}{2}\right)^2+x^2}$$

ចំពោះ $i = 1, 2, \cdots, n-1$

គេបាន $\sum_{i=1}^{n}\frac{a_i}{\left(a_i^2+x^2\right)^2}\leqslant\frac{1}{2}\sum_{i=1}^{n}\left[\frac{1}{\left(a_i-\frac{1}{2}\right)^2+x^2}-\frac{1}{\left(a_{i+1}-\frac{1}{2}\right)^2+x^2}\right]$

$\leqslant\frac{1}{2}\cdot\frac{1}{\left(a_1-\frac{1}{2}\right)^2+x^2}\leqslant\frac{1}{2}\cdot\frac{1}{a_1(a_1-1)+x^2}$

ដូចនេះ $\left(\sum_{i=1}^{n}\frac{1}{a_i^2+x^2}\right)^2\leqslant\frac{1}{2}\cdot\frac{1}{a_1(a_1-1)+x^2}$ ។

លំហាត់ទី២៨:គណនាលីមីតដូចខាងក្រោមៈ

$$A=\lim_{x\to0}\frac{e^{-2x}-3x-1}{\sin^2 x}\ ,$$

ដំណោះស្រាយ

$$\lim_{x\to0}\frac{e^{3x}-3x-1}{\sin^2 x}=\lim_{x\to0}\frac{\frac{e^{3x}-3x-1}{9x^2}}{\frac{\sin^2 x}{9x^2}}=\frac{\lim\limits_{x\to0}\frac{e^{3x}-3x-1}{(3x)^2}}{\frac{1}{9}\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^2}=\frac{\lim\limits_{y\to0}\frac{e^{y}-y-1}{y^2}}{\frac{1}{9}\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^2}=\frac{\frac{1}{2}}{\frac{1}{9}\cdot1^2}=\frac{9}{2}$$

ដែល

$$L=\underbrace{\lim_{y\to0}\frac{e^{y}-y-1}{y^2}}_{y=2z}=\lim_{t\to0}\frac{e^{2t}-2t-1}{4t^2}\Rightarrow 4L=\lim_{y\to0}\frac{e^{2y}-2y-1}{y^2}$$

$$2L=\lim_{y\to0}\frac{e^{2y}-2y-1}{y^2}-2\lim_{y\to0}\frac{e^{y}-y-1}{y^2}=\lim_{y\to0}\frac{e^{2y}-2e^{y}+1}{y^2}=\left(\lim_{y\to0}\frac{e^{y}-1}{y}\right)^2=1^2=1$$

$$=\lim_{y\to0}\frac{e^{y}-y-1}{y^2}=L=\frac{1}{2}$$

ដូច្នេះ $A=\lim\limits_{x\to0}\frac{e^{-2x}-3x-1}{\sin^2 x}=\frac{9}{2}$ ។

លំហាត់ទី២៩: ដោះស្រាយសមីការខាងក្រោម៖

$$\begin{cases} 56x+33y=-\dfrac{y}{x^2+y^2} \\ 33x-56y=\dfrac{x}{x^2+y^2} \end{cases}$$ ។

ដំណោះស្រាយ

យើងបាន $\dfrac{56x+33y}{33x-56y}=-\dfrac{y}{x}$

$56x^2+33xy--33xy+56y^2$ ចំណាំ $\vee$ អានថាឬ

$56x^2+66xy-56y^2=0$

$28x^2+33xy-28y^2=0$

$(7x-4y)(7y+4x)=0$

$y=\dfrac{7\text{x}}{4}$ V $y=-\dfrac{4\text{x}}{7}$

$$\Rightarrow y=\frac{7x}{4}\rightarrow\begin{cases} 56x+\dfrac{231x}{4}=-\dfrac{\frac{7x}{4}}{x^2+\frac{49x^2}{16}} \\ 33x-98x=\dfrac{x}{x^2+\frac{49x^2}{16}} \end{cases}\leftrightarrow\begin{cases} \dfrac{455x}{4}=-\dfrac{28}{65x} \\ -65x=\dfrac{16}{65x} \end{cases}\leftrightarrow x=\pm\frac{4i}{65}\notin R$$

$$y=-\frac{4x}{7}\rightarrow=\begin{cases} 56x-\dfrac{132x}{7}=-\dfrac{\frac{4x}{7}}{x^2+\frac{16x^2}{49}} \\ 33x+32x=\dfrac{x}{x^2+\frac{16x^2}{49}} \end{cases}\Leftrightarrow\begin{cases} \dfrac{260x}{7}=\dfrac{28}{65x} \\ 65x=\dfrac{49}{65x} \end{cases}$$

$$\leftrightarrow\left[x=\pm\frac{7}{65}\right]\to\left[y=\mp\frac{4}{65}\right],|x|+|y|=\frac{11}{65}$$

ដូចនេះ $x=\pm\frac{7}{65},y=\frac{\mp4}{65}$ ។

លំហាត់ទី៣០: គេឲ $\begin{cases}\left(\log_x^2 y+\log_z^2 t\right)\cdot\left(\log_y^2 z+\log_t^2 x\right)=37\\ \log_y t+\log_t y=5\end{cases}$ គណនា $\log_x z+\log_z x=?$

ដំណោះស្រាយ

$$\begin{cases}\log_x^2 y\cdot\log_y^2 z+\log_x^2 y\cdot\log_t^2 x+\log_z^2 t\cdot\log_y^2 z+\log_z^2 t\cdot\log_t^2 x=37\\ \left(\log_y t+\log_t y\right)^2=5^2\end{cases}$$

$$\Leftrightarrow\begin{cases}\log_x^2 z+\log_t^2 y+\log_y^2 t+\log_z^2 x=37\\ \log_y^2 t+2\log_y t\cdot\log_t y+\log_t^2 y=25\end{cases}$$

$$\Leftrightarrow\begin{cases}\log_x^2 z+\log_t^2 y+\log_y^2 t+\log_z^2 x=37\\ \log_y^2 t+\log_t^2 y=23\end{cases}\Rightarrow$$

$$\Rightarrow\log_x^2 z+\log_z^2 x=14$$

$$x=\log_x z+\log_z x\Leftrightarrow x^2=\log_x^2 z+2\log_x z\cdot\log_z x+\log_z^2 x=\log_x^2 z+\log_z^2 x+2=16$$

$$x^2=16\Leftrightarrow x_{1,2}=\pm4$$

ដូចនេះ $x_{1,2}=\pm4$ ។

លំហាត់ទី៣១ គេឲ $A=\sqrt{666666666^2-4444444444^2-2222222222^2}$ ។

គណនា A=?

ដំណោះស្រាយ

តាង 22222222222=a, 4444444444=2 a, 6666666666=3 a

គេបាន $A=\sqrt{666666666^2-4444444444^2-22222222222^2}$

$A=\sqrt{9a^2-4a^2-a^2}=\sqrt{4a^2}=2a=4444444444$

ដូចនេះ $A=44444444444$ ។

លំហាត់ទី៣២: គេដឹងថា $x^4=7-\cfrac{1}{7-\cfrac{1}{7-\cfrac{1}{\ddots}}}$ រកតម្លៃ $P=\sqrt[4]{\dfrac{(x^6-1)\cdot(x^4+x^2+1)}{x^5}}$

ដំណោះស្រាយ

ដោយ $x^4+\dfrac{1}{x^4}=7$

$\Leftrightarrow x^4+2x^2\dfrac{1}{x^2}+\dfrac{1}{x^4}=7+2$

$\left(x^2+\dfrac{1}{x^2}\right)^2=9$

$x^2+\dfrac{1}{x^2}=3$

$\left(x-\dfrac{1}{x}\right)^2=x^2+\dfrac{1}{x^2}-2$

$\left(x-\dfrac{1}{x}\right)^2=3-2 \qquad \Rightarrow x-\dfrac{1}{x}=1$

$p=\sqrt[4]{\dfrac{(x^6-1)(x^4+x^2+1)}{x^5}}$ $p=\sqrt[4]{\left(x^3-\dfrac{1}{x^3}\right)\left(x^2+1+\dfrac{1}{x^2}\right)}$

$p=\sqrt[4]{\left(x-\dfrac{1}{x}\right)\left(x^2+1+\dfrac{1}{x^2}\right)\left(x^2+1+\dfrac{1}{x^2}\right)}=\sqrt[4]{\left(x-\dfrac{1}{x}\right)\left(x^2+1+\dfrac{1}{x^2}\right)^2}$

នាំឲ $P=\sqrt[4]{16}=2$

ដូចនេះ $P=2$ ។

លំហាត់ទី៣៣: ចូរដោះស្រាយសមីការ $x^2+\frac{9x^2}{(x-3)^2}=40$ ។

ដំណោះស្រាយ

ដោយ $x^2+\frac{9x^2}{(x-3)^2}=40$

$$x^2+\frac{(3x)^2}{(x-3)^2}=40 \Rightarrow x^2+\frac{(3x)^2}{(x-3)^2}+2.x.\frac{3x}{x-3}-2.x.\frac{3x}{x-3}=40$$

$$\left(x+\frac{3x}{x-3}\right)^2-2.x.\frac{3x}{x-3}=4 \Rightarrow \left(\frac{x^2-3x+3x}{x-3}\right)^2-\frac{6x^2}{x-3}=40$$

$$\left(\frac{x^2}{x-3}\right)^2-6.\frac{x^2}{x-3}-40=0 \Rightarrow \left(\frac{x^2}{x-3}+4\right)\left(\frac{x^2}{x-3}-10\right)=0$$

$$\frac{x^2}{x-3}+4\vee\frac{x^2}{x-3}-10=0$$

$x^4+4x-12=0$ ឬ $x^2-10x+30=0$

$(x+6)(x-2)=0$ ឬ $\vee\Delta<0\Rightarrow x\notin\mathbb{R}$

$$\Rightarrow x_1=-6\vee x_2=2$$

លំហាត់ទី៣៤: រក n^2+1 គេដឹងថា $\sqrt{\frac{\sqrt[3]{\sqrt{\frac{1}{x}}}}{x}}=\sqrt[6]{x^4\sqrt{x^{2n+1}}}$ ។

ដំណោះស្រាយ

គេមាន $\sqrt{\dfrac{\sqrt[3]{\sqrt{\frac{1}{x}}}}{x}} = \sqrt[6]{x^4\sqrt{x^{2n+1}}}$

$$\Leftrightarrow \left(\dfrac{\sqrt[3]{\sqrt{\frac{1}{x}}}}{x}\right)^6 = \left(x\sqrt[4]{\sqrt{x^{2n+1}}}\right)^4$$

$$\Leftrightarrow \dfrac{\left(\sqrt[3]{\sqrt{\frac{1}{x}}}\right)^6}{x^6} = x^4\left(\sqrt[4]{\sqrt{x^{2n+1}}}\right)^4 \Leftrightarrow \dfrac{\left(\dfrac{\sqrt{\frac{1}{x}}}{x}\right)^2}{x^6} = x^4.x^{2n+1}$$

$$\Leftrightarrow \dfrac{\frac{1}{x}}{x^2} = x^6.x^4.x^{2n+1}$$

$$\Leftrightarrow \dfrac{1}{x^3} = x^{11}.x^{2n} \Leftrightarrow 1 = x^{14+2n}$$

$\Rightarrow n = -7 \quad \Rightarrow n^2 + 1 = (-7)^2 + 1 = 50$ ។

លំហាត់ទី៣៥ រកតម្លៃ $P = \sqrt[n]{\dfrac{\sqrt[n]{\dfrac{\sqrt[n]{\dfrac{\cdot^{\cdot^{\cdot^{\infty}}}}{x^{a+2r}}}}{x^{a+r}}}}{x^a}}$ ។

ដំណោះស្រាយ

យើងមាន $P = \sqrt[n]{\dfrac{\sqrt[n]{\dfrac{\sqrt[n]{\dfrac{\cdot^{\cdot^{\cdot^{\infty}}}}{x^{a+2r}}}}{x^{a+r}}}}{x^a}}$

គេបាន:

$$P^{-1}=\sqrt[n]{x^a\sqrt[n]{x^{a+r}\sqrt[n]{x^{a+2r}\ldots}}} \quad\Rightarrow\quad P^{-1}=x^{\frac{a}{n}+\frac{a}{n^2}+\frac{a}{n^3}+\ldots\infty+\frac{r}{n^2}+\frac{2r}{n^3}+\frac{3a}{n^4}+\ldots\infty}$$

$$p=x^{\frac{a}{n-1}+\frac{r}{n}\left(\frac{1}{n}+\frac{2}{n^2}+\frac{3}{n^3}\right)}$$

$$P^{-1}=x^{\frac{a}{n-1}+\frac{r}{n}\left(\frac{1}{n-1}+\frac{1}{(n-1)^2}\right)}$$

$$p^{-1}=x^{\frac{a}{a-1}+\frac{r}{n}\left(\frac{1}{n-1}+\frac{1}{(n-1)^2}\right)}$$

$$p=\left(x^{\frac{a}{a-1}+\frac{r}{n}\frac{n}{(n-1)^2}}\right)^{-1}$$

ដូចនេះ $p=x^{\frac{a(1-n)-r}{(n-1)^2}}$ ។

លំហាត់ទី៣៥: ដោះស្រាយសមីការក្នុង $\mathbb{R}$ $(x+1)2^{-3x^3-4x+1}=x^2+2x+2$ ។

ដំណោះស្រាយ

ដោយ $(x+1)2^{-3x^3-4x+1}=x^2+2x+2$

$$\Rightarrow 2^{-3x^3-4x+1}=+1+\frac{1}{x+1}$$

តាមវិសមភាព $AM\geq GM$

$$\frac{x+1+\frac{1}{x+1}}{2}\geq\sqrt{(x+1).\frac{1}{(x+1)}}$$

$$x+1+\frac{1}{x+1}\geq 2$$

$$x+1+\frac{1}{x+1}=2$$

$$2^{-3x^3-4x+1}=2^1$$

$$\Leftrightarrow -3x^3-4x+1=1$$

$$x\left(3x^2+4\right)=0$$

$$x=0 \vee 3x^2+4=0$$

$$3x^2+4=0 \Rightarrow \Delta<0 \Rightarrow x \notin \mathbb{R}$$

$x=0$ ។

លំហាត់ទី៣៥: រកតម្លៃ x គេដឹងថា $x^{\log_2 \sqrt[3]{x^2}}-2x^{\log_8 x}=\frac{16}{3}\log_4 8$

ដំណោះស្រាយ

យើងមាន $c^{\log_b a}=a^{\log_b c}$

$$l\,og_a\, a=1 \Rightarrow x^{\log_2 \sqrt[3]{x^2}}-2x^{\log_8 x}=\frac{16}{3}\log_4 8$$

$$\sqrt[3]{x^2}^{\log x}-2x^{\log_{2^3} x}=\frac{16}{3}\log_{2^2} 2^3$$

$$x^{\frac{2}{3}\log_2 x}-2x^{\frac{1}{3}\log_2 x}=8$$

$$\log_2 x=k \Rightarrow x=2^k$$

$$x^{\frac{2}{3}k}-2x^{\frac{1}{3}k}-8=0$$

$$\left(x^{\frac{1}{3}k}-4\right)\left(x^{\frac{1}{3}k}+2\right)=0 \qquad x^{\frac{1}{3}k}=4 \vee x^{\frac{1}{3}k}=-2 \qquad \left(2^k\right)^{\frac{1}{3}k}=2^2$$

$$\frac{1}{3}k^2=2 \Rightarrow k^2=6 \Rightarrow k=\sqrt{6}$$

$x = 2^k = 2^{\sqrt{6}}$ ។

លំហាត់ទី៣៦: គេដឹងថា $\frac{a}{b+c-3a} = \frac{b}{a\_+c-3b} = \frac{c}{a+b-3c}$ ។ រកតម្លៃ P $\frac{3b}{a} + \frac{3c}{a} + \frac{a}{c} + \frac{b}{c}$

ដំណោះស្រាយ

គេមាន $\frac{a}{b+c-3a} = \frac{b}{a\_+c-3b} = \frac{c}{a+b-3c}$

នាំឲ $\frac{4a}{b+c-3a} = \frac{4b}{a\_+c-3b} = \frac{4c}{a+b-3c}$

$$\frac{4a}{b+c-3a} + 1 = \frac{4b}{a\_+c-3b} + 1 = \frac{4c}{a+b-3c} + 1$$

$$\frac{a+b+c}{b+c-3a} = \frac{a+b+c}{a+c-3b} = \frac{a+b+c}{a+b-3c}$$

$$b+c-3a = b+c-3a = b+c-3a$$

$$\frac{1}{b+c-3a} = \frac{1}{a\_+c-3b} = \frac{1}{a+b-3c}$$

$$4b = 4a \qquad 4b = 4c$$

$a = b$ និង $b = c \Rightarrow a = b = c$

+ករណីទី១

$$\frac{3b}{a} + \frac{3c}{a} + \frac{a}{c} + \frac{b}{c} \Rightarrow \frac{3a}{a} + \frac{3a}{a} + \frac{a}{a} + \frac{a}{a} = 8$$

+ករណីទី២

$$\frac{3(b+c)}{a} + \frac{a+b}{c} \Rightarrow \frac{3(-a)}{a} + \frac{-c}{c} = -4$$

លំហាត់ទី៣៧: ចំពោះ $a,b,c\in\mathbb{R}$ ដែល $\frac{ac}{a+b}+\frac{ba}{b+c}+\frac{bc}{c+a}=-9$ និង $\frac{bc}{a+b}+\frac{ca}{b+c}+\frac{ba}{c+a}=10$

រកតម្លៃ L នោះ $L=\frac{b}{a+b}+\frac{c}{b+c}+\frac{a}{c+a}$ ។

ដំណោះស្រាយ

$$+\begin{cases}\frac{ac}{a+b}+\frac{ba}{b+c}+\frac{bc}{c+a}=-9\\ \frac{bc}{a+b}+\frac{ca}{b+c}+\frac{ba}{c+a}=10\end{cases}\Rightarrow\frac{ac+bc}{a+b}+\frac{ba+ca}{b+c}+\frac{+bcba}{c+a}=1$$

$$\Rightarrow\frac{c(a+b)}{a+b}+\frac{a(b+c)}{b+c}+\frac{b(c+a)}{c+a}=1\qquad\Rightarrow a+b+c=1$$

យើងមាន

$$L=\frac{b}{a+b}+\frac{c}{b+c}+\frac{a}{c+a}$$

$$L.1=\left(\frac{b}{a+b}+\frac{c}{b+c}+\frac{a}{c+a}\right)(a+b+c)$$

$$L=\frac{b(a+b+c)}{a+b}+\frac{c(a+b+c)}{b+c}+\frac{a(a+b+c)}{c+a}$$

$$L=\frac{(a+b)b+bc}{a+b}+\frac{c(b+c)+ac}{b+c}+\frac{a(a+c)+ac}{c+a}$$

$$L=b+\frac{bc}{a+b}+c+\frac{ac}{b+c}+a+\frac{ac}{c+a}$$

$$L=(a+b+c)+\frac{bc}{a+b}+\frac{ac}{b+c}+\frac{ac}{c+a}=1+10=11\text{ ។}$$

ដូចនេះ $L=\frac{b}{a+b}+\frac{c}{b+c}+\frac{a}{c+a}=11$ ។

លំហាត់ទី៣៨: គណនា $L = \sqrt[x^{\log_{abc} y}]{\left[\dfrac{\log_y \sqrt[x]{x}}{\log \sqrt[xy]{y}}\right]^{y^{\log_{abc} x}}}$ ។

ដំណោះស្រាយ

រូបមន្ត $\sqrt[n]{a^n} = a$ និង $\sqrt[n]{a} = a^{\frac{1}{n}}$, $\dfrac{1}{\log_b a} = \log_a b$, $a^{\log_a b} = b, a^{\log_b c} = a^{\log_b c}$

គេបាន $L = \sqrt[y^{\log_{abc} x}]{\left[\dfrac{\log_y \sqrt[x]{x}}{\log \sqrt[xy]{y}}\right]^{y^{\log_{abc} x}}} = \dfrac{\log_y \sqrt[x]{x}}{\log \sqrt[xy]{y}} = \dfrac{x^{\frac{1}{\log_y x}}}{y^{\log_x y}} = \dfrac{x^{\log_x y}}{y^{\log_y x}} = \dfrac{y}{x}$

ដូចនេះ គេបាន $L = \sqrt[x^{\log_{abc} y}]{\left[\dfrac{\log_y \sqrt[x]{x}}{\log \sqrt[xy]{y}}\right]^{y^{\log_{abc} x}}} = \dfrac{y}{x}$ ។

លំហាត់ទី៣៩: គណនា $S = 6 + 66 + 666 + \ldots + \underbrace{666\ldots6}_{n}$

ដំណោះស្រាយ

យើងមាន $S = 6 + 66 + 666 + \ldots + \underbrace{666\ldots6}_{n}$

គេបាន $6 = \dfrac{6}{9}(10-1)$, $66 = \dfrac{6}{9}(10^2-1)$, $666 = \dfrac{6}{9}(10^3-1)$, ... , $\underbrace{666\ldots6}_{n} = \dfrac{6}{9}(10^n-1)$

$s = \dfrac{6}{9}(10-1) + \dfrac{6}{9}(10^2-1) + \dfrac{6}{9}(10^3-1) + \ldots. + \dfrac{6}{9}(10^n-1)$

$= \dfrac{6}{9}\left[10 + 10^2 + 10^3 + \ldots. + 10^n \underbrace{-1-1-\ldots.-1}_{n}\right]$

$$\frac{6}{9}\left(\frac{10.(10^n-1)}{10-1}-n\right)=\frac{6}{9}\left(\frac{10^{n+1}-10-9n}{9}\right)=\frac{6(10^{n+1}-10-9n)}{81}$$

ដូចនេះ $S=6+66+666+...+\underbrace{666...6}_{n}=\dfrac{6(10^{n+1}-10-9n)}{81}$ ។

លំហាត់ទី៤០: គេឲ $A=7+7^2+7^3+....+7^{2008}$ ។បង្ហាញថា A ចែកដាច់នឹង 2800 ។

ដំណោះស្រាយ

$$A=7+7^2+7^3+7^4+7^5+7^6+7^7+7^8....+7^{2005}+7^{2006}+7^{2007}+7^{2008}$$

$$A=(7+7^2+7^3+7^4)+7^4(7+7^2+7^3+7^4)+....+7^{2004}(7+7^2+7^3+7^4)$$

$$A=(7+7^2+7^3+7^4)(1+7^4+....+7^{2004})$$

$$=2800.(1+7^4+....+7^{2004})$$

នាំឲ A ចែកដាច់នឹង 2800 ។

ដូចនេះ A ចែកដាច់នឹង 2800 ។

លំហាត់ទី៤១: គេឲ $E=\sqrt{1+2017+\sqrt{1+2018\sqrt{1+2019\sqrt{1+......}}}}$ ។ ចូររកតម្លៃ E ។

ដំណោះស្រាយ

គេមាន $1+a=\sqrt{(1+a)^2}=\sqrt{1+a(a+2)}=\sqrt{1+a\sqrt{(2+a)^2}}=\sqrt{1+a\sqrt{1+4a+3+a^2}}$

$$1+a=\sqrt{1+a\sqrt{1+(a+1)(a+3)}}=\sqrt{1+a\sqrt{1+(a+1)\sqrt{(a+3)^2}}}$$

$$1+a=\sqrt{1+a\sqrt{1+(a+1)\sqrt{1+a^2+6a+8}}}=\sqrt{1+a\sqrt{1+(a+1)\sqrt{1+(a+2)\sqrt{(a+4)^2}}}}$$

$$\sqrt{1+a\sqrt{1+(a+1)\sqrt{1+(a+2)\sqrt{(a+4)^2}}}}=\sqrt{1+a\sqrt{1+(a+1)\sqrt{1+(a+2)\sqrt{1+......}}}}$$ ចំពោះ $a=2017$

នោះ $1+2017=\sqrt{1+a\sqrt{1+(a+1)\sqrt{1+(a+2)\sqrt{1+.....}}}}$

$$E=\sqrt{1+a\sqrt{1+(a+1)\sqrt{1+(a+2)\sqrt{1+.....}}}}=2018$$

ដូចនេះ $E=\sqrt{1+a\sqrt{1+(a+1)\sqrt{1+(a+2)\sqrt{1+.....}}}}=2018$ ។

លំហាត់ទី៤២: គណនាផលបូក $S=\dfrac{1}{\sin 2x}+\dfrac{1}{\sin 4x}+.....+\dfrac{1}{\sin 2^n x}$ ចំពោះ $n\in\mathbb{N}$

$x\neq\dfrac{\pi k}{2^t}(t=0,1,...,;k\in\mathbb{Z})$ ។

ដំណោះស្រាយ

ចំណាំ $ctg2^{m-1}-ctg2^m=\dfrac{1}{\sin 2^m x}$

ចំពោះ $m=1,2,....n$ គេបាន

$$\begin{cases} ctgx-ctg2x=\dfrac{1}{\sin 2x}\\ ctg2x-ctg4x=\dfrac{1}{\sin 4x}\\ ....\\ ctg2^{n-1}x-ctg2^n x=\dfrac{1}{\sin 2^n x}\end{cases}$$

បូកអង្គទាំងពីរគេបាន $\dfrac{1}{\sin 2x}+\dfrac{1}{\sin 4x}+.....+\dfrac{1}{\sin 2^n x}=ctgx-ctg2^n x$

ដូចនេះ $S=\dfrac{1}{\sin 2x}+\dfrac{1}{\sin 4x}+.....+\dfrac{1}{\sin 2^n x}=ctgx-ctg2^n x$ ។

លំហាត់ទី៤៣: គណនា $\cos\dfrac{\pi}{7}-\cos\dfrac{2\pi}{7}+\cos\dfrac{3\pi}{7}$ ។

ដំណោះស្រាយ

ដោយ $\cos\frac{2\pi}{7}=\cos\left(\pi-\frac{5\pi}{7}\right)=-\cos\frac{5\pi}{7}$

នោះ $\cos\frac{\pi}{7}-\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}=\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}$

តាមរូបមន្ត $2\cos a\sin b=\sin(a+b)-\sin(a-b)$

ដោយ $(a,b)=\left(\frac{\pi}{7},\frac{\pi}{7}\right),\left(\frac{3\pi}{7},\frac{\pi}{7}\right),\left(\frac{5\pi}{7},\frac{\pi}{7}\right)$ គេបាន:

$$+\begin{cases}2\cos\frac{\pi}{7}\sin\frac{\pi}{7}=\sin\frac{2\pi}{7}-\sin 0\\ 2\cos\frac{3\pi}{7}\sin\frac{\pi}{7}=\sin\frac{4\pi}{7}-\sin\frac{2\pi}{7}\\ 2\cos\frac{5\pi}{7}\sin\frac{\pi}{7}=\sin\frac{6\pi}{7}-\sin\frac{4\pi}{7}=\sin\frac{\pi}{7}-\sin\frac{4\pi}{7}\end{cases}$$

$$\Rightarrow 2\sin\frac{\pi}{7}\left(\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}\right)=\sin\frac{\pi}{7}$$

$$\Rightarrow \cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}=\frac{1}{2}$$

ដូចនេះ $\cos\frac{\pi}{7}-\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}=\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}=\frac{1}{2}$ ។

លំហាត់ទី៤៤: គេអោយ $\begin{cases}x^3y+xy^3=\frac{10}{9}(x+y)^2\\ x^4y+xy^4=\frac{2}{3}(x+y)^3\end{cases}$ ។ ចូរដោះស្រាយសមីការនេះ

ដំណោះស្រាយ

គេបាន $x^2+y^2=(x+y)^2-2xy$, $x^3+y^3=(x+y)^3-3xy(x+y)$

$$\begin{cases} xy\left(x^2+y^2\right)=\frac{10}{9}(x+y)^2 \\ xy\left(x^3+y^3\right)=\frac{2}{3}(x+y)^3 \end{cases} \Rightarrow \begin{cases} xy\left[(x+y)^2-2xy\right]=\frac{10}{9}(x+y)^2 \\ xy\left[(x+y)^3-3xy(x+y)\right]=\frac{2}{3}(x+y)^3 \end{cases}$$

តាង $a=x+y$, $b=xy$ គេបាន $\begin{cases} b\left(a^2-2b\right)=\frac{10}{9}a^2 \\ b\left(a^3-3ab\right)=\frac{2}{3}a^3 \end{cases} \Rightarrow \begin{cases} a^2=\frac{18b^2}{9b-10} (*) \\ \left[\begin{array}{l} a^2=\frac{9b^2}{3b-2} (**) \\ a=0 \Rightarrow b=0 \end{array}\right. \end{cases}$

$$(*)=(**) \Rightarrow \frac{18b^2}{9b-10}=\frac{9b^2}{3b-2} \Rightarrow 6b-4=9b-10 \Rightarrow 3b=6 \Rightarrow b=2$$

យក b ជំនួស ក្នុង $(*)$ $a^2=\frac{18b^2}{9b-10}=\frac{18.4}{8}=9 \Rightarrow a=\pm 3$

នាំឲ $\begin{cases} \left[\begin{array}{l} a=0 \\ b=0 \end{array}\right. \\ \left[\begin{array}{l} a=3 \\ b=2 \end{array}\right. \\ \left[\begin{array}{l} a=-3 \\ b=2 \end{array}\right. \end{cases} \Rightarrow \begin{cases} \left[\begin{array}{l} x+y=0 \\ xy=0 \end{array}\right. \\ \left[\begin{array}{l} x+y=3 \\ xy=2 \end{array}\right. \\ \left[\begin{array}{l} x+y=-3 \\ xy=2 \end{array}\right. \end{cases} \Rightarrow \begin{cases} \left[\begin{array}{l} x_1=0 \\ y_1=0 \end{array}\right. \\ \left[\begin{array}{l} x_2=1, x_3=2 \\ y_2=2, y_3=1 \end{array}\right. \\ \left[\begin{array}{l} x_4=-1, x_5=-2 \\ y_4=-2, x_5=-1 \end{array}\right. \end{cases}$

ដូចនេះ $(-2;-1)$, $(-1;-2)$, $(0;0)$, $(1;2)$, $(2;1)$ ។

លំហាត់ទី៤៥: គណនា $\left(1+2+2^2\right)\left(1+2^3+2^6\right)\left(1+2^9+2^{18}\right)\left(1+2^{27}+2^{54}\right)$ ។

ដំណោះស្រាយ

យើងមាន $\left(1+2+2^2\right)\left(1+2^3+2^6\right)\left(1+2^9+2^{18}\right)\left(1+2^{27}+2^{54}\right)$

តាមរូបមន្ត $(a-b)\left(a^2+ab+b^2\right)=a^3-b^3$

គេបាន :

$$B=\frac{\left[(1-2)\left(1+2+2^2\right)\right]\left(1+2^3+2^6\right)\left(1+2^9+2^{18}\right)\left(1+2^{27}+2^{54}\right)}{1-2}$$

$$=\frac{\left[\left(1-2^3\right)\left(1+2^3+2^6\right)\right]\left(1+2^9+2^{18}\right)\left(1+2^{27}+2^{54}\right)}{1-2}$$

$$=\frac{\left[\left(1-2^9\right)\left(1+2^9+2^{18}\right)\right]\left(1+2^{27}+2^{54}\right)}{1-2}$$

$$=\frac{\left[\left(1-2^{27}\right)\right]\left(1+2^{27}+2^{54}\right)}{1-2}$$

$$=\frac{\left(1-2^{81}\right)}{1-2}=2^{81}-1$$

ដូចនេះ $\left(1+2+2^2\right)\left(1+2^3+2^6\right)\left(1+2^9+2^{18}\right)\left(1+2^{27}+2^{54}\right)=2^{81}-1$ ។

លំហាត់ទី៤៦ គេដឹងថា $a+b+c=0$ ។ គណនា $\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}$ ។

ដំណោះស្រាយ

តាង $A=\frac{a^2}{a^2-bc}$, $B=\frac{b^2}{b^2-ca}$, $C=\frac{c^2}{c^2-ab}$, $-b=a+c$

$$A=\frac{a^2}{a^2-bc}=\frac{a^2}{a^2+(a+c)c}=\frac{a^2}{a^2+ac+c^2}$$

$$B=\frac{b^2}{b^2-ca}=\frac{(a+c)^2}{b^2+ac+c^2}=\frac{a^2+2ab+c^2}{b^2+ac+c^2}$$

$$C=\frac{c^2}{c^2-ab}=\frac{c^2}{c^2+a(c+a)}=\frac{c^2}{c^2+ac+a^2}$$

$$A+B+C=\frac{a^2}{a^2+ac+c^2}+\frac{a^2+2ab+c^2}{b^2+ac+c^2}+\frac{c^2}{c^2+ac+a^2}$$

$$A+B+C=\frac{2a^2+2ab+2c^2}{b^2+ac+c^2}=\frac{2\left(b^2+ac+c^2\right)}{b^2+ac+c^2}=2$$

ដូចនេះ $\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}=2$ ។

លំហាត់ទី៤៧: ចំពោះ $x,y,z\in\mathbb{N}$ ដោយ $x+\frac{y}{z}=\frac{2020^4+2020^2+1}{2020^3+1}$ ។ រកតម្លៃ $z=?$

ដំណោះស្រាយ

តាង $a=2020$

នោះ $x+\frac{y}{z}=\frac{a^4+a^2+1}{a^3+1}=\frac{(a^2+a+1)(a^2-a+1)}{(a+1)(a^2-a+1)}$

$$\frac{a^2+a+1}{a+1}=a+\frac{1}{a+1}$$

$x+\frac{y}{z}=2020+\frac{1}{2021}\Leftrightarrow x=2020,\ y=1,\ z=2021$

ដូចនេះ តម្លៃ $z=2021$ ។

លំហាត់ទី៤៨: គេឲ្យអនុគមន៍ $f(x)=x.(x-1)(x-2)....(x-10)$ ។ រក $f'(0)=?$

ដំណោះស្រាយ

តាមរូបមន្តដេរីវេគេបាន $[u(x).v(x)]=u'(x).v(x)+u(x).v'(x)$

តាង $\begin{cases}u(x)=x\\v(x)=(x-1)(x-2)....(x-10)\end{cases}\Rightarrow f(x)=u(x).v(x)$

$$\begin{cases}u(0)=0\\v(0)=(-1)(-2)....(-10)\end{cases}$$

$f'(x)=u'(x).v(x)+u(x).v'(x)=1.v(x)+u(x).v'(x)$

$f'(0)=1.v(0)+u(0).v'(0)=10!+0.v'(0)=10!$

ដូចនេះ $f'(0)=10!$ ។

លំហាត់ទី៤៩: គណនា $\frac{3^2+1}{3^2-1}+\frac{5^2+1}{5^2-1}+\frac{7^2+1}{7^2-1}+....+\frac{21^2+1}{21^2-1}$ ។

ដំណោះស្រាយ

គេមាន $\frac{n^2+1}{n^2-1}=\frac{n^2-1+2}{n^2-1}=1+\frac{2}{n^2-1}=1+\frac{1}{n-1}-\frac{1}{n+1}$

$$+\begin{cases}\frac{3^2+1}{3^2-1}=1+\frac{1}{3-1}-\frac{1}{3+1}=1+\frac{1}{2}-\frac{1}{4}\\ \frac{5^2+1}{5^2-1}=1+\frac{1}{5-1}-\frac{1}{5+1}=1+\frac{1}{4}-\frac{1}{6}\\ \frac{7^2+1}{7^2-1}=1+\frac{1}{7-1}-\frac{1}{7+1}=1+\frac{1}{6}-\frac{1}{8}\\ \dots\dots\dots\dots\dots\dots\\ \frac{21^2+1}{21^2-1}=1+\frac{1}{21-1}-\frac{1}{21+1}=1+\frac{1}{20}-\frac{1}{22}\end{cases}$$

$$\Rightarrow \frac{3^2+1}{3^2-1}+\frac{5^2+1}{5^2-1}+\frac{7^2+1}{7^2-1}+....+\frac{21^2+1}{21^2-1}=10+\frac{1}{2}-\frac{1}{22}$$

ដូចនេះ $\frac{3^2+1}{3^2-1}+\frac{5^2+1}{5^2-1}+\frac{7^2+1}{7^2-1}+....+\frac{21^2+1}{21^2-1}=10\frac{5}{11}$ ។

លំហាត់ទី ៥០: គណនា $\left[\frac{2^1}{1!}+\frac{2^2}{2!}+....+\frac{2^{100}}{100!}\right]$ ។

ដំណោះស្រាយ

ចំណាំ $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+\frac{x}{1!}+\frac{x^2}{2!}+.....+\frac{x^n}{n!}+....$

ចំពោះ $x=2$, $n=100$ $\Rightarrow$ $e^2=1+\frac{2^1}{1!}+\frac{2^2}{2!}+....+\frac{2^{100}}{100!}+...$

$$\Leftrightarrow \frac{2^1}{1!}+\frac{2^2}{2!}+....+\frac{2^{100}}{100!}+...=e^2-1$$

ដូចនេះ $\left[\frac{2^1}{1!}+\frac{2^2}{2!}+....+\frac{2^{100}}{100!}\right]=e^2-1$ ។

លំហាត់ទី៥១: $4-\cfrac{4}{4-\cfrac{4}{4-\cfrac{4}{4-\cfrac{4}{....}}}}+x-\cfrac{6}{x-\cfrac{6}{x-\cfrac{6}{x-\cfrac{6}{....}}}}=4$ ។ រកតម្លៃ x

ដំណោះស្រាយ

យើងមាន $4-\cfrac{4}{4-\cfrac{4}{4-\cfrac{4}{4-\cfrac{4}{....}}}}+x-\cfrac{6}{x-\cfrac{6}{x-\cfrac{6}{x-\cfrac{6}{....}}}}=4\Rightarrow A+B=4$

$$A=\cfrac{4}{4-\cfrac{4}{4-\cfrac{4}{4-\cfrac{4}{....}}}}$$

$$\Rightarrow A=4-\frac{4}{A}\Rightarrow A^2-4A+4=0 \Rightarrow A=2$$

$$\Rightarrow 2+B=4\Rightarrow B=2$$

$$\Rightarrow \Rightarrow x\cfrac{6}{x-\cfrac{6}{x-\cfrac{6}{....}}} \qquad x-\frac{6}{2}=2\Rightarrow x=5$$

ដូចនេះ $x=5$ ។

លំហាត់ទី៥២: រក x,y,z, $(x,y,z\in\mathbb{R})$ ដែល $(1-x)^2+(x-y)^2+(y-z)^2+z^2=\frac{1}{4}$ ។

ដំណោះស្រាយ

យើងមាន:

$$a^2+b^2+c^2+d^2 \geq \frac{(a+b+c+d)^2}{4}$$

បើ $a=b=c$, $a^2+b^2+c^2+d^2=\dfrac{(a+b+c+d)^2}{4}$

$$(1-x)^2+(x-y)^2+(y-z)^2+z^2=\frac{(1-x+x-y+y-z+z)}{4}=\frac{1}{4}$$

គេបាន $1-x=x-y=y-z=z \Rightarrow x=\dfrac{3}{4},\ y=\dfrac{1}{2},\ z=\dfrac{1}{4}$

ដូចនេះ $x=\dfrac{3}{4},\ y=\dfrac{1}{2},\ z=\dfrac{1}{4}$ ។

លំហាត់ទី៥៣: រកផលបូក $\left(2+\dfrac{1}{2}\right)^2+\left(4+\dfrac{1}{4}\right)^2+\ldots+\left(2^n+\dfrac{1}{2^n}\right)^2$ ។

ដំណោះស្រាយ

យើងមាន $S=2^2+2+\dfrac{1}{2^2}+4^2+2+\dfrac{1}{4^2}+\ldots.+2^{2n}+2+\dfrac{1}{2^{2n}}$

$$S=\left(2^0+2^2+4^2+\ldots.+2^{2n}\right)+\left(2+2+\ldots.+2\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}+\ldots.+\frac{1}{2^{2n}}\right)$$

$$S=\frac{2^2\left[\left(2^2\right)^n-1\right]}{2^2-1}+2n+\frac{2^{-2}\left[1-\left(2^{-2}\right)^n\right]}{1-2^{-2}}=\frac{\left(2^2\right)^n-1}{3}\left(\frac{2^{2n+2}+1}{2^n}\right)+2n$$

ដូចនេះ $\left(2+\dfrac{1}{2}\right)^2+\left(4+\dfrac{1}{4}\right)^2+\ldots+\left(2^n+\dfrac{1}{2^n}\right)^2=\dfrac{\left(2^2\right)^n-1}{3}\left(\dfrac{2^{2n+2}+1}{2^n}\right)+2n$ ។

លំហាត់ទី៥៤: គណនា $A=\dfrac{\frac{1}{2}}{1+\frac{1}{2}}+\dfrac{\frac{1}{3}}{\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)}+\ldots+\dfrac{\frac{1}{1009}}{\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdot\ldots\cdot\left(1+\frac{1}{1009}\right)}$ ។

ដំណោះស្រាយ

យើងមាន

$$A=\frac{\frac{1}{2}}{\frac{3}{2}}+\frac{\frac{1}{3}}{\frac{3}{2}\cdot\frac{4}{3}}+\frac{\frac{1}{4}}{\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}}+\frac{\frac{1}{5}}{\frac{4}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}}+\ldots+\frac{\frac{1}{1009}}{\frac{4}{2}\cdot\frac{4}{3}\cdot\ldots\cdot\frac{1010}{1009}}$$

$$A=\frac{2}{2\cdot3}+\frac{2}{3\cdot4}+\frac{2}{4\cdot5}+\frac{2}{5\cdot6}+\ldots+\frac{2}{1009\cdot1010}$$

$$A=2\cdot\left(\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\frac{1}{5\cdot6}+\ldots+\frac{1}{1009\cdot1010}\right)$$

$$A=2\cdot\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\ldots+\frac{1}{1009}-\frac{1}{1010}\right)$$

$$=2\cdot\left(\frac{1}{2}-\frac{1}{1010}\right)$$

ដូចនេះ $A=1-\dfrac{1}{505}$ ។

លំហាត់ទី៥៥:

ចំពោះ $|x|\neq|y|\neq|z|$ ដែល $\dfrac{x^2}{y+x}+\dfrac{y^2}{y+z}+\dfrac{z^2}{x+z}=3$ គណនា $S=\dfrac{y^2}{y+x}+\dfrac{z^2}{y+z}+\dfrac{x^2}{x+z}$ ។

ដំណោះស្រាយ

យើងមាន:

$$S=\frac{y^2}{y+x}+\frac{z^2}{y+z}+\frac{x^2}{x+z}$$ ហើយ $-x^2+x^2=-y^2+y^2=-z^2+z^2=0$

$$\Rightarrow S=\frac{y^2}{y+x}+\frac{z^2}{y+z}+\frac{x^2}{x+z}=\frac{y^2-x^2+x^2}{y+x}+\frac{z^2-y^2+y^2}{y+z}+\frac{x^2-z^2+z^2}{x+z}$$

$$S=\left(\frac{y^2-x^2}{y+x}+\frac{x^2}{y+x}\right)+\left(\frac{z^2-y^2}{y+z}+\frac{y^2}{y+z}\right)+\left(\frac{x^2-z^2}{x+z}+\frac{z^2}{x+z}\right)$$

$$S=\left(y-x+\frac{x^2}{y+x}\right)+\left(z-y+\frac{y^2}{y+z}\right)+\left(x-z+\frac{z^2}{x+z}\right)=\frac{x^2}{y+x}+\frac{y^2}{y+z}+\frac{z^2}{x+z}$$

$$S=(y-x+z-y+x-z)+\left(\frac{x^2}{y+x}+\frac{y^2}{y+z}+\frac{z^2}{x+z}\right)=\frac{x^2}{y+x}+\frac{y^2}{y+z}+\frac{z^2}{x+z}$$

$$S=\frac{x^2}{y+x}+\frac{y^2}{y+z}+\frac{z^2}{x+z}=3$$

ដូចនេះ $S=\dfrac{x^2}{y+x}+\dfrac{y^2}{y+z}+\dfrac{z^2}{x+z}=3$ ។

លំហាត់ទី៥៦: បើ $a+b+c=20$ និង $\dfrac{1}{b+c}+\dfrac{1}{a+c}+\dfrac{1}{a+b}=1$ ។ រក $\dfrac{a}{b+c}+\dfrac{b}{a+c}+\dfrac{c}{a+b}=?$

ដំណោះស្រាយ

យើងបាន $(a+b+c)\left(\dfrac{1}{b+c}+\dfrac{1}{a+c}+\dfrac{1}{a+b}\right)=1\cdot(a+b+c)$

$$\Leftrightarrow \frac{a+b+c}{b+c}+\frac{a+b+c}{a+c}+\frac{a+b+c}{a+b}=a+b+c$$

$$\frac{a+(b+c)}{b+c}+\frac{b+(a+c)}{a+c}+\frac{c+(a+b)}{a+b}=a+b+c$$

$$\frac{a}{b+c}+1+\frac{b}{a+c}+1+\frac{c}{a+b}+1=20$$

$$\Rightarrow \frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}=17$$

ដូចនេះ $\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}=17$ ។

លំហាត់ទី៥៧ សម្រួលកន្សោម $\frac{x^8+x^6y^2+x^4y^4+x^2y^6+y^8}{x^4+x^3y+x^2y^2+xy^3+y^4}$ ។

ដំណោះស្រាយ

យើងមាន $\frac{x^8+x^6y^2+x^4y^4+x^2y^6+y^8}{x^4+x^3y+x^2y^2+xy^3+y^4}=\frac{A}{B}$

គេបាន $\frac{x^8+x^6y^2+x^4y^4+x^2y^6+y^8}{x^4+x^3y+x^2y^2+xy^3+y^4}=\frac{\left(x^8+x^4y^4+y^8\right)+\left(x^6y^2+x^2y^6\right)}{\left(x^4+x^2y^2+y^4\right)+\left(x^3y+xy^3\right)}$

$$=\frac{\left(x^8+x^4y^4+y^8\right)+x^2y^2\left(x^4+y^4\right)}{x^4+x^2y^2+y^4+xy\left(x^2+y^2\right)}$$

$$\begin{cases}\frac{A}{B}=\frac{\left(x^4-y^4\right)\times A}{\left(x^4-y^4\right)\times B}=\frac{\left(x^4-y^4\right)\times A}{\left(x^2+y^2\right)\left(x^2-y^2\right)\times B}\end{cases}$$

$$\frac{\left(x^4-y^4\right)\left(x^8+x^4y^4+y^8\right)+x^2y^2\left(x^4+y^4\right)\Big]}{\left(x^2+y^2\right)\left(x^2-y^2\right)\left(\left(x^4+x^2y^2+y^4\right)+xy\left(x^2+y^2\right)\right)}$$

$$=\frac{x^{12}-y^{12}+x^2y^2\left(x^8-y^8\right)}{\left(x^2+y^2\right)\left(x^6-y^6+xy\left(x^4-y^4\right)\right)}\Bigg)=\frac{x^{12}+x^{10}y^2-x^2y^{10}-y^{12}}{\left(x^2+y^2\right)\left(x^6+x^5y-xy^5-y^6\right)}$$

$$=\frac{x^{10}\left(x^2+y^2\right)-y^{10}\left(x^2+y^2\right)}{\left(x^2+y^2\right)\left(x^5(x+y)-y^5(x+y)\right)\Big]}=\frac{\left(x^2+y^2\right)\left(x^{10}-y^{10}\right)}{\left(x^2+y^2\right)(x+y)\left(x^5-y^5\right)}$$

$$=\frac{\left(x^2+y^2\right)\left(x^5-y^5\right)\left(x^5+y^5\right)}{\left(x^2+y^2\right)(x+y)\left(y^5-y^5\right)}=\frac{x^5+y^5}{y+y}=x^4-x^3y+x^2y^2-xy^3+y^4$$

ដូចនេះ $\frac{x^8+x^6y^2+x^4y^4+x^2y^6+y^8}{x^4+x^3y+x^2y^2+xy^3+y^4}=x^4-x^3y+x^2y^2-xy^3+y^4$ ។

លំហាត់ទី៥៨ គណនា $arctg\frac{1}{2}+arctg\frac{1}{8}+arctg\frac{1}{18}+arctg\frac{1}{32}+arctg\frac{1}{50}+arctg\frac{1}{72}=?$

ដំណោះស្រាយ

$$arctg(x)-\mathrm{arctg}(y)=\mathrm{acrtg}\frac{x-y}{1+x\cdot y}$$

$$arctg3-\mathrm{arctg}1=\mathrm{arctg}\frac{3-1}{1+1\cdot 3}=\mathrm{arctg}\frac{1}{2}$$

$$arctg5-\mathrm{arctg}\,3=\mathrm{arctg}\frac{5-3}{1+5\cdot 3}=\mathrm{arctg}\frac{1}{8}$$

$$\mathrm{arctg}\,7-\mathrm{arctg}\,5=\mathrm{arctg}\frac{7-5}{1+7\cdot 3}=\mathrm{arctg}\frac{1}{18}$$

$$\mathrm{arctg}\,9-\mathrm{arctg}\,7=\mathrm{arctg}\frac{9-7}{1+9\cdot 7}=\mathrm{arctg}\frac{1}{32}$$

$$\mathrm{arctg}11-\mathrm{arctg}11=\mathrm{arctg}\frac{13-11}{1+13\cdot 11}=\mathrm{arctg}\frac{1}{72}$$

$$\mathrm{arctg}13-\mathrm{arctg}11=\mathrm{arctg}\frac{13-11}{1+13\cdot 11}=\mathrm{arctg}\frac{1}{72}$$

$$\mathrm{arctg}13-\mathrm{arctg}1=\mathrm{arctg}\frac{13-1}{1+13\cdot 1}=\mathrm{arctg}\frac{6}{7}$$

$$arctg13-\mathrm{arctg}1=\mathrm{arctg}\frac{13-1}{1+1\cdot 13}=\mathrm{arctg}\frac{6}{7}$$

លំហាត់ទី៥៩ គណនា $\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\right)^{\frac{1}{4}}\cdot\frac{\sqrt[4]{5}-1}{\sqrt[4]{5}+1}$ ។

ដំណោះស្រាយ

តាមរូបមន្ត $a^{\frac{1}{n}}b=\left(a\cdot b^{n}\right)^{\frac{1}{n}}$

គេបាន $A=\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\right)^{\frac{1}{4}}\cdot\frac{\sqrt[4]{5}-1}{\sqrt[4]{5}+1}=\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\cdot\frac{(\sqrt[4]{5}-1)^{4}}{(\sqrt[4]{5}+1)^{4}}\right)^{\frac{1}{4}}=\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\cdot B\right)^{\frac{1}{4}}$

$$B=\frac{(\sqrt[4]{5}-1)^4}{(\sqrt[4]{5}+1)^4}=\frac{(\sqrt{5}+1-2\sqrt[4]{5})^2}{(\sqrt{5}+1+2\sqrt[4]{5})^2}=\frac{5+1+4\sqrt{5}+2(\sqrt{5}-2\sqrt[4]{125}-2\sqrt[4]{5})}{5+1+4\sqrt{5}+2(\sqrt{5}+2\sqrt[4]{125}+2\sqrt[4]{5})}$$

$$B=\frac{6+6\sqrt{5}-4(\sqrt[4]{125}+\sqrt[4]{5})}{6+6\sqrt{5}+4(\sqrt[4]{125}+\sqrt[4]{5})}=\frac{6(\sqrt{5}+1)-4\sqrt[4]{5}(\sqrt{5}+1)}{6(\sqrt{5}+1)+4\sqrt[4]{5}(\sqrt{5}+1)}=\frac{2(\sqrt{5}+1)(3-2\sqrt[4]{5})}{2(\sqrt{5}+1)(3+2\sqrt[4]{5})}$$

$$A=\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\cdot B\right)^{\frac{1}{4}}=\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\cdot\frac{2(\sqrt{5}+1)(3-2\sqrt[4]{5})}{2(\sqrt{5}+1)(3+2\sqrt[4]{5})}\right)^{\frac{1}{4}}=(1)^{\frac{1}{4}}=1$$

លំហាត់ទី៦០ គណនា $x\cdot\left(\left(x^2\right)^2\right)^2\cdot\left(\left(x^3\right)^3\right)^3\cdot\left(\left(x^4\right)^4\right)^4\cdot\ldots\cdot\left(\left(x^n\right)^n\right)^n$ បើ $x=\sqrt[n]{\sqrt[n(n+1)]{\sqrt[(n+1)]{1024}}}$ ។

ដំណោះស្រាយ

នាំឲ $x=\sqrt[n]{\sqrt[n(n+1)]{\sqrt[(n+1)]{1024}}}=2^{\frac{10}{[n(n+1)]^n}}$

យើងមាន $E=x\cdot\left(\left(x^2\right)^2\right)^2\cdot\left(\left(x^3\right)^3\right)^3\cdot\left(\left(x^4\right)^4\right)^4\cdot\ldots\cdot\left(\left(x^n\right)^n\right)^n$

$$\Rightarrow E=x^1\cdot x^8\cdot x^{27}\cdot x^{64}\cdot\ldots\cdot x^{n^3}$$

$$=x^{1+2^3+3^3+4^3+\ldots+n^3}$$

$$=x^{\left[\frac{n(n+1)}{2}\right]^2}$$

ចំណាំ $1^3+2^2+3^3+\ldots+n^3=\left[\frac{n(n+1)}{2}\right]^2$

$$E=\left(2^{\frac{10}{n(n+1)]^2}}\right)^{\left[\frac{n(n+1)}{2}\right]^2}$$

$$=2^{\frac{10}{4}}=4\sqrt{2}$$

លំហាត់ទី៦១ បើ $a+b+c=3$ និង $ab+ac+bc=2$ ។ រកតម្លៃ $a^3+b^3+c^3-3abc$?

ដំណោះស្រាយ

បើ $a+b+c=3$

យើងបាន $(x+y)^3=x^3+y^3+3xy(x+y)$,$x=a+b, y=c$ តែ $(a+b+c)^3=3^3$

$\Rightarrow (a+b)^3+c^3+3(a+b)c(a+b+c)=27$

$a^3+b^3+3ab(a+b)+c^3+3(a+b)c\cdot 3=27 \quad \Rightarrow a^3+b^3+3ab(3-c)+c^3+9(a+b)c=27$

ដោយ x, y, z គេបាន :

$a^3+b^3+c^3+9ab-3abc+9ac+9bc=27 \quad \Rightarrow a^3+b^3+c^3-3abc+9(ab+ac+bc)=27$

$a^3+b^3+c^3-3abc=27-9(ab+ac+bc) \Rightarrow a^3+b^3+c^3-3abc=27-9.2=9$

តាមទម្រង់ $a^3+b^3+c^3-3abc=(a+b+c)^3-9(ab+ac+bc)$

ដូចនេះ $a^3+b^3+c^3-3abc=9$ ។

លំហាត់ទី៦២: គេឲ x, y, z ជាចំនួនពិតផ្ទៀងផ្ទាត់ថា៖ $\begin{cases} x^2+4y^2+16z^2=48 \\ xy+4yz+2xz=24 \end{cases}$ ។

គណនាតម្លៃនៃ $x^2+y^2+z^2$ ។

ដំណោះស្រាយ

យើងមាន $x^2+4y^2+16z^2=48$

$xy+4yz+2xz=24 \quad \ldots(2)$

$2xy+8yz+4xz=48 \quad \ldots(2)\times 2$

$$x^2+4y^2+16z^2=2xy+8yz+4xz$$

$$2x^2+8y^2+32z^2=4xy+16yz+8xz$$

$$x^2-4xy+4y^2+4y^2-16yz+16z^2+16z^2-8xz+x^2=0$$

$$(x-2y)^2+(2y-4z)^2+(4z-x)^2=0$$

$$x=2y;\quad y=2z;$$

$$x^2=4y^2;\quad 4y^2=4\left(4z^2\right);\quad 16z^2=x^2;4z=x$$

$$x^2+4y^2+16z^2=48$$

$$16z^2+16z^2+16z^2=48$$

$$48z^2=48\to z=\pm1$$

$$\Rightarrow 4z=x\to x=\pm4\quad;\quad y=2z\to y=\pm2$$

$$x^2+y^2+z^2=(\pm4)^2+(\pm2)^2+(\pm1)^2$$

$$x^2+y^2+z^2=16+4+1$$

$$x^2+y^2+z^2=21$$

ដូចនេះ $x^2+y^2+z^2=21$ ។

លំហាត់ទី៦២

ដោះស្រាយសមីការ $x^3+\left(\log_2 5+\log_3 2+\log_5 3\right)x=\left(\log_2 3+\log_3 5+\log_5 2\right)x^2+1$ ។

ដំណោះស្រាយ

យើងមាន $x^3+\left(\log_2 5+\log_3 2+\log_5 3\right)x=\left(\log_2 3+\log_3 5+\log_5 2\right)x^2+1$

ហើយប្តូរអថេរគេបាន $\log_2 3=a\quad;\quad \log_3 5=b\quad;\quad \log_5 2=c$

តាមសមីការគេបាន៖

$$x^3+\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)x=(a+b+c)x^2+1$$

$$x^3-(a+b+c)x^2+\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)x-1=0$$

$$x^3-x^2(a+b+c)+x\left(\frac{ab+bc+ac}{abc}\right)-1=0$$

$$x^3-x^2(a+b+c)+x\left(\frac{ab+bc+ac}{abc}\right)-1=0$$

$$x^3\cdot abc-x^2(a+b+c)\cdot abc+x(ab+bc+ac)-abc=0$$

$$x^3-x^2(a+b+c)+x(ab+bc+ac)-abc=0$$

$$x=a \quad x=b \quad x=c$$

$$x_1=\log_2 3; x_2=\log_3 5; x_3=\log_5 2$$

ដូចនេះ $x_1=\log_2 3; x_2=\log_3 5; x_3=\log_5$ ។

លំហាត់ទី៦៣ គេឲ $x_1,x_2,\ldots,x_{2019}$ ជាចំនួនពិតដែលកំណត់ដោយ $x_1+x_2+\cdots+x_{2019}=1$ ហើយ $\frac{x_1}{1-x_1}+\frac{x_2}{1-x_2}+\cdots+\frac{x_{2019}}{1-x_{2019}}=1$ ។ គណនា $\frac{x_1^2}{1-x_1}+\frac{x_2^2}{1-x_2}+\cdots+\frac{x_{2019}^2}{1-x_{2019}}$

ដំណោះស្រាយ

ដោយ $x_1+x_2+\cdots+x_{2019}=1$

$$\frac{x_1}{1-x_1}+\frac{x_2}{1-x_2}+\cdots+\frac{x_{2019}}{1-x_{2019}}=1$$

តាង $F=\frac{x_1^2}{1-x_1}+\frac{x_2^2}{1-x_2}+\cdots+\frac{x_{2019}^2}{1-x_{2019}}$

$$1+x_1+1+x_2+\cdots+1+x_{2019}=1+1+1+\cdots+1$$

ដោយ $$1+x_1+1+x_2+\cdots+1+x_{2019}=1+1(2019)$$

$$1+x_1+1+x_2+\cdots+1+x_{2019}=2020\ldots(A)$$

$$\frac{x_1}{1-x_1}+1+\frac{x_2}{1-x_2}+1+\cdots+\frac{x_{2019}}{1-x_{2019}}+1=1+1+1+\cdots+1$$

និងដែល $$\frac{x_1+1-x_1}{1-x_1}+\frac{x_2+1-x_2}{1-x_2}+\cdots+\frac{x_{2019}+1-x_{2019}}{1-x_{2019}}=1+1(2019)$$

$$\frac{1}{1-x_1}+\frac{1}{1-x_2}+\cdots+\frac{1}{1-x_{2019}}=2020\ldots\ldots(B)$$

តាម $(A)\&(B)$ គេបាន

$$F=\frac{1-1+x_1^2}{1-x_1}+\frac{1-1+x_2^2}{1-x_2}+\cdots+\frac{1-1+x_{2019}^2}{1-x_{2019}}$$

$$F=\frac{1-\left(1-x_1^2\right)}{1-x_1}+\frac{1-\left(1-x_2^2\right)}{1-x_2}+\cdots+\frac{1-\left(1-x_{2019}^2\right)}{1-x_{2019}}$$

$$F=\frac{1}{1-x_1}-\frac{\left(1-x_1^2\right)}{1-x_1}+\frac{1}{1-x_2}-\frac{\left(1-x_2^2\right)}{1-x_2}+\cdots+\frac{1}{1-x_{2019}}-\frac{\left(1-x_{2019}^2\right)}{1-x_{2019}}$$

$$F=\frac{1}{1-x_1}-\frac{(1+x_1)(1-x_1)}{1-x_1}+\frac{1}{1-x_2}-\frac{(1+x_2)(1-x_2)}{1-x_2}+\cdots+\frac{1}{1-x_{2019}}-\frac{(1+x_{2019})(1-x_{2019})}{1-x_{2019}}$$

$$F=\frac{1}{1-x_1}-(1+x_1)+\frac{1}{1-x_2}-(1+x_2)+\cdots+\frac{1}{1-x_{2019}}-(1+x_{2019})$$

$$F=\frac{1}{1-x_1}+\frac{1}{1-x_2}+\cdots+\frac{1}{1-x_{2019}}-\left[(1+x_1)+(1+x_2)+\cdots+(1+x_{2019})\right]$$ ។

លំហាត់ទី៦៤ ចូររកតម្លៃ $(x,y\in Z):55\left(x^3y^3+x^2+y^2\right)=229\left(xy^3+1\right)$ ។

ដំណោះស្រាយ

យើងមាន $55\left[x^2\left(xy^3+1\right)+y^2\right]=229\left(xy^3+1\right)$

$$\Rightarrow \frac{x^2\left(xy^3+1\right)+y^2}{xy^3+1}=\frac{229}{55}\Rightarrow x^2+\frac{y^2}{xy^3+1}=4+\frac{9}{55}$$

$$x^2+\frac{1}{\frac{xy^3+1}{y^2}}=4+\frac{1}{\frac{55}{9}}\Rightarrow x^2+\frac{1}{xy+\frac{1}{y^2}}=4+\frac{1}{6+\frac{1}{9}}$$

លំហាត់ទី៦៥ គណនា $\sum_{n=1}^{1.000.000}\left\lfloor\sqrt{n}\right\rfloor$ ។

ដំណោះស្រាយ

តាង $k=1000$ គេបានៈ

$$=k+\sum_{i=1}^{k-1}\sum_{n=i^2}^{(i+1)^2-1}\left\lfloor\sqrt{n}\right\rfloor$$

$$=k+\sum_{i=1}^{k-1}\sum_{n=i^2}^{(i+1)^2-1} i=k+\sum_{i=1}^{k-1}(2i+1)i$$

$$=k+2\sum_{i=1}^{k-1}i^2+\sum_{i=1}^{k-1}i$$

$$=k+\frac{k(k-1)(2k-1)}{3}+\frac{k(k-1)}{2}$$

$$=\frac{k(4k^2-3k+5)}{6}$$

ចំពោះ $k=1000 \quad \Rightarrow 666167500$ ។

លំហាត់ទី៦៦ គណនា $\frac{sin1}{cos0cos1}+\frac{sin1}{cos1cos2}+...+\frac{sin1}{cos44cos45}=?$

ដំណោះស្រាយ

យើងមាន $\tan(k+1)-\tan k=\frac{\sin(k+1-k)}{\cos(k)\cos(k+1)}=\frac{\sin 1}{\cos(k)\cos(k+1)}$

គេបានៈ

$$\sum_{k=0}^{44}\frac{\sin 1}{\cos k\cos(k+1)}=\sum_{k=0}^{44}\left(\tan(k+1)-\tan k\right)=\tan 45-\tan 0=1$$

លំហាត់ទី៦៧ រកគ្រប់អនុគមន៍ $f:\mathbb{R}\to\mathbb{R}$ ដែលផ្ទៀងផ្ទាត់លក្ខខ័ណ្ឌថា ៖

$$f(f(x))=x$$

$$f(f(x)f(y)+x+y))=f(x)+f(y)+xy$$ ។

ដំណោះស្រាយ

បើ $x=0, y=f(0)$ និង $0=f(0)$ ហើយ $y=f(-1)$

ជំនួស x ដោយ $f(x)$ គេបាន $-1=f(x)+f(-1)+xf(-1)$

$\Rightarrow f(x)=f(-1)-xf(-1)+1$

នោះ $x=f(f(x))=f(-1)-f(-1)(f(1)-xf(1)+1)+1=xf(-1)^2-f(-1)^2+1$ គ្រប់តម្លៃ x

បញ្ជាក់ថា $f(-1)=\pm1$, $f(-1)=-1$

បញ្ជាក់ថា $f(x)=x$ ដោយ $f(-1)=1$

បញ្ជាកនូវ $f(x)=-x+2$ បន្តែនៅក្នុងផ្ទុយជាមួយ $f(0)=0$

, គេបាន $f(x)=x$

ដូចនេះ $f(x)=x$ ។

លំហាត់ទី៦៨ គណនា $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots+\frac{1}{98\cdot99\cdot100}$ ។

ដំណោះស្រាយ

យើងមាន $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots+\frac{1}{98\cdot99\cdot100}$

គេបាន $a_n = \dfrac{1}{n\cdot(n+1)\cdot(n+2)} = \dfrac{1}{2}\cdot\dfrac{2}{n\cdot(n+1)\cdot(n+2)}$

$$a_n = \frac{1}{2}\cdot\frac{(n+2)-n}{n\cdot(n+1)\cdot(n+2)} = \frac{1}{2}\left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right)$$

$$= \frac{1}{2}\left(\left(\frac{1}{n} - \frac{1}{n+1}\right) - \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right)$$

$$\sum_{n=1}^{n} a_n = \sum_{n=1}^{n} \frac{1}{2}\left(\left(\frac{1}{n} - \frac{1}{n+1}\right) - \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right)$$

$$= \frac{1}{2}\left(\left(\frac{1}{1} - \frac{1}{99}\right) - \left(\frac{1}{2} - \frac{1}{100}\right)\right)$$

$$\sum_{n=1}^{n} a_n = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{99\cdot 100}\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{99\cdot 100}\right)$$

លំហាត់ទី៦៨ គណនា $N = \lim\limits_{x\to 0} \dfrac{\sin(ax)+\sin(bx)-\sin((a+b)x)}{\ln\left(abx^3+(a+b)x+1\right)-\ln((a+b)+1)}$ ។

ដំណោះស្រាយ

យើងមាន $\lim\limits_{x\to 0} \dfrac{\sin(ax)-ax+\sin(bx)-bx-\sin((a+b)x)+(a+b)x}{\ln\left(abx^3+(a+b)x+1\right)-\ln((a+b)+1)} \div \dfrac{x^3}{x^3}$

តាង $I = \lim\limits_{x\to 0} \dfrac{(\sin(ax)-ax)+(\sin(bx)-bx)-(\sin((a+b)x)-(a+b)x)}{x^3}$

$$I = \lim_{x\to 0} \frac{a^3(\sin ax - ax)}{(ax)^3} + \frac{b^3(\sin bx - bx)}{(bx)^3} - \frac{(a+b)^3(\sin(a+b)x-(a+b)x)}{((a+b)x)^3}$$

$$I = \frac{-1}{6}\left(a^3+b^3-(a+b)^3\right) = \frac{1}{2}ab(a+b)$$

តាង $M = \lim\limits_{x\to 0} \dfrac{\ln\left(abx^3+(a+b)x+1\right)-\ln((a+b)x+1)}{x^3}$

$$M=\lim_{x\to 0}\frac{\ln\left(\frac{\left(abx^3+(a+b)x+1\right)}{(a+b)x+1}\right)}{x^3}$$

$$M=\lim_{x\to 0}\frac{\ln\left(\frac{\left(abx^3\right)}{(a+b)x+1}+1\right)}{\left(\frac{\left(abx^3\right)}{(a+b)x+1}\right)}\times\frac{\left(\frac{abx^3}{(a+b)x+1}\right)}{x^3}=1\times ab=ab$$

$$I=\frac{1}{2}ab(a+b)\quad ,M=ab$$

$$N=\frac{I}{M}=\frac{\frac{1}{2}ab(a+b)}{ab}=\frac{1}{2}(a+b)$$

លំហាត់ទី៦៩ បើ $abc=1$ ។ បង្ហាញថា $\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}=\frac{1+b^{-1}}{1+a+b^{-1}}$ ។

ដំណោះស្រាយ

គេមាន $\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}$

គេបាន $\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}=\frac{1}{1+a+b^{-1}}+\frac{ac}{ac\left(1+b+c^{-1}\right)}$

$=\frac{1}{1+a+b^{-1}}+\frac{ac}{ac+abc+a}$ ព្រោះ $abc=1$

$=\frac{1}{1+a+b^{-1}}+\frac{b^{-1}}{b^{-1}+1+a}$

$=\frac{1}{1+a+b^{-1}}+\frac{b^{-1}}{1+a+b^{-1}}$

$\Rightarrow\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}=\frac{1+b^{-1}}{1+a+b^{-1}}$

ដូចនេះ $\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}=\frac{1+b^{-1}}{1+a+b^{-1}}$ ។

លំហាត់ទី៧០

ចំពោះ $a,b,c>0$.។ បង្ហាញថា $\left(1+\frac{b^3c^3}{a^3}\right)\left(1+\frac{c^3a^3}{b^3}\right)\left(1+\frac{a^3b^3}{c^3}\right)\geq\left(1+\frac{bc^3}{a}\right)\left(1+\frac{ca^3}{b}\right)\left(1+\frac{ab^3}{c}\right)$ ។

ដំណោះស្រាយ

$$(1)\Leftrightarrow\left(a^3+b^3c^3\right)\left(b^3+c^3a^3\right)\left(c^3+a^3b^3\right)\geq a^2b^2c^2\left(a+bc^3\right)\left(b+ca^3\right)\left(c+ab^3\right)$$

តាមវិសមភាព Holder គេបាន

$$\begin{cases}\left(a^3+b^3c^3\right)^2\left(b^3+c^3a^3\right)^{1/3}\geq a^2b+b^2c^3a=ab\left(a+bc^3\right)\\ \left(b^3+c^3a^3\right)^{2/3}\left(c^3+a^3b^3\right)^{1/3}\geq b^2c+c^2a^3b=bc\left(b+ca^3\right)\\ \left(c^3+a^3b^3\right)^{2/3}\left(a^3+b^3c^3\right)^{1/3}\geq c^2a+a^2b^3c=ca\left(c+ab^3\right)\end{cases}$$

$$\Rightarrow\left(a^3+b^3c^3\right)\left(b^3+c^3a^3\right)\left(c^3+a^3b^3\right)\geq a^2b^2c^2\left(a+bc^3\right)\left(b+ca^3\right)\left(c+ab^3\right)$$

$$\Leftrightarrow\left(1+\frac{b^3c^3}{a^3}\right)\left(1+\frac{c^3a^3}{b^3}\right)\left(1+\frac{a^3b^3}{c^3}\right)\geq\left(1+\frac{bc^3}{a}\right)\left(1+\frac{ca^3}{b}\right)\left(1+\frac{ab^3}{c}\right)$$

ដូចនេះ $\left(1+\frac{b^3c^3}{a^3}\right)\left(1+\frac{c^3a^3}{b^3}\right)\left(1+\frac{a^3b^3}{c^3}\right)\geq\left(1+\frac{bc^3}{a}\right)\left(1+\frac{ca^3}{b}\right)\left(1+\frac{ab^3}{c}\right)$ ។

លំហាត់ទី៧១ គេឲ $n\in IN^*$ ស្វីត (S_n) កំណត់ដោយ: $S_n=\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\sin\frac{3\pi}{n}+.......$

1. គេមាន : $z=\cos\frac{\pi}{n}+i\sin\frac{\pi}{n}$

$a.$ គណនា $S=1+z+z^2+z^3+\cdots+z^{n-2}+z$ ។

2. បង្ហាញថា $S_n=\frac{1}{\tan\frac{\pi}{2n}}$ និង 3. គណនា $\lim_{n\to+\infty}\left(\frac{S_n}{n}\right)$ ។

ដំណោះស្រាយ

$a,$ គណនា $S=1+z+z^2+z^3+....+z^{n-2}+z^{n-1}$

យើងមាន $S=1+z+z^2+z^3+\cdots+z^{n-2}+z^{n-1}$

គេបាន $z=1\times\dfrac{1-z^n}{1-z}$

$$=\frac{1-\left(\cos\frac{\pi}{n}+i\sin\frac{\pi}{n}\right)^n}{1-\left(\cos\frac{\pi}{n}+i\sin\frac{\pi}{n}\right)}$$

$$=\frac{1-\cos\pi-\sin\pi}{1-\cos\frac{\pi}{n}-i\sin\frac{\pi}{n}}$$

$$=\frac{1-(-1)}{1-\cos\frac{\pi}{n}-i\sin\frac{\pi}{n}}=\frac{2}{2\sin^2\frac{\pi}{2n}-2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}$$

$$=\frac{2}{2\left(\sin\frac{\pi}{2n}\right)\left(\sin\frac{\pi}{2n}-i\cos\frac{\pi}{2n}\right)}=$$

$$\frac{1}{\left(\sin\frac{\pi}{2n}\right)\left(\sin\frac{\pi}{2n}-i\cos\frac{\pi}{2n}\right)}$$

$$=\frac{\left(\sin\frac{\pi}{2n}+i\cos\frac{\pi}{2n}\right)}{\left(\sin\frac{\pi}{2n}\right)\left(\sin\frac{\pi}{2n}-i\cos\frac{\pi}{2n}\right)\left(\sin\frac{\pi}{2n}+i\cos\frac{\pi}{2n}\right)}=\frac{\left(\sin\frac{\pi}{2\pi}+i\cos\frac{\pi}{2n}\right)}{\left(\sin\frac{\pi}{2n}\right)\left(\sin^2\frac{\pi}{2n}+\cos^2\frac{\pi}{2n}\right)}$$

$$=\frac{\left(\sin\frac{\pi}{2n}+i\cos\frac{\pi}{2n}\right)}{\left(\sin\frac{\pi}{2n}\right)\left(\sin^2\frac{\pi}{2n}+\cos^2\frac{\pi}{2n}\right)}=\frac{\sin\frac{\pi}{2n}+i\cos\frac{\pi}{2n}}{\sin\frac{\pi}{2n}}=\int_{-\infty}^{\infty}\frac{\cos\frac{\pi}{2n}}{\sin\frac{\pi}{2n}}=1+\frac{1}{\tan\frac{\pi}{2n}}$$

ដូចនេះ $S_n=\dfrac{1}{\tan\frac{\pi}{2n}}$ ។

2.

2. បង្ហាញថា $S_n = \dfrac{1}{\tan\dfrac{\pi}{2n}}$

យើងមាន $S_n = \dfrac{1}{\tan\dfrac{\pi}{2n}}$

គេបាន:

$$S_n = \sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \sin\frac{3\pi}{n} + \cdots + \sin\frac{(n-1)\pi}{n}$$
$$S = 1 + z + z^2 + z^3 + \cdots + z$$
$$= 1 + \left(\cos\frac{\pi}{n} + i\sin\frac{\pi}{n}\right) + \left(\cos\frac{\pi}{n} + i\sin\frac{\pi}{n}\right)^2 + \left(\cos\frac{\pi}{n} + i\sin\frac{\pi}{n}\right)^3 + \ldots + \left(\cos\frac{\pi}{n} + \sin\frac{\pi}{n}\right)$$
$$= 1 + \cos\frac{\pi}{n} + i\sin\frac{\pi}{n} + \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n} + \cos\frac{3\pi}{n} + i\sin\frac{3\pi}{n} + \cdots + \cos\frac{(n-1)\pi}{n} + i\sin\frac{(n-1)\pi}{n}$$
$$= \cos\frac{\pi}{n} + \cos\frac{2\pi}{n} + \cos\frac{3\pi}{n} + \cdots + \cos\frac{(n-1)\pi}{n} + i\left(\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \sin\frac{3\pi}{n} + \cdots + \sin\frac{(n-1)\pi}{n}\right)$$
$$= 1 + \frac{1}{\tan\dfrac{\pi}{2n}} i$$
$$\Rightarrow S_n = \sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \sin\frac{3\pi}{n} + \cdots + \sin\frac{(n-1)\pi}{n} = \frac{1}{\tan\dfrac{\pi}{2n}}$$

3. គណនា $\lim\limits_{n\to+\infty}\left(\dfrac{S_n}{n}\right)$

$$\lim_{n\to+\infty}\left(\frac{S_n}{n}\right) = \lim_{n\to+\infty}\left(\frac{\dfrac{1}{\tan\dfrac{\pi}{2n}}}{n}\right) = \sin_{n\to+\infty}\left(\frac{1}{n\tan\dfrac{\pi}{2n}}\right)$$

តាង $N = \dfrac{1}{n}\sin\cdots+\infty \Rightarrow N \to 0$

$$\lim_{n\to+\infty}\left(\frac{1}{n\tan\frac{\pi}{2n}}\right)=l\,\text{im}_{x\to 0}\left(\frac{N}{\tan\frac{\pi N}{2}}\right)$$

$$=\lim_{N\to 0}\left(\frac{\frac{\pi N}{2}}{\tan\frac{\pi N}{2}}\times\frac{2}{\pi}\right)=\frac{2}{\pi}\lim_{N\to 0}\left(\frac{\frac{\pi N}{2}}{\tan\frac{\pi N}{2}}\right)=\frac{2}{\pi}\times 1=\frac{2}{\pi}$$

លំហាត់ទី៣

គេឲចំនួនពិត $a\neq 0$ ។ បង្ហាញថា $\sqrt{a^2+\sqrt{a^2+\ldots+\sqrt{a^2}}}<\frac{1}{2}+\frac{1}{8}(\sqrt{1+16a^2}+\sqrt{9+16a^2})$ ។

ដំណោះស្រាយ

ដោយ $x_n=\sqrt{a^2+\sqrt{a^2+\ldots+\sqrt{a^2}}}$

$$\Rightarrow x_n^2=a^2+x_{n-1}$$

យើងត្រូវស្រាយ (x_n) ។ដូច្នេះ

$$x_n^2<a^2+x_n \Leftrightarrow x_n^2-x_n-a^2<0$$

$$\Leftrightarrow \frac{1}{2}(1-\sqrt{1+4a^2})<x_n<\frac{1}{2}(1+\sqrt{1+4a^2})$$

$$\Rightarrow x_n<\frac{1}{2}+\sqrt{1+4a^2}\,(1)$$

ហើយតាមវិសមភាព៖

$$\sqrt{(a_1+a_2)^2+(b_1+b_2)^2}\leq\sqrt{a_1^2+b_1^2}+\sqrt{a_2^2+b_2^2}\quad(a_1,a_2,b_1,b_2\in R)$$

យើងមាន $\sqrt{1+4a^2}=\sqrt{\left(\frac{1}{4}+\frac{3}{4}\right)^2+(a+a)^2}$

$$\leq\sqrt{\frac{1}{16}+a^2}+\sqrt{\frac{9}{16}+a^2}=\frac{1}{4}(\sqrt{1+16a^2}+\sqrt{9+16a^2})(2)$$

តាម (1) & (2) គេបាន $\sqrt{a^2+\sqrt{a^2+\ldots+\sqrt{a^2}}}<\frac{1}{2}+\frac{1}{8}(\sqrt{1+16a^2}+\sqrt{9+16a^2})$ ។

លំហាត់ទី៧៤ គណនា $\frac{1}{1-10+50}+\frac{4}{4-20+50}+\frac{9}{9-30+50}+\ldots+\frac{100}{100-100+50}$ ។

ដំណោះស្រាយ

យើងមាន $\frac{1}{1-10+50}+\frac{4}{4-20+50}+\frac{9}{9-30+50}+\ldots+\frac{100}{100-100+50}$

យើងបាន : $\sum_{n=1}^{10}\frac{n^2}{n^2-10n+50}$

$$\Rightarrow A_n=\frac{n^2}{(n-5)^2+5^2}$$

$$A_{5-k}=\frac{(5-k)^2}{k^2+5^2}$$

$$A_{5-k}=\frac{(5+k)^2}{k^2+5^2}$$

$$A_{5-k}+A_{5+k}=\frac{(5-k)^2+(5+k)^2}{k^2+5^2}=2$$

$$A_{5-k}+A_{5+k}=\frac{(5-k)^2+(5+k)^2}{k^2+5^2}=2$$

$$nA_1+A_2+A_3+\ldots+A_{10}n=\left(A_1+A_9\right)+\left(A_2+A_8\right)+\left(A_3+A_7\right)+\left(A_4+A_6\right)+A_5+A_{10}\text{ d}$$

$$=2+2+2+2+\frac{5^2}{5^2}+\frac{10^2}{2\times5^2}=2+2+2+2+1+2=11$$

ដូចនេះ $\frac{1}{1-10+50}+\frac{4}{4-20+50}+\frac{9}{9-30+50}+\ldots+\frac{100}{100-100+50}=11$ ។

លំហាត់ទី៧៥ គណនា $\frac{1}{2020}+\frac{2}{2020^2}+\ldots+\frac{2019}{2020^{2019}}$ ។

ដំណោះស្រាយ

យើងមាន $1+x+x^2+\ldots+x^n=\frac{1-x^{n+1}}{1-x}\Rightarrow 1+2x+3x^2\ldots+nx^{n-1}=\frac{1-x^{n+1}}{(1-x)^2}-\frac{(n+1)x^n}{1-x}$

$$x+2x^2+3x^3\ldots+nx^n=\frac{x\left(1-x^{n+1}\right)}{(1-x)^2}-\frac{(n+1)x^{n+1}}{1-x}$$ ដែល $x=\frac{1}{2020}$, $n=2019$

គេបាន:

$$\frac{1}{2020^{+}}+\frac{2}{2020^2}+\frac{3}{2020^3}+\ldots+\frac{2019}{2020^{2019}}=\frac{1-\frac{1}{2020^{2020}}}{2020\left(1-\frac{1}{2020}\right)^2}-\frac{2020}{\left(1-\frac{1}{2020}\right)2020^{2020}}$$

$$\frac{1}{2020}+\frac{2}{2020^2}+\frac{3}{2020^3}+\ldots+\frac{2019}{2020^{2019}}=\frac{2020^{2020}-1}{2019^2\times2020^{2019}}-\frac{1}{2019\times2020^{2018}}n$$

$$\Rightarrow\frac{1}{2020}+\frac{2}{2020^2}+\frac{3}{2020^3}+\ldots+\frac{2019}{2020^{2019}}=\frac{2020\left(2020^{2019}-2019\right)-1}{2019^2\times2020^{2019}}$$

$$S=\sum_{k=1}^{2019}k\left(\frac{1}{2020}\right)^k=w\left(\frac{1}{2020}\right)$$

$$w(x)=\sum_{k=1}^{2019}kx^k\quad,\ x\neq1$$

$$\sum_{k=0}^{2019}x^k=\frac{x^{2020}-1}{x-1}$$

$$\Rightarrow\sum_{k=1}^{2019}kx^{k-1}=\frac{d}{dx}\left(\frac{x^{2020}-1}{x-1}\right)=\frac{2020x^{2019}(x-1)-\left(x^{2020}-1\right)}{(x-1)^2}$$

$$=\frac{2019x^{2020}-2020x^{2019}+1}{(x-1)^2}$$

$$\Rightarrow\sum_{k=1}^{2019}kx^k=\frac{2019x^{2021}-2020x^{2020}+x}{(x-1)^2}$$

$$S=\frac{2019\left(\frac{1}{2020}\right)^{2021}-2020\left(\frac{1}{2020}\right)^{2020}+\frac{1}{2020}}{\left(1-\frac{1}{2020}\right)^2}$$

លំហាត់ទី៣៦ គណនា $\lim_{n\to\infty}(1+x)\left(1+x^2\right)\left(1+x^4\right)\ldots\left(1+x^{2^n}\right)$ ។ បើ $x=\frac{3}{2}$ ។

ដំណោះស្រាយ

$$=\lim_{n\to\infty}\frac{(1-x)(1+x)\left(1+x^2\right)\left(1+x^4\right)\ldots\left(1+x^{2^n}\right)}{(1-x)}$$

$$=\lim_{n\to\infty}\frac{\left(1-x^2\right)\left(1+x^2\right)\left(1+x^4\right)\ldots\left(1+x^{2^n}\right)}{(1-x)}$$ យក $x=\frac{3}{2}$

$$=\lim_{n\to\infty}\frac{\left(1-x^{2^{n+1}}\right)}{(1-x)}$$ ព្រោះ $\lim_{n\to\infty}\left(\frac{2}{3}\right)^{2^{n+1}}=0$

$=3$ ។

លំហាត់ទី៧៧ បើ $(1!+2!+3!+\cdots+2000!)^{2001}\equiv x(\bmod 7)$ ។ រកតម្លៃ x?

ដំណោះស្រាយ

គេបាន :

$1!\equiv 1(\bmod 7)$

$4!\equiv 3(\bmod 7)$

$2!\equiv 2(\bmod 7)$

$5!\equiv 1(mod 7)$

$3!\equiv -1(\bmod 7)$

$6!\equiv -1\bmod 7)$

$1!+2!+3!+\ldots+2000!=5(\bmod 7)=-2(\bmod 7)$

$(11+2!+\cdots+2000)^{2001}\equiv(-2)^{2001}(\bmod 7)\equiv(-2)^{3\cdot 667}(\bmod 7)$

$\equiv(-1)^{667}(\bmod 7)\equiv(-1)(\bmod 7)=6(\bmod 7)$

$x=6$

លំហាត់ទី៧៨ គណនា $\sqrt{2!^2+2!\sqrt{3!^2+3!\sqrt{4!^2+4!\sqrt{\ldots..}}}}$ ។

ដំណោះស្រាយ

យើងបាន $8=3!+2!=\sqrt{(3!+2!)^2}$

$= \sqrt{2!(2!+2.3!+3.3!)}$

$= \sqrt{2!\{2!+2.3!+3!(4-1)\}}$

$= \sqrt{2!\{2!+3!+4!} = \sqrt{2!\{2!+\sqrt{(3!+4!)^2}}$

$= \sqrt{2!^2+2!\sqrt{3!(3!+2.4!+4.4!)}}$

$= \sqrt{2!^2+2!\sqrt{3!\{3!+2.4!+4!(5-1)\}}}$

$= \sqrt{2!^2+2!\sqrt{3!\{3!+(4!+5!)}}$

$= \sqrt{2!^2+2!\sqrt{3!^2+3!\sqrt{(4!+5!)^2}}}$ ......

$= \sqrt{2!^2+2!\sqrt{3!^2+3!\sqrt{4!^2+4!\sqrt{......}}}}$

ដូចនេះ $8 = \sqrt{2!^2+2!\sqrt{3!^2+3!\sqrt{4!^2+4!\sqrt{......}}}}$ ។

លំហាត់ទី៣៩ $53^{\log_{\frac{1}{\sqrt{e^{\pi}}}}\left[\sqrt[9999999]{(x+11)!}\right]} = 1$ ។ គណនា $\frac{x_1}{x_2}+0,9$ ។

ដំណោះស្រាយ

$\Rightarrow \log_{\frac{1}{\sqrt{e^{\pi}}}}[(\sqrt[9999999]{(x+11)!}] = 0$

$\Rightarrow \sqrt[9999999]{(x+11)!} = 1$

$\Rightarrow (x+11)! = 1$

$\Rightarrow x+11 = 0 \text{ or } 1$

$\Rightarrow x = -11 \text{ or } -10$

$\Rightarrow x_1 = -11, x_2 = -10 \text{ or } x_1 = -10, x_2 = -11$

$\Rightarrow \frac{x_1}{x_2}+0.9 = \frac{11}{11}+\frac{9}{10} = 2 or$

$\Rightarrow \frac{x_1}{x_2}+0.9 = \frac{10}{11}+\frac{9}{10} = \frac{199}{110}$

ដូចនេះ $\frac{x_1}{x_2}+0,9=\frac{199}{110}$ ។

លំហាត់ទី៨០ គណនា

$$A=\cos\frac{\pi}{100}+\cos\frac{2\pi}{100}+\cos\frac{3\pi}{100}+\cdots+\cos\frac{50\pi}{100}=???$$

$$B=\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots=???$$

ដំណោះស្រាយ

$A,$ យើងមាន $\sum_{k=0}^{n}\cos kx=\frac{1}{2}\left(1+\frac{\sin(n+1/2)x}{\sin x/2}\right)$

$\Rightarrow\sum_{k=0}^{n}\cos\frac{k\pi}{2n}=\frac{1}{2}\left(1+\cot\frac{\pi}{4n}\right)$ ចំពោះ $n=50$

$$\Rightarrow 1+\cos\frac{\pi}{100}+\cos\frac{2\pi}{100}+\cdots+\cos\frac{50\pi}{100}\approx 32$$

$$\Rightarrow \quad \cos\frac{\pi}{100}+\cos\frac{2\pi}{100}+\cdots+\cos\frac{50\pi}{100}\approx 31$$

$B,$ យើងមាន $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$ ចំពោះ $x=1$

$$\Rightarrow\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\cdots=e$$

ដូចនេះ $A=31, B=e$ ។

លំហាត់ទី៨១ គណនា $S=1^2\cdot 2!+2^2\cdot 3!+3^2\cdot 4!+4^2\cdot 5!+\ldots+2019^2\cdot 2020!-2018\cdot 2021!$ ។

ដំណោះស្រាយ

គេបាន $1^2\cdot 2!=2$

$2^2\cdot 3!=(1\cdot 4-0)\cdot 3!=4!$

$3^2 \cdot 4! = (2\cdot 5-1)\cdot 4! = 2\cdot 5! - 4!$

$4^2 \cdot 5! = (3\cdot 6-2)\cdot 5! = 3\cdot 6! - 2\cdot 5!$

$5^2 \cdot 6! = (4\cdot 7-3)\cdot 6! = 4\cdot 7! - 3\cdot 6!$

$\vdots$

$2019^2 \cdot 2020! = (2018\cdot 2021-2017)\cdot 2020! = 2018\cdot 2021! - 2017\cdot 2020!$

$\Rightarrow S = 2 + 2018\cdot 2021! - 2018\cdot 2021! = 2$

ដូចនេះ $S = 2$ ។

លំហាត់ទី៨២ គណនា $S = \sqrt{2016 + 2007\sqrt{2018 + 2009\sqrt{2020 + 2011\sqrt{2022 + \ldots\ldots}}}}$ ។

ដំណោះស្រាយ

ចំពោះ $a = 2016$

$$(a-6) = \sqrt{(a-6)^2} = \sqrt{a + \left(a^2 - 13a + 36\right)}$$

$$= \sqrt{a + (a-9)(a-4)} = \sqrt{a + (a-9)\sqrt{(a-4)^2}}$$

$$= \sqrt{a + (a-9)\sqrt{(a+2) + (a-7)(a-2)}}$$

$$= \sqrt{a + (a-9)\sqrt{(a+2) + (a-7)\sqrt{(a-2)^2}}}$$

$$= \sqrt{a + (a-9)\sqrt{(a+2) + (a-7)/(a+4) + (a-5)a}}$$

$$= \sqrt{a + (a-9)\sqrt{(a+2) + (a-7)/(a+4) + (a-5)\sqrt{(a+6) + (a-3)(a+2)}}}$$

$$\sqrt{2016 + 2007\sqrt{2018 + 2009\sqrt{2020 + 2011\sqrt{2022 + 2013\ldots}}}} = 2010$$

ដូចនេះ $S = \sqrt{2016 + 2007\sqrt{2018 + 2009\sqrt{2020 + 2011\sqrt{2022 + \ldots\ldots}}}} = 2010$ ។

លំហាត់ទី៨២ បង្ហាញថា $\sin\left(\sum_{k=1}^{n} x_k\right)| \le \sum_{k=1}^{n} \sin x_k \; (0 \le x_k \le \pi, k = 1, 2, \dots, n)$ ។

ដំណោះស្រាយ

ចំពោះ $\forall x, y \in [0, \pi]$ ហើយ $\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$

$$\Rightarrow \sin(x+y) = \leqslant |\sin(x+y) = |\sin(x)\cos(y) + \cos(x)\cos(y)||$$

$$\leqslant \sin(x)|\cos(y)| + |\cos(x)|\sin(y) \leqslant \sin(x)\times 1 + \sin(y)\times 1$$

$$\Rightarrow \sin(x+y) \leqslant \sin(x) + \sin(y)$$

ដោយ $\sin\left(\sum_{k=1}^{n+1} X_k\right) = \sin\left(\sum_{k=1}^{n} X_k + X_{n+1}\right) = \sin\left(\sum_{k=1}^{n} X_k\right)\sin\left(X_{n+1}\right) + \mathrm{Cos}\left(\sum_{k=1}^{n} X_k\right)\sin\left(X_{n+1}\right)$

$\leqslant \left|\sin\left(\sum_{k=1}^{n} X_k\right|\sin\left(X_{n+1}\right)\right| + \left|\cos\left(\sum_{k=1}^{n} X_k\right)\sin\left(X_{n+1}\right)\right|$ តែ $|\sin(x)|, |\cos(x)| \leqslant 1$

$$\leqslant \left|\sum_{k=1}^{n} \sin\left(X_k\right)\right| + \left|\sin\left(x_{n+1}\right)\right| = \sum_{k=1}^{n} \sin\left(X_k\right) + \sin\left(X_{n+1}\right) = \sum_{k=1}^{n+1} \sin\left(x_k\right)$$

$\left|\sum \sin\left(x_k\right)\right| = \sum \left|\sin\left(x_k\right)\right| = \sum \sin\left(x_k\right)$ ករណី $x_k \in [0, \pi], \sin \ge 0$

ដូចនេះ $\sin\left(\sum_{k=1}^{n} x_k\right)| \le \sum_{k=1}^{n} \sin x_k \; (0 \le x_k \le \pi, k = 1, 2, \dots, n)$ ។

លំហាត់ទី៨៣ បង្ហាញថា $\cos\frac{2\pi}{13} + \cos\frac{6\pi}{13} + \cos\frac{8\pi}{13} = \frac{\sqrt{13}-1}{4}$ ។

ដំណោះស្រាយ

យើងមាន $x = \cos\left(\frac{2\pi}{13}\right) + \cos\left(\frac{6\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right)$

$$y=\cos\left(\frac{4\pi}{13}\right)+\cos\left(\frac{10\pi}{13}\right)+\cos\left(\frac{12\pi}{13}\right)$$ តាមរូបមន្ត $\cos(a)\cos(b)=\frac{1}{2}\cos(a-b)+\frac{\cos(a+b)}{2}$

$$x+y=\sum_{k=1}^{6}\cos\left(\frac{2k\pi}{13}\right)=\sum_{k=0}^{6}\cos\left(\frac{2k\pi}{13}\right)-1$$

$$=\frac{\cos\left(\frac{6\pi}{13}\right)\sin\left(\frac{\pi}{13}\right)}{\sin\left(\frac{\pi}{13}\right)}-1=\frac{1}{2}\frac{\sin\left(\frac{\pi-6\pi}{13}\right)+\sin\left(\frac{7\pi+6\pi}{13}\right)}{\sin\left(\frac{\pi}{13}\right)}-1=\frac{1}{2}-1=-\frac{1}{2}$$

$$xy=\frac{1}{2}\left\{\cos\left(\frac{\pi}{13}\right)+\cos\left(\frac{2\pi}{13}\right)+\cos\left(\frac{12\pi}{13}\right)+\cos\left(\frac{8\pi}{13}\right)+\cos\left(\frac{14\pi}{13}\right)+\cos\left(\frac{10\pi}{13}\right)\right.$$

$$+\cos\left(\frac{10\pi}{13}\right)+\cos\left(\frac{2\pi}{13}\right)+\cos\left(\frac{16\pi}{13}\right)+\cos\left(\frac{4\pi}{13}\right)+\cos\left(\frac{18\pi}{13}\right)+\cos\left(\frac{6\pi}{13}\right)$$

$$\left.+\cos\left(\frac{12\pi}{13}\right)+\cos\left(\frac{4\pi}{13}\right)+\cos\left(\frac{18\pi}{13}\right)+\cos\left(\frac{2\pi}{13}\right)+\cos\left(\frac{20\pi}{13}\right)+\cos\left(\frac{4\pi}{13}\right)\right\}$$

$$=\frac{1}{2}\left\{3\cos\left(\frac{2\pi}{13}\right)+3\cos\left(\frac{4\pi}{13}\right)+2\cos\left(\frac{6\pi}{13}\right)+\cos\left(\frac{8\pi}{13}\right)+2\cos\left(\frac{10\pi}{13}\right)+2\cos\left(\frac{12\pi}{13}\right)\right.$$

$$\left.+\cos\left(\frac{14\pi}{13}\right)+\cos\left(\frac{16\pi}{13}\right)+2\cos\left(\frac{18\pi}{13}\right)+\cos\left(\frac{20\pi}{13}\right)\right\}$$

$$\cos\left(\frac{20\pi}{13}\right)=\cos\left(\frac{6\pi}{13}\right),\cos\left(\frac{18\pi}{13}\right)=\cos\left(\frac{8\pi}{13}\right),\cos\left(\frac{16\pi}{13}\right)=\cos\left(\frac{10\pi}{13}\right),$$

$$\cos\left(\frac{14\pi}{13}\right)=\cos\left(\frac{12\pi}{13}\right)$$

$$xy=\frac{1}{2}\left[3\cos\left(\frac{2\pi}{13}\right)+3\cos\left(\frac{4\pi}{13}\right)+3\cos\left(\frac{6\pi}{13}\right)+\operatorname{sen}\left(\frac{8\pi}{13}\right)+3\cos\left(\frac{10\pi}{13}\right)+3\cos\left(\frac{12\pi}{13}\right)\right)=\frac{3}{4}$$

$$\Rightarrow\begin{cases}x+y=-\frac{1}{2}\\ xy=-\frac{3}{4}\end{cases}$$

$$\Rightarrow x^2+\frac{1}{2}x-\frac{3}{4}$$

$$x_1=\frac{-2-\sqrt{13}}{2}=-\frac{1-\sqrt{13}}{4}$$

$$x_2 = \frac{-\frac{1}{2}+\sqrt{\frac{13}{2}}}{2} = \frac{\sqrt{13}-1}{4}$$

$$x = \cos\left(\frac{2\pi}{13}\right)+\cos\left(\frac{6\pi}{13}\right)+\cos\left(\frac{8\pi}{13}\right) = \cos\left(\frac{2\pi}{13}\right)+2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{7\pi}{13}\right)$$

$$= 2\cos^2\left(\frac{\pi}{13}\right)-1+2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{7\pi}{13}\right) = -1+2\cos\left(\frac{\pi}{13}\right)\left(\cos\left(\frac{\pi}{13}\right)+\cos\left(\frac{7\pi}{13}\right)\right)$$

$$= -1+2\cos\left(\frac{\pi}{13}\right)2\cos\left(\frac{4\pi}{13}\right)\cos\left(\frac{3\pi}{13}\right) = -1+4\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{4\pi}{13}\right)\cos\left(\frac{3\pi}{13}\right)$$

$$\cos\left(\frac{\pi}{13}\right)\geqslant\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2},\cos\left(\frac{\pi}{13}\right)\geqslant\cos\left(\frac{\pi}{3}\right)=\frac{1}{2},\cos\left(\frac{3\pi}{13}\right)\geqslant\cos\left(\frac{\pi}{4}\right)=-\frac{3}{2}$$

$$\Rightarrow x\geqslant -1+\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}\cdot 4 = -1+\frac{\sqrt{6}}{2} > 0$$

$$x = \cos\left(\frac{2\pi}{13}\right)+\cos\left(\frac{\pi}{13}\right)+\cos\left(\frac{8\pi}{13}\right) = \frac{\sqrt{13}-1}{4}$$

ដូចនេះ $\cos\frac{2\pi}{13}+\cos\frac{6\pi}{13}+\cos\frac{8\pi}{13} = \frac{\sqrt{13}-1}{4}$ ។

លំហាត់ទី៨៤   បើ $\sqrt[x^x]{x} = 10$ ។គណនា $E = \sqrt[\log x]{\log \sqrt[\log x]{\log \sqrt[\log x]{\log x}}}$

ដំណោះស្រាយ

បើ $\sqrt[x^x]{x} = 10 \Rightarrow 10^{x^x} = x \Rightarrow \log x = \log 10^{x^x} = x^x$

$$\Rightarrow E = \sqrt[\log x]{\log \sqrt[\log x]{\log \sqrt[\log x]{\log x}}} = \sqrt[x^x]{\log \sqrt[\log x]{\log\left(x^{x^{\frac{1}{\log x}}}\right)}}$$

$$\sqrt[\log x]{\log \sqrt[\log x]{\log x^{10}}} = \sqrt[\log x]{\log\left(\log x^{10}\right)^{\frac{1}{\log x}}} = \sqrt[\log x]{\log\left(\log x^{\frac{10}{\log x}}\right)}$$

$$\sqrt[\log x]{\log\left(\log x^{10\log_x^{10}}\right)} = \sqrt[\log x]{\log\left(\log 10^{10}\right)} = \sqrt[\log x]{\log 10} = \left(\log 10\right)^{\frac{1}{\log x}} = 1^{\log_x 10} = 1$$

ដូចនេះ $E = \sqrt[\log x]{\log \sqrt[\log x]{\log \sqrt[\log x]{\log x}}} = 1$ ។

លំហាត់ទី៨៥ បើ $\operatorname{arctg} x+\operatorname{arctg} y+\operatorname{arctg} z=\pi$ ។ រកតម្លៃ $x+y+z=?$

ដំណោះស្រាយ

យើងដឹងថា $\operatorname{arctg} x+\operatorname{arctg} y+\operatorname{arctg} z=\pi$ រក $x+y+z=?$

យើងបាន $\operatorname{tg}(\operatorname{arctg} x+\operatorname{arctg} y+\operatorname{arctg} z)=\operatorname{tg}\pi$

$$\Rightarrow \frac{\operatorname{tg}(\operatorname{arctg} x+\operatorname{arctg} y)+\operatorname{tg}(\operatorname{arctg} z)}{1-\operatorname{tg}(\operatorname{arctg} x+\operatorname{arctg} y)\cdot\operatorname{tg}(\operatorname{arctg} z)}=0$$

$$\operatorname{tg}(\operatorname{arctg} x+\operatorname{arctg} y)+z=0$$

$$\frac{\operatorname{tg}(\operatorname{arctg} x)+\operatorname{tg}(\operatorname{arctg} y)}{1-\operatorname{tg}(\operatorname{arctg} x)\cdot\operatorname{tg}(\operatorname{arctg} y)}+z=0$$

$$\frac{x+y}{1-x\cdot y}+z=0$$

$$\frac{x+y+z-x\cdot y\cdot z}{1-x\cdot y}=0$$

$$x+y+z-x\cdot y\cdot z=0$$

$$x+y+z=x\cdot y\cdot z$$

ដូចនេះ $x+y+z=x\cdot y\cdot z$ ។

លំហាត់ទី៨៦ ដោះស្រាយសមីការ $\sqrt[3]{(2-x)^2}+\sqrt[3]{(7+x)^2}-\sqrt[3]{(7+x)(2-x)}$ ។

ដំណោះស្រាយ

យើងមាន $\sqrt[3]{(2-x)^2}+\sqrt[3]{(7+x)^2}-\sqrt[3]{(7+x)(2-x)}$

តាង $\sqrt[3]{2-x}=a;\sqrt[3]{7+x}=b$

$$\begin{cases} a^2+b^2-ab=3 \\ a^3+b^3=9 \end{cases}$$

$$\begin{cases} a^2+b^2-ab=3 \\ (a+b)\left(a^2+b^2-ab\right)=9 \end{cases}$$

$$\begin{cases} (a+b)^2-3ab=3 \\ a+b=3 \end{cases}$$

$$\begin{cases} ab=2 \\ a+b=3 \end{cases}$$

$$\begin{pmatrix} a=2 \\ x=-6 \end{pmatrix} \quad \begin{pmatrix} a=1 \\ x=1 \end{pmatrix}$$

ដូចនេះ $x=-6,\ \ x=1$ ។

លំហាត់ទី៨៧ សមីការ $x+y+z=3, \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$ និង $x^2+y^2+z^2=6$ ។រកតម្លៃនៃ $xyz=?$ ។

ដំណោះស្រាយ

ដោយ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$

គេបាន $xy+yz+zx=2xyz$

នាំឲ $(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

$9=6+2+2(xyz)$

$\Rightarrow xyz=\frac{3}{4}$

ដូចនេះ $xyz=\frac{3}{4}$ ។

លំហាត់ទី៨៨ បើ $\sin 2x=\frac{1}{5}$ ។គណនាតម្លៃ នៃ $(\sin x+\cos x)$ ។

ដំណោះស្រាយ

បើ $\sin 2x = \frac{1}{5}$

$\Rightarrow 1+\sin 2x = 1+\frac{1}{5}$

$\Rightarrow \sin^2 x+\cos^2 x+2\sin x\cos x = \frac{6}{5}$

$\Rightarrow (\sin x+\cos x)^2 = \frac{6}{5}$

$(\sin x+\cos x) = \sqrt{\frac{6}{5}}$

ដូចនេះ $(\sin x+\cos x)$ ។

លំហាត់ទី៨៩ បើ $T_n = \sin^n\theta+\cos^n\theta$, នោះ $\frac{T_1-T_n}{T_1} = ?$

ដំណោះស្រាយ

យើងមាន $\frac{T_1-T_2}{T_1} = \left(\frac{\left(\sin^2\theta+\cos^3\theta\right)-\left(\sin^2\theta+\cos^3\theta\right)}{\sin\theta+\cos\theta}\right)$

$= \left(\frac{\left(\sin^3\theta-\sin^5\theta\right)+\left(\cos^3\theta-\cos^5\theta\right)}{\sin\theta+\cos\theta}\right)$

$= \frac{\sin^3\theta\left(1-\sin^2\theta\right)+\cos^3\theta\left(1-\cos^2\theta\right)}{\sin\theta+\cos\theta}$

$= \frac{\sin^3 0\cos^2\theta+\cos^3\theta\sin^2\theta}{\sin\theta+\cos\theta}$

$= \frac{\sin^2\theta\cos^2\theta(\sin\theta+\cos\theta)}{\sin\theta+\cos\theta}$

$=\sin^2\theta\cdot\cos^2\theta$ ។

លំហាត់ទី៩០ បើ $x=(\sqrt{3}+\sqrt{2})^{-3}\cdot y=(\sqrt{3}-\sqrt{2})^{-3}$ ។ រកតម្លៃ $(x+1)^{-1}+(y+1)^{-1}$ ។

ដំណោះស្រាយ

យើងមាន $x=(\sqrt{3}+\sqrt{2})^{-3}$

$\Rightarrow \frac{1}{x}=(\sqrt{3}-\sqrt{2})^{-3}$; $y=(\sqrt{3}-\sqrt{2})^{-3}$

$\Rightarrow \frac{1}{x}=y$

$=(x+1)^{-1}+(y+1)^{-1}$

$=\frac{1}{x+1}+\frac{1}{y+1}$

$=\frac{1}{x+1}+\frac{1}{\frac{1}{x}+1}=\frac{1}{x+1}+\frac{1}{1+x}$

លំហាត់ទី៩១ បើ $x+\frac{1}{x}=\sqrt{3}$ ហើយរកតម្លៃ $x^{96}+\frac{1}{x^{96}}$ ។

ដំណោះស្រាយ

យើងមាន $x+\frac{1}{x}=\sqrt{3}\Rightarrow x^6=-1$

$\Rightarrow x^{66}+\frac{1}{x^{66}}=\left(x^6\right)^{16}+\frac{1}{\left(x^6\right)^{16}}$

$=(-1)^{16}+(-1)^{16}=2$

ដូចនេះ $x^{96}+\frac{1}{x^{96}}=2$ ។

លំហាត់ទី៩២ $S=\frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+\frac{4444}{19^4}+\ldots(*)$

ដំណោះស្រាយ

$$19S=4+\frac{44}{19}+\frac{444}{19^2}+\frac{4444}{19^3}+\ldots.(**)$$

យក $(**)-(*)$ គេបាន

$$18S=4+4\left(\frac{10}{19}\right)+4\left(\frac{100}{19^2}\right)+4\left(\frac{1000}{19^3}\right)+\ldots$$

$$18S=4\left(\frac{1}{1-\frac{10}{19}}\right)=\frac{4\times19}{9}$$

$S=\frac{38}{81}$ ។

លំហាត់ទី៩២ ចូរដោះស្រាយប្រព័ន្ធសមីការដូចខាងក្រោម៖ $\begin{cases}30\sqrt{x_1}+4\sqrt{x_2}=\sqrt[2000]{x_3^{2001}}\\30\sqrt{x_2}+4\sqrt{x_3}=\sqrt[2000]{x_4^{2008}}\\30\sqrt{x_{2003}}+4\sqrt{x_1}=\sqrt[2000]{x_2^{2008}}\\x_1>0;x_2>0;\ldots;x_{2000}>0\end{cases}$

ដំណោះស្រាយ

យើងដឹងថា $\begin{cases}30\sqrt{x_1}+4\sqrt{x_2}=\sqrt[2000]{x_3^{2004}}\\30\sqrt{x_2}+4\sqrt{x_3}=\sqrt[2000]{x_4^{2006}}\\30\sqrt{x_{2003}}+4\sqrt{x_1}=\sqrt[2000]{x_2^{2008}}\\x_1>0;x_2>0;\ldots;x_{2000}>0\end{cases}$

ដោយ $(x_1,x_2,\ldots,x_{2008})$ (1)

តាង $a=\text{Max}\left|x_1,x_2,\ldots,x_{2008}\right|$

$b=\text{Min}\{x_1,x_2,\ldots,x_{2008}\}$

$\Rightarrow a_2 \ge b > 0$

គេបាន $\begin{cases} 34\sqrt{a} \ge 30\sqrt{x_1} + 4\sqrt{x_2} = \sqrt[2000]{x_3^{2004}} \\ 34\sqrt{a} \ge 30\sqrt{x_2} + 4\sqrt{x_2} = \sqrt[200]{x_4^{2008}} \\ \ldots\ldots\ldots\ldots \\ 34\sqrt{a} \ge 30\sqrt{x_{2n+3}} + 4\sqrt{x_1} = \sqrt[2000]{x_2^{20018}} \end{cases}$

$34\sqrt{a} \ge \text{Max}\left\{2wd\sqrt{x_1^{2208}};\ldots, x_{000}\sqrt{x_{2008}^{2008}}\right\}$

$\Rightarrow 34\sqrt{a} \ge \sqrt[2000]{a^{2008}}$

$\Rightarrow 34^{4018}a^{2009} \ge a^{4016} \Leftrightarrow a^{2007} \le 34^{4018} \Leftrightarrow a \le^{2007} \sqrt{34^{4018}}$

នោះ $\begin{cases} 34b \le 30\sqrt{x_1} + 4\sqrt{x_2} = 2000\sqrt{x_3^{2000}} \\ 34b \le 30\sqrt{x_2} + 4\sqrt{x_3} = \sqrt[2000]{x_1^{2006}} \\ \dfrac{1}{34b} \le 30\sqrt{x_{2000}} + 4\sqrt{x_1} = 200\sqrt{x_2^{2005}} \end{cases}$

$34b \le Min\left\{\sqrt[2009]{x_1^{2008}};\ldots.;\sqrt[2009]{x_{2008}^{2008}}\right\}$

$34b \le \sqrt[2009]{b^{2008}}$

$b \ge \sqrt[2009]{34^{2008}}(2)$

តាម $(1)\&(2)$ គេបាន :

$a = b = \sqrt[2007]{34^{4018}}$

$\Rightarrow x_1 = x_2 = \ldots. = x_{2006} = \sqrt[2007]{34^{4018}}$ ។

លំហាត់ទី៩៣ គេមានស្វ៊ីត U_n កំណត់ដោយ: $\begin{cases} u_o = 3 \\ u_{n+1} = 2 + \ln u_n \end{cases}$ ។បង្ហាញថា $3 \le u_n \le 4, \forall n \in \mathbb{N}$ ។

ដំណោះស្រាយ

យើងមាន $u_o = 3$

$\Rightarrow 3 \le u_0 \le 4$ ពិត

ឧបមាថាថាពិតដល់តួទី n គឺ $3 \le u_n \le 4$ ពិត

យើងត្រូវស្រាយថាវាពិតដល់តួទី $n+1$ គឺ $3 \le u_{n+1} \le 4$ ពិត

យើងមាន $u_{n+1} = 2 + \ln u_n$

ដោយ $3 \le u_n \Rightarrow \ln 3 \le \ln u_n$

ហើយ $3 \succ e$

$\Rightarrow \ln 3 \succ \ln e = 1 \Rightarrow \ln 3 \succ \sim 1 \Rightarrow 3 \le \ln u_n$

គេមាន $u_{n+1} = 2 + \ln u_n$

$u_{n+1} \ge 2 + 1 = 3 \Rightarrow u_n \ge 3 (1)$

ម្យាងទៀត

$u_n \le 4 \Rightarrow \ln u_n \le \ln 4$

ដោយ $2 \prec e \Rightarrow 2^2 \prec e^2 \Rightarrow 4 \prec e^2 \Rightarrow \ln 4 \prec \ln e = 1$

$\ln u_n \le 2$

$u_{n+1} = 2 + \ln u_n$

$u_n \le 4 (2)$

តាម (១) & (២) យើងបាន $3 \le u_{n+1} \le 4$ ពិត

ការឧបមាខាងលើពិត គឺ $3 \le u_n \le 4$ ។

ដូចនេះ $3 \le u_n \le 4, \forall n \in \mathbb{N}$ ។

លំហាត់ទី៥២ គណនា $\sqrt{1 + \underbrace{99...9^2}_{n} + \underbrace{0,99...9^2}_{n}}$ ។

ដំណោះស្រាយ

ដោយ $99...9^2 = (10^n - 1)^2$

$0,99...9^2 = \dfrac{(10^n - 1)^2}{10^{2n}}$

$\Rightarrow P = \sqrt{1 + (10^n - 1)^2 + \dfrac{(10^n - 1)^2}{10^{2n}}}$

តាង $a = 10^n - 1$ គេបាន

$P = \sqrt{1 + a^2 + \dfrac{a^2}{(a+1)^2}}$

$P = \dfrac{1}{a+1}\sqrt{(a+1)^2 + a^2(a+1)^2 + a^2}$

$P = \dfrac{1}{a+1}\sqrt{(a^2 + a + 1)^2}$

$P = \dfrac{a^2 + a + 1}{a+1}$

គេបាន $P = \dfrac{10^{2n} - 10^n - +1}{10^n}$

$P = 10^n - \dfrac{10^n - 1}{10^n} = 10^n - 0,\underbrace{99...9}_{n}$

លំហាត់ទី៩៥ ដោះស្រាយសមីការ $3^{\cos 2x} + 4^{\cos^2 x} = 3$ ចំពោះ $x \in \left[\dfrac{3}{4}, 1\right]$ ។

ដំណោះស្រាយ

គេមាន $4^{\cos 2x} + 4^{\cos^2 x} = 3$

$\Leftrightarrow 4^{2\cos^2 x - 1} + 4^{\cos^2 x} = 3$

$\Leftrightarrow 4^{2\cos^2 x}+4.4^{\cos^2 x}=3$

$\Leftrightarrow 4^{2\cos^2 x}+4.4^{\cos^2 x}-12=0$

តាង $t=4^{\cos^2 x}$ $1\le t\le 4$ គេបាន

$t^2+4t-12=0 \Rightarrow \begin{cases} t=2 \\ t=-6\prec 0 \end{cases}$

$t=2\Rightarrow 4^{\cos^2 x}=2$

$2^{2\cos^2 x}=2\Rightarrow 2\cos^2 x-1=0$

$\cos 2x=0\Rightarrow 2x=\dfrac{\pi}{2}+k\pi, k\in Z$

$x=\dfrac{\pi}{4}+\dfrac{k\pi}{2}, k\in Z$

ដោយ $x\in\left[\dfrac{3}{4},1\right]$ នោះគេត្រូវឲ្យ $k=0$

ដូចនេះ ឫសនៃសមីការខាងលើគឺ $x=\dfrac{\pi}{4}$ ។

លំហាត់ទី ៩៦ គេឲ្យស្វ៊ីត (u_n) ដែលកំណត់ដោយ $:(\sqrt{n+1}+\sqrt{n})u_n=\dfrac{2}{2n+1};n=1,2,3,\ldots$

។ បង្ហាញថា $:u_1+u_2+u_3+\ldots+u_{2008}<\dfrac{1004}{1005},\forall k\in N$ ។

ដំណោះស្រាយ

ដោយ $u_k=\dfrac{2}{(2k+1)\cdot(\sqrt{k+1}+\sqrt{k})}=\dfrac{2(\sqrt{k+1}-\sqrt{k})}{(2k+1)}$

$\Rightarrow u_k<\dfrac{2((\sqrt{k+1}-\sqrt{k})}{2\sqrt{k(k+1)}}$

$\sqrt{k(k+1)} < \frac{k+(k+1)}{2} = \frac{2k+1}{2}$

$\Rightarrow u_k < \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}$

នោះ $u_1 + u_2 + \ldots + u_k < \left(1 - \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right) + \ldots + \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}\right)$

$\Rightarrow u_1 + u_2 + \ldots + u_k < 1 - \frac{1}{\sqrt{k+1}}$

តែ $1 - \frac{1}{\sqrt{k+1}} = 1 - \frac{2}{\sqrt{4k+4}} < 1 - \frac{2}{\sqrt{k^2+4k+4}}$

$$= 1 - \frac{2}{k+2} = \frac{k}{k+2}$$

$\Rightarrow u_1 + u_2 + \ldots + u_k < \frac{k}{k+2}$ ចំពោះ $k = 2008$

ដូចនេះ : $u_1 + u_2 + u_3 + \ldots + u_{2008} < \frac{1004}{1005}, \forall k \in N$ ។

លំហាត់ទី៩៧ បើ $x+y+z=0$, ។ស្រាយបញ្ជាក់ថា $\frac{x^2}{2x^2+yz} + \frac{y^2}{2y^2+zx} + \frac{z^2}{2z^2+xy} = 1$ ។

ដំណោះស្រាយ

យើងមាន $x+y+z=0 \Rightarrow x = -y-z$

$y = -x-z$

$z = -x-y$

$\frac{x^2}{2x^2+yz} + \frac{y^2}{2y^2+zx} + \frac{z^2}{2z^2+xy}$

$= \frac{x^2}{x^2+x\cdot x+yz} + \frac{y^2}{y^2+y\cdot y+zx} + \frac{z^2}{z^2+z\cdot z+xy}$

$$=\frac{x^2}{x^2+x(-y-z)+yz}+\frac{y^2}{y^2+y(-x-z)+z}+\frac{z^2}{z^2+z(-x-y)+xy}$$

$$=\frac{x^2}{x^2-xy-xz+yz}+\frac{y^2}{y^2-yx-zy+zx}+\frac{z^2}{z^2-x-zy+xy}$$

$$=\frac{x^2}{x(x-y)-z(x-y)}+\frac{y^2}{y(y-x)-z(y-x)}+\frac{z^2}{z(z-x)-y(z-x)}$$

$$=\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-x)(y-z)}+\frac{z^2}{(z-x)(z-y)}$$

$$=-\frac{x^2}{(x-y)(z-x)}-\frac{y^2}{(x-y)(y-z)}-\frac{z^2}{(z-x)(y-z)}$$

$$=-\left[\frac{x^2(y-z)+y^2(z-x)+z^2(x-y)}{(x-y)(y-z)(z-x)}\right]$$

$$=-\left[\frac{-(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)}\right]=-(-1)=1$$

ដូចនេះ $\frac{x^2}{2x^2+yz}+\frac{y^2}{2y^2+zx}+\frac{z^2}{2z^2+xy}=1$ ។

លំហាត់ទី៥៨ គណនាតម្លៃកន្សោម $A=3\tan^2\frac{\pi}{6}+\frac{4}{3}\cos^2\frac{\pi}{6}-\frac{1}{2}\cot^3\frac{\pi}{4}-\frac{2}{3}\sin^2\frac{\pi}{3}+\frac{1}{8}\sec^4\frac{\pi}{3}$ ។

ដំណោះស្រាយ

យើងមាន $3\tan^2\frac{\pi}{6}+\frac{4}{3}\cos^2\frac{\pi}{6}-\frac{1}{2}\cot^3\frac{\pi}{4}-\frac{2}{3}\sin^2\frac{\pi}{3}+\frac{1}{8}\sec^4\frac{\pi}{3}$

គេបាន $3\tan^2 30^\circ+\frac{4}{3}\cos^2 30^\circ-\frac{1}{2}\cot^3 45^\circ-\frac{2}{3}\sin^2 60^\circ+\frac{1}{8}\sec^4 60^\circ$

$$=3\times\left(\frac{1}{\sqrt{3}}\right)^2+\frac{4}{3}\times\left(\frac{\sqrt{3}}{2}\right)^2-\frac{1}{2}\times(1)^3-\frac{2}{3}\times\left(\frac{\sqrt{3}}{2}\right)^2+\frac{1}{8}\times(2)^4$$

$$=3\times\frac{1}{3}+\frac{4}{3}\times\frac{3}{4}-\frac{1}{2}-\frac{2}{3}\times\frac{3}{4}+\frac{1}{8}\times16=1+1-\frac{1}{2}-\frac{1}{2}+2=3$$

ហេតុដូចនេះ $A=3$ ។

លំហាត់ទី៩៩ បង្ហាញថា

$$T_n = \frac{1}{A_2^2} + \frac{1}{A_3^2} + \ldots + \frac{1}{A_n^2}, \text{ ចំពោះ } n \geq 2 \text{ ។}$$

$$S_n = \frac{1}{A_n^3} + \frac{1}{A_{n+1}^3} + \ldots + \frac{1}{A_{n+m}^3}, \text{ ចំពោះ } n \geq 3 \text{ ។}$$

ដំណោះស្រាយ

$a)$ យើងមាន $\frac{1}{A_k^2} = \frac{(k-2)!}{k!} = \frac{1}{k(k-1)} = \frac{1}{k-1} - \frac{1}{k}$

$$\Rightarrow T_n = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{n-1} - \frac{1}{n}\right) = 1 - \frac{1}{n} = \frac{n-1}{n}$$

$b)$ យើងបាន $\frac{1}{A_k^3} = \frac{(k-3)!}{k!} = \frac{1}{k(k-1)(k-2)}$

$$= \frac{1}{2} \cdot \frac{1}{(k-1)(k-2)} - \frac{1}{2} \cdot \frac{1}{(k-1)k} = \frac{1}{2}\left(\frac{1}{k-2} - \frac{2}{k-1} + \frac{1}{k}\right)$$

$$\Rightarrow S_n = \frac{1}{2}\left(\frac{1}{n+m} - \frac{1}{n+m-1} - \frac{1}{n-1} + \frac{1}{n-2}\right) \text{ ។}$$

លំហាត់ទី១០០ ស្រាយបញ្ជាក់ថា $\sqrt{2+\sqrt{3}} = \frac{\sqrt{2}+\sqrt{6}}{2}$ ។

ដំណោះស្រាយ

យើងមាន $\sqrt{2+\sqrt{3}} = \frac{\sqrt{2}+\sqrt{6}}{2}$

ចំណាំ $\sqrt{a+b} = \left(\sqrt{\sqrt{a+b}}\right)^2$

នាំឲ

$$2+\sqrt{3}=2\cdot\frac{4}{4}+\sqrt{3}\cdot\frac{4}{4}$$

$$=2\cdot\frac{1+3}{4}+\sqrt{3}\cdot\frac{4}{4}$$

$$=2\cdot\frac{1+(\sqrt{3})^2}{4}+\sqrt{3}\cdot\frac{4}{4}$$

$$=\frac{2+2(\sqrt{3})^2}{4}+4\cdot\sqrt{3}\cdot\frac{1}{4}$$

$$=\frac{2+2(\sqrt{3})^2}{4}+2\cdot2\cdot\sqrt{3}\cdot\frac{1}{4}$$

$$=\frac{2+2(\sqrt{3})^2+2\cdot2\cdot\sqrt{3}}{4}$$

$$=\frac{2+2(\sqrt{3})^2+2\cdot(\sqrt{2}\sqrt{2})\cdot\sqrt{3}}{4}$$

$$=\frac{(\sqrt{2})^2+(\sqrt{2}\sqrt{3})^2+2\cdot(\sqrt{2})\cdot(\sqrt{2}\sqrt{3})}{4}$$

$$=\frac{(\sqrt{2})^2+2\cdot(\sqrt{2})\cdot\sqrt{2}\sqrt{3}+(\sqrt{2}\sqrt{3})^2}{4}$$

$$=\frac{(\sqrt{2}+\sqrt{2}\sqrt{3})^2}{4}=\frac{(\sqrt{2}+\sqrt{6})^2}{4}$$

$$=\frac{(\sqrt{2}+\sqrt{6})^2}{2^2}=\left(\frac{\sqrt{2}+\sqrt{6}}{2}\right)^2$$

$$\sqrt{2+\sqrt{3}}=\sqrt{\left(\frac{\sqrt{2}+\sqrt{6}}{2}\right)^2}=\frac{\sqrt{2}+\sqrt{6}}{2}$$

ហេតុដូចនេះ $\sqrt{2+\sqrt{3}}=\dfrac{\sqrt{2}+\sqrt{6}}{2}$ ។

លំហាត់ទី១០១ គណនាផលគុណ $P=\dfrac{\left(1^4+\frac{1}{4}\right)\left(3^4+\frac{1}{4}\right)\ldots\left(99^4+\frac{1}{4}\right)}{\left(2^4+\frac{1}{4}\right)\left(4^4+\frac{1}{4}\right)\ldots\left(100^4+\frac{1}{4}\right)}$ ។

ដំណោះស្រាយ

យើងបាន $P=\prod_{n=1}^{50}\frac{\left(4(2n-1)^4+1\right)}{\left(4(2n)^4+1\right)}$

$$=\prod_{n=1}^{50}\left(\frac{(\sqrt{2}(2n-1))^4+1)}{\left((\sqrt{2}(2n))^4+1\right)}\right.$$

$$=\prod_{n=1}^{50}\frac{\left(2(2n-1)^2-2(2n-1)+1\right)\left(2(2n-1)^2+2(2n-1)+1\right)}{\left(2(2n)^2-2(2n)+1\right)\left(2(2n)^2+2(2n)+1\right)}n$$

$$=\prod_{n=1}^{50}\frac{(2(2n-1)(2n-2)+1)(2(2n-1)(2n)+1)}{(2(2n)(2n-1)+1)(2(2n)(2n+1)+1)}$$

$$=\prod_{n=1}^{50}\frac{(4(n-1)(2n-1)+1)}{(4n(2n+1)+1)}=\frac{1}{4.3+1}\cdot\frac{4.3+1}{4.2.5+1}\cdot\frac{4.2.5+1}{4.3.7+1}\cdots\frac{4.49.(2.49-1)+1}{4.50.(2.50+1)+1}=\frac{1}{20201}$$

ដូចនេះ $P=\frac{\left(1^4+\frac{1}{4}\right)\left(3^4+\frac{1}{4}\right)\ldots\left(99^4+\frac{1}{4}\right)}{\left(2^4+\frac{1}{4}\right)\left(4^4+\frac{1}{4}\right)\ldots\left(100^4+\frac{1}{4}\right)}=\frac{1}{20201}$ ។

លំហាត់ទី១០២ គណនា $S=\frac{3^2+1}{3^2-1}+\frac{5^2+1}{5^2-1}+\ldots+\frac{2019^2+1}{2019^2-1}$ ។

ដំណោះស្រាយ

ដោយទម្រង់ $2k+1, k\in\{1,2,3\ldots,1009\}$

គេបាន $\frac{(2k+1)^2+1}{(2k+1)^2-1}=\frac{4k^2+4k+2}{4k^2+4k}$

$$=\frac{2k^2+2k+1}{2k^2+2k}=1+\frac{1}{2k^2+2k}$$

$$=1+\frac{1}{2}\left[\frac{1}{k(k+1)}\right]=1+\frac{1}{2}\left[\frac{1}{k}-\frac{1}{k+1}\right]$$

ចំពោះ $k=1,2,.....,1009$ គេបាន

$$S = 1009 \times 1 + \frac{1}{2}\left[\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \cdots - \frac{1}{1010}\right]$$

$$= 1009 + \frac{1}{2} \times \frac{1009}{1010}$$

$$S = 1009\frac{1009}{2020}$$

លំហាត់ទី១០៣ រកតម្លៃ x ។ គេដឹងថា $\log^{\log x}\sqrt{z} = x$ ហើយ $x^{x^{x^{x+1}}}.\log_z x = x^{\frac{1}{2\sqrt{2}\sqrt[]{2}}-1}$ ។

ដំណោះស្រាយ

តាម $\log^{\log x}\sqrt{z} = x$ (1)

និង $x^{x^{x^{x+1}}}.\log_z x = x^{\frac{1}{2\sqrt{2}\sqrt[]{2}}-1}$ (2)

តែ $\frac{\log z}{\log x} = x \quad \log_x z \quad \log_z x = x^{-1}$

$$\Rightarrow x^{x^{x^x}}.x^{-1} = x^{\sqrt[\sqrt{2}]{\frac{1}{\sqrt{2}}}}.x^{-1}$$

$$\left(x^x\right)^{\left(x^x\right)} = \left(\frac{1}{\sqrt{2}}\right)^{\left(\frac{1}{\sqrt{2}}\right)}$$

$$x^x = \left(\frac{1}{\sqrt{2}}\right)$$

$$x^x = \left(\frac{1}{2}\right)^{\left(\frac{1}{2}\right)}$$

$x = \left\{\frac{1}{4}, \frac{1}{2}\right\}$ ។

ដូចនេះ $x = \left\{\frac{1}{4}, \frac{1}{2}\right\}$ ។

លំហាត់ទី១០៤ រកគ្រប់អនុគមន៍ $f:\mathbb{R}\to\mathbb{R}$ ចំពោះ $x,y\in\mathbb{R}$

ដែល $f\left(f\left(y+f(x)\right)\right)=f\left(x+y\right)+f(x)+y$ ។

ដំណោះស្រាយ

តាង $P(x,y)$ នោះ $f\left(f\left(y+f(x)\right)\right)=f\left(x+y\right)+f(x)+y$

$P\left(x,f(y)\right)\Rightarrow f\left(f(f(x)+f(y))\right)=f\left(x+f(y)\right)+f(x)+f(y)$

$P\left(y,f(x)\right)\Rightarrow f\left(f(f(x)+f(y))\right)=f\left(y+f(x)\right)+f(x)+f(y)$

គេបាន $f\left(x+f(y)\right)=f\left(y+f(x)\right)$

$\Rightarrow f(f(x+f(y))=f\left(f\left(y+f(x)\right)\right)$

ដោយប្រើ $P(x,y)\&P(y,x); f(x+y)+f(y)+x=f(x+y)+f(x)+y$

នោះ $f(x)-x=f(y)-y$.និង $f(x)=x+a$

ដូចនេះ $f(x)=x+a$

លំហាត់ទី១០៥

នៅក្នុងត្រីកោណ a,b,c ។បង្ហាញថា $\cos^2 a+\cos^2 b+\cos^2 c=1-2\cos a\cdot\cos b\cdot\cos c$ ។

ដំណោះស្រាយ

តាមរូបមន្ត $\cos a \cos b = \dfrac{\cos(a+b)+\cos(a-b)}{2}$

តែ $a+b+c=180\Leftrightarrow a+b=180-c$

គេបាន $1-2\cos a\cdot\cos b\cdot\cos c$

$=1+2\cos(a+b)\cdot\cos a\cdot\cos b=1+2\cos(a+b)\cdot\dfrac{(\cos(a+b)+\cos(a-b)}{2}$

$=1+\cos^2(a+b)+\cos(a+b)\cdot\cos(a-b)$

$=1+\cos^2 c+\dfrac{\cos 2a+\cos 2b}{2}$

$=1+\cos^2 c+\dfrac{2\cos^2 a-1+2\cos^2 b-1}{2}$

$=\cos^2 a+\cos^2 b+\cos^2 c$

ដូចនេះ $\cos^2 a+\cos^2 b+\cos^2 c=1-2\cos a\cdot\cos b\cdot\cos c$ ។

លំហាត់ទី១០៦	គណនាតម្លៃនៃកន្សោមដូចខាងក្រោម $A=\log_{1/3}\sqrt[4]{\sqrt{729}\cdot\sqrt[3]{9^{-1}27^{-4/3}}}$ ។

ដំណោះស្រាយ

យើងមាន $A=\log_{1/3}\sqrt[4]{\sqrt{729}\cdot\sqrt[3]{9^{-1}27^{-4/3}}}$

$\Rightarrow \log_{\frac{1}{3}}(\sqrt[4]{\sqrt{\sqrt{729}\sqrt[3]{9^{-1}(27)^{-4/3}}}})$

$=\log_{1/3}\left(3^{6/8}\left(3^{-2}\cdot 3^{-4}\right)^{1/12}\right)$

$=\log_{1/3}\left(3^{6/8-1/2}\right)$

$=\log_{\left(3^{-1}\right)} 3^{2/8}$

$=\dfrac{1/4}{-1}\log_3 3$

$=-\dfrac{1}{4}$

ដូចនេះ $A=-\dfrac{1}{4}$ ។

លំហាត់ទី ១០៧	គណនា $(a)\dfrac{\log_2 24}{\log_{96} 2}-\dfrac{\log_2 192}{\log_{12} 2}$ ។ $(c)7\log\dfrac{16}{15}+5\log\dfrac{25}{24}+3\log\dfrac{81}{80}$ ។

ដំណោះស្រាយ

$(a)\dfrac{\log_2 24}{\log_{96} 2}-\dfrac{\log_2 192}{\log_{12} 2}$

$=\log_2 24\log_2 96-\log_2 192\log_2 12$

$=\log_2\left(2^3\times 3\right)\log_2\left(2^5\times 3\right)-\log_2\left(2^6\times 3\right)\log_2\left(2^2\times 3\right)$

$=\left[3+\log_2^3\right]\left[5+\log_2^3\right]-\left[6+\log_2^3\right]\left[2+\log_2^3\right]$

$=15+8\log_2 3+\left(\log_2 3\right)^2-12-8\log_2 3-\left(\log_2 3\right)^2=3$

$(c)7\log\dfrac{16}{15}+5\log\dfrac{25}{24}+3\log\dfrac{81}{80}$

$=\log\left(\dfrac{16}{15}\right)^7+\log\left(\dfrac{25}{24}\right)^5\log\left(\dfrac{81}{80}\right)^3$

$=\log\left(\dfrac{16}{15}\right)^7\times\left(\dfrac{25}{24}\right)^5\times\left(\dfrac{81}{80}\right)^3$

$=\log\dfrac{4^{14}}{3^7\times 5^7}\times\dfrac{5^{10}}{3^5\times 2^{15}}\times\dfrac{3^{12}}{5^3\times 2^{12}}$

$=\log\dfrac{2^{18}\times 5^{10}\times 3^{12}}{3^{12}\times 5^{10}\times 2^{27}}$

$=\log\dfrac{2^{18}}{2^{27}}$

លំហាត់ទី១០៨ បង្ហាញថា 0.4232323..... ជាចំនួនអសនិទាន។

ដំណោះស្រាយ

តាង $S=0.4232323.....$

គេបាន :

$$S=\frac{4}{10}+\frac{23}{1000}+\frac{23}{100000}+\ldots\ldots=\frac{4}{10}+\frac{23}{10^3}+\frac{23}{10^5}+\ldots.$$

$$S=\frac{4}{10}+\frac{23}{10^3}\left[1+\frac{1}{10^2}+\frac{1}{10^4}+\ldots..\right]$$

$$=\frac{4}{10}+\frac{23}{10^3}\left[\frac{1}{1-1/10^2}\right]$$

$$=\frac{4}{10}+\frac{23}{10^3}\times\frac{100}{99}=\frac{4}{10}+\frac{23}{990}=\frac{419}{990}$$

ដូចនេះ 0.4232323..... ជាចំនួនអសនិទាន។

លំហាត់ទី១០៩ បង្ហាញថា 0.2578239 ជាចំនួនសនិទាន ។

ដំណោះស្រាយ

តាង $X=0.25782398239$

$\Rightarrow 10^3 X=257.82398239$

$\Rightarrow 10^7 X=2578239.82398239$

តាមសមីការ $(2)-(1)$ គេបាន :

$10^7 X-10^3 X=2578239-257$

$$\Rightarrow X=\frac{2578239-257}{10^7-10^3}=\frac{2577982}{9999000}$$

ដូចនេះ 0.2578239 ជាចំនួនសនិទាន ។

លំហាត់ទី១១០ គណនាតម្លៃនៃសេរី $\frac{1}{2.4}+\frac{1.3}{2.4.6}+\frac{1.3.5}{2.4.6.8}+.....$ ។

ដំណោះស្រាយ

$$S_n=\frac{1}{2.4}+\frac{1.3}{2.4.6}+\frac{1.3.5}{2.4.6.8}+\ldots\ldots+\frac{1.3.5}{2.4.6.8.(2n+2)}$$

តែ $T_n=\dfrac{1.3.5\ldots\ldots(2n-1)}{2.4.6\ldots\ldots(2n+2)}$

$$=\frac{1.3.5\ldots\ldots(2n-1)[(2n+2)-(2n+1)]}{2.4.6\ldots\ldots.(2n+2)}$$

$$\Rightarrow T_n = \frac{1.3.5\ldots\ldots(2n-1)}{2.4.6\ldots\ldots.2n} - \frac{1.3.5\ldots\ldots(2n-1)\cdot(2n+1)}{2.4.6\ldots\ldots(2n+2)}$$

$$S_n = \sum_{n=1}^{n} T_n = \begin{bmatrix} \left(\frac{1}{2} - \frac{1.3}{2.4}\right) + \left(\frac{1.3}{2.4} - \frac{1.3.5}{2.4.6}\right) + \left(\frac{1.3.5}{2.4.6} - \frac{1.3.5.7}{2.4.6.8}\right) \\ +\ldots\ldots\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots.2n} - \frac{1.3.5\ldots(2n-1)(2n+1)}{2.4.6\ldots(2n+2)}\right) \end{bmatrix}$$

$$S_n = \left[\frac{1}{2} - \frac{1.3.5\ldots(2n+1)}{2.4.6\ldots\ldots(2n+2)}\right]$$

ដូចនេះ $S_n = \left[\frac{1}{2} - \frac{1.3.5\ldots(2n+1)}{2.4.6\ldots\ldots(2n+2)}\right]$ ។

លំហាត់ទី១១១ រកតម្លៃ minimum នៃ $\dfrac{\left(x+\frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + x^3 + \frac{1}{x^3}}$ ចំពោះ $x > 0$ ។

ដំណោះស្រាយ

$$y = \frac{\left[\left(x+\frac{1}{x}\right)^3\right]^2 - \left[x^3 + \frac{1}{x^3}\right]^2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3 + \frac{1}{x^3}\right)}$$

$$= \left(x+\frac{1}{x}\right)^3 - x^3 - \frac{1}{x^3}$$

$$\Rightarrow 3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow y_m = 6 \text{ ព្រោះ } x + \frac{1}{x} \geq 2, x > 0$$

ដូចនេះ $y_m = 6$ ។

លំហាត់ទី១១២ រកគ្រប់ចំនួនគត់វិជ្ជមាន n ដើម្បីឲ $n^2 - 19n + 99$ ជាការេប្រាកដ ។

ដំណោះស្រាយ

រកគ្រប់ចំនួនគត់វិជ្ជមាន n ដើម្បីឲ $n^2-19n+99$ ជាការេប្រាកដ លុះត្រាតែ

$n^2-19n+99=k^2$, $k\in\mathbb{N}$

$n^2-19n+99-k^2=0$

$$\Delta=(-19)^2-4(1)(99-k^2)$$
$$=361-396+4k^2$$
$$=4k^2-35$$

វាជាចំនួនការេលុះត្រាតែ $4k^2-37=t^2$, $t\in\mathbb{N}$

$$\Leftrightarrow 4k^2-37-t^2=0$$
$$\Leftrightarrow (2k-t)(2k+t)=35$$

ដោយ k , $t\in\mathbb{N}\Rightarrow 2k-t<2k+t$

គេបាន $35=1\times35=5\times7$

ចំពោះ $35=1\times35$ គេបាន $\begin{cases}2k-t=1\\2k+t=35\end{cases}\Rightarrow k=9$

$$\Delta=4(9)^2-35=17^2$$
$$n_1=\frac{19-17}{2}=1\quad n_2=\frac{19+17}{2}=18$$

គេបាន $35=1\times35=5\times7$

ចំពោះ $35=5\times7$ គេបាន $\begin{cases}2k-t=5\\2k+t=7\end{cases}\Rightarrow k=3$

$$\Delta=1$$
$$n_1=\frac{19-1}{2}=9\quad n_2=\frac{19+1}{2}=10$$

ដូចនេះ $n\in\{1,9,10,18\}$ ។

លំហាត់ទី១១៣ បង្ហាញថា

$a, 5 < \sqrt{5} + \sqrt[3]{5} + \sqrt[4]{5}$ ។

$b, 8 < \sqrt{8} + \sqrt[3]{8} + \sqrt[4]{8}$ ។

ដំណោះស្រាយ

$a, 5 < \sqrt{5} + \sqrt[3]{5} + \sqrt[4]{5}$

ដោយ $\sqrt{5} > \sqrt{4} = 2 \Rightarrow \sqrt{5} > 2(1)$

$3^3 \times 5 \times 5^3 \Rightarrow 3 \times \sqrt[3]{5} > 5 \Rightarrow \sqrt[3]{5} > \frac{5}{3}(2)$

$3^4 \times 5 \times 5^4 \Rightarrow 3 \times \sqrt[4]{5} > 4 \Rightarrow \sqrt[3]{5} > \frac{4}{3}(3)$

$(1)+(2)+(3) \Rightarrow \sqrt{5} + \sqrt[3]{5} + \sqrt[4]{5} > 2 + \frac{5}{3} + \frac{4}{3} = 5$

ដូចនេះ $5 < \sqrt{5} + \sqrt[3]{5} + \sqrt[4]{5}$ ។

$b, 8 < \sqrt{8} + \sqrt[3]{8} + \sqrt[4]{8}$

ដោយ

$3 = \sqrt{9} > \sqrt{8} \Rightarrow \quad 3 > \sqrt{8}(4)$

$3^3 > 8 \Rightarrow 3 > \sqrt[3]{8}$

$\Rightarrow 3 > \sqrt[3]{8}(5)$

$2^4 > 8 \Rightarrow 2 > \sqrt[4]{8}$

$\Rightarrow = 2 > \sqrt[4]{8}(6)$

$(4)+(5)+(6) \Rightarrow 8 = 3+3+2 > \sqrt{8} + \sqrt[3]{8} + \sqrt[4]{8}$

ដូចននេះ $8 < \sqrt{8} + \sqrt[3]{8} + \sqrt[4]{8}$ ។

លំហាត់ទី១១៤ គេអោយអនុគន៍ f កំណត់ដោយ ៖ $f(x)=\frac{x^2}{2020}+x+2020$ និង ពីរចំនួនពិត $a>0$,$b>0$ ។ចូរស្រាយថា $f\left(\frac{a}{1+a}+\frac{b}{1+b}\right)>f\left(\frac{a+b}{1+a+b}\right)$ $\forall a,b>0$ ។

ដំណោះស្រាយ

គេមាន $f(x)=\frac{2x}{2020}+1=\frac{x}{1010}+1$ មានឬស $x=-1010$

ចំពោះ $x\in(-\infty,-1010)$ នោះ $f'(x)<0$ ហើយ f ជាអនុគមន៍ចុះ

ចំពោះ $x\in(-1010,+\infty)$ នោះ $f'(x)>0$ ហើយ f ជាអនុគមន៍កើន

គេតាង $\alpha=\frac{a}{1+a}+\frac{b}{1+b}>0$, $\beta=\frac{a+b}{1+a+b}>0$

គេបាន $\frac{a}{1+a}>\frac{a}{1+a+b}$

$\frac{b}{1+b}>\frac{b}{1+a+b}$ នោះ $\frac{a}{1+a}+\frac{b}{1+b}>\frac{a+b}{1+a+b}$ ឬ $\alpha>\beta$

គេបាន $f\left(\frac{a}{1+a}+\frac{b}{1+b}\right)>f\left(\frac{a+b}{1+a+b}\right)$ ឬ $f(\alpha)>f(\beta)$

ដូចនេះ $f\left(\frac{a}{1+a}+\frac{b}{1+b}\right)>f\left(\frac{a+b}{1+a+b}\right)$ $\forall a,b>0$ ។

លំហាត់ទី១១៥ គេឲ $x_1+1=1, x_2+2=4$,..., $x_n+n=n^2$ ។ តើតម្លៃ $x_1+...+x_n$ ស្មើប៉ុន្មាន?

ដំណោះស្រាយ

$$\begin{cases} x_1+1=1 \\ x_2+2=4 \\ x_3+3=9 \\ \dots\dots\dots\dots\dots \\ x_n+n=n^2 \end{cases}$$

$$x_1+\dots+x_n+\frac{n(n+1)}{2}=\frac{n(n+1)(2n+1)}{6}$$

$$x_1+\dots+x_n=\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}$$

$$x_1+\dots+x_n=\frac{n^3-n}{3}$$

លំហាត់ទី១១៦ គណនាកន្សោម $\frac{a^6+a^{-6}}{a^2+a^{-2}}$ ។ បើ $a^4=\sqrt{29-12\sqrt{3+2\sqrt{2}}}$ ដែល $a>0$ ។

ដំណោះស្រាយ

ដោយ $3+2\sqrt{2}=(1+\sqrt{2})^2$

$$a^4=\sqrt{29-12(1+\sqrt{2})}=\sqrt{17-12\sqrt{2}}$$

$$17-12\sqrt{2}=(3-2\sqrt{2})^2$$

$$a^4=3-2\sqrt{2}$$

តែ $\frac{a^6+\frac{1}{a^6}}{a^2+\frac{1}{a^2}}=\frac{(a^2+\frac{1}{a^2})(a^4-1+\frac{1}{a^4})}{a^2+\frac{1}{a^2}}=a^4+\frac{1}{a^4}-1$

លំហាត់ទី១១៧ រក $10^{33}(\mathrm{mod}47)$?

ដំណោះស្រាយ

យើងអាចប្រើ *Modulo* ស្វ័យគុណគេបាន:

$10^1 \equiv 10 \pmod{47}$

$10^2 \equiv 6 \pmod{47}$

$10^4 \equiv (10^2)^2 \equiv 6^2 \equiv 36 \pmod{47}$

$10^8 \equiv (10^4)^2 \equiv 36^2 \equiv 27 \pmod{47}$

$10^{16} \equiv (10^8)^2 \equiv 27^2 \equiv 24 \pmod{47}$

$10^{32} \equiv (10^{16})^2 \equiv 24^2 \equiv 12 \pmod{47}$

$10^{33} \equiv 10^{32} \cdot 10^1 \equiv 12 \cdot 10 \equiv 26 \pmod{47}$

លំហាត់ទី១១៨

គេឲ $x, y, z \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$។ បង្ហាញថា $\left|\frac{\sin x - \sin y}{\sin z} + \frac{\sin y - \sin z}{\sin x} + \frac{\sin z - \sin x}{\sin y}\right| \le \left(1 - \frac{1}{\sqrt{2}}\right)^2$ ។

ដំណោះស្រាយ

តាង $\sin x = a; \sin y = b; \sin z = c$ ដែល $a, b, c \in \left[\frac{1}{2}, 1\right]$

គេបាន $\frac{a-b}{c} + \frac{b-c}{a} + \frac{c-a}{b} = \frac{(a-b)(b-c)(c-a)}{abc}$

តែ $\frac{1}{2} \le a \le b \le c \le 1$, $u = \frac{a}{c}; v = \frac{b}{c}$ ចំពោះ $\frac{1}{2} \le u \le v \le 1$

$$\Leftrightarrow \frac{(v-u)(1-u)(1-v)}{uv} \le \left(1 - \frac{1}{\sqrt{2}}\right)^2$$

$$\frac{(v-u)(1-u)(1-v)}{uv} \le \frac{\left(v - \frac{1}{2}\right)\left(1 - \frac{1}{2}\right)(1-v)}{\frac{1}{2}v}$$

$$1 + \frac{1}{2} - v - \frac{1}{2v} \le 1 + \frac{1}{2} - 2\sqrt{v \cdot \frac{1}{2v}} = \left(1 - \frac{1}{\sqrt{2}}\right)^2$$

ដូចនេះ $\left|\frac{\sin x-\sin y}{\sin z}+\frac{\sin y-\sin z}{\sin x}+\frac{\sin z-\sin x}{\sin y}\right|\leq\left(1-\frac{1}{\sqrt{2}}\right)^2$ ។

លំហាត់ទី១១៩ គណនា $A=\sqrt[3]{\sqrt{9+\frac{125}{27}}+3}-\sqrt[3]{\sqrt{9+\frac{125}{27}}-3}$ ។

ដំណោះស្រាយ

គេដឹងថា $x^3-y^3=(x-y)^3+3xy(x-y)$

តាង $x=\sqrt[3]{\sqrt{9+\frac{125}{27}}+3}$ និង $y=\sqrt[3]{\sqrt{9+\frac{125}{27}}-3}$

$\Rightarrow z=(x-y)$

គេបាន: $x^3-y^3=(x-y)^3+3xy(x-y)$

$6=z^3+3z(\sqrt[3]{9+\frac{126}{27}-9})\Leftrightarrow 6=z^3+3z\times\frac{5}{3}$

$z^3+5z-6=0$

$(z-1)\left(z^2+z+6\right)=0$

នាំឲ $z=1$

ដូចនេះ $A=1$ ។

លំហាត់ទី១២០ បើ $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}$ ។ បង្ហាញថា $x^{Y+Z}+y^{x+z}+z^{x+Y}\geq 3$ ។

ដំណោះស្រាយ

ពិនិត្យ :

$\log x=(y-z)p$ និង $\log y=(z-x)p$, $\log z=(x-y)p$

គេបាន $x^{Y+Z}+y^{X+Z}+z^{X+Y}\geq 3\sqrt[3]{x^{Y+Z}\times y^{X+Z}\times z^{X+Y}}$

$=3\sqrt[3]{e^{(Y+Z)\ln x}\times e^{(X+Z)\ln y}\times e^{(X+Y)\ln z}}$

$=3\sqrt[3]{e^{(Y+Z)(Y-Z)P}\times e^{(X+Z)(Z-X)P}\times e^{(X+Y)(X-Y)P}}$

$=3\sqrt[3]{e^{\left(Y^2-Z^2\right)P}\times e^{\left(Z^2-X^2\right)P}\times e^{\left(X^2-Y^2\right)P}}$

$=3\sqrt[3]{e^0}=3$

ដូចនេះ $x^{Y+Z}+y^{x+z}+z^{x+Y}\geq 3$ ។

លំហាត់ទី១២១ ដោះស្រាយវិសមីការ $x+\dfrac{x}{\sqrt{x^2-1}}>\dfrac{35}{12}$ ។

ដំណោះស្រាយ

+បញ្ជាក់ $|x|>1$ បើ $x<-1$ នោះ $x+-\dfrac{x}{\sqrt{x^2-1}}<0$

នាំឲវិសមីការគ្មានចម្លើយ

+ វិសមីការ $\Leftrightarrow\begin{cases}x>1\\ x^2+\dfrac{x^2}{x^2-1}+\dfrac{2x^2}{\sqrt{x^2-1}}-\dfrac{1225}{144}>0\end{cases}$

$\Leftrightarrow\begin{cases}x>1\ (1)\\ \dfrac{x^4}{x^2-1}+2\cdot\dfrac{x^2}{\sqrt{x^2-1}}-\dfrac{1225}{144}>0\ \ (2)\end{cases}$

តាង $t=\dfrac{x^2}{\sqrt{x^2-1}}>0$

តាម(2) គេបាន $t^2+2t-\dfrac{1225}{144}>0\quad\Rightarrow t>\dfrac{25}{12}$

តាម $(1)\&(2)$ លុះត្រាតែ $\begin{cases} x>1 \\ \dfrac{x^2}{\sqrt{x^2-1}}>\dfrac{25}{12} \end{cases} \Leftrightarrow \begin{cases} x>1 \\ \dfrac{x^4}{x^2-1}>\dfrac{625}{144} \end{cases}$

$$\Leftrightarrow \begin{cases} x>1 \\ 144x^4-625x^2+625>0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x>1 \\ x^2<\dfrac{25}{9} \quad \vee \quad x^2<\dfrac{25}{16} \end{cases} \Leftrightarrow x>\frac{5}{3} \vee 1<x<\frac{1}{4}$$

លំហាត់ទី១២២

សម្រាយបញ្ជាក់ថាពហុធា: $x^{9999}+x^{8888}+x^{7777}+x^{6666}+x^{5555}+x^{4444}+x^{3333}+x^{2222}+x^{1111}+1$

ចែកដាច់ដោយពហុធា $x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1$ ។

ដំណោះស្រាយ

យើងមាន $A=x^{9999}+x^{8888}+x^{7777}+x^{6666}+x^{5555}+x^{4444}+x^{3333}+x^{2222}+x^{1111}+1$

$$B=x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1$$

គេបាន $A-B=\left(x^{9999}-x^9\right)+\left(x^{8888}-x^8\right)+\ldots+\left(x^{111}-x\right)$

$$=x^9\left(\left(x^{10}\right)^{999}-1\right)+x^8\left(\left(x^{10}\right)^{888}-1\right)+\ldots+x\left(\left(x^{10}\right)^{111}-1\right)$$

សន្និដ្ឋានថា $\mathbf{A-B}$ ចែកដាច់ $\mathbf{x}^{10}-1$

នោះ $\mathbf{A-B}$ ចែកដាច់ដោយពហុធា $\mathbf{B}=\dfrac{\mathbf{x}^{10}-1}{\mathbf{x}-1}$

ដូចនេះ A ចែកដាច់ដោយ B ។

លំហាត់ទី១២៣

គេឲស្វ៊ីត a_n កំណត់ដោយៈ $a_n = 1+\frac{1}{2^2}+\frac{1}{3^2}+.....+\frac{1}{n^2}, n\geq 1$ ។ បង្ហាញថា a_n ជាស្វ៊ីតទាល់លើ។

ដំណោះស្រាយ

គេមាន $a_n = 1+\frac{1}{2^2}+\frac{1}{3^2}+.....+\frac{1}{n^2}, n\geq 1$

គេបាន $a_n = 1+\frac{1}{2^2}+\frac{1}{3^2}+.....+\frac{1}{n^2} < 1+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n-1)}$

$$< 1+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+....+\left(\frac{1}{n}-\frac{1}{n-1}\right)$$

$$< 1+1-\frac{1}{n}$$

$$< 2-\frac{1}{n}$$

$$< 2$$

$\Rightarrow a_n$ ជាស្វ៊ីតទាល់លើ ។

លំហាត់ទី១២៤ គណនា $S = C_3^0 + xC_3^1 + x^2C_3^2 + \ldots + x^5C_5^5$ ។

ដំណោះស្រាយ

ដោយ

$$(1+x)^5 = C_5^0 + xC_5^1 + x^2C_5^2 + \ldots + x^5C_5^5$$

$$\Rightarrow (1+2)^5 = C_5^0 + 2C_5^1 + 2^2C_5^2 + \ldots + 2^5C_5^5$$

$$\Rightarrow 3^5 = C_5^0 + 2C_5^1 + 2^2C_5^2 + \ldots + 2^5C_5^5$$

$$\Rightarrow S = 3^5 = 243$$

ដូចនេះ $C_3^0 + xC_3^1 + x^2C_3^2 + \ldots + x^5C_5^5 = 243$ ។

លំហាត់ទី១២៥ គណនា $\mathbf{C}=C_n^0+\dfrac{C_n^1}{2}+\ldots+\dfrac{C_n^n}{n+1}$ ។

$\mathbf{D}=C_n^1-2C_n^2+\ldots+(-1)^{n-1}\cdot n\cdot C_n^n$ ។

ដំណោះស្រាយ

$$\mathbf{C}=C_n^0+\frac{C_n^1}{2}+\ldots+\frac{C_n^n}{n+1}$$

គេបាន

$$\int_0^1(1+x)^n dx=\int_0^1\left(C_n^0+C_n^1x+\ldots+C_n^nx^n\right)dx$$

$$=\left.\frac{(1+x)^{n-1}}{n+1}\right|_0^1=\frac{2^{n+1}-1}{n+1}$$

$$\mathrm{C}=\frac{2^{n+1}-1}{n+1}$$

ដូចនេះ $\mathrm{C}=\dfrac{2^{n+1}-1}{n+1}$ ។

$$\mathbf{D}=C_n^1-2C_n^2+\ldots+(-1)^{n-1}\cdot n\cdot C_n^n$$

គេបាន

$$\left((1-x)^n\right)'=\left[C_n^0-C_n^1x+C_n^2x^2-C_n^3x^3+\ldots+(-1)^nC_n^nx^n\right]$$

$$-n(1-x)^{n-1}=-C_n^1+2C_n^2x-3C_n^3x^2+\ldots+(-1)^n nC_n^nx^{n-1}$$

$$n(1-1)^{n-1}=\mathrm{D}\Rightarrow \mathrm{D}=0$$

ដូចនេះ $C_n^1-2C_n^2+\ldots+(-1)^{n-1}\cdot n\cdot C_n^n=0$ ។

លំហាត់ទី១២៦ គណនា $A+B$ ។គេមាន $\mathrm{A}=C_{2\mathrm{n}}^1+C_{2\mathrm{n}}^3+\ldots+C_{2n}^{2n-1}$

$\mathrm{B}=C_{2n}^0+C_{2n}^2+\ldots+C_{2n}^{2n}$

ដំណោះស្រាយ

$$A+B=C_{2n}^{1}+C_{2n}^{3}+\ldots+C_{2n}^{2n-1}+C_{2n}^{0}+C_{2n}^{2}+\ldots+C_{2n}^{2n}$$

$=(1+1)^{n}$

$=2^{n}$

ដូចនេះ $A+B=2^{n}$ ។

លំហាត់ទី១២៧ ដោះស្រាយសមីការ

$$\log\sqrt{x^{\frac{4}{3}}}+3\log_{x}(16x)=7$$

ដំណោះស្រាយ

តាង $\log x=y$

ដោយ $\log\sqrt{x^{\frac{4}{3}}}+3\log_{x}(16x)=7$

$\Rightarrow \frac{2}{3}\log_{4}x+3\log_{x}16+3\log_{x}x=7$

$\Rightarrow \frac{2}{3}y+\frac{6}{\log x}+4=0$

$\Rightarrow y^{2}-6y+9=0$

$\Rightarrow (y-3)=0 \Leftrightarrow \therefore y=3$

$\log_{4}x=3 \Rightarrow x=64$

ដូចនេះ $x=64$ ។

លំហាត់ទី១២៨ ដោះស្រាយសមីការ ៖

$$(x^{2}+2x+1)^{2}+(x^{2}+3x+2)^{2}+(x^{2}+4x+3)^{2}+\ldots+(x^{2}+1996x+1995)^{2}=0$$ ។

ដំណោះស្រាយ

យើងបាន $(x^2+2x+1)^2+(x^2+3x+2)^2+(x^2+4x+3)^2+...+(x^2+1996x+1995)^2=0$

$=(x+1)^4+(x+1)^2(x+2)^2+(x+1)^2(x+3)^2+...+(x+1)^2(x+1995)^2=0$

$=(x+1)^2[(x+1)^2+(x+2)^2+(x+3)^2+...+(x+1995)^2]=0$

ព្រោះ $(x+1)^2+(x+2)^2+(x+3)^2+...+(x+1995)^2>0\forall x$

$\Rightarrow x=-1$

ដូចនេះ $x=-1$ ។

លំហាត់ទី១២៩ ដោះស្រាយសមីការៈ $\frac{4x-3}{3x-1}=\frac{8x-7}{5x-3}$ ។

ដំណោះស្រាយ

$\Leftrightarrow (4x-3)(5x-3)=(8x-7)(3x-1)$

$\Rightarrow 20x^2-12x-15x+9=24x^2-8x-21x+7$

$\Rightarrow 24x^2-20x^2-29x+27x-9+7=0$

$\Rightarrow 4x^2-2x-2=0 \Rightarrow 2x^2-x-1=0$

$\Rightarrow 2x^2-2x+x-1=0$

$\Rightarrow 2x(x-1)+1(x-1)=0$

$\Rightarrow (x-1)(2x+1)=0$

$\Rightarrow \quad x-1=0$ រឺ $2x+1=0$ រឺ $2x=-1$

ដូចនេះ $x=-\frac{1}{2}$

$$x=1,-\frac{1}{2}$$

លំហាត់ទី១៣០ រកតម្លៃ $\sum_{r=0}^{2017}\frac{1}{3^r+\sqrt{3^{2017}}}$ ។

ដំណោះស្រាយ

$$=\sum_{r=0}^{2017}\frac{1}{3^r+3^{\frac{2017}{2}}}$$

$$=\sum_{r=0}^{2017}\frac{1}{3^{\frac{2017}{2}}(1+3^{(r-\frac{2017}{2})})}$$

$$3^{-\frac{2017}{2}}\cdot\sum_{r=0}^{2017}\frac{1}{(1+3^{(r-\frac{2017}{2})})}$$

$$=3^{\frac{-2017}{2}}\cdot\sum_{r=0}^{1008}\left(\frac{1}{(1+3^{(\frac{2n+1}{2})})}+\frac{1}{(1+3^{(-(\frac{2n+1}{2}))})}\right)$$

ព្រោះ $\frac{1}{1+3^n}+\frac{1}{1+3^{(-n)}}=\frac{1}{1+3^n}+\frac{3^n}{3^n+1}=\frac{3^n+1}{3^n+1}=1$

$$=3^{\frac{-2017}{2}}\cdot(1009\times 1)$$

$$=\frac{1009}{\sqrt{3^{2017}}}$$

ដូចនេះ $\sum_{r=0}^{2017}\frac{1}{3^r+\sqrt{3^{2017}}}=\frac{1009}{\sqrt{3^{2017}}}$ ។

លំហាត់ទី១៣១ បើ $x_1,x_2,....,x_{2019}$ are roots of $P(x)=x^{2019}+2019x-1$ ។កំណត់តម្លៃនៃ

$$\sum_{i=1}^{2019}\frac{x_i}{x_i-1}$$ ។

ដំណោះស្រាយ

តែ $\sum_{i=1}^{2019}\frac{x_i}{x_i-1}$

គេបាន : $\sum_{i=1}^{2019}\frac{x_i}{x_i-1}=\sum_{i=1}^{2019}\frac{x_i-1+1}{x_i-1}$

$=2019+\sum_{i=1}^{2019}\frac{1}{x_i-1}$

$=2019-\frac{P'(1)}{P(1)}$

ដោយ $P'(x)=2019x^{2018}+2019$

$\Rightarrow P'(1)=2019+2019=2.2019$

$P(x)=x^{2019}+2019x-1 \quad \Rightarrow P(1)=1+2019-1=2019$

$=2019-\frac{2.2019}{2019}=2017$

លំហាត់ទី១៣២ គណនាផលបូក $S=+(2\cdot 4)-(6\cdot 8)+(10\cdot 12)-\ldots+(2018\cdot 2020)$

ដំណោះស្រាយ

ដោយ

$\sum_{k=0}^{252}(8k+2)\cdot(8k+4)-\sum_{k=0}^{252}(8k-2)\cdot 8k=\sum_{k=0}^{252}(64k^2+48k+8)-\sum_{k=0}^{252}(64k^2-16k)$

$=\sum_{k=0}^{252}(64k+8)=64\sum_{k=0}^{252}k+253\cdot 8=64\cdot\frac{252\cdot 253}{2}+253\cdot 8=253(64\cdot 126+8)=2,042,216$

ដូចនេះ $S=2,042,216$ ។

លំហាត់ទី១៣៣

គេឲពហុធា $P(x)=x^{15}-2009x^{14}+2009x^{13}-\ldots..-2009x^2+2009x$ ។ រកតម្លៃ P(2009)?

ដំណោះស្រាយ

ចំពោះ $n \in \mathbf{Z}^{+}$

ដោយ $x^{n+2} - 2009x^{n+1} + 2008x^{n} = x^{n}(x-1)(x-2008)$

$$P(x) = \left(x^{15} - 2009x^{14} + 2008x^{13}\right) + \left(x^{13} - 2009x^{12} + 2008x^{11}\right) + \ldots + \left(x^{3} - 2009x^{2} + 2008x\right) + x$$

$$\Rightarrow P(x) = \left(x^{13} + x^{11} + \ldots.. + x\right)(x-1)(x-2008) + x$$

$$\Rightarrow \mathrm{P}(2008) = 2008$$

ដូចនេះ $\mathrm{P}(2008) = 2008$

លំហាត់ទី១៣៤

គេឲស្វ៊ីត $(\mathbf{a_n})$ និង $(\mathbf{b_n})$ កំណត់ដោយដូចខាងក្រោម; $\mathbf{a_1} > \mathbf{0}, \mathbf{b_1} > \mathbf{0}$ ចំពោះ $\mathbf{n} = \mathbf{1}, \mathbf{2}, \mathbf{3},$

ដែល $a_{n+1} = a_n + \dfrac{1}{b_n}, b_{n+1} = b_n + \dfrac{1}{a_n}$ ។បង្ហាញថា . $\mathrm{a}_{2008} + b_{2008} > 4\sqrt{1004}$

ដំណោះស្រាយ

ដោយ $C_n = \left(a_n + b_n\right)^2$

$$C_{n+1} = \left(a_{n+1} + b_{n+1}\right)^2 = \left(a_n + \frac{1}{b_n} + b_n + \frac{1}{a_n}\right)^2$$

$$= \left[\left(a_n + b_n\right) + \left(\frac{1}{a_a} + \frac{1}{b_n}\right)\right]^2 > \left(a_n + b_n\right)^2 + 2\left(a_n + b_n\right)\left(\frac{1}{a_n} + \frac{1}{b_n}\right)$$

$$= C_n + 4 + 2\left(\frac{a_n}{b_n} + \frac{b_1}{a_n}\right) \geq C_n + 8 C_2 = \left(\left(a_1 + \frac{1}{a_1}\right)^6 + \left(b_1 + \frac{1}{b_1}\right)\right)^2 \geq 16$$

តែ $C_3 > C_2 + 8 \geq 16 + 8 = 3.8, C_4 > C_3 + 8 > 24 + 8 = 4.8$

$$C_{2008} > 2008.8 = 164.251 = 16.1004$$

$$\Rightarrow a_{2008} + b_{2008} > 4\sqrt{1004}$$

ដូចនេះ . $a_{2008}+b_{2008}>4\sqrt{1004}$ ។

លំហាត់ទី១៣៥ បង្ហាញថា	$a) e^x > 1+x+\frac{x^2}{2!}+\ldots+\frac{x^n}{n!}, \forall x>0$ ។ $b) \sum_{n=1}^{m}\frac{1}{n(n+1)\cdot 2^n} < 1-\ln 2, \quad \forall m\in N, m>1$ ។

ដំណោះស្រាយ

$a) e^x > 1+x+\frac{x^2}{2!}+\ldots+\frac{x^n}{n!}, \forall x>0$

ស្រាយថាតាមវិចារកំណើន;

ចំពោះ $n=1$ នោះ $e^x > 1+x, \forall x>0$

ព្រោះតែ $e^y > 1, \forall y\in(0,x) \Rightarrow \int_0^x e^y dy > \int_0^x dy$

$\Rightarrow e^x - 1 > x$ ពិត

ឧបមាថាវាពិតដល់ $n=k$ គឺ $e^x > 1+x+\frac{x^2}{2!}+\ldots+\frac{x^k}{k!}, \forall x>0$ ពិត

យើងត្រូវតែស្រាយថាវាពិតដល់ $n=k+1$ គឺត្រូវស្រាយថា: $e^x > 1+x+\frac{x^2}{2!}+\ldots+\frac{x^{k+1}}{(k+1)!}, \forall x>0$?

គេមាន $e^x > 1+x+\frac{x^2}{2!}+\ldots+\frac{x^k}{k!}, \forall x>0$

គេបាន : $e^y > 1+y+\frac{y^2}{2!}+\ldots+\frac{y^k}{k!}, \forall y\in(0;x)$

$$\Rightarrow \int_0^x e^y dy > \int_0^x \left(1+y+\frac{y^2}{2!}+\ldots+\frac{y^k}{k!}\right)dy$$

$\Rightarrow e^x - 1 > x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^{k+1}}{(k+1)!}$ ពិត

ដូចនេះ $e^x > 1+x+\frac{x^2}{2!}+\ldots+\frac{x^n}{n!}, \forall x>0$ ។

$b)\sum_{n=1}^{m}\frac{1}{n(n+1)\cdot 2^n}<1-\ln 2,\quad \forall m\in N, m>1$

+ចំពោះ $k\in \mathbf{N}^*, \mathbf{x}\in(0;1): 1+\mathbf{x}+\mathbf{x}^2+\ldots+\mathbf{x}^k=\frac{1-\mathbf{x}^{k+1}}{1-\mathbf{x}}<\frac{1}{1-\mathbf{x}}$

+ចំពោះ $y\in(0;1)$

យើងបានៈ $\int_0^y\left(1+x+\ldots x^k\right)dx<\int_0^1\frac{dx}{1-x}\Rightarrow y+\frac{1}{2}y^2+\ldots+\frac{1}{k+1}y^{k+1}<-\ln(1-y)$

+ចំពោះ $z\in(0;1)$

យើងបានៈ $\int_0^2\left(y+\frac{1}{2}y^2+\ldots+\frac{1}{k+1}y^{k+1}\right)dy<\int_0^z(-\ln(1-y)dy$

$\Rightarrow\frac{1}{1.2}z^2+\frac{1}{2.3}z^3+\ldots+\frac{1}{k(k+1)}.z^{k+2}<z+(1-z)\ln(1-z)$

តែ $z=\frac{1}{2}$ និង $k=m-1$ នោះ $\sum_{n=1}^{m}\frac{1}{n(n+1)\cdot 2^n}<1-\ln 2,\quad \forall m\in N, m>1$

ដូចនេះ $\sum_{n=1}^{m}\frac{1}{n(n+1)\cdot 2^n}<1-\ln 2,\quad \forall m\in N, m>1$ ។

លំហាត់ទី១៣៦

គេឲ n ជាចំនួនគត់វិជ្ជមាន $n\geq 2$ និង $a,b>0$ ។បង្ហាញថា $\frac{(a+b)^n-a^n-b^n}{2^n-2}\geq\sqrt{(ab)^n}$ ។

ដំណោះស្រាយ

យើងបាន : $C_n^0+C_n^1+C_n^2+\ldots+C_n^n=2^n$ ហើយពន្លាតទ្វេធាៈ

$$\frac{(a+b)^n-a^n-b^n}{2^n-2}=\frac{\sum_{i=0}^{n}C_n^i a^{n-i}\cdot b^i-a^n-b^n}{2^n-2}$$

$$=\frac{\sum_{i=1}^{n-1}C_n^i\cdot a^{n-i}\cdot b^i}{2^n-2}=\frac{1}{2^n-2}\cdot\frac{1}{2}\left(\sum_{i=1}^{n-1}C_n^i\left(a^{n-i}b^i+\sum_{i=1}^{n-1}C_n^i\cdot b^{n-i}\cdot a^i\right)\right)$$

$$\geq \frac{1}{2^n-2}\cdot\sum_{i=1}^{n} C_n^i\sqrt{a^n\cdot b^n} = \frac{1}{2^n-2}\left(2^n-2\right)\cdot\sqrt{a^nb^n} = \sqrt{(ab)^n}$$ ។

លំហាត់ទី១៣៧ គេឲ្យស្វ៊ីត $\{a_n\}$ កំណត់ដោយ: $\begin{cases} a_0 = 1999 \\ a_{a+1} = \dfrac{a_n^2}{1+a_n}, \forall n\geq 0 \end{cases}$ ។រកផ្នែកគត់នៃ a_n ចំពោះ $0\leq n\leq 999$ ។

ដំណោះស្រាយ

ចំពោះ $\mathbf{a}_n > 0, \forall n\geq 0$, នោះ : $a_n - a_{n+1} = a_n - \dfrac{a_n^2}{1+a_n} = \dfrac{a_n}{1+a_n} > 0, \forall n\geq 0$

$\Rightarrow \{a_n\}$ ជាស្វ៊ីតកើន (1)

$$\Rightarrow a_{n+1} = \frac{a_n^2}{1+a_n} = a_n - \frac{a_n}{1+a_n} > a_n - 1, \forall n\geq 0$$

$$\Rightarrow a_{n+1} > a_0 - (n+1), \forall n\geq 0$$

$$\Rightarrow a_{n-1} > a_0 - (n-1), \forall n\geq 2$$

$$\Rightarrow a_{n-1} > 2000 - n, \forall n\geq 2 \qquad (2)$$

$$a_n = a_0\left(a_1-a_0\right)+\left(a_2-a_1\right)+\ldots+\left(a_n-a_{n-1}\right)$$

$$= 1999 - \left(\frac{a_0}{1+a_0}+\frac{a_1}{1+a_1}+\ldots+\frac{a_{n-1}}{1+a_n-1}\right)$$

$$= 1999 - n + \left(\frac{1}{1+a_0}+\frac{1}{1+a_1}+\ldots+\frac{1}{1+a_{n-1}}\right) \qquad (3)$$

តាម $(1)\,\&\,(2)$ គេបាន: $0 < \dfrac{1}{1+a_0}+\dfrac{1}{1+a_1}+\ldots+\dfrac{1}{1+a_{n-1}} < 1+a_{n-1}$

$< \dfrac{\mathrm{n}}{2001-\mathrm{n}} < \dfrac{\mathrm{n}}{1998-\mathrm{n}} \leq 1$ ចំពោះ $2\leq \mathrm{n}\leq 1999$ (4)

តាម (3) & (4) គេបាន :

$1999-n<a_n<1999-n\pm 1$ ចំពោះ $612\le n<999$

$\Rightarrow [a_n]=1999-n$ ចំពោះ $2\le n\le 999$

ដោយ

+ $a_0=1999\Rightarrow [a_0]=1999$

+ $a_1=\dfrac{a_0^2}{1+a_0}=a_0-\dfrac{a_0}{1+a_0}=1999-\dfrac{1999}{2000}$

$\Rightarrow a_1=1998+\dfrac{1}{2000}$

$\Rightarrow [a_1]=1998$

ដូច្នេះ $[a_n]=1999-n$ ចំពោះ $0\le n\le 999$ ។

លំហាត់ទី១៣៨ បង្ហាញថា $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)+\cos^{-1}\left(\frac{3}{\sqrt{5}}\right)$ ។

ដំណោះស្រាយ

តាមទម្រង់ $\tan^{-1}x=\sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)=\cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$

គេបាន $\tan^{-1}\left(\dfrac{1}{2}\right)=\sin^{-1}\left(\dfrac{\frac{1}{2}}{\sqrt{1+\frac{1}{4}}}\right)=\sin^{-1}\left(\dfrac{\frac{1}{2}}{\frac{\sqrt{5}}{2}}\right)$

$\tan^{-1}\left(\dfrac{1}{2}\right)=\sin^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$ (1)

គេបាន $\tan^{-1}\left(\dfrac{1}{3}\right)=\cos^{-1}\left(\dfrac{1}{\sqrt{1+\frac{1}{9}}}\right)=\cos^{-1}\left(\dfrac{1}{\frac{\sqrt{10}}{3}}\right)$

$\tan^{-1}\left(\frac{1}{3}\right)=\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$ (2)

តាម (1) & (2) យើងបាន : $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)+\cos^{-1}\left(\frac{3}{\sqrt{5}}\right)$

ដូចនេះ $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)+\cos^{-1}\left(\frac{3}{\sqrt{5}}\right)$ ។

លំហាត់ទី១៣៩ គេអោយស្វ៊ីត (a_n) កំណត់ដោយ :

$$\begin{cases} a_1=1 \\ a_{n+1}=\sqrt{a^2{}_n+\frac{1}{2^n}} \end{cases}, n\geq 1$$

។បង្ហាញថា $a_{n+1}<a_n+\frac{1}{2^{n+1}}, \forall n\geq 1$

ទាញបញ្ជាក់ថា (a_n) ជាស្វ៊ីតរួមហើយគណនាលីមីតរបស់វា។

ដំណោះស្រាយ

បង្ហាញថា $a_{n+1}<a_n+\frac{1}{2^{n+1}}, \forall n\geq 1$ (2)

$+a_n>0 \ , \forall n\geq 1$

គេបានៈ $a_n=\sqrt{a^2{}_{n-1}+\frac{1}{2^{n-1}}}>0 \quad , \forall n\geq 1$

$+(a_n)$ ជាស្វ៊ីតកើន ហើយ $a\geq 1 \ , \forall n\geq 1$

$a_{n+1}=\sqrt{a^2{}_n+\frac{1}{2^n}}>\sqrt{a^2{}_n}=a_n$

$\Rightarrow (a_n)$ ជាស្វ៊ីតកើន

តែ $a_1=1 \Rightarrow , \forall n\geq 1 : a_n\geq a_1=1$

គេមាន $a_{n+1}<a_n+\frac{1}{2^{n+1}}$

$$\Leftrightarrow a_{n+1}^2 < \left(a_n + \frac{1}{2^{n+1}}\right)^2$$

$$\Leftrightarrow a^2{}_n + \frac{1}{2^n} < a_n^2 + \frac{a_n}{2^n} + \frac{1}{2^{2n+2}}$$

$\Leftrightarrow \frac{1}{2^n} < \frac{a_n}{2^n} + \frac{1}{2^{2n+2}}$ ពិត តាម (1)

ព្រោះ $a_n > 1$

ដូចនេះ $a_{n+1} < a_n + \frac{1}{2^{n+1}}, \forall n \geq 1$ ។

+ ទាញបញ្ជាក់ថា (a_n) ជាស្វីតរួម

យើងបាន : $\begin{cases} a_n < a_{n-1} + \frac{1}{2^n} \\ a_{n-1} < a_{n-2} + \frac{1}{2^{n-1}} \\ a_{n-2} < a_{n-3} + \frac{1}{2^{n-2}} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_2 < a_1 + \frac{1}{2^2} \end{cases}$

$$a_n < a_1 + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^n}$$

$$a_n < 1 + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^n}$$

$$a_n < 1 + \frac{1}{4} \cdot \frac{1 - \left(\frac{1}{2}\right)^{n-1}}{\frac{1}{2}}$$

$$a_n < 1 + \left(\frac{1}{2} - \frac{1}{2^n}\right)$$

$$a_n < \frac{3}{2}$$

ដូច្នេះ $\{a_n\}$ ជាស្វ៊ីតទាល់លើ

គេបាន $\{a_n\}$ ជាស្វ៊ីតរួម ។

+ គណនាលីមីត

តាង $l = \lim_{n\to\infty} a_n$, $u_n = a_n^2$

គេបាន:

$$u_n = u_{n-1} + \frac{1}{2^{n-1}}$$
$$u_{n-1} = u_{n-2} + \frac{1}{2^{n-2}}$$
$$\dots\dots\dots$$
$$\dots\dots\dots$$
$$u_2 = u_1 + \frac{1}{2^1}$$

$$u_n = u_1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^n}$$

$$= 1.\frac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$$

$$= 2 - \frac{1}{2^{n-1}}$$

$$\lim_{n\to\infty} u_n = 2$$

$\lim_{n\to\infty} a_n^2 = 2$ ។

លំហាត់ទី១៤០ សម្រាយបញ្ជាក់ថាកន្សោមខាងក្រោមមិនអាស្រ័យអថេរ x, y :

$A = \cos^2(\alpha + x) + \cos^2 x - 2\cos\alpha\cos x\cos(\alpha + x)$ ។

$\mathrm{B} = \sin 6x \cot 3x - \cos 6x)$ ។

$C = \left(\cot\frac{x}{3} - \tan\frac{x}{3}\right)\tan\frac{2x}{3} - \tan y \cdot \tan\left(\frac{\pi}{2} + y\right)$ ។

ដំណោះស្រាយ

$$A=\cos(\alpha+x)[\cos(\alpha+x)-2\cos\alpha\cos x]+\cos^2 x$$

$$=\cos(\alpha+x)(-\cos\alpha\cos x-\sin\alpha\sin x)+\cos^2 x$$

$$=-\cos(\alpha+x)\cos(\alpha-x)+\cos^2 x=\frac{1}{2}(\cos 2\alpha+\cos 2x)+\cos^2 x$$

$$=\frac{1}{2}\cos 2\alpha-\frac{\cos 2x}{2}+\cos^2 x=-\frac{1}{2}\cos 2\alpha+\frac{1}{2}=\sin^2\alpha$$

$$B=2\sin 3x\cos 3x\cdot\frac{\cos 3x}{\sin 3x}-\left(2\cos^2 3x-1\right)$$

$$=2\cos^2 3x-2\cos^2 3x+1=1$$

$$C=\left(\frac{\cos\frac{x}{3}}{\sin\frac{x}{3}}-\frac{\sin\frac{x}{3}}{\cos\frac{x}{3}}\right)\frac{\sin\frac{2x}{3}}{\cos\frac{2x}{3}}+\tan y\cdot\cot y$$

$$=\frac{\cos^2\frac{x}{3}-\sin^2\frac{x}{3}}{\sin\frac{x}{3}\cos\frac{x}{3}}\cdot\frac{\sin\frac{2x}{3}}{\cos\frac{2x}{3}}+1=\frac{\cos\frac{2x}{3}}{\frac{1}{2}\sin\frac{2x}{3}}\cdot\frac{\sin\frac{2x}{3}}{\cos\frac{2x}{3}}+1=3$$

លំហាត់ទី១៤១ គណនា $P=\cos 10^\circ\cos 30^\circ\cos 50^\circ\cos 70^\circ$ ។

$$A=\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$$ ។

ដំណោះស្រាយ

$$P=\cos 30^\circ\left(\cos 10^\circ\cdot\cos 50^\circ\right)\cdot\cos 70^\circ$$

$$=\frac{\sqrt{3}}{4}\left(\cos\left(-40^\circ\right)+\cos 60^\circ\right)\cdot\cos 70^\circ$$

$$=\frac{\sqrt{3}}{4}\cos 40^\circ\cdot\cos 70^\circ+\frac{\sqrt{3}}{8}\cos 70^\circ$$

$$=\frac{\sqrt{3}}{8}\cos\left(-30^{\circ}\right)+\frac{\sqrt{3}}{8}\cos 110^{\circ}+\frac{\sqrt{3}}{8}\cos 70^{\circ}=\frac{3}{16}$$

$$A\sin\frac{\pi}{7}=\cos\frac{2\pi}{7}\sin\frac{\pi}{7}+\cos\frac{4\pi}{7}\sin\frac{\pi}{7}+\cos\frac{6\pi}{7}\sin\frac{\pi}{7}$$

$$=\frac{1}{2}\left(\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}\right)+\frac{1}{2}\left(\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}\right)+\frac{1}{2}\left(\sin\alpha-\sin\frac{5\pi}{7}\right)$$

$$=-\frac{1}{2}\sin\frac{\pi}{7}\cdot$$

$$A=-\frac{1}{2}$$

លំហាត់ទី១៤១ គេអោយ x,y,z ជាចំនួនវិជ្ជមានដែល $xz-zy-yx=1$ ។

រកតម្លៃតូចបំផុតនៃ $P=\frac{2x^2}{1+x^2}-\frac{2y^2}{1+y^2}+\frac{3z^2}{1+z^2}$ ។

ដំណោះស្រាយ

ចំពោះ $x,y,z>0$

$$xz-zy-yx=1$$

$$\Rightarrow\frac{1}{y}-\frac{1}{x}-\frac{1}{z}=\frac{1}{xyz}$$

$a=\frac{1}{x}$; $b=\frac{1}{y}$; $c=\frac{1}{z}$ ព្រោះ $a,b,c>0$

$$b-a-c=abc$$

$a+c=b(1+ac)\quad\Rightarrow b=\frac{a+c}{1+ac}\quad;\begin{cases}a=\tan\alpha\\ b=\tan\beta\end{cases}$ ចំពោះ $\beta,\alpha\in\left(0,\frac{\pi}{2}\right)$

ដោយ $b=\tan(\alpha+\beta)\quad b>0\ \ \alpha,\beta>0\ \ 0<\alpha+\beta<\frac{\pi}{2}$

$$P=\frac{1}{1+a^2}+\frac{1}{1+b^2}+\frac{1}{1+c^2}$$

$= 2\cos^2\alpha - 2\cos^2(\alpha+\beta) + 3\cos^2\beta$

$= \cos 2\alpha - \cos 2(\alpha+\beta) + 3\cos^2\beta$

$= 2\sin\beta.\sin(\alpha+\beta) + 3\cos^2\beta$

$= -3\sin^2\beta + 2\sin\beta.\sin(\alpha+\beta) + 3$

$= -3\left[\sin\beta - \frac{1}{3}\sin(2\alpha+\beta)\right]^2 - \frac{1}{3}\cos^2(2\alpha+\beta) + \frac{10}{3} \le \frac{10}{3}$

$$\begin{cases} \cos(2\alpha+\beta) = 0 \\ \sin\beta = \frac{1}{3} \end{cases} \Leftrightarrow \begin{cases} x = \sqrt{2} \\ y = \frac{\sqrt{2}}{2} \\ z = 2\sqrt{2} \end{cases}$$

$P = \frac{10}{3}$ ។

លំហាត់ទី១៤២ គេអោយ n ជាចំនួនគត់ដែល $n \ge 2$ ។បង្ហាញថា $\left(1+\frac{1}{n}\right)^n < 3$

ដំណោះស្រាយ

យើងមាន $\left(1+\frac{1}{n}\right)^n = \sum_{k=0}^{n} C_n^k \left(\frac{1}{n}\right)^k$

គេបាន $\left(1+\frac{1}{n}\right)^n = \sum_{k=0}^{n} C_n^k \left(\frac{1}{n}\right)^k = C_n^0\left(\frac{1}{n}\right)^0 + C_n^1\left(\frac{1}{n}\right)^1 + \ldots\ldots\ldots$

$= 1 + 1 + \frac{n!}{2!(n-2)!}\cdot\left(\frac{1}{n}\right)^2 + \frac{n!}{3!(n-3)!}\cdot\left(\frac{1}{n}\right)^3 + \ldots$

$= 2 + \frac{1}{2!}\cdot\frac{n-1}{n} + \frac{1}{3!}\cdot\frac{(n-1)(n-2)}{n^2} + \ldots < 2 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} < 2 + \frac{1}{1.2} + \frac{1}{2.3} + \ldots + \frac{1}{(n-1)n}$

$= 2 + \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{n-1} - \frac{1}{n}\right)$

$=3-\frac{1}{n}<3$

ដូចនេះ $\left(1+\frac{1}{n}\right)^n<3$ ។

លំហាត់ទី១៤៣ បង្ហាញថាចំនួនគត់ធម្មជាតិ $n\geq 2$ កំណត់ដោយ: $\sqrt[n]{1+\frac{\sqrt[n]{n}}{n}}+\sqrt[n]{1-\frac{\sqrt[n]{n}}{n}}<2$ ។

ដំណោះស្រាយ

តាមវិសមភាព Cauchy គេបាន:

$$\frac{1+1+1+....+\left(1+\frac{\sqrt[n]{n}}{n}\right)}{n}>\sqrt[n]{1+\frac{\sqrt[n]{n}}{n}}$$

$$\Leftrightarrow \frac{n+\frac{\sqrt[n]{n}}{n}}{n}>\sqrt[n]{1+\frac{\sqrt[n]{n}}{n}}$$

$$\Leftrightarrow 1+\frac{\sqrt[n]{n}}{n}>\sqrt[n]{1+\frac{\sqrt[n]{n}}{n}} \quad (1)$$

តាមវិសមភាព Cauchy គេបាន:

$$\frac{1+1+1+....+\left(1+\frac{\sqrt[n]{n}}{n}\right)}{n}>\sqrt[n]{1-\frac{\sqrt[n]{n}}{n}}$$

$$\Leftrightarrow \frac{n-\frac{\sqrt[n]{n}}{n}}{n}>\sqrt[n]{1-\frac{\sqrt[n]{n}}{n}}$$

$$\Leftrightarrow 1-\frac{\sqrt[n]{n}}{n}>\sqrt[n]{1-\frac{\sqrt[n]{n}}{n}} \quad (2)$$

តាម $(1)\&(2)$ គេបាន : $\sqrt[n]{1+\frac{\sqrt[n]{n}}{n}}+\sqrt[n]{1-\frac{\sqrt[n]{n}}{n}}<2$

ដូចនេះ $\sqrt[n]{1+\frac{\sqrt[n]{n}}{n}}+\sqrt[n]{1-\frac{\sqrt[n]{n}}{n}}<2$ ។

លំហាត់ទី១៤៤ គណនា $\frac{(4\times7+2)(6\times9+2)(8\times11+2)\ldots(100\times103+2)}{(5\times8+2)(7\times10+2)(9\times12+2)\ldots(99\times102+2)}$

ដំណោះស្រាយ

តាមរូបមន្ត $n(n+3)+2=n^2+3n+2=(n+1)(n+2)$

$$\frac{(4\times7+2)(6\times9+2)(8\times11+2)\cdots\cdots(100\times103+2)}{(5\times8+2)(7\times10+2)(9\times12+2)\cdots(99\times102+2)}$$
$$=\frac{(5\times6)(7\times8)(9\times10)\cdots(101\times102)}{(6\times7)(8\times9)(10\times11)\cdots(100\times101)}$$
$$=5\times102=510$$

លំហាត់ទី១៤៥ គណនា $\frac{83^2+17^3}{83\times66+17^2}$

ដំណោះស្រាយ

តាមរូបមន្ត $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$

$$\frac{83^2+17^3}{83\times66+17^2}$$
$$=\frac{(83+17)\left(83^2-83\times17+17^2\right)}{83\times66+17^2}$$
$$=\frac{100\times\left(83\times66+17^2\right)}{83\times66+17^2}=100$$

លំហាត់ទី១៤៦ គេអោយ $x+\frac{1}{x}=3$ ។រកតម្លៃ $(i)x^3+\frac{1}{x^3};\quad(ii)x^4+\frac{1}{x^4}$

ដំណោះស្រាយ

(i) $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

$=3\left[\left(x+\frac{1}{x}\right)^2-3\right]$

$=3\left(3^2-3\right)=18$

(ii) $x^4+\frac{1}{x^4}=\left[\left(x^2\right)^2+2+\left(\frac{1}{x^2}\right)^2\right]-2$

$=\left(x^2+\frac{1}{x^2}\right)^2-2$

$=\left[\left(x+\frac{1}{x}\right)^2-2\right]^2-2=\left(3^2-2\right)^2-2=47$

លំហាត់ទី១៧ គេអោយ |q|<1 ។ គណនាផលបូកអនន្ត

$a)A=1+2q+3q^2+\ldots+nq^{n-1}+\ldots$ ។

$b)B=1+4q+9q^2+\ldots+n^2q^{n-1}+\ldots$ ។

ដំណោះស្រាយ

យើងមាន $a)A=1+2q+3q^2+\ldots+nq^{n-1}+\ldots$

នោះ $Aq=q+2q^2+3q^3+\ldots+nq^n+\ldots$

គេបាន $A-Aq=1+q+q^2+\ldots+q^n+\ldots$

នោះ $A(1-q)=\frac{1}{1-q}$.

$\Rightarrow A=\frac{1}{(1-q)^2}$

ដូចនេះ $A=\frac{1}{(1-q)^2}$ ។

$b)B=1+4q+9q^2+\ldots+n^2q^{n-1}+\ldots$

យើងមាន $B=1+4q+9q^2+\ldots+n^2q^{n-1}+\ldots$

នោះ $\mathrm{Bq}=q+4q^2+9q^3+\ldots+n^2q+$

គេបាន $\mathrm{B(1-q)=1+3q+5q^2+\ldots+(2n+1)q^n+\ldots}$

នោះ $B(1-q)q=q+3q^2+5q^3+\ldots+(2n+1)q^{n+1}+\ldots$

$\Rightarrow B(1-q)-B(1-q)^q=1+2q+2q^2+\ldots+2q^n+\ldots B(1-q)^2$

$=1+2q\left(1+q+q^2+\ldots\right)=1+2q\cdot\dfrac{1}{1-q}=\dfrac{1+q}{1-q}$

$\Rightarrow B=\dfrac{1+q}{(1-q)^3}$

លំហាត់ទី១៤៨ គណនា $\dfrac{1}{3-\sqrt{8}}-\dfrac{1}{\sqrt{8}-\sqrt{7}}+\dfrac{1}{\sqrt{7}-\sqrt{6}}-\dfrac{1}{\sqrt{6}-\sqrt{5}}+\dfrac{1}{\sqrt{5}-2}$

ដំណោះស្រាយ

យើងបាន :

$\dfrac{1}{3-\sqrt{8}}=3+\sqrt{8}$

$\dfrac{1}{\sqrt{8}-\sqrt{7}}=\sqrt{8}+\sqrt{7}$

$\dfrac{1}{\sqrt{7}-\sqrt{6}}=\sqrt{7}+\sqrt{6}$

$\dfrac{1}{\sqrt{6}-\sqrt{5}}=\sqrt{6}+\sqrt{5}$

$\dfrac{1}{\sqrt{5}-2}=\sqrt{5}+2$

គេបាន $\dfrac{1}{3-\sqrt{8}}-\dfrac{1}{\sqrt{8}-\sqrt{7}}+\dfrac{1}{\sqrt{7}-\sqrt{6}}-\dfrac{1}{\sqrt{6}-\sqrt{5}}+\dfrac{1}{\sqrt{5}-2}$

$=3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2=3+2=5$

លំហាត់ទី១៤៩   គណនា $E=\left[\sqrt[\sqrt{\sqrt{\sqrt{\sqrt{2}}}}]{\sqrt[\sqrt{\sqrt{\sqrt{2}}}]{\sqrt[\sqrt{\sqrt{2}}]{\sqrt[\sqrt{2}]{2^{\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}}}}}\right]^{\frac{1}{2}}$

ដំណោះស្រាយ

គេបាន $a=\sqrt{\sqrt{\sqrt{\sqrt{2}}}}=2^{\frac{1}{16}}$

$b=\sqrt{\sqrt{\sqrt{2}}}=2^{\frac{1}{8}}$

$c=\sqrt{\sqrt{2}}=2^{\frac{1}{4}}$

$c=\sqrt{\sqrt{2}}=2^{\frac{1}{4}}$

$c=\sqrt{2}=2^{\frac{1}{2}}$

$e=\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}=2^{\frac{15}{16}}$

គេបាន $E=\left[\sqrt[a]{\sqrt[b]{\sqrt[c]{\sqrt[d]{2^{e}}}}}\right]^{\frac{1}{2}}$

$$=\left[\sqrt[\sqrt[16]{2^{15}}]{2^{2^{\frac{15}{16}}}}\right]^{\frac{1}{2}}=\left[\left(2^{2^{\frac{15}{16}}}\right)^{2^{\frac{-15}{16}}}\right]^{\frac{1}{2}}=\left[2^{2^{\frac{15}{16}-\frac{15}{16}}}\right]^{\frac{1}{2}}=\left[2^{2^{0}}\right]^{\frac{1}{2}}=\sqrt{2}$$

លំហាត់ទី១៥០   ចូរស្រាយបញ្ជាក់ថា $e^{ix}=\cos x+i\sin x$ ។

ដំណោះស្រាយ

គេមាន $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

នោះ $e^{ix} = \sum_{n=0}^{\infty} \frac{(ix)^n}{n!} = \frac{(ix)^0}{0!} + \frac{(ix)^1}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + ...+ \frac{(ix)^n(-1)^n}{n!} + R(x)$

$$= 1 + \frac{ix}{1!} - \frac{ix^2}{2!} + \frac{ix^3}{3!} + ...+ \frac{(-1)^n ix^{2n}}{2n!} + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + R(x)$$

$$= \underbrace{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ...+ \frac{(-1)^n x^{2n+1}}{(2n+1)!}\right)}_{\cos x} + i\underbrace{\left(\frac{x}{1!} + \frac{x^3}{3!} + ...+ \frac{(-1)^n x^{2n}}{2n!}\right)}_{\sin x}$$

$$\Rightarrow e^{ix} = \cos x + i\sin x$$

លំហាត់ទី១៨១ បង្ហាញថា $\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + ...+ \frac{1}{n!} < 3$

ដំណោះស្រាយ

គេមាន $\frac{1}{0!} = 1$

$\frac{1}{1!} = 1$

$\frac{1}{2!} = 1 - \frac{1}{2}$

$\frac{1}{3!} = \frac{1}{2} - \frac{1}{3}$

$\frac{1}{4!} = \frac{1}{3} - \frac{1}{4}$

....

....

$\frac{1}{n!} < \frac{1}{n(n-1)} = \frac{1}{n-1} - \frac{1}{n}$

$s_n < 1+1+1-\frac{1}{n}$

$s_n < 3-\frac{1}{n} \Rightarrow s_n < 3$ ពិត

លំហាត់ទី១៥២

បង្ហាញថា ក. $44442^{44442}+49996^{49996} \vdots 5$ (លំហាត់សិស្សពូកែទី១២ ឆ្នាំ២០២០២)

ខ. $8049^{8049^{8049}}+3\times 13335^{88888} \vdots 4$

ដំណោះស្រាយ

ក. $44442^{44442}+49996^{49996} \vdots 5$

$44442^{44442}=\left(4444+2\right)^{44442} \equiv 2^{44442} \equiv \left(5-1\right)^{22221} (\text{mod}\,5) \equiv -1(\text{mod}\,5)$ (1)

$49996^{49996}=\left(49995+1\right)^{49996} \equiv 1(\text{mod}\,5)$ (2)

តាម (1) & (2) គេបាន : $44442^{44442}+49996^{49996} \equiv 0(\text{mod}\,5)$ ។

ដូចនេះ $44442^{44442}+49996^{49996} \vdots 5$ ។

ខ. $8049^{8049^{8049}}+3\times 13335^{88888} \vdots 4$

$8049^{8049^{8049}}=\left(8048+1\right)^{8049^{8049}} \equiv 1(\text{mod}\,4)$ (1)

$3\times 13335^{88888}=3\left(13336-1\right)^{88888} \equiv 3(\text{mod}\,4)$ (2)

តាម (1) & (2) គេបាន : $8049^{8049^{8049}}+3\times 13335^{88888} \vdots 4$

ដូចនេះ $8049^{8049^{8049}}+3\times 13335^{88888} \vdots 4$ ។

លំហាត់ទី១៥៣

គេឲ្យស្វីតចំនួនពិត $\{x_n\}$ និង $\{y_n\}$ កំណត់ដោយ $x_n = 1!1 + 2!2 + 3!3 + \ldots\ldots\ldots\ldots + n!n$

និង $y_n = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots\ldots\ldots + \frac{n}{(n+1)!}$ ។គេដឹងថា $\frac{1}{1.2} + \frac{2}{2.3} + \ldots\ldots\ldots + \frac{n}{2018.2019} = \frac{2018}{2019}$

ចូរប្រៀបធៀបចំនួន $\left(\frac{x_{2019}}{2020y_{2019}}\right)^2$ និង 2019^{2019} ។ (លំហាត់សិស្សពូកែ២០២០)

ដំណោះស្រាយ

ចំណាំ

$$(k+1)! = (k+1)k! = kk! + k!$$
$$\Rightarrow kk! = (k+1)! - k!$$

គេមាន $x_n = 1!1 + 2!2 + 3!3 + \ldots\ldots\ldots\ldots + n!n$

$$y_n = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots\ldots\ldots + \frac{n}{(n+1)!}$$

តាង $A = \left(\frac{x_{2019}}{2020y_{2019}}\right)^2$ និង $B = 2019^{2019} \Rightarrow B = 2019 \times 2019 \times \ldots\ldots\ldots \times 2019 \quad (1)$

គេបាន : $$A = \left(\frac{x_{2019} = 1!1 + 2!2 + 3!3 + \ldots\ldots\ldots\ldots + 2019!2019}{2020.y_{2019} = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots\ldots\ldots + \frac{n}{(n+1)!}}\right)^2 = \left(\frac{x_{2019} = \sum_{n=1}^{2019} n!n}{2020.y_{2019} = \sum_{n=1}^{2019} \frac{n}{(n+1)!}}\right)^2$$

$$= \left(\frac{x_{2019} = \sum_{n=1}^{2019} n!n = \sum_{n=1}^{2019} (n+1)! - n!}{2020.y_{2019} = \sum_{n=1}^{2019} \frac{n}{(n+1)!} = \sum_{n=1}^{2019} \frac{n+1-1}{(n+1)!} = \sum_{n=1}^{2019} \frac{1}{n!} - \sum_{n=1}^{2019} \frac{1}{(n+1)!}}\right)^2 = \left(\frac{x_{2019} = 2020! - 1!}{2020.y_{2019} = 1 - \frac{1}{2020!}}\right)^2$$

$$= \left(\frac{x_{2019} = 2020! - 1}{2020.y_{2019} = \frac{2020! - 1}{2020!}}\right)^2 \Rightarrow A = \left(\frac{2020! - 1}{2020 \frac{2020! - 1}{2020.2019!}}\right)^2 = (2019!)^2$$

តែ $\Rightarrow A = (2019 \times 2018 \times \ldots\ldots\ldots \times 1)^2 = 2019^2 \times 2018^2 \times \ldots\ldots \times 1^2 \quad (2)$

តាម (1) និង (2) គេបាន : $A > B$

ដូចនេះ $A=\left(\frac{x_{2019}}{2020y_{2019}}\right)^2 > \text{B}=2019^{2019}$ ។

លំហាត់ទី១៩៤ គេអោយ x,y,z ជាចំនួនពិតវិជ្ជមាន ។ស្រាយបញ្ជាក់ថា

$$\left(\frac{x}{y}+\frac{z}{\sqrt[3]{xyz}}\right)^2+\left(\frac{y}{z}+\frac{x}{\sqrt[3]{xyz}}\right)^2+\left(\frac{z}{x}+\frac{y}{\sqrt[3]{xyz}}\right)^2\geq 12$$

ដំណោះស្រាយ

គេមាន x,y,z ជាចំនួនពិតវិជ្ជមាន គេបានៈ

$$\left(\frac{x}{y}+\frac{z}{\sqrt[3]{xyz}}\right)^2+\left(\frac{y}{z}+\frac{x}{\sqrt[3]{xyz}}\right)^2+\left(\frac{z}{x}+\frac{y}{\sqrt[3]{xyz}}\right)^2$$

$$=\left(\frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}\right)+\left(\frac{x}{\sqrt{xyz}}+\frac{y}{\sqrt[3]{xyz}}+\frac{z}{\sqrt[3]{xyz}}\right)+2\left(\frac{yz}{x\sqrt[3]{xyz}}+\frac{xz}{y\sqrt[2]{xyz}}+\frac{xy}{z\sqrt[3]{xyz}}\right)$$

+ តាមវិសមភាព *Chauchy* បីតួ

$$\frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}\geq 3\sqrt{\left(\frac{x^2}{y^2}\right)\left(\frac{y^2}{z^2}\right)\left(\frac{z^2}{x^2}\right)}$$

$$\frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}\geq 3 \quad (1)$$

$$\frac{x}{\sqrt[3]{xyz}}+\frac{y}{\sqrt[3]{xyz}}+\frac{z}{\sqrt{xyz}}\geq 3\sqrt[3]{\left(\frac{x}{\sqrt[3]{xyz}}\right)\left(\frac{y}{\sqrt[3]{xyz}}\right)\left(\frac{z}{\sqrt[3]{xyz}}\right)}$$

$$\frac{x}{\sqrt{xyz}}+\frac{y}{\sqrt{xyz}}+\frac{z}{\sqrt{xyz}}\geq 3 \quad (2)$$

$$\frac{x}{\sqrt[3]{xyz}}+\frac{y}{\sqrt[3]{xyz}}+\frac{z}{\sqrt[3]{xyz}}\geq 3\sqrt[3]{\frac{yz}{x\sqrt[3]{xyz}}\frac{xz}{y\sqrt[3]{xyz}}\frac{xy}{z\sqrt[3]{xyz}}}$$

$$\frac{yz}{x\sqrt{xyz}}+\frac{xz}{y\sqrt{xyz}}+\frac{xy}{z\sqrt{xyz}}\geq 3 \quad (3)$$

តាម (1) & (2) & (3) គេបាន : $\left(\frac{x}{y}+\frac{z}{\sqrt[3]{xyz}}\right)^2+\left(\frac{y}{z}+\frac{x}{\sqrt[3]{xyz}}\right)^2+\left(\frac{z}{x}+\frac{y}{\sqrt[3]{xyz}}\right)^2 \geq 3+3+3.2=12$

ដូចនេះ $\left(\frac{x}{y}+\frac{z}{\sqrt[3]{xyz}}\right)^2+\left(\frac{y}{z}+\frac{x}{\sqrt[3]{xyz}}\right)^2+\left(\frac{z}{x}+\frac{y}{\sqrt[3]{xyz}}\right)^2 \geq 12$ ។

លំហាត់ទី១៥៥ តាង m,n ជាចំនួនគត់ដែល $1\leq m\leq n$ កំណត់ដោយ :

$f(m,n)=\left(1-\frac{1}{m}\right)\left(1-\frac{1}{m+1}\right)\left(1-\frac{1}{m+2}\right)\ldots\left(1-\frac{1}{n}\right)$ ។

បើ $S=f(2,2020)+f(3,2020)+\ldots+f(2020,2020)$ ។ រកតម្លៃនៃ $2S$ ។

ដំណោះស្រាយ

មាន m,n ជាចំនួនគត់ដែល $1\leq m\leq n$ និង $f(m,n)$

$=\left(1-\frac{1}{m}\right)\left(1-\frac{1}{m+1}\right)\left(1-\frac{1}{m+2}\right)\cdots\left(1-\frac{1}{n}\right)$

$=\left(\frac{m-1}{m}\right)\left(\frac{m}{m+1}\right)\left(\frac{m+1}{m+2}\right)\ldots\left(\frac{n-1}{n}\right)=\frac{m-1}{n}$

$f(2,n)+f(3,n)+\ldots+f(n,n)$

$=\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{n-1}{n}=\frac{n-1}{2}$

$S=f(2,2020)+(3,2020)+\ldots+f(2020,2020)=\frac{2019}{2}$

ដូចនេះ $2S-2019$ ។

លំហាត់ទី១៥៦ រកពហុធាដឺក្រេតូចបំផុត ដោយមានមេគុណជាចំនួនគត់ហើយរឺសមួយរបស់វាគឺ

$\sqrt{2}+\sqrt[3]{3}$ ។

ដំណោះស្រាយ

រកពហុធាដឺក្រេតូចបំផុតរឺសមួយគឺ $x=\sqrt{2}+\sqrt[3]{3}$

$\Rightarrow \sqrt[2]{3^3}=(x-\sqrt{2})^3$

$\Rightarrow 3 \quad = x^3-3x^2\sqrt{2}+6x-2\sqrt{2}$

$\Rightarrow \left(x^3+6x-3\right)=\sqrt{2}\left(3x^2+2\right)$

លើកអង្គទាំងពីរជាការេ

$\Rightarrow \left(x^3+6x-3\right)^2=2\left(3x^2+2\right)^2$

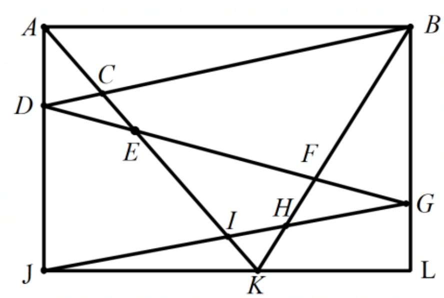
$\Rightarrow x^6+36x^2+9+12x^4-6x^3-36x=18x^4+24x^2+8$

គេបាន $x^6-6x^4-6x^3+12x^2-36x+1=0$

ដូចនេះ សមីការដឺក្រេតូចបំផុតគឺ $x^6-6x^4-6x^3+12x^2-36x+1=0$ ។

លំហាត់ទី១៥៧

ក្នុងរូបខាងស្តាំ បង្ហាញថាពីចតុកោណកែង $ABLJ$ ដែលផ្ទៃក្រឡានៃត្រីកោណ ACD ចតុកោណ $BCEF$ ចតុកោណ $DEIJ$ និង ត្រីកោណ FGH ស្មើនឹង $22cm^2, 500cm^2$ $482cm^2$ និង $22cm^2$ រៀងគ្នា ។ រកផ្ទៃក្រឡាត្រីកោណ HIK ជា cm^2 ។ តាមរូបដូចខាងក្រោម៖



ដំណោះស្រាយ

តាង $S=S_{\text{ABLJ}}$

$$\Rightarrow S_{ABC}=\frac{1}{2}S=S_{ABD}+S_{DGJ}$$

ដោយ $S_{ABK}=S_{ABC}+S_{BCEF}+S_{EFHI}+S_{IHK}$ (1) និង

$$S_{ABD}+S_{DGJ}=S_{ABC}+S_{ACD}+S_{DEIJ}+S_{EFHI}+S_{FGH} \quad (2)$$

$(1)=(2)$

$$\Rightarrow S_{BCEF}+S_{IHK}=S_{ACD}+S_{DEIJ}+S_{FGH}$$

$$\Rightarrow \quad 500+S_{IHK}=22+482+22$$

$$\Rightarrow S_{IHK}=526-500=26$$

ដូចនេះ $S_{IHK}=26cm^2$ ។

លំហាត់ទី១៥៨

រកអនុគមន៍ $f(x)$ កំណត់គ្រប់ $x\neq 0$ ដែលផ្ទៀងផ្ទាត់ $f\left(\frac{1}{x}\right)+\frac{1}{x}f(-x)=2x$ ។ គណនា $f(2020)$

ដំណោះស្រាយ

រកអនុគមន៍ $f(x)$

គេមាន $f\left(\frac{1}{x}\right)+\frac{1}{x}f(-x)=2x \quad (1)$

ជំនួស x ដោយ $\frac{1}{x}$ គេបាន $f\left(\frac{1}{\frac{1}{x}}\right)+xf\left(-\frac{1}{x}\right)=\frac{2}{x}$

ជំនួស x ដោយ $-x$ គេបាន $f(-x)-xf\left(\frac{1}{x}\right)=-\frac{2}{x} \quad (2)$

តាម (1) និង (2) $\begin{cases} f\left(\frac{1}{x}\right)+\frac{1}{x}f(-x)=2x(1) \\ f(-x)-xf\left(\frac{1}{x}\right)=-\frac{2}{x}(2)\times\frac{1}{x} \end{cases}$

គេបាន $2f\left(\frac{3}{x}\right)=2x+\frac{2}{x^2}\Rightarrow f\left(\frac{2}{x}\right)=x+\frac{1}{x^2}\Rightarrow f(x)=\frac{1}{x}+x^2=\frac{x^2+1}{x}$

+គណនា $f(2020)$

$$\Rightarrow f(2020)=\frac{2020^3+1}{2020}$$ ។

លំហាត់ទី១៥៩ ស្រាយបញ្ជាក់ថា ចំនួន $\frac{a_1}{a_1+a_2};\frac{a_2}{a_2+a_3};\frac{a_1}{a_3+a_4}$ ជាបីតួតគ្នានៃស្វ៊ីតនព្វន្តមួយ។

ដំណោះស្រាយ

ដោយ

$$a_1=C(n,0)=\frac{n!}{n!0!}=1 \quad , \quad a_2=C(n,1)=\frac{n!}{(n-1)!1!}=n \quad , \quad a_3=C(n,2)=\frac{n!}{(n-2)!2!}=\frac{n(n-1)}{2}$$

$$,a_4=C(n,3)=\frac{n!}{(n-3)!3!}=\frac{n(n-1)(n-2)}{6}$$

$$\Rightarrow \frac{a}{a+a_2}=\frac{1}{n+1}$$

$$\Rightarrow \frac{a_2}{a_2+a_3}=\frac{n}{n+\frac{n(n-1)}{2}}=\frac{1}{1+\frac{n-1}{2}}=\frac{2}{n+1}$$

$$\Rightarrow \frac{a_3}{a_3+a_4}=\frac{\frac{n(n-1)}{2}}{\frac{n(n-1)}{2}+\frac{n(n-1)(n-2)}{6}}=\frac{1}{1+\frac{n-2}{3}}=\frac{3}{n+1}$$

$$\Leftrightarrow \frac{a}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{1}{n+1}+\frac{3}{n+1}=\frac{4}{n+1}$$

$$\Leftrightarrow 2\frac{a_1}{a_2+a_3}=\frac{4}{n+1}$$

$\Rightarrow \frac{a_1}{a_1+a_2}+\frac{a_1}{a_3+a_4}=2\frac{a_2}{a_2+a_3}$ តាមទំនាក់ទំនងនេះបញ្ជាក់ឲឃើញថា $\frac{a_1}{a_1+a_2};\frac{a_2}{a_2+a_2};\frac{a_1}{a_3+a_4}$ ជាបីតួនៃស្វ៊ីតនព្វន្ត។

ដូចនេះ $\frac{a_1}{a_1+a_2};\frac{a_2}{a_2+a_2};\frac{a_1}{a_3+a_4}$ ជាបីតួនៃស្វ៊ីតនព្វន្ត។

លំហាត់ទី១៦០ បើ $a_1+a_2+a_3+\ldots+a_n=n$ ។ចូរស្រាយបញ្ជាក់ថា $a_1^4+a_2^4+a_3^4+\ldots+a_n^4\geq n$

ដំណោះស្រាយ

តាមវិសមភាព Chauchy - Schwarz គេបាន

$(a_1+a_2+a_3+\ldots+a_n)^2\leq(1^2+1^2+1^2+\ldots+1^2)(a_1^2+a_2^2+a_1^2+\ldots+a_n^2)$

$n^2\leq n(a_1^2+a_2^2+a_3^2+\ldots+a_n^2)\Rightarrow a_1^2+a_2^2+a_3^2+\ldots+a_n^2\geq n$ ពិត

ដូច្នេះ $\Rightarrow a_1^2+a_2^2+a_3^2+\ldots+a_n^2\geq n$ ។

លំហាត់ទី១៦១ រកចំនួនគត់ដែលមានលេខបួនខ្ទង់ខុសគ្នានៅចន្លោះ 2000 និង 5000 ។

ដំណោះស្រាយ

រកចំនួនគត់គូដែលមានលេខ 4 ខ្ទង់ចំនួនគត់គូដែលមានលេខ 4 ខ្ទង់ខុសគ្នានៅន្លោះ 2000 និង

5000

លេខខ្ទង់ពាន់ $2,3,4$ និងលេខខ្ទង់រាយ $0,2,4,6,8$

+ ករណីចំនួនចន្លោះផ្តើមដោយលេខ 2 គេបាន

ព្រឹត្តិការណ៍ជ្រើសរើសលេខខ្ទង់ពាន់មាន 1 របៀប

រាយមាន 4 របៀប

ដប់មាន 8 របៀប

រយមាន 7 របៀប

តាមគោការណ៍ផលគុណ $1.4.8.7=224$ ចំនួន

+ ករណីចំនួនចន្លោះផ្តើមដោយលេខ 3 គេបាន

ព្រឹត្តិការណ៍ជ្រើសរើសលេខខ្ទង់ពាន់មាន 1 របៀប

រាយមាន 5 របៀប

ដប់មាន 8 របៀប

រាយមាន 7 របៀប

តាមគោការណ៍ផលគុណ $1.5.8.7=280$ ចំនួន

+ ករណីចំនួនចន្លោះផ្តើមដោយលេខ 3 គេបាន

ដូចករណីចំនួនចន្លោះផ្តើមដោយលេខ 2 ដែរ

ព្រឹត្តិការណ៍ជ្រើសរើសលេខខ្ទង់ពាន់មាន 1 របៀប

ចំនួន 224 ចំនួន

គេបាន ចំនួនគត់គូសរុបគឺ $224+280+224=728$ ចំនួន ។

លំហាត់ទី១៦២ គេឲចំនួនពិត $x, y\in(-2,2)$ និង $xy=-1$ ។ ស្រាយបំភ្លឺថា $\dfrac{4}{4-x^2}+\dfrac{9}{9-y^2}\geq\dfrac{12}{5}$

ដំណោះស្រាយ

ចំពោះគ្រប់ $x, y\in(-2,2)$ គេបាន

$$\frac{4}{4-x^2}>0 \quad \frac{9}{9-y^2}>0$$

តាមវិសមភាពកូស៊ី

$$\frac{4}{4-x^2}+\frac{9}{9-y^2}\geqslant 2\sqrt{\left(\frac{4}{4-x^2}\right)\left(\frac{9}{9-y^2}\right)}$$

$$\geqslant\frac{12}{\sqrt{36-9x^2-4y^2+x^2y^2}}$$

$$\geqslant \frac{12}{\sqrt{37-\left(9x^2+4y^2\right)}} (1)$$

(ព្រោះ $xy=-1$)

ដោយ

$$9x^2+4y^2 \geqslant 2\sqrt{\left(9x^2\right)\left(4y^2\right)}$$
$$\geqslant 12$$

$$-\left(9x^2+4y^2\right)\leq -12$$
$$37-\left(9x^2+4y^2\right)\leqslant 25$$
$$\sqrt{37-\left(9x^2+4y^2\right)}\leqslant 5$$

$$\Rightarrow \frac{1}{\sqrt{37-(9x^2+4y^2)}}\geq \frac{1}{5}$$

$$\Rightarrow \frac{4}{4-x^2}+\frac{9}{9-y^2}\geq \frac{12}{5}$$

ដូចនេះ $\frac{4}{4-x^2}+\frac{9}{9-y^2}\geq \frac{12}{5}$ ។

លំហាត់ទី១៦៣ គេអោយអនុគមន៍ f កំណត់លើ $\mathbb{N}$ f(1)=11, f(2)=22, f(3)=33, f(4)=44

និង f(n)=f(n-1)-f(n-2)+f(n-3)-f(n-4) ចំពោះ $n\geq 5$ ។

ដំណោះស្រាយ

យើងមាន $\begin{cases} f(n)=f(n-1)-f(n-1)+f(n-3)-f(n-4) \\ f(n-1)=f(n-2)-f(n-3)+f(n-4)-f(n-5) \end{cases}$

$$f(n)=-f(n-5)$$
$$=-\left[-f\left(n-5-5\right)\right]$$

$$=(-1)^2 f(n-5\times 2)=(-1)^2\left[-f(n-5\times 2-5)\right]=(-1)^3 f\left(n-5\times 3\right)=.......$$

ជាទូទៅ $f(n)=(-1)^k f(n-5k) \Rightarrow f(2020)=(-1)^{403} f(2020-5\times 403)=-f(5)$

$=-\left[f(4)-f(3)+f(2)-f(1)\right]=\left(44-33+22-11\right)=-22$

ដូចនេះ $f(2020)=-22$ ។

លំហាត់ទី១៦៤ គេអោយ $a>1,b>1$ ។បង្ហាញថា $\frac{a^2}{a-1}+\frac{b^2}{a-1}\geq 8$

ដំណោះស្រាយ

ចំពោះ $a>1,b>1$ ដែល $a=1+x$ $b=1+y$ $x,y>0$

គេបាន $\frac{(x+1)^2}{x}+\frac{(y+1)^2}{y}=\frac{x^2+2x+1}{x}+\frac{y^2+2y+1}{y}$

$\left(x+\frac{1}{x}\right)+\left(y+\frac{1}{y}\right)+4\geq 2\sqrt{x\cdot\frac{1}{x}}+2\sqrt{y\cdot\frac{1}{y}}+4=8$

$\Rightarrow \frac{a^2}{a-1}+\frac{b^2}{a-1}\geq 8$

លំហាត់ទី១៦៥ $A=\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{8}\right)+\left(1+\frac{1}{15}\right)+.......+\left(1+\frac{1}{n^2+2n}\right)$ ។គណនា $\lim_{n\to\infty} A$

ដំណោះស្រាយ

គេមាន $A=\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{8}\right)+\left(1+\frac{1}{15}\right)+.......+\left(1+\frac{1}{n^2+2n}\right)$

$\Rightarrow A=\frac{2^2}{3}\cdot\frac{3^2}{8}\cdot.........\cdot\frac{(n+1)^2}{n^2+2n}$

$=\frac{2.2}{1.3}\cdot\frac{3.3}{2.4}\cdot\frac{4.4}{3.4}\cdot.........\cdot\frac{(n+1)(n+1)}{n(n+2)}$

$=\frac{2}{1}\cdot\frac{(n+1)}{n+2}=\frac{2n+2}{n+2}$

\+ គណនា $\lim_{n\to\infty} A$

$$\lim_{n\to\infty}\frac{2n+2}{n+2}=\lim_{n\to\infty}\frac{n\left(2+\frac{2}{n}\right)}{n\left(1+\frac{2}{n}\right)}=2$$

លំហាត់ទី១៦៦ ចូរស្រាយបញ្ជាក់ថា

$$\frac{1}{3\left(1+\sqrt{2}\right)}+\frac{1}{5\left(\sqrt{2}+\sqrt{3}\right)}+.......+\frac{1}{40399\left(\sqrt{2019}+\sqrt{2020}\right)}<\frac{22}{25}$$

ដំណោះស្រាយ

គេមាន $\frac{1}{(2n+1)\left(\sqrt{n}+\sqrt{n+1}\right)}=\frac{\sqrt{n}-\sqrt{n+1}}{2n+1}$

$$=\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{4n^2+4n+1}}<\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{4n^2+4n}}$$

$$=\frac{\sqrt{n+1}-\sqrt{n}}{2\sqrt{n}\cdot\sqrt{n+1}}=\frac{1}{2}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$$

គេបាន $\frac{1}{(2n+1)\left(\sqrt{n}+\sqrt{n+1}\right)}<\frac{1}{2}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$

ចំពោះ $n=1$ នោះ $\frac{1}{3\left(1+\sqrt{2}\right)}<\frac{1}{2}\left(\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{2}}\right)$

ចំពោះ $n=2$ នោះ $\frac{1}{5\left(\sqrt{2}+\sqrt{3}\right)}<\frac{1}{2}\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)$

....

....

ចំពោះ $n=2019$ នោះ $\frac{1}{40399\left(\sqrt{2019}+\sqrt{2020}\right)}<\frac{1}{2}\left(\frac{1}{\sqrt{2019}}-\frac{1}{\sqrt{2020}}\right)$

គេបាន : $\frac{1}{3(1+\sqrt{2})}+\frac{1}{5(\sqrt{2}+\sqrt{3})}+.......+\frac{1}{40399(\sqrt{2019}+\sqrt{2020})}<\frac{1}{2}\left(\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{2020}}\right)$

$<\frac{1}{2}\left(\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{2025}}\right)=\frac{1}{2}\left(1-\frac{1}{45}\right)=\frac{22}{45}$

ដូចនេះ $\frac{1}{3(1+\sqrt{2})}+\frac{1}{5(\sqrt{2}+\sqrt{3})}+.......+\frac{1}{40399(\sqrt{2019}+\sqrt{2020})}<\frac{22}{25}$ ។

លំហាត់ទី១៦៧

គេឲ n ជាចំនួនមានលេខដប់ខ្ទង់ មានទម្រង់ $n=\overline{2019x2020y}\vdots 33$ ដែល x,y ជាលេខ $0,1,2,3,4,5,...,9$ ។ រកគ្រប់គូ (x,y) ដែល n ចែកដាច់នឹង 33 ។

ដំណោះស្រាយ

រកចំនួន n ដែលចែកដាច់នឹង 33 មាន $n=\overline{2019x2020y}\vdots 33$

$n\vdots 3\Leftrightarrow 16+x+y\vdots 3$

$\Leftrightarrow 1+x+y\vdots 3$

$\Rightarrow 1+x+y=3k_1,k_1\in\mathbb{Z}\quad(1)$

$n:11\Leftrightarrow(2+1+x)-(9+2+2+y):11$

$\Leftrightarrow 1+x-y:11$

$\Rightarrow 1+x-y=11k_2,k_2\in\mathbb{Z}(2)$

$\Rightarrow 1+x-y=11k_2,k_2\in$

$(2)\Rightarrow -8\le 1+x-y=11k_2\le 10$

$\Rightarrow k_2=0\Leftrightarrow y=x+1$

$(1)\ \Rightarrow 1+x+x+1=3k$

យើ $2x+2=3k$

តែ $2\le 2x+2=3k_1\le 20$

$\Rightarrow \frac{2}{3} \le k_1 \le \frac{20}{3}$
$\Rightarrow k_1 = 2,4,6$

គេបាន $x = \frac{3k_1 - 2}{2} = 2,5,8$

ដោយ $y = x + 1 \Rightarrow y = 3,6,9$

$(x, y) = (2,3),(5,6),(8,9)$

ដូចនេះ មានចំនួន n ដែលចែកដាច់នឹង 33 ។

លំហាត់ទី១៦៨	បង្ហាញថា បើ $\left(\overline{ab} + \overline{cd} + \overline{eg}\right) \vdots 11$ នោះ $\overline{abc\deg} \vdots 11$

ដំណោះស្រាយ

ដោយ $\overline{abc\deg} = 10000\overline{ab} + 100\overline{cd} + \overline{eg} = (9999\overline{ab} + 99\overline{cd}) + (\overline{ab} + \overline{cd} + \overline{eg})$

$9999 \vdots 11$, $99 \vdots 11$ $\Rightarrow \left(9999\overline{ab} + 99\overline{cd}\right)$

ប៉ុន្តែ $\left(\overline{ab} + \overline{cd} + \overline{eg}\right) \vdots 11$ នោះ $\overline{abc\deg} \vdots 11$

លំហាត់ទី១៦៩	គេអោយ $A = 2 + 2^2 + 2^3 + \ldots + 2^{00}$ ។ បង្ហាញថា : $A \vdots 3 \vdots 7 \vdots 15$ ។

ដំណោះស្រាយ

$*A = \left(2 + 2^2\right) + \left(2^3 + 2^4\right) + \left(2^3 + 2^4\right) + \ldots + \left(2^{59} + 2^{60}\right)$
$= 2(1+2) + 2^3(1+2) + \ldots + 2^{59}(1+2)$
$= 3\left(2 + 2^3 + \ldots + 2^{59}\right) : 3$

$*A = \left(2 + 2^2 + 2^3\right) + \left(2^4 + 2^5 + 2^6\right) + \ldots + \left(2^{55} + 2^{59} + 2^{60}\right)$
$= 2 \cdot \left(1 + 2 + 2^2\right) + 2^4 \cdot \left(1 + 2 + 2^2\right) + \ldots + 2^{58} \cdot \left(1 + 2 + 2^2\right)$
$= 7\left(2 + 2^4 + \ldots + 2^{58}\right); 7$

$*A = \left(2+2^2+2^3+2^4\right)+\left(2^5+2^6+2^7+2^8\right)+\ldots+\left(2^{57}+2^{55}+2^{59}+2^{60}\right)$

$= 2\left(1+2+2^2+2^3\right)+2^5\left(1+2+2^2+2^3\right)+\ldots+2^{57}\left(1+2+2^2+2^3\right)=$

$= 15.\left(2+2^5+\ldots+2^{57}\right) \vdots 15$

ដូចនេះ : $A \vdots 3 \vdots 7 \vdots 15$ ។

លំហាត់ទី១៩០ ស្រាយបញ្ជាក់ថា $\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots+\frac{1}{n^2}<1$ ។

ដំណោះស្រាយ

គេមាន $\frac{1}{n^2}<\frac{1}{n(n-1)}=\frac{1}{n-1}-\frac{1}{n}$

ដោយ $\frac{1}{2^2}<1-\frac{1}{2};\frac{1}{3^2}<\frac{1}{2}-\frac{1}{3};\ldots;\frac{1}{n^2}<\frac{1}{n-1}-\frac{1}{n}$

$\Rightarrow \frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots+\frac{1}{n^2}<1-\frac{1}{n}<1$

លំហាត់ទី១៩១ គេអោយ $S=5+5^2+5^3+\ldots\ldots\ldots+5^{2006}$ ។

ក. គណនា S

ខ.បង្ហាញថា $SM126$

ដំណោះស្រាយ

ក. គេបាន $5S=5^2+5^3+5^4+\ldots\ldots\ldots\ldots+5^{200}$

$\Rightarrow 5S-S=\left(5^2+5^3+5^4+\ldots\ldots\ldots+5^{2007}\right)-\left(5+5^2+5^3+\ldots\ldots\ldots+5^{2006}\right)$

$\Rightarrow 4S=5^{2007}-5$

$S=\frac{5^{2007}-5}{4}$

ខ. $S=\left(5+5^4\right)+\left(5^2+5^5\right)+\left(5^3+5^6\right)+\ldots\ldots\ldots\ldots+\left(5^{2003}+5^{2006}\right)$

$S=126.\left(5+5^2+5^3+\ldots\ldots\ldots+5^{2003}\right)$

$\Rightarrow$ S M126 ។

លំហាត់ទី១៩២ រកគូមានលំដាប់នៃចំនួនគត់វិជ្ជមាន (x,y) ដែលបំពេញសមីការ :

$$x\sqrt{y}+y\sqrt{x}+\sqrt{2020xy}-\sqrt{2020x}-\sqrt{2020y}-2020=0$$

ដំណោះស្រាយ

រកគូមានលំដាប់នៃចំនួនគត់វិជ្ជមានមានសមីការ :

$$x\sqrt{y}+y\sqrt{x}+\sqrt{2020xy}-\sqrt{2020x}-\sqrt{2020y}-2020=0$$

គេបាន

$$\sqrt{xy}(\sqrt{x}+\sqrt{y}+\sqrt{2020})-\sqrt{2020}(\sqrt{x}+\sqrt{y}+\sqrt{2020})=0$$

$$(\sqrt{x}+\sqrt{y}+\sqrt{2020})(\sqrt{xy}-\sqrt{2020})=0$$

ដោយ $\sqrt{x}+\sqrt{y}+\sqrt{2020}>0$

$$\Rightarrow \sqrt{xy}-\sqrt{2020}=0$$

$$\Rightarrow xy=2020=2^2\times 5\times 101$$

ដូចនេះ គូមានលំដាប់ នៃចំនួនគត់វិជ្ជមាន (x,y) គឺ

$(x,y)=(1,2020),(2,1010),(4,505),(5,404)$

$(10,202),(20,101),(101,20),(202,10)$

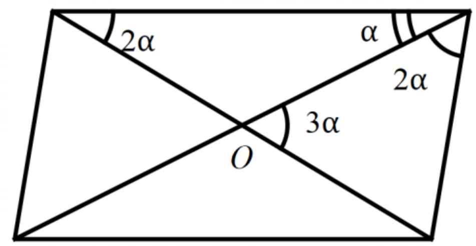
$(404,5),(505,4),(1010,2),(2020,1)$ ។

លំហាត់ទី១៩៣ គេអោយ O ជាចំណុចប្រសព្វរវាងអង្កត់ទ្រូង AC និង BD នៃប្រលេឡូក្រាម $ABCD$ ដែលមាន $\angle CAB=\angle DBC=2\angle DBA$ ។ គណនាផលធៀបនៃមុំ $\angle ACB$ និង $\angle AOB$ ។

ដំណោះស្រាយ

គណនា $\frac{\angle ACB}{\angle AOB}$

តាង $LDBA=\alpha$ យើងបាន



$\angle EAC=\angle DBC=2\alpha$

$\angle AOB=180^{\circ}-3\alpha$

$LACB=180-5\alpha$

តាមទ្រឹស្តីបទស៊ីនុសចំពោះ ΔAOB និង ΔBOC

$$\frac{OA}{\sin\alpha}=\frac{OB}{\sin 2\alpha}\Rightarrow\frac{OA}{CB}=\frac{\sin\alpha}{\sin 2\alpha}$$

$$\frac{OC}{\sin 2\alpha}=\frac{OB}{\sin(180^{\circ}-5\alpha)}\Rightarrow\frac{OC}{OB}=\frac{\sin 2\alpha}{\sin 5\alpha}$$

ដោយ $OC=OA$

$$\Rightarrow\frac{\sin\alpha}{\sin 2\alpha}=\frac{\sin 2\alpha}{\sin 5\alpha}$$

$$\sin^2 2\alpha=\sin\alpha\sin 5\alpha$$

$$1-\cos^2 2\alpha=\frac{1}{2}(\cos 4\alpha-\cos 6\alpha)$$

$$\cdot 2-2\cos^2 2d=2\cos^2 2\alpha-1-\left(4\cos^3 2\alpha-3\cos 2\alpha\right)$$

$$2-2\cos^2 2\alpha=2\cos^2 2\alpha-1-4\cos^2 2\alpha+3\cos 2\alpha$$

$$4\cos^3 2\alpha-4\cos^2 2\alpha-3\cos 2\alpha+3=0$$

$$4\cos^2 2\alpha(\cos 2\alpha-1)-3(\cos 2\alpha-1)=0$$

$$\left(4\cos^2 2\alpha-3\right)(\cos 2\alpha-2)=0$$

គេទាញបាន

$$\cos 2\alpha-1=0$$
$$\cos 2\alpha=1=\cos 0$$

$2\alpha = 0 \Rightarrow \alpha = 0$ (មិនយក)

$4\cos^2 2\alpha - 3 = 0$

$\cos^2 2\alpha = \frac{3}{4}$

$\cos 2\alpha = \frac{\sqrt{3}}{2} = \cos 30^\circ$ នាំឲ $\alpha = 15^\circ$

$2\alpha = 30^\circ$

គេបាន : $\frac{\angle ACB}{\angle AOB} = \frac{180^\circ - 5\times 15^\circ}{180^\circ - 3\times 15^\circ} = \frac{105^\circ}{135^\circ}$

$= \frac{7}{9}$

ដូចនេះ $\frac{\angle ACB}{\angle AOB} = \frac{7}{9}$ ។

លំហាត់ទី១៥៤ បើ ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$ ។ រក n

ដំណោះស្រាយ

យើងមាន ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$

$$\Rightarrow \frac{(2n+1)!}{\{(2n+1)-(n-1)\}!} : \frac{(2n-1)!}{\{(2n-1)-n\}!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)!}{(n+2)!} : \frac{(2n-1)!}{(n-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5} \Rightarrow \frac{(2n+1)\times 2n\times[(2n-1)!]}{(n+2)\times(n+1)\times n\times[(n-1)!]} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\Rightarrow 10(2n+1) = 3(n+2)(n+1) \Rightarrow 20n+10 = 3\left(n^2+3n+2\right)$$

$$\Rightarrow 3n^2 - 11n - 4 = 0$$

$$\Rightarrow (n-4)(3n+1) = 0 \Rightarrow n = 4 \quad \Rightarrow n = 4$$ ។

លំហាត់ទី១៩៥ .ក.ស្រាយបញ្ជាក់ថា $\dfrac{(\sin x-\sin y)}{(\cos x+\cos y)}=\tan\left(\dfrac{x-y}{2}\right)$

ក.ស្រាយបញ្ជាក់ថា $(\cos x+\cos y)^2+(\sin x+\sin y)^2=4\cos^2\left(\dfrac{x-y}{2}\right)$

ដំណោះស្រាយ

ក. គេមាន $\dfrac{(\sin x-\sin y)}{(\cos x+\cos y)}$

$$\Rightarrow \frac{\sin x-\sin y}{\cos x+\cos y}=\frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}=\tan\left(\frac{x-y}{2}\right)$$ ពិត

ខ. គេមាន $(\cos x+\cos y)^2+(\sin x+\sin y)^2$

$$=\left\{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right\}^2+\left\{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right\}^2$$

$$=4\cos^2\left(\frac{x+y}{2}\right)\cos^2\left(\frac{x-y}{2}\right)+4\sin^2\left(\frac{x+y}{2}\right)\cos^2\left(\frac{x-y}{2}\right)$$

$$=4\cos^2\left(\frac{x-y}{2}\right)\cdot\left\{\cos^2\left(\frac{x+y}{2}\right)+\sin^2\left(\frac{x+y}{2}\right)\right\}$$

$$=4\cos^2\left(\frac{x-y}{2}\right)$$

លំហាត់ទី១៩៦ រកតម្លៃនៃ n ចំពោះ $\dfrac{a^{(n+1)}+b^{(n+1)}}{a^n+b^n}$ ជាមធ្យមធរណីមាត្រមានន័យរវាង a និង b ដែល $a\neq b$ ។

ដំណោះស្រាយ

យើងមាន $\dfrac{a^{(n+1)}+b^{(n+1)}}{a^n+b^n}$

$$\Rightarrow \quad \frac{a^{(n+1)}+b^{(n+1)}}{a^n+b^n}=a^{\frac{1}{2}}b^{\frac{1}{2}}$$

$$\Rightarrow \quad \left[a^{(n+1)}+b^{(n+1)}\right]=\left(a^n+b^n\right)\left(a^{\frac{1}{2}}b^{\frac{1}{2}}\right)$$

$$\Rightarrow \quad \left[a^{(n+1)}+b^{(n+1)}\right]=a^{\left(n+\frac{1}{2}\right)}b^{\frac{1}{2}}+a^{\frac{1}{2}}b^{\left(n+\frac{1}{2}\right)}$$

$$\Rightarrow \quad \left\{a^{(n+1)}-a^{\left(n+\frac{1}{2}\right)}b^{\frac{1}{2}}\right\}=a^{\frac{1}{2}}b^{\left(n+\frac{1}{2}\right)}-b^{(n+1)}$$

$$\Rightarrow \quad a^{\left(n+\frac{1}{2}\right)\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)=b^{\left(n+\frac{1}{2}\right)}\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)}$$

$$\Rightarrow \quad a^{\left(n+\frac{1}{2}\right)}=b^{\left(n+\frac{1}{2}\right)} \qquad \left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)\neq 0,\text{ចំពោះ } a\neq b$$

$$\Rightarrow \quad \left(\frac{a}{b}\right)^{\left(n+\frac{1}{2}\right)}=1=\left(\frac{a}{b}\right)^0 \quad \Rightarrow \quad n+\frac{1}{2}=0\Rightarrow n=-\frac{1}{2}$$

លំហាត់ទីទី១៩៧

បើ $x, y,\ z$ ខុសពីចំនួនគត់វិជ្ជមាននោះស្រាយបញ្ជាក់ថា (x+y)(y+z)(z+x)>8 x y z ។

ដំណោះស្រាយ

ប្រើ AM > GM គេបាន :

$$\frac{x+y}{2}>\sqrt{xy},\quad \frac{y+z}{2}>\sqrt{yz} \quad \text{និង } \frac{z+x}{2}>\sqrt{zx}$$

$$\Rightarrow \quad x+y>2\sqrt{xy}, y+z>2\sqrt{yz} \ \text{ និង } z+x>2\sqrt{zx}$$

$$\Rightarrow \quad (x+y)(y+z)(z+x)>2\sqrt{xy}\times 2\sqrt{yz}\times 2\sqrt{zx}$$

$$\Rightarrow \quad (x+y)(y+z)(z+x)>8xyz$$

ដូចនេះ (x+y)(y+z)(z+x)>8 x y z ។

លំហាត់ទី១៩៨

គេអោយស្វ៊ីត $\{u_n\}$ កំណត់ដោយ: $u_n = (-1)^n \sin\sqrt{n} \quad \forall n \in \mathbb{N}$ ។បង្ហាញថា$\{u_n\}$ ជាស្វ៊ីតទាល់។

ដំណោះស្រាយ

ចំពោះ $\{u_n\} n \in \mathbb{N} \quad |u_n| = \left|(-1)^n \sin\sqrt{n}\right|$

$= \left|(-1)^n\right| |\sin\sqrt{n}| = |\sin\sqrt{n}| \le 1$

$|u_n| \le 1 \quad \forall n \in \mathbb{N}$

ដូចនេះ $\{u_n\}$ ជាស្វ៊ីតទាល់។

លំហាត់ទី១៩៩ គេអោយស្វ៊ីត $\{u_n\}$ កំណត់ដោយ: $u_n = \dfrac{2+\cos n}{3-\sin\sqrt{n}} \quad \forall n \in \mathbb{N}$ ។ បង្ហាញថា$\{u_n\}$ ជាស្វ៊ីតទាល់។

ដំណោះស្រាយ

ចំពោះ $n \in \mathbb{N}$ គេបាន:

$-1 \le \cos n \le 1 \quad \forall n \in \mathbb{N} \qquad -1 \le \sin\sqrt{n} \le 1$

$1 \le 2+\cos n \le 3$ និង $-1 \le -\sin\sqrt{n} \le 1$ ហើយ $1 \le 2+\cos n \le 3$ និង $2 \le 3-\sin\sqrt{n} \le 4$

នាំឲ $1 \le 2+\cos n \le 3$ និង $\dfrac{1}{4} \le \dfrac{1}{3-\sin\sqrt{n}} \le \dfrac{1}{2}$

$\dfrac{1}{4} \le \dfrac{2+\cos n}{3-\sin\sqrt{n}} \le \dfrac{3}{2}$

$1/4 \le \dfrac{2+\cos n}{3-\sin\sqrt{n}} \le 3/2 \quad 1/4 \le u_n \le 3/2 \quad \Rightarrow (u_n)$ ជាស្វ៊ីតទាល់។

លំហាត់ទី២០០ បើ $2[x] = x + 2\{x\}$ នោះ x ស្មើប៉ុន្មាន ?

ដំណោះស្រាយ

គេមាន :

$x=[x]+\{x\}$

$\Rightarrow 2[x]=[x]+3\{x\}$

$\Rightarrow [x]=3\{x\}$

$0<\{x\}\leqslant 1$

$\Rightarrow [x]=3\{x\}<3$

$\begin{cases}[x]=0,1,2\\ \{x\}=0,\frac{1}{3},\frac{2}{3}\end{cases}\Rightarrow x=[x]+\{x\}=0,1\frac{1}{3},2\frac{2}{3}$

លំហាត់ទី២០១

គេអោយ $\{x_n\}$ ជាស្វ៊ីតកំណត់ដោយ : $x_1=2$ និង $x_{n+1}=\sqrt{x_n+8}-\sqrt{x_n+3},\quad \forall n\geq 1$

ក.បង្ហាញថា$\{x_n\}$ ជាស្វ៊ីតរួម និង រកលីមីតរបស់វា ។

ដំណោះស្រាយ

ដោយ $x_n>0$ ចំពោះ $n\in\mathbb{N}^*$

គេដឹងថា $|x_{n+1}-1|=|\sqrt{x_n+8}-3+2-\sqrt{x_n+3}|$

$$=\left|(x_n-1)\left(\frac{1}{\sqrt{x_n+8}+3}+\frac{1}{\sqrt{x_n+3}+2}\right)\right|$$

$$\leq|x_n-1|\left(\frac{1}{\sqrt{x_n+8}+3}+\frac{1}{\sqrt{x_n+3}+2}\right)$$

$$\leq|x_n-1|\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{5}{6}|x_n-1|$$

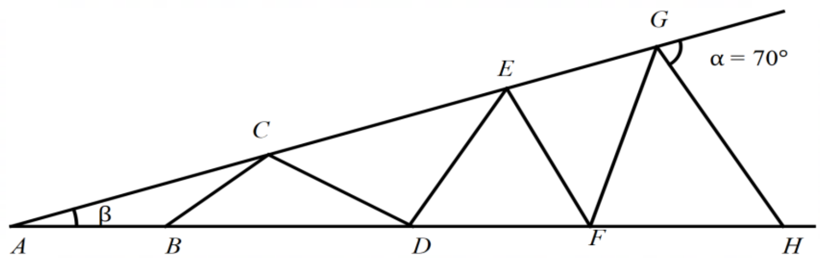
គេបាន $|x_n-1|\leq\frac{5}{6}|x_{n-1}-1|\leq\cdots\leq\left(\frac{5}{6}\right)^{n-1}|x_1-1|=\left(\frac{5}{6}\right)^n$

+ $\lim_{n\to\infty}\left(\frac{5}{6}\right)^n = 0 \Rightarrow \lim_{n\to\infty} x_n = 1$ ។

លំហាត់ទី២០២

នៅក្នុងរូបភាពដែល $AB = BC = CD = DE = EF = FG = GH, \angle\alpha = 70^\circ$ ។

រកទំហំនៃ $\angle\beta$ នៅក្នុងដីក្រេ។



ដំណោះស្រាយ

$\angle A = \beta \Rightarrow \angle ACB = \beta \Rightarrow \angle CBD = 2\beta \Rightarrow \angle CDB = 2\beta$

$\Rightarrow \angle ECD = 3\beta \Rightarrow \angle CED = 3\beta \Rightarrow \angle EDF = 4\beta \Rightarrow \angle EFD = 4\beta$

$\Rightarrow \angle GEF = 5\beta \Rightarrow \angle EGF = 5\beta \Rightarrow \angle GFH = 6\beta \Rightarrow \angle GHF = 6\beta$

$\Rightarrow \angle\alpha = 7\beta$

នោះ $\beta = 10^\circ$ ។

លំហាត់ទី២០៣ ក.បង្ហាញថា $81^6 - 9\cdot 27^7 - 9^{11}$ ចែកដាច់ដោយ 45 ។

ខ.បង្ហាញថា $\underbrace{33....33}_{2n} - \underbrace{66...66}_{n}$ ជាចំនួនការេប្រាកដ ។

ដំណោះស្រាយ

ក.គេបាន :

$81^6 - 9\cdot 27^7 - 9^{11} = 9^{12} - 3\cdot 9^{11} - 9^{11} = 9^{11}(9-3-1) = 5\cdot 9^{11} = 45\cdot 9^{10} \vdots 45$ ។

ខ. $= \underbrace{33\cdots 33}_{n}\cdot 10^n + \underbrace{33\cdots 33}_{n} - \underbrace{66\cdots 66}_{n}$

$$=\underbrace{33\cdots 33}_{n\ \text{ដង}}\cdot 10^n-\underbrace{33\cdots 33}_{n\ \text{ដង}}$$

$$=\underbrace{33\cdots 33}_{n\ \text{ដង}}\left(10^n-1\right)=\underbrace{33\cdots 33}_{n\ \text{ដង}}\cdot\underbrace{99\cdots 99}_{n\ \text{ដង}}=(\underbrace{33\cdots 33}_{n\ \text{ដង}})^2\cdot 9=(\underbrace{66\cdots 66}_{n\ \text{ដង}})^2$$ ។

លំហាត់ទី២០៤

គេមានម៉ាសទ្រីស $A=\begin{bmatrix}1 & -4\\ 0 & 5\\ 6 & 7\end{bmatrix}$ និង $B=\begin{bmatrix}2 & 3 & -1\\ 1 & 0 & -7\end{bmatrix}$ ផ្ទៀងផ្ទាត់ថា $(AB)'=B'A'$ ។

ដំណោះស្រាយ

គេបាន

$$AB=\begin{bmatrix}1 & -4\\ 0 & 5\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 3 & -1\\ 1 & 0 & -7\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3+0 & -1+28\\ 0+5 & 0+0 & 0-35\\ 12+7 & 18+0 & -6-49\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 3 & 27\\ 5 & 0 & -35\\ 19 & 18 & -55\end{bmatrix}$$

$$(AB)'=\begin{bmatrix}-2 & 5 & 19\\ 3 & 0 & 18\\ 27 & -35 & -55\end{bmatrix}$$

ដោយ

$$B'=\begin{bmatrix}2 & 1\\ 3 & 0\\ -1 & -7\end{bmatrix}$$ និង $$A'=\begin{bmatrix}1 & 0 & 6\\ -4 & 5 & 7\end{bmatrix}$$

គេបាន

$$B'A'=\begin{bmatrix}2&1\\3&0\\-1&-7\end{bmatrix}\begin{bmatrix}1&0&6\\-4&5&7\end{bmatrix}$$

$$=\begin{bmatrix}2-4&0+5&12+7\\3+0&0+0&18+0\\-1+28&0-35&-6-49\end{bmatrix}=\begin{bmatrix}-2&5&19\\3&0&18\\27&-35&-55\end{bmatrix}$$

ដូចនេះ $(AB)'=B'A'$ ។

> លំហាត់ទី២០៥
>
> គេអោយ $a_0=\sqrt{2}+\sqrt{3}+\sqrt{6}$ និង $a_{n+1}=\dfrac{(a_n)^2-5}{2(a_n+2)}$ គ្រប់ $n\geq 0$ ។ស្រាយបញ្ជាក់ថាចំពោះ $n\in\mathbb{N}$
>
> ដែល $a_n=\cot\left(\dfrac{2^{n-2}\pi}{3}\right)-2$ ។

ដំណោះស្រាយ

យើងមាន $\cot\dfrac{\pi}{24}=\dfrac{\cos\frac{\pi}{24}}{\sin\frac{\pi}{24}}=\dfrac{2\cos^2\frac{\pi}{24}}{2\sin\frac{\pi}{24}\cos\frac{\pi}{24}}=\dfrac{1+\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}}=\dfrac{1+\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}=2+\sqrt{2}+\sqrt{3}+\sqrt{6}$

យើងបាន $a_0=\cot\dfrac{\pi}{24}-2=\cot\left(\dfrac{2^{0-3}\pi}{3}\right)-2$ ពិតចំពោះ $n=0$

ឧបមាថាវាពិតរហូតដល់គឺ $n=k, a_k=\cot\left(\dfrac{2^{k-3}\pi}{3}\right)-2$

យើងត្រូវស្រាយថាវាពិតរហូតដល់ $n=k+1$ គឺ ស្រាយថា $a_{k+1}=\cot\left(\dfrac{2^{k-2}\pi}{3}\right)-2$

តាង $b_k=\cot\left(\dfrac{2^{2n}-n}{3}\right)\Rightarrow b_k=a_k+2, k\geq 1$

តាម $a_{n+1}=\dfrac{(a_n)^2-5}{2(a_n+2)}$ សមមូល $b_{k+1}=\dfrac{b_k^{\ 2}-1}{2b_k}=\ldots=a_{k+1}+2$ ពិត

លំហាត់ទី២០៦

បង្ហាញថាគ្រប់ចំនួនគត់ធម្មជាតិ n ដែល $\frac{1}{2}<\frac{1}{n^2+1}+\frac{2}{n^2+2}+\ldots+\frac{n}{n^2+n}<\frac{1}{2}+\frac{1}{2n}$ ។

ដំណោះស្រាយ

ដោយ $\frac{1}{n^2+1}+\frac{2}{n^2+2}+\frac{3}{n^2+3}+\ldots\ldots\ldots\ldots+\frac{n}{n^2+n}>\frac{1}{n^2+n}+\frac{2}{n^2+n}+\frac{3}{n^2+n}+\ldots\ldots\ldots\ldots+\frac{n}{n^2+n}$

ដោយ : $\frac{1}{n^2+1}>\frac{1}{n^2+n}$

$\frac{2}{n^2+2}>\frac{2}{n^2+n}$

$\ldots\ldots\ldots\ldots\ldots\ldots$

$\Rightarrow \frac{1}{n^2+1}+\frac{2}{n^2+2}+\frac{3}{n^2+3}+\ldots\ldots\ldots\ldots+\frac{n}{n^2+n}$

$>\frac{1}{n^2+n}\sum_{k=1}^{n}k=\frac{1}{n^2+n}\cdot\frac{n(n+1)}{2}=\frac{1}{2}$

គេមាន

$\frac{1}{n^2+1}+\frac{2}{n^2+2}+\frac{3}{n^2+3}+\ldots\ldots\ldots\ldots+\frac{n}{n^2+n}$

$<\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\ldots\ldots\ldots\ldots+\frac{n}{n^2}$

ដោយ : $\frac{1}{n^2+1}<\frac{1}{n^2}$

$\frac{2}{n^2+2}<\frac{2}{n^2}$

$\ldots\ldots\ldots\ldots\ldots$

$\Rightarrow \frac{1}{n^2+1}+\frac{2}{n^2+2}+\frac{3}{n^2+3}+\ldots\ldots\ldots\ldots+\frac{n}{n^2+n}$

$<\frac{1}{n^2}\sum_{k=1}^{n}k=\frac{1}{n^2}\cdot\frac{n(n+1)}{2}=\frac{n+1}{2n}=\frac{1}{2}+\frac{1}{2n}$

គេបាន

$$\Rightarrow \frac{1}{2} < \frac{1}{n^2+1} + \frac{2}{n^2+2} + \frac{3}{n^2+3} + \ldots\ldots\ldots\ldots + \frac{n}{n^2+n} < \frac{1}{2} + \frac{1}{2n}$$ ។

លំហាត់ទី២០៧ គេអោយចំនួនកុំផ្លិច $z = \left[\dfrac{1+\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}}\right]^8$ សរសេរ z ជាទម្រង់ពិជគណិត។

ដំណោះស្រាយ

គេបាន:

$$= \left(\frac{2\cos^2\frac{\pi}{16} + 2i\sin\frac{\pi}{16}\cos\frac{\pi}{16}}{2\cos^2\frac{\pi}{16} - 2i\sin\frac{\pi}{16}\cos\frac{\pi}{16}}\right)^8$$

$$= \left(\frac{\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}}{\cos\frac{\pi}{16} - i\sin\frac{\pi}{16}}\right)^8 = \left[\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right)\left(\cos\frac{\pi}{16} - i\sin\frac{\pi}{16}\right)^{-1}\right]^8$$

$$= \left[\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right)\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right)\right]^8$$

$$= \left[\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right)^2\right]^8 = \cos\pi + i\sin\pi = -1$$

លំហាត់ទី២០៨ បើ $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \cdots + {}^nC_nx^n$, ។ គណនាផលបូកសេរីនៃ

$${}^nC_0 + 3\,{}^nC_1 + 5\,{}^nC_2 + \cdots + (2n+1)\,{}^nC_n$$

ដំណោះស្រាយ

$$C_0^n + 3C_1^n + 5C_2^n + \ldots\ldots + (2n+1)C_n^n = \left(C_0^n + C_1^n + C_2^n + \ldots\ldots + C_n^n\right) + 2C_1^n + 4C_2^n + \ldots\ldots + 2nC_n^n$$

$$= 2^n + 2\left(C_0^n + C_1^n + C_2^n + \ldots\ldots + nC_n^n\right)$$

$$= 2^n + 2\left(n + 2\cdot\frac{n(n-1)}{2!} + 3\cdot\frac{n(n-1)(n-2)}{3!} + \cdots + n\cdot 1\right)$$

$$= 2^n + 2\left(n + n(n-1) + \frac{n(n-1)(n-2)}{2!} + \cdots + n\right)$$

$$= 2^n + 2n\left(1 + (n-1) + \frac{(n-1)(n-2)}{2!} + \cdots + 1\right)$$

$$= 2^n + 2n(n-1C_0 + {}^{n-1}C_1 + {}^{n-1}C_2 + \cdots + {}^{n-1}C_{n-1})$$

$$= 2^n + 2n\cdot 2^{n-1} = 2^n + n\cdot 2^n = 2^n(n+1)$$

លំហាត់ទី២០៩ គណនាលីមីត

ក. $\lim\limits_{x\to\infty} x^2\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\ldots\infty}}}}$

ខ. $\lim\limits_{x\to\frac{\pi}{2}}\left\{\frac{1^{\cos^2 x} + 2^{\cos^2 x} + \ldots + n^{\cos^2 x}}{n}\right\}^{\frac{1}{\cos^2 x}}$

ដំណោះស្រាយ

$$\lim_{x\to\infty} x^2\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\ldots\ldots..\infty}}}}}$$

តាង $y = \sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\left(1-\cos\frac{1}{x}\right)\sqrt{\ldots\ldots..\infty}}}}}$

$$\Rightarrow y = \sqrt{\left(1-\cos\frac{1}{x}\right)y} \Leftrightarrow y^2 - \left(1-\cos\frac{1}{x}\right)y = 0$$

$y = 0$ ឬ $y = \left(1-\cos\frac{1}{x}\right)$

$$\lim_{x\to\infty} x^2 y = \lim_{x\to\infty} x^2\left(1-\cos\frac{1}{x}\right)$$

តាង $t = \frac{1}{x}$ នោះ $x\to\infty, t\to 0$

$$L=\lim_{t\to 0}\frac{1}{t^2}\left(1-\cos t\right)=\lim_{t\to 0}\frac{2\sin^2\left(\frac{t}{2}\right)}{\left(\frac{t}{2}\right).4}=\frac{1}{2}\lim_{t\to 0}\left[\frac{\sin\left(\frac{t}{2}\right)}{\left(\frac{t}{2}\right)}\right]^2=\frac{1}{2}\times 1^2$$

$$=\frac{1}{2}$$

ខ. $\lim_{x\to\frac{\pi}{2}}\left\{\frac{1^{\cos^2 x}+2^{\cos^2 x}+3^{\cos^2 x}+....+n^{\cos^2 x}}{n}\right\}^{\frac{1}{\cos^2 x}}$

តាង $f(x)=\frac{1^{\cos^2 x}+2^{\cos^2 x}+3^{\cos^2 x}+....+n^{\cos^2 x}}{n}$

តាង $\lim_{x\to\frac{\pi}{2}}f(x)=\frac{1^0+2^0+.....+n^0}{n}$

$$=\frac{1+1+....+1}{n}=\frac{n}{n}=1$$

$$\lim_{x\to\frac{\pi}{2}}\left[f(x)\right]^{g(x)}=e^{\lim_{x\to\frac{\pi}{2}}\left[f(x)-1\right]g(x)}$$

$$\lim_{x\to\frac{\pi}{2}}\left\{\frac{1^{\cos^2 x}+2^{\cos^2 x}+3^{\cos^2 x}+....+n^{\cos^2 x}}{n}\right\}^{\frac{1}{\cos^2 x}}$$

$$=e^{\lim_{x\to\frac{\pi}{2}}\left[\frac{1^{\cos^2 x}+2^{\cos^2 x}+.....+n^{\cos^2 x}-n}{\cos^2 x}\right]\frac{1}{n}}$$

$$=e^{\lim_{x\to\frac{\pi}{2}}\left[\frac{1^{\cos^2 x}-1}{\cos^2 x}+\frac{2^{\cos^2 x}-1}{\cos^2 x}+.....+\frac{n^{\cos^2 x}-1}{\cos^2 x}\right]\frac{1}{n}}$$

$$=e^{\frac{1}{n}\left[\lim_{\cos^2 x\to 0}\left(\frac{1^{\cos^2 x}-1}{\cos^2 x}\right)+\lim_{\cos^2 x\to 0}\left(\frac{2^{\cos^2 x}-1}{\cos^2 x}\right)+.....+\lim_{\cos^2 x\to 0}\left(\frac{n^{\cos^2 x}-1}{\cos^2 x}\right)\right]}$$

$$=e^{\frac{1}{n}\left(\log 1+\log 2+.....+\log n\right)}=e^{\frac{1}{n}\log(1.2.3...n)}$$ ។

លំហាត់ទី៣០០ ឧបមាថាពហុធា $p(x)=x^5+x^2+1$ មានឬសប្រាំ r_1,r_2,r_3,r_4,r_5 និង $q(x)=x^2-2$

ចូរគណនា $q(r_1)q(r_2)q(r_3)q(r_4)q(r_5)$

ដំណោះស្រាយ

យើងមាន r_1,r_2,r_3,r_4,r_5 មានឬសនៃ $p(x)$

$P(x)=(x-r_1)(x-r_2)(x-r_3)(x-r_4)(x-r_5)$

ដោយ

$$q(x)=x^2-2$$
$$q(r_1)=r_1^2-2$$
$$q(r_2)=r_2^2-2$$
$$q(r_3)=r_3^2-2$$
$$q(x_4)=r_4^2-2$$
$$q(r)=r_5^2-2$$

$$\Rightarrow g(r_1)g(r_2)q(r_3)g(r_4)g(r_5)$$
$$=(r_1^2-2)(r_2^2-2)(r_3^2-2)(r_4^2-2)(r_5^2-2)$$
$$=(r_1-\sqrt{2})(r_1+\sqrt{2})(r_2-\sqrt{2})(r_2+\sqrt{2})(r_3+\sqrt{2})(r_3-\sqrt{2})$$
$$(r_4-\sqrt{2})(r_4+\sqrt{2})(r_5-\sqrt{2})(r_5+\sqrt{2})$$
$$=((\sqrt{2}-r_1)(-\sqrt{2}-r_1)(\sqrt{2}-r_2)(-\sqrt{2}-r_2)(\sqrt{2}-r_3)$$
$$(-\sqrt{2}-r_3)(\sqrt{2}-r_4)(-\sqrt{2}-r_4)(\sqrt{2}-r_5)(-\sqrt{2}-r_5)$$

$$=\left[(\sqrt{2}-r_1)(\sqrt{2}-r_2)(\sqrt{2}-r_3)(\sqrt{2}-r_4)(\sqrt{2}-r_5)\right]$$
$$\left[(-\sqrt{2}-r_1)(-\sqrt{2}-r_2)(-\sqrt{2}-r_3)(-\sqrt{2}-r_4)(-\sqrt{2}-r_5)\right]$$
$$=\quad P(\sqrt{2})\times P(-\sqrt{2})$$

$$=\left[(\sqrt{2})^5-(\sqrt{2})^2+1\right]\left[(-\sqrt{2})^5+(-\sqrt{2})^2+1\right]$$

ព្រោះ $p(x)=x^5+x^2+1$

$= (a(2+2+1)(-4i2+2+1)$
$= (3+112)(3-1\sqrt{2})$
$= 9-32 = -23$

ដូចនេះ $q(r_1)q(r_2)q(r_3)q(r_4)q(r_5) = -23$ ។

លំហាត់ទី៣០១ គេឲ្យផ្នែកទសភាគនៃ $M = \left(\sqrt{13}+\sqrt{11}\right)^6$ គឺ p ។ រកតម្លៃ $M(1-p)$ ។

ដំណោះស្រាយ

រកតម្លៃ $M(1-p)$

មាន $M = \left(\sqrt{13}+\sqrt{11}\right)^6$ ផ្នែកទសភាគនៃ M គឺ p

ដំបូង $\lfloor M \rfloor$ ដែល $\{M\} = p$

តាង $A = \sqrt{13}+\sqrt{11}$, $B = \sqrt{13}-\sqrt{11}$

$\Rightarrow M = A^6, A+B = 2\sqrt{13}, A\cdot B = 2$

$A^2+B^2 = (A+B)^2 - 2AB$
$= 4\times 13 - 2\times 2$
$= 48$
$A^6+B^6 = \left(A^2\right)^3 + \left(B^2\right)^3$
$= \left(A^2+B^2\right)\left(A^4 - A^2B^2 + B^4\right)$

$= 48\left(A^2+B^2\right)^2 - 3A^2B^2 \Big]$

$= 48\left(48^2-12\right) \Rightarrow \left(A^4+B^4\right) \in \mathbb{N}$

ដោយ $0 < B = \sqrt{13}-\sqrt{11} < 1 \Rightarrow 0 < B^4 < 1$

$\Rightarrow \lfloor M \rfloor = A^6+B^6-1 p = \{M\} = M - \lfloor M \rfloor = A^6 - \left(A^6+B^6-1\right)$
$\Rightarrow 1-p = B^6$

$p = \{M\} = M - \lfloor M \rfloor = A^6 - \left(A^6+B^4-1\right)$

$\Rightarrow 1-p=B^6$

$\Rightarrow M(1-p)=A^6B^6=2^6=64$

ដូចនេះ $M(1-p)=64$ ។

លំហាត់ទី៣០២ តាង $x=1+\sqrt[6]{2}+\sqrt[6]{4}+\sqrt[6]{8}+\sqrt[6]{16}+\sqrt[6]{32}$ ។ រកតម្លៃ $B=\left(1+\frac{1}{x}\right)^{30}$ ។

ដំណោះស្រាយ

គេមាន $x=1+\sqrt[6]{2}+\sqrt[6]{4}+\sqrt[6]{8}+\sqrt[6]{16}+\sqrt[6]{32}$

រកតម្លៃ $B=\left(1+\frac{1}{x}\right)^{30}$

តាង $a=\sqrt[6]{2} \Rightarrow x=1+a+a^2+a^3+a^4+a^5$

$$\Rightarrow x=\frac{a^6-1}{a-1}=\frac{2-1}{\sqrt[3]{2}-1}=\frac{1}{\sqrt[3]{2}-1}$$

$$\Rightarrow B=\left(1+\frac{1}{x}\right)^{10}=(1+\sqrt[6]{2}-1)^{30}$$

ដូចនេះ $B=32$ ។

លំហាត់ទី៣០៣ ឧបមាថាចំនួនពិត $a_1,a_2,\cdots,a_n$ ផ្ទៀងផ្ទាត់ថា $a_1+a_2+\cdots+a_n=0$ ។បង្ហាញថា

$$\max_{1<i<n} a_i^2 \leqslant \frac{n}{3}\sum_{i=1}^{n-1}(a_i-a_{i+1})^2$$ ។

ដំណោះស្រាយ

ចំពោះ $k\in\{1,2,\cdots,n\}$ គេមាន :

$$a_k^2 \leqslant \frac{n}{3}\sum_{i=1}^{n-1}(a_i-a_{i+1})^2$$

តាង $d_k = a_k - a_{k+1}, k = 1,2,\cdots,n-1,$ នោះ $a_k = a_k$

$$a_{k+1} = a_k - d_k, a_{k+2} = a_k - d_k - d_{k+1}, \cdots$$
$$a_n = a_k - d_k - d_{k+1} - \cdots - d_{n-1}$$
$$a_{k-1} = a_k + d_{k-1}, a_{k-2} = a_k + d_{k-1} + d_{k-2}, \cdots$$
$$a_1 = a_k + d_{k-1} + d_{k-2} + \cdots + d_1$$

តែ $a_1 + a_2 + \cdots + a_n = 0$ គេបាន ;

$$na_k - (n-k)d_k - (n-k-1)d_{k+1} - \cdots - d_{n-1} + (k-1)d_{k-1} + (k-2)d_{k-2} + \cdots + d_1 = 0$$

តាមវិសមភាព Cauchy គេបាន:

$$\left(na_k\right)^2 = \left((n-k)d_k + (n-k-1)d_{k+1} + \cdots + d_{n-1} - (k-1)d_{k-1} - (k-2)d_{k-2} - \cdots - d_1\right)^2$$

$$\leqslant \left(\sum_{i=1}^{k-1} i^2 + \sum_{i=1}^{n-k} i^2\right)\left(\sum_{i=1}^{n-1} d_i^2\right)$$
$$\leqslant \left(\sum_{i=1}^{n-1} i^2\right)\left(\sum_{i=1}^{n-1} d_i^2\right) = \frac{n(n-1)(2n-1)}{6}\left(\sum_{i=1}^{n-1} d_i^2\right)$$
$$\leqslant \frac{n^3}{3}\left(\sum_{i=1}^{n-1} d_i^2\right)$$

គេបាន : $a_k^2 \leqslant \frac{n}{3}\sum_{i=1}^{n-1}\left(a_i - a_{i+1}\right)^2$ ពិត

ដូចនេះ $\max_{1<i<n} a_i^2 \leqslant \frac{n}{3}\sum_{i=1}^{n-1}\left(a_i - a_{i+1}\right)^2$ ។

លំហាត់ទី៣០៤

គេអោយ ABC ជាត្រីកោណមួយ។ចំណុច D និង F នៅលើផ្នែក AB និង AC រៀងគ្នា, និង

និងចំណុច F នៅលើអង្កត់បន្ទាត់ DE ។ គេឲ $\frac{AD}{AB} = x,\ \frac{AE}{AC} = y, \frac{DF}{DE} = z$

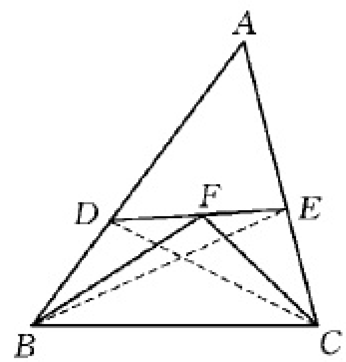
បង្ហាញថា (157 IMO)

(1), $S_{\triangle BDF} = (1-x)yzS_{\triangle ABC}$ និង $S_{\triangle CEF} = x(1-y)(1-z)S_{\triangle ABC}$

(2), $\sqrt[3]{S_{\triangle BDF}} + \sqrt[3]{S_{\triangle CEF}} \leqslant \sqrt[3]{S_{\triangle ABC}}$

ដំណោះស្រាយ

យើងមាន



(1) $S_{\triangle BDF} = zS_{\triangle BDE} = z(1-x)S_{\triangle ABE}$

$= z(1-x)yS_{\triangle ABC}$ and $S_{\triangle CEF} = (1-z)S_{\triangle CDE}$

$= (1-z)(1-y)S_{\triangle ACD}$

$= (1-z)(1-y)xS_{\triangle ABC}$ ។

ដូចនេះ $S_{\triangle CEF} = x(1-y)(1-z)S_{\triangle ABC}$ ។

(2), $\sqrt[3]{S_{\triangle BDF}} + \sqrt[3]{S_{\triangle CEF}} \leqslant \sqrt[3]{S_{\triangle ABC}}$

តាម (1) គេបាន

$\sqrt[3]{S_{\triangle BDF}} + \sqrt[3]{S_{\triangle CEF}}$

$= (\sqrt[3]{(1-x)yz} + \sqrt[3]{x(1-z)(1-y)})^3\sqrt{S_{\triangle ABC}}$

$\leqslant \left(\dfrac{(1-x)+y+z}{3} + \dfrac{x+(1-y)+(1-z)}{3}\right)\sqrt[3]{S_{\triangle ABC}} = \sqrt[3]{S_{\triangle ABC}}$

ដូចនេះ $\sqrt[3]{S_{\triangle BDF}} + \sqrt[3]{S_{\triangle CEF}} \leqslant \sqrt[3]{S_{\triangle ABC}}$

លំហាត់ទី៣០៥ គណនាអាំងតេក្រាល $\displaystyle\int \frac{x}{\sqrt{\dfrac{x}{\sqrt{\dfrac{x}{\sqrt{\dfrac{x}{\dots}}}}}}}dx$ ។

ដំណោះស្រាយ

យើងមាន $\dfrac{x}{\sqrt{\dfrac{x}{\sqrt{\dfrac{x}{\sqrt{\dfrac{x}{\dots}}}}}}} = a \quad \Leftrightarrow \sqrt{\dfrac{x}{a}} = a \Leftrightarrow x = a^3 \Leftrightarrow a = x^{\frac{1}{3}}$

$$\Rightarrow I=\int\sqrt{\frac{x}{\sqrt{\frac{x}{\ddots}}}}dx=\int x^{\frac{1}{3}}dx=\frac{x^{\frac{1}{3}+1}}{3}+C=\frac{3}{4}x^{\frac{4}{3}}+C$$

លំហាត់ទី៣០៥

k ជាចំនួនគត់ $k\geq 2$ ។ ស្វ៊ីត $\{x_n\}$ កំណត់ដោយ $x_0=x_1=1$ និង $x_{n+1}=\dfrac{x_n^k+1}{x_{n+1}}$ ចំពោះ $n\geq 1$។ បង្ហាញថាចំពោះគ្រប់ចំនួនគត់ $k\geq 2$ ស្វ៊ីត$\{x_n\}$ ជាស្វ៊ីតនៃចំនួនគត់ ។

ដំណោះស្រាយ

បង្ហាញថា $\{x_n\}$ ជាចំនួនគត់ $k\geq 2$

ដោយ $x_0=x_1=1$ និង $x_{n+1}=\dfrac{x_n^k+1}{x_{n+1}}$ $\quad n\geq 1$

មាន $x_1=\dfrac{1^k+1}{x_1=1}=2$ ជាចំនួនគត់

$x_1=\dfrac{2^k+1}{1}=2^k+1$ជាចំនួនគត់

ឧបមាថា $x_1;x_2;x_3;.......;x_n$ ជាចំនួនគត់

ស្រាយថា x_{n+1} ជាចំនួនគត់

មាន $x_{n+1}=\dfrac{x_n^k+1}{x_{n-1}}$ និង $x_n=\dfrac{x_{n-1}^k+1}{x_{n-2}}$

$$x_{n+1}=\frac{\left(x_{n-1}^x+1\right)^x+x_{n-2}^k}{x_{n-2}^k\cdot x_{n-1}}$$

តាង $N=\left(x_{n-1}^k+1\right)^k+x_{n-2}^k$

$N:x_{n-2}^{k}$ ព្រោះ $x_n=\dfrac{x_{n-1}^k+1}{x_{n-2}}$ គត់

រឺ $x_{n-1}^{k}+1$ ចែកដាច់នឹង x_{n-2}

$\left(x_{n-1}^{k}+1\right)^{k}$ ចែកដាច់នឹង $\left(x_{n-2}\right)^{k}$

$\left(x_{n-1}^{x}+1\right)^{x}+x_{n-2}^{k}$ ចែកដាច់នឹង $x^{k}{}_{n-2}$ (១)

$N:x_{n-1}$ ព្រោះ $x_{n-1}^{x}+1\equiv 1(\bmod x_{n-1})$

នាំអោយ $\left(x_{n-1}^{x}+1\right)^{k}\equiv 1(\bmod x_{n-1})$

$\left(x_{n-1}^{x}+1\right)^{x}+x_{n-2}^{k}\equiv 1+x_{n-2}^{k}(\bmod x_{n-1})$ តែ $x_{n-1}=\dfrac{x_{n-2}^{k}+1}{x_{n-3}}$

នាំអោយ $x_{n-1}\cdot x_{n-3}=x_{n-2}^{k}+1=0(\bmod x_{n-1})$

គេបាន $N\equiv 0(\bmod x_{n-1})$ (២)

តាម (១) & (២) x_{n+1} ជាចំនួនគត់ ។

លំហាត់ទី៣០៦

បង្ហាញថា $F=\dfrac{21n+4}{14n+3}$ ជាប្រភាគសម្រួលមិនបានចំពោះគ្រប់ចំនួនគត់ធម្មជាតិ n ។

ដំណោះស្រាយ

ឧបមាថាមាន d ជាតួចែករួមធំបំផុតនៃ 21 n+4&14 n+3 នោះគេបាន

$$21n+4=da, a\in M$$
$$14n+3=db \quad , b\in M$$
$$21n+4-(14n+3)=(a-b)d$$
$$7n+1 \quad =(a-b)d$$
$$21n+3 \quad =3(a-b)d$$

$$21n+4-(21a+3)=ad-3(a-b)d$$
$$1=(3b-2a)d$$

ដោយ 3 b-2 a ជាចំនួនគត់នោះ d⋮1គឺ d=1

ដូចនេះ $F=\dfrac{21n+4}{14n+3}$ ជាប្រភាគសម្រួលមិនបាន ។

លំហាត់ទី៣០៥

(x_n) ជាស្វ៊ីតនៃចំនួនដែលកំណត់ដោយ : $x_1=2003, x_1=2004$ និង $x_{n+1}=x_n(x_n-1)+2$

ដែលចំពោះ $n\geq 2$ ។បង្ហាញថា $\left(x_1^2+1\right)\left(x_2^2+1\right)\ldots\left(x_{2004}^2+1\right)-1=\left(x_{2005}-1\right)^2$

ដំណោះស្រាយ

តាង $S_n=\left(x_1^2+1\right)\left(x_2^2+1\right)\ldots\left(x_n^2+1\right)-1$

គេនឹងបង្ហាញថា $S_n=\left(x_{n+1}-1\right)^2$

ចំពោះ $n=1$ គេបាន $S_1=\left(2003\right)^2=\left(2004-1\right)^2=(x_2-1)^2$ ពិត

ឧបមាថាទំនាក់ទំនងពិតដល់ $n=p$ $S_p=\left(x_{p+1}-1\right)^2$

ពិនិត្យចំពោះ $n=p+1$

$$\left[\left(x_1^2+1\right)\left(x_2^2+1\right)\ldots\left(x_{p+1}^2+1\right)-1\right]$$
$$=\left[\left(x_1^2+1\right)\left(x_2^2+1\right)\ldots\left(x_p^2+1\right)-1+1\right]\left(x_{p+1}^2+1\right)-1$$

$$=\left[S_p+1\right]\left(x_{p+1}^2+1\right)-1=\left[\left(x_{p+1}-1\right)^2+1\right]\left(x_{p+1}^2+1\right)-1$$
$$=\left[\left(x_{p+1}^2+1\right)-2x_{p+1}+1\right]\left(x_{p+1}^2+1\right)-1$$

$$=\left[\left(x_0^2+1\right)^2-2x_{m+1}\left(x_{m+1}^2+1\right)+\left(x_{m+1}^2+1\right)\right]-1$$
$$=\left[\left(x_{p+1}^2+1\right)-x_{p+1}\right]^2=\left[x_{p+1}\left(x_{p+1}-1\right)+2-1\right]^2=\left(x_{p+2}-1\right)^2$$

$S_{p+1}=\left(x_{p+2}-1\right)^2$ ឧបមាថាទំនាក់ទំនងពិតដល់ $n=p+1$

ដូចនេះ $S_n=\left(x_{n+1}-1\right)^2$ ចំពោះ $n:n\geq 1$ ។

ដោយយក $n = 2004$ គេបាន $\left(x_1^2+1\right)\left(x_2^2+1\right)\ldots\left(x_{2004}^2+1\right)-1=\left(x_{2005}-1\right)^2$ ។

លំហាត់ទី៣០៦ a, b, c, d ជាបួនចំនួនគត់ដែលចែកមិនដាច់នឹង 5 ។ m ជាចំនួនគត់មួយដែលធ្វើឲ am^3+bm^2+cm+d ចែកដាច់នឹង 5 ។បង្ហាញថា មានចំនួនគត់ n ដែលធ្វើអោយ an^3+bn^2+cn+a ចែកដាច់នឹង 5 ដែរ ។

ដំណោះស្រាយ

តាង $A = am^3+bm^2+cm+d$

$B = an^3+bn^2+cn+a$

A ចែកដាច់នឹង 5 នោះ am^3+bm^2+cm+d ចែកមិនដាច់នឹង 5

ដោយ $am^3+bm^2+cm = m(am^2+bm+c)$ ចែកមិនដាច់នឹង 5 នោះគេបាន m ចែកមិនដាច់នឹង 5

គេបាន $m = 5q+r$ ដែល q ជាចំនួនគត់ ហើយ $r = 1,2,3,4$ ។

គេសិក្សា

$$\begin{aligned}
&An^3 - B = am^3n^2 + bm^2a^3 + cmn^3 - dn^2 - bn - a \\
&= a\left(m^3n^3-1\right) + bn\left(m^2n^2-1\right) + cn^2(mn-1) \\
&= (mn-1)\left(am^2n^2 + amn + a + bmn^2 + bn + cn^2\right)
\end{aligned}$$

ដើម្បីអោយ B ចែកដាច់នឹង 5 គេគ្រាន់តែចង់ជ្រើសរើសចំនួនគត់ n ដែលធ្វើឲ $mn-1$ ចែកដាច់នឹង

5

បើ $m = 5q+r; n = 5p+s$ នោះគេបាន m n-1=5 k+r s-1 ដែល $k = 5pq+qs+pr$

ដើម្បីអោយ $mn-1$ ចែកដាច់នឹង 5 លុះត្រាតែ $rs-1$ ចែកដាច់នឹង 5 ។

ដោយ $1 \le r \le 4$ នោះគេអាចរក s បាន តាមរបៀបដូចខាងក្រោមៈ

បើ $r = 1$ នោះ $s = 1$; $m = 5p+1$ ដែល p ជាចំនួនគត់

បើ $r=2$ នោះ $s=2$; $m=5p+2$ ដែល p ជាចំនួនគត់

បើ $r=3$ នោះ $s=3$; $m=5p+3$ ដែល p ជាចំនួនគត់

បើ $r=4$ នោះ $s=4$; $m=5p+4$ ដែល p ជាចំនួនគត់

ដូចនេះ គេអាចរកបាន n ដែលធ្វើឲ B ចែកដាច់នឹង 5 ។

លំហាត់៣០៧

រកចំនួនគត់ធម្មជាតិតូចបំផុតដែលមាន 6 ជាលេខខ្ទង់ឯកតា ហើយបើគេប្តូរលេខ 6 ទៅខាងមុខគេបំផុតវិញនោះគេនឹងបានចំនួនថ្មីដែលស្មើ 4 ដងនៃចំនួនដើមនោះ។

ដំណោះស្រាយ

តាង $x=\overline{x_1x_2\cdots x_n}$ នោះ $\overline{x_1x_2\cdots x_n6}$ ជាចំនួនដែលសួររក

គេបាន $\overline{6x_1x_2\cdots x_n}=4(\overline{x_1x_2\cdots x_n6})$

$$6\times10^n+x=4(10x+6)$$
$$2\left(10^n-4\right)=13x$$

ដោយ $(2;13)=1$ នោះ $13|\left(10^n-4\right)$

បើ $n=1$ នោះ $10^n-4=10-4=6$ ចែកមិនដាច់នឹង 13

បើ $n=2$ នោះ $10^n-4=10^2-4=96$ ចែកមិនដាច់នឹង 13

បើ $n=3$ នោះ $10^n-4=10^3-4=996$ ចែកមិនដាច់នឹង 13

បើ $n=4$ នោះ $10^n-4=10^4-4=9996$ ចែកមិនដាច់នឹង 13

បើ $n=5$ នោះ $10^n-4=10^5-4=99996$ ចែកដាច់នឹង 13

ករនីនេះគេបាន $x=\dfrac{2(99996)}{13}=15384$ ហើយ $\overline{x_1x_2\cdots x_n6}=153846$ ។

លំហាត់ទី៣០៨ រកតួចែករួមធំបំផុតនៃ $\left(2^{2007}-1\right)$ និង $\left(2^{2^{2007}}-1\right)$ ។

ដំណោះស្រាយ

តាង d ជាតួចែករួមធំបំផុតនៃ $\left(2^{2007}-1\right)$ និង $\left(2^{2^{2007}}-1\right)$

គេបាន $2^{2007}-1=md$ ដែល m ជាចំនួនគត់វិជ្ជមាន

$2^{2^{2007}}-1=nd$ ដែល n ជាចំនួនគត់វិជ្ជមាន ហើយ $(m;n)=1$

នោះគេទាញបាន $2^{2007}=md+1$: $2^{2^{2007}}=nd-1$

ហើយ $\left(2^{3007}\right)^{2^{2001}}=(md+1)^{2^{2001}}=kd+1$ ដែល k ជាចំនួនគត់វិជ្ជមាន (1)

$\left(2^{2^{2007}}\right)^{2007}=(nd-1)^{2001}=pd-1$ ដែល p ជាចំនួនគត់វិជ្ជមាន (2)

តាម (1) និង (2) : $pd-1=kd+1$ នោះ $(p-k)d=2$ ដែល $p-k$ ជាចំនួនគត់វិជ្ជមាន ។

គេទាញបាន $d\vdots 2$

តែ d ជាតួចែករួមធំបំផុតនៃ $\left(2^{2007}-1\right)$ និង $\left(2^{2^{2007}}+1\right)$ នោះ d ជាចំនួនគត់សេស។

ដូចនេះ $d=1$ ។

លំហាត់ទី៣០៩ ពីរចំនួនពិតគត់វិជ្ជមាន a និង b ជាចំនួនបឋមរវាងគ្នា ,

ចូររកគ្រប់តួចែករួមធំបំផុតដែលអាចនៃ $a+b$ និង $\dfrac{a^{2005}+b^{2005}}{a+b}$ ។

ដំណោះស្រាយ

តាង $d=\gcd\left(a+b,\dfrac{a^{2005}+b^{2005}}{a+b}\right)$

$\Rightarrow a+b\equiv 0(\bmod d)\Leftrightarrow a\equiv -b(\bmod d)$

និង $\dfrac{a^{2005}+b^{2005}}{a+b}\equiv 0(\bmod d)$

$$\Leftrightarrow a^{2004}-a^{2003}b+a^{2002}b^{2}-\cdots+b^{2004}\equiv 0(\bmod d)$$
$$\Rightarrow b^{2004}+b^{2004}+\cdots+b^{2004}\equiv 0(\bmod d)$$
$$\Rightarrow 2005b^{2004}\equiv 0(\bmod d) \quad \Rightarrow 2005a^{2004}\equiv 0(\bmod d)$$
$$\Rightarrow d\left|2005a^{2004}\wedge d\right|2005b^{2004}$$
$$\Rightarrow d\mid \gcd\left(2005a^{2004},2005b^{2004}\right)$$
$$=2005\gcd\left(a^{2004},b^{2004}\right)=2005(\gcd(a,b))^{2004}=2005 \quad () \text{ ព្រោះ } \gcd(a,b)=1$$
$$\Rightarrow d\in\{1,5,401,2005\}$$

ដូចនេះ តម្លៃដែលអាចនៃ d គឺ $\Rightarrow d\in\{1,5,401,2005\}$ ។

លំហាត់ទី៣១០ ចូរកំណត់គ្រប់ស្វីតនៃចំនួនពិត $a_1,a_2,\ldots\ldots,a_{1995}$ ដែលផ្ទៀងផ្ទាត់ថា: $2\sqrt{a_n-(n-1)}\geq a_{n+1}-(n+1)$ ចំពោះ $n=1,2,3,\ldots\ldots,1994$ និង $2\sqrt{a_n-1994}\geq a_1+1$ ។

ដំណោះស្រាយ

គេមាន

$$\sum_{n=1}^{1995}2\sqrt{a_n-(n-1)}=\sum_{n=1}^{1994}2\sqrt{a_n-(n-1)}+2\sqrt{a_{1995}-1994}$$
$$\geq\sum_{n=1}^{1994}\left[a_{n+1}-(n-1)\right]+a_1+1\geq\sum_{n=1}^{1994}a_{n+1}+a_1-\sum_{n=1}^{1994}(n-1)+1$$
$$=\sum_{n=1}^{1995}a_n-\sum_{n=1}^{1995}\left[(n-1)-1\right]$$
$$=\sum_{n=1}^{1995}\left[a_n-(n-1)-1\right]$$
$$\Rightarrow\sum_{n=1}^{1995}\left[a_n-(n-1)+1-2\sqrt{a_n-1994}\geq a_1+1\right]$$
$$\Leftrightarrow\sum_{n=1}^{1995}\left(\sqrt{a_n-(n-1)}-1\right)^2\leq 0$$

គេទាញបាន: $\sqrt{a_n-(n-1)}-1=0$ ចំពោះ គ្រប់ $n=1,2,3,\ldots,1995$

$\Leftrightarrow a_n=n$ ចំពោះ គ្រប់ $n=1,2,3,\ldots,1995$

ដូចនេះ ស្វ៊ីតដែលត្រូវរកគឺ $a_n = n, n = 1, 2, 3, \dots, 1995$ ។

លំហាត់ទី៣១១

គេតាង $[x]$ ជាផ្នែកគត់នៃចំនួនពិត x ហើយដែលកំណត់ដោយ $[x] \le x \le [x]+1$ ។

បង្ហាញថា បើ b ជាចំនួនគត់ វិជ្ជមាន នោះគេបាន $\left[\frac{x}{b}\right] = \left[\frac{[x]}{b}\right]$ ។

ដំណោះស្រាយ

តាង $a = [x]$ ហើយតាមវិធីចែកអឺគ្លីតនៃ $a \& b$ គេបាន $a = bq + r, 0 \le r < b$

ដោយ $, 0 \le r < b \Rightarrow bq \le bq + r < b(q+1)$ ហើយ $bq \le a < b(q+1) \Rightarrow q \le \frac{a}{b} < q+1$

កេទាញបាន $\left[\frac{a}{b}\right] = q$ ហើយ $\left[\frac{a}{b}\right] = \left[\frac{x}{b}\right] = q$ (១)

ម្យ៉ាងទៀត $[x] \le x <| x | +1 \Rightarrow x = a + \varepsilon = bq + r + \varepsilon, 0 \le \varepsilon < 1$

គេបាន $[x/b] = [(bq + r + \varepsilon)/b] = [q + (r + \varepsilon)/b]$

តែ $0 \le \varepsilon < 1 \& 0 \le r \le b-1 \Rightarrow 0 \le r + \varepsilon < b$ ហើយ $0 \le \frac{(r+\varepsilon)}{b} < 1$

គេបាន $q \le \frac{q+(r+\varepsilon)}{b} < q+1$ ហើយ $\Rightarrow q \le \left[\frac{x}{b}\right] < (q+1) \Rightarrow \left[\frac{x}{b}\right] = q$ (២)

តាម (១) និង (២) $\Rightarrow \left[\frac{x}{b}\right] = \left[\frac{[x]}{b}\right]$ ។

លំហាត់ទី៣១២

អនុគមន៍ f កំណត់ដោយគ្រប់ចំនួនពិត x ដោយ $f(x) = \cos x + \cos(\sqrt{p}x)$ ដែល p ជាចំនួនបឋម។បង្ហាញថា f មិនមែនជាអនុគមន៍ខួបលើចសុំណុំចំនួនពិតទេ ។

ដំណោះស្រាយ

ឧបមាថា f មានខួប T នោះគេបាន $f(x+T)=f(x)$ ចំពោះគ្រប់ចំនួនពិត x ។

បានន័យថា $]\cos(x+T)+\cos(\sqrt{p}+T\sqrt{p})=f(x)=\cos x+\cos(\sqrt{p}x)$ ចំពោះគ្រប់ចំនួនពិត x ។

ពិនិត្យចំពោះ $x=0$ គេបាន : $]\cos T+\cos T\sqrt{p}=2$

លុះត្រាតែ $\cos T=1 \wedge \cos T\sqrt{p}=1$

$T=2k\pi \wedge T=2m\pi, k\in\mathbb{Z}, m\in\mathbb{Z}$ ហើយ $\sqrt{p}=\frac{2m\pi}{2k\pi}=\frac{m}{k}$

ដោយ $\sqrt{p}\in\mathbb{Q}_+ \wedge \frac{m}{k}\in\mathbb{Q}_+$ នោះគេបាន $\sqrt{p}=\frac{m}{k}$ ជាករណីមិនអាចមាន ។

លំហាត់ទី៣១៣ $\forall n\in\mathbb{N}$: f ជាអនុគមន៍កំណត់ដូចខាងក្រោម៖

$f(n+2)=2f(n+1)-f(n)+1$ ដែល $f(0)=f(1)=0$ ។ គណនា $f(2020)$ ។

ដំណោះស្រាយ

យើងមាន $f(n+2)=2f(n+1)-f(n)+1$ តែ $\forall n\in\mathbb{N}$

ដែល $f(0)=f(1)=0$

គេបាន
$$\begin{cases} f(n+2)-2f(n+1)+f(n)-1=0 \\ f(n+1)-2f(n)+f(n-1)-1=0 \\ f(n)-2f(n-1)+f(n-2)-1=0 \\ \dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots \\ f(3)-2f(2)+f(1)-1=0 \\ f(2)-2f(1)+f(0)-1=0 \end{cases}$$

$f(n+2)-f(n+1)-f(1)+f(0)-(n+1)=0$ ដោយ $f(0)=f(1)=0$

យើងបាន
$$\begin{cases} f(n+2)-f(n+1)=n+1 \\ f(n+1)-f(n)=n \\ f(n)-f(n-1)=n-1 \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ f(4)-f(3)=3 \\ f(3)-f(2)=2 \\ f(2)-f(1)=1 \end{cases}$$

$f(n+2)-f(1)=1+2+3+\dots\dots+n+(n+1)$

$f(n+2)-f(1)=\dfrac{(n+1)(n+2)}{2}$ ហើយ $f(n)=\dfrac{(n-1)n}{2}$

បើ $n=2020$ នោះ $f(2021)=\dfrac{20202021}{2}=1010.2021$

លំហាត់ទី៣១៤ គេអោយស្វ៊ីត x_n កំណត់ដោយ: $x_1=2, x_1+x_2+x_3+\dots+x_n=n^2x_n, n\geq 2$ ។ គណនា x_{2020} ។

.

ដំណោះស្រាយ

យើងមាន $x_1+x_2+x_3+\dots+x_n=n^2x_n$

គេជំនួស n ដោយ $n+1$ គេទាញបាន : $x_1+x_2+x_3+\dots+x_n+x_{n+1}=(n+1)^2x_n$?

ដោយយើងមានស្វ៊ីត $x_1+x_2+x_3+\dots+x_n=n^2x_n, n\geq 2$

បើ $x_1+x_2+x_3+\dots+x_n+x_{n+1}=(n+1)^2x_n$

គេបាន : $n^2x_n+x_{n+1}=(n+1)^2x_n$

$n^2x_n=(n+1)^2x_{n+1}-x_{n+1}$

$\Leftrightarrow nx_n=(n+2)x_{n+1}$

$\Leftrightarrow x_{n+1}=\dfrac{nx_n}{(n+2)}$

ដោយផលគុណពីរអង្គគេបាន : $\prod_{i=1}^{n} x_{i+1} = \prod_{i=1}^{n} \frac{i}{i+2} x_i \Leftrightarrow x_{n+1} = \frac{n!}{(n+2)!/(1\cdot 2)} x_1 = \frac{4}{(n+1)(n+2)}$

$$\Rightarrow x_n = \frac{4}{n(n+1)}$$

ចំពោះ $n = 2020$ គេបាន $x_{2020} = \frac{4}{2020 \cdot 2021}$ ។

លំហាត់ទី៣១៥ គេអោយ $\frac{x^4}{a} + \frac{y^4}{b} = \frac{1}{a+b}$ ដែល $x^2 + y^2 = 1$ ។

ក.បង្ហាញថា $bx^2 = ay^2$ ។

ខ.ស្រាយបញ្ជាក់ថា $\frac{x^{2000}}{a^{2000}} + \frac{y^{2000}}{b^{2000}} = \frac{2}{(a+b)^{2000}}$ ។

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ដំណោះស្រាយ

ក. យើងបាន $\frac{x^4}{a} + \frac{y^4}{b} = \frac{(x^2+y^2)^2}{a+b}$

$$\Rightarrow (a+b)(bx^4 + ay^4) = ab(x^2+y^2)^2$$

$$\Rightarrow (ay^2 - bx^2) = 0 \Rightarrow bx^2 = ay^2$$

ដូចនេះ $bx^2 = ay^2$ ។

ខ. ដោយ $bx^2 = ay^2 \Rightarrow \frac{x^2}{a} = \frac{y^2}{b} = \frac{x^2+y^2}{a+b} = \frac{1}{a+b}$

តែ $\frac{x^4}{a} + \frac{y^4}{b} = \frac{1}{a+b} \Rightarrow \left(\frac{x^2}{a}\right)^{1000} = \left(\frac{1}{a+b}\right)^{1000} \Rightarrow \left(\frac{x^2}{a}\right)^{1000} = \frac{1}{(a+b)^{1000}}$

ហើយ $\Rightarrow \left(\frac{y^2}{a}\right)^{1000} = \left(\frac{1}{a+b}\right)^{1000} \Rightarrow \left(\frac{y^2}{a}\right)^{1000} = \frac{1}{(a+b)^{1000}}$

គេបាន $\frac{x^{2000}}{a^{2000}} + \frac{y^{2000}}{b^{2000}} = \frac{2}{(a+b)^{2000}}$

ដូចនេះ $\frac{x^{2000}}{a^{2000}}+\frac{y^{2000}}{b^{2000}}=\frac{2}{(a+b)^{2000}}$ ។

លំហាត់ទី៣១៦

1. គេអោយកន្សោម $A=\sum_{n=1}^{10000}\frac{1}{\sqrt{n}}$ ។ដោយមិនប្រើម៉ាស៊ីនគិតលេខ ចូលកំណត់ $\lfloor A\rfloor$

2. គេអោយស្វ៊ីតចំនួនពិត $(u_n)=\{7,8,11,16,23,.....\}$ ។

   ចូររកតួទី n និង ផលបូក n តួដំបូងនៃស្វ៊ីតនេះ ។

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ដំណោះស្រាយ

គណនាផ្នែកគត់នៃ $\lfloor A\rfloor$

យើងមាន
$$A=\frac{2}{2\sqrt{k}}<\frac{2}{\sqrt{k}+\sqrt{k}}<\frac{2}{\sqrt{k}-\sqrt{k-1}}=2\left(\sqrt{k}-\sqrt{k-1}\right)$$
$$\Rightarrow\frac{1}{\sqrt{k}}<2\left(\sqrt{k}-\sqrt{k-1}\right)$$

យើងបាន
$$A=\sum_{k=1}^{10000}\frac{1}{\sqrt{k}}=1+\sum_{k=2}^{10000}\frac{1}{\sqrt{k}}<1+\sum_{k=2}^{10000}2\left(\sqrt{k}-\sqrt{k-1}\right)$$
$$A<1+2\left(\sqrt{10000}-\sqrt{1}\right)=1+2(99)=199(i)$$

តែ
$$A=\frac{2}{2\sqrt{k}}<\frac{2}{\sqrt{k}+\sqrt{k}}>\frac{2}{\sqrt{k}+\sqrt{k+1}}=2\left(\sqrt{k+1}-\sqrt{k}\right)$$
$$\Rightarrow\frac{1}{\sqrt{k}}>2\left(\sqrt{k+1}-\sqrt{k}\right)$$

យើងបាន $\Rightarrow A>\sum_{k=1}^{10000}2\left(\sqrt{k+1}-\sqrt{k}\right)=2\left(\sqrt{10000}-\sqrt{1}\right)=2(100-1)=198(ii)$

តាម (១) & (2) គេបាន $198<A<199$

ដូចនេះ $\lfloor A \rfloor = 198$ ។

២.កំណត់តួទី n

យើងមាន $(u_n) = \{7,8,11,16,23,.....\}$

យក (v_n) ជាផលសងដំដាប់ I នៃ (u_n) គឺ $v_n = u_{n+1} - u_n$

គេបាន $v_n = \{1,3,5,7,........\}$

តែ $v_n = v_1 + 2(n-1)d = 2 + 2(n-1)d = 2 + 2(n-1)2 = 2n-1$

$$(u_n) = u_1 + \sum_{k=1}^{n} v_k \quad \text{ចំពោះ } n \ge 2$$

$$= 7 + \sum_{k=1}^{n-1}(2n-1) = 7 + 2\sum_{k=1}^{n-1} n - \sum_{k=1}^{n-1} 1$$

$$= 7 + 2\left(\frac{n(n-1)}{2}\right) - (n-1) = 7 + n^2 - n - n + 1$$

$$= n^2 - 2n + 8$$

ចំពោះ $n = 1 \Rightarrow u_1 = 1 - 2 + 8 = 7$ ពិត

ដូចនេះ $(u_n) = n^2 - 2n + 8$ ចំពោះ $\forall n \ge 1$ ។

+ គណនាផលបូក n តួដំបូងនៃស្វ៊ីត

យើងមាន $(u_n) = n^2 - 2n + 8$

នោះ $S_n = \sum_{k=1}^{n} u_k = \sum_{k=1}^{n}\left(k^2 - 2k + 8\right)$

$$= \sum_{k=1}^{n} k^2 - 2\sum_{k=1}^{n} k + 8\sum_{k=1}^{n} 1 = \frac{n(n+1)(2n+1)}{6} - 2\frac{n(n+1)}{2} + 8n$$

$$= \frac{n(n+1)(2n+1) - 6n(n+1) + 48n}{6} = \frac{n\left[(n+1)(2n+1) - 6(n+1) + 48\right]}{6}$$

$$= \frac{n\left(2n^2 - 3n + 43\right)}{6}$$

ដូចនេះ $S_n = \dfrac{n\left(2n^2-3n+43\right)}{6}$ ។

លំហាត់៣១៧ 1.សម្រួលកន្សោម $\left(1+\dfrac{1}{a}\right)\left(1+\dfrac{1}{a^2}\right)..........\left(1+\dfrac{1}{a^{2^n}}\right)$ ។

2.គេអោយ $\sum_{k=0}^{n-1}\left(\sqrt{ak^3+bk^2+ck+1}-\sqrt{ak^3+bk^2+ck}\right)^{\frac{1}{3}}=\sqrt{n}$ ដែល $n=1,2,3,4,.....$

ចូរកំណត់ចំនួនថេរ a,b និង c ។

ដំណោះស្រាយ

.គេមាន $\left(1+\dfrac{1}{a}\right)\left(1+\dfrac{1}{a^2}\right)..........\left(1+\dfrac{1}{a^{2^{2n}}}\right)$

តាមរូបមន្ត

$a^2-b^2=(a-b)(a+b)$

$\Rightarrow a+b=\dfrac{a^2-b^2}{a-b}$

ដោយ $\left(1+\dfrac{1}{a}\right)=\dfrac{\left(1-\dfrac{1}{a^2}\right)}{\left(1-\dfrac{1}{a}\right)}$; $\left(1+\dfrac{1}{a^2}\right)=\dfrac{\left(1-\dfrac{1}{a^4}\right)}{\left(1-\dfrac{1}{a^2}\right)}$;..........; $\left(1+\dfrac{1}{a^{2n}}\right)=\dfrac{\left(1-\dfrac{1}{a^{2^{4n}}}\right)}{\left(1-\dfrac{1}{a^{2^{2n}}}\right)}$

គេបាន : $\left(1+\dfrac{1}{a}\right)\left(1+\dfrac{1}{a^2}\right)..........\left(1+\dfrac{1}{a^{2^{2n}}}\right)$

$$=\frac{\left(1-\dfrac{1}{a^2}\right)}{\left(1-\dfrac{1}{a}\right)}\cdot\frac{\left(1-\dfrac{1}{a^4}\right)}{\left(1-\dfrac{1}{a^2}\right)}\cdot........................\cdot\frac{\left(1-\dfrac{1}{a^{2^{4n}}}\right)}{\left(1-\dfrac{1}{a^{2^{2n}}}\right)}$$

$$= \frac{1-\frac{1}{a^{2^{2n}}}}{1-\frac{1}{a}} = \frac{\frac{a^{2^{2n}}-1}{a^{2^{2n}}}}{\frac{a-1}{a}}$$

១. គេមាន $\sum_{k=0}^{n-1}\left(\sqrt{ak^3+bk^2+ck+1}-\sqrt{ak^3+bk^2+ck}\right)^{\frac{1}{3}}=\sqrt{n}$

យើងត្រូវរក a,b និង c

យើងមាន $\left(\sqrt{k+1}-\sqrt{k}\right)^3=\sqrt{(k+1)^3}-3\sqrt{(k+1)^2k}+k\sqrt{k+1}-\sqrt{k^3}$

$$=(k+1)\sqrt{(k+1)}-3(k+1)\sqrt{k}+3k\sqrt{k+1}-k\sqrt{k}$$

$$=(4k+1)\sqrt{(k+1)}-(4k+3)\sqrt{k}$$
$$=\sqrt{(4k+1)^2(k+1)}-\sqrt{(4k+3)^2k}$$
$$=\sqrt{(16k^2+8k+1)(k+1)}-\sqrt{(16k^2+24k+9)k}$$

$$=\sqrt{(16k^3+24k^2+9k+1)}-\sqrt{(16k^3+24k^2+9k)}=\sqrt{k+1}-\sqrt{k}$$

គេបាន $\sum_{k=0}^{n-1}\left(\sqrt{ak^3+bk^2+ck+1}-\sqrt{ak^3+bk^2+ck}\right)^{\frac{1}{3}}=\sum_{n=0}^{n-1}\left(\sqrt{k+1}-\sqrt{k}\right)=\sqrt{n}$

គេបាន $a=16,\ b=24\ ,\ b=9$

ដូចនេះ $a=16,\ b=24\ ,\ b=9$ ។

លំហាត់ទី៣១៨ បង្ហាញថា អនុគមន៍មានដេរីវេជាអនុគមនជាប់ ។

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ដំណោះស្រាយ

ឧបមាថា f ជាអនុគមន៍មានដេរីវេជាអនុគមន៍លើ $\mathbb{R}$

យើងបាន $\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$ កំណត់ចំពោះ គ្រប់ $x_0\in\mathbb{R}$

ហេតុនេះ $\lim_{x \to x_0} f(x) - f(x_0) = \lim_{x \to x_0} \left[\frac{f(x) - f(x_0)}{(x - x_0)} \times (x - x_0) \right]$

$$= \lim_{x \to x_0} \frac{f(x) - f(x_0)}{(x - x_0)} \times \lim_{x \to x_0} (x - x_0)$$

$$= \lim_{x \to x_0} \frac{f(x) - f(x_0)}{(x - x_0)} \times 0 = 0$$

$$\lim_{x \to x_0} f(x) - \lim_{x \to x_0} f(x_0) = 0$$

$$\lim_{x \to x_0} f(x) = f(x_0)$$

សមភាពនេះ បញ្ជាក់ថា f ជាប់ចំពោះគ្រប់ $x_0 \in \mathbb{R}$ ។

ដូចនេះ គ្រប់ អនុគមន៍មានដេរីវេសុទ្ធតែជាអនុគមនជាប់ ។

លំហាត់ទី៣១៨

គេប្រើប័ណ្ណដែលមានចុះលេខ $0,1,2,3,4,5,6,7,8,9$ ដើម្បីបង្កើតលេខពីរខ្ទង់ដែលខ្ទង់ទីមួយមិនមានលេខសូន្យ ។ រកប្រូបាបដែលលេខបង្កើតបាននោះជាចំនួនបឋម។

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ដំណោះស្រាយ

រកប្រូបាបដែលលេខបង្កើតបាននោះជាចំនួនបឋម

តាមបំរាប់៖

ចំនួនដែលគេអាចបង្កើតបាន $S = \{10, 11, 12, ..., 99\}$

ចំនួនបឋម មាន $A = \{11, 13, 15, ..., 97\}$

ដូចនេះ $n(S) = 90$ និង $n(A) = 21$

ដូចនេះ $P(A) = \frac{n(A)}{n(S)} = \frac{21}{90} = \frac{7}{30}$ ។

លំហាត់ទី៣១៩ ចំពោះគ្រប់ចំនួនគត់ធម្មជាតិ n គេមានស្វ៊ីត $u_n = \frac{2n^2 - 3\sin n}{n^2+1}$ ។

បង្ហាញថា $\frac{2n^2-3}{n^2+1} \le u_n \le \frac{2n^2+3}{n^2+1}$ រួចទាញរកលីមីតនៃស្វ៊ីត u_n កាលណា n ខិតជិត ∞ ។

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ដំណោះស្រាយ

គេមាន $u_n = \frac{2n^2 - 3\sin n}{n^2+1}$

ដោយ $-1 \le \sin n \le 1 \Rightarrow -3 \le -3\sin n \le 3 \Rightarrow 2n^2 - 3 \le 2n^2 - 3\sin n \le 2n^2 + 3$

$$\frac{2n^2-3}{n^2+1} \le \frac{2n^2-3\sin n}{n^2+1} \le \frac{2n^2+3}{n^2+1}$$

$$\Rightarrow \frac{2n^2-3}{n^2+1} \le u_n \le \frac{2n^2+3}{n^2+1}$$

+ទាញរកលីមីតនៃ u_n កាលណា n ខិតជិត ∞

$$\frac{2n^2-3}{n^2+1} \le u_n \le \frac{2n^2+3}{n^2+1}$$

$$\lim_{x\to+\infty} \frac{2n^2-3}{n^2+1} \le \lim_{n\to+\infty} u_n \le \lim_{n\to+\infty} \frac{2n^2+3}{n^2+1}$$

$$\lim_{n\to+\infty} \frac{n^2\left(2-\frac{3}{n^2}\right)}{n^2\left(1+\frac{1}{n^2}\right)} \le \lim_{n\to+\infty} u_n \le \lim_{n\to+\infty} \frac{n^2\left(2+\frac{3}{n^2}\right)}{n^2\left(1+\frac{1}{n^2}\right)}$$

$$2 \le \lim_{n\to\infty} u_n \le 2$$ ។

លំហាត់ទី៣២០

គេអោយ $g(a)$ ដែលគ្រប់ a,b ជាចំនួនគត់វិជ្ជមាន ផ្ទៀងផ្ទាត់លក្ខណខាងក្រោម:

$g(a+b) - g(a) - g(b) = 0 \quad or, g(2) = 0, g(3) > 0$ និង $g(9999) = 3333$

គណនាតម្លៃ $g(2010)$ ។

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ដំណោះស្រាយ

គណនាតម្លៃ $g(2010)$

តាមលក្ខខណ្ឌយើងមានៈ

$g(a+b)\geq g(a)+g(b)$ បើ $a=b=1$ នោះ

$g(1+1)\geq g(1)+g(1)=2g(1)$
$g(2)\geq 2g(1)$

បើ $a=1,b=2$ នោះ $g(3)=g(1)+g(2)+0=0$

ដោយ $g(3)>0$ គេយក $g(3)=1$

គេបាន $g(1\times 3)=g(3)=1\geq 1$

និង $g(2\times 3)=g(3\times 3)\geq 2g(3)=2$

ដោយប្រើវិចារអនុមានរួមយើងបានៈ

$g(n\times 3)\geq n$ ចំពោះ n ដោយ $g(9999)=g(3333)=3333$

តម្លៃនេះ ស្មើរហូតដល់ យ៉ាងហោចណាស់ខូចជាង ឬស្មើ $n=3333$

$g(n\times 3)=n$ ចំពោះ $n\leq 3333$ នោះ

ម្យ៉ាងទៀត

$2010=g(2010\times 3)\geq g(2\times 2010)+g(2010)\geq 3g(2010)$

$3g(2010)\leq 2010$

$g(2010)\leq \dfrac{2010}{3}=670$

គេបាន : $g(2010)\leq 670$ (1)

តែ $g(2010)\geq g(2007)+g(3)=g(669\times 3)=669+1=670$ (2)

តាម (1) និង (2) គេបាន ; $g(2010)=670$

ដូចនេះ $g(2010)=670$ ។

លំហាត់ទី៣២១

គេអោយចំនួនកុំផ្លិច $z\neq 1$ ជាឫសសមីការ $x^7-1=0$ និង $f(x)=\dfrac{x}{1+x^2}+\dfrac{x^2}{1+x^4}+\dfrac{x^3}{1+x^6}$

ចូររកតម្លៃ $f(z)$ ។

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ដំណោះស្រាយ

ដោយ $z^7=1\Rightarrow 1+z+z^2+z^3+....+z^6$

$$f(z)=\frac{z}{1+z^2}+\frac{z^2}{1+z^4}+\frac{z^3}{1+z^6}$$

$$f(z)=\frac{z}{z^7+z^2}+\frac{z^2}{z^7+z^4}+\frac{z^3}{z^7+z^6}$$

$$f(z)=\frac{z}{z^2\left(z^5+1\right)}+\frac{z^2}{z^4\left(z^3+1\right)}+\frac{z^3}{z^6\left(z+1\right)}$$

គេបាន ; $f(z)=\dfrac{1}{z\left(z^5+1\right)}+\dfrac{1}{z^2\left(z^3+1\right)}+\dfrac{1}{z^3\left(z+1\right)}$

$$=\frac{z^2+z^3+z^5+z^6+z+z^2+z^6+z^7+1+z^3+z^5+z^8}{z^3(z+1)\left(z^3+1\right)\left(z^5+1\right)}$$

$$=\frac{-2z^4}{z^3\left(1+z+z^3+z^5+z^4+z^6+z^8+z^9\right)}$$

$$=\frac{-2z}{1+z-z^7}=-2$$

ដូចនេះ តម្លៃ $f(z)=-2$ ។

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លំហាត់ទី៣២២

ស្រាយបញ្ជាក់ថា $2\left(\sin^4 a+\cos^4 a+\sin^2 a\cos^2 a\right)^2-\left(\sin^8 a+\cos^8 a\right)=1$ ។

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ដំណោះស្រាយ

គេមាន $\sin^2 a+\cos^2 a=1$ យើងបាន:

$\sin^4 a+\cos^4 a+\sin^2 a\cos^2 a$

$=\left(\sin^2 a+\cos^2 a\right)^2-\sin^2 a\cos^2 a=1-\sin^2 a\cos^2 a$

នោះ $2\left(\sin^4 a+\cos^4 a+\sin^2 a\cos^2 a\right)^2$

$=2\left(1-\sin^2 a\cos^2 a\right)^2$

$=2+2\sin^4 a\cos^4 a-4\sin^2 a\cos^2 a$

ម្យ៉ាងទៀត៖

$\sin^8 a+\cos^8 a=\left(\sin^4 a+\cos^4 a\right)^2-2\sin^4 a\cos^4 a$

$=\left(1-2\sin^2 a\cos^2 a\right)^2-2\sin^4 a\cos^4 a$

$=1+2\sin^4 a\cos^4 a-4\sin^2 a\cos^2 a$

គេបាន : $2\left(\sin^4 a+\cos^4 a+\sin^2 a\cos^2 a\right)^2-\left(\sin^8 a+\cos^8 a\right)$

$=2+2\sin^4 a\cos^4 a-4\sin^2 a\cos^2 a-\left(1+2\sin^4 a\cos^4 a-4\sin^2 a\cos^2 a\right)=1$

ដូចនេះ $2\left(\sin^4 a+\cos^4 a+\sin^2 a\cos^2 a\right)^2-\left(\sin^8 a+\cos^8 a\right)=1$ ។

លំហាត់ទី៣២៣ ចំពោះ គ្រប់ចំនួនពិតវិជ្ជមាន x, y, z ។ ចូរស្រាយបញ្ជាក់ថា

$$\frac{x}{\sqrt[3]{yz}}+\frac{y}{\sqrt[3]{xz}}+\frac{z}{\sqrt[3]{xy}}\geq\sqrt[3]{x+y+z+24\sqrt[3]{xyz}}$$ ។

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ដំណោះស្រាយ

តាមលក្ខណៈភាព $(a+b+c)^3=a^3+b^2+c^2+3(a+b)(b+c)(c+a)$

តាមវិសមភាពកូស៊ីចំពោះ $a,b,c>0$ គេមាន $\begin{cases} a+b\ge 2\sqrt{ab} \\ b+c\ge 2\sqrt{bc} \\ c+a\ge 2\sqrt{ac} \end{cases} \Rightarrow (a+b)(b+c)(c+a)\ge 8abc$

គេទាញបាន $(a+b+c)^3\ge a^3+b^3+c^3+24abc$ ឬ $a+b+c\ge\sqrt[3]{a^2+b^3+c^3+24abc}$

យក $\begin{cases} a=\dfrac{x}{\sqrt[3]{yz}} \\ b=\dfrac{y}{\sqrt[3]{xz}} \\ c=\dfrac{z}{\sqrt[3]{xy}} \end{cases}$

គេបាន : $\dfrac{x}{\sqrt[3]{yz}}+\dfrac{y}{\sqrt[3]{xz}}+\dfrac{z}{\sqrt[3]{xy}}\ge\sqrt[3]{\dfrac{x^2}{yz}+\dfrac{y^2}{xz}+\dfrac{z^3}{xy}+24\dfrac{x}{\sqrt[3]{yz}}\dfrac{y}{\sqrt[3]{xz}}\dfrac{z}{\sqrt[3]{xy}}}=\sqrt[3]{\dfrac{x^2}{yz}+\dfrac{y^3}{xz}+\dfrac{z^2}{xy}+24\sqrt[3]{xyz}}$ (1)

ម្យ៉ាងទៀតតាមវិសមភាពកូស៊ី $u+v+w\ge 3\sqrt[3]{uvw}, u,v,w>0$ គេបាន ;

$$\begin{cases} \dfrac{x^3}{yz}+y+z\ge 3x \\ \dfrac{y^3}{xz}+x+z\ge 3y \\ \dfrac{z^3}{xy}+x+y\ge 3z \end{cases}$$

បូកវិសមភាពទាំងនេះអង្គ និង អង្គគេបាន : $\dfrac{x^3}{yz}+\dfrac{y^3}{xz}+\dfrac{z^3}{xy}+2(x+y+z)\ge 3(x+y+z)$

$\dfrac{x^2}{yz}+\dfrac{y^3}{xz}+\dfrac{z^3}{xy}\ge x+y+z$ (2)

តាម (1) និង (2) គេបាន : $\dfrac{x}{\sqrt[3]{yz}}+\dfrac{y}{\sqrt[3]{xz}}+\dfrac{z}{\sqrt[3]{xy}}\ge\sqrt[3]{x+y+z+24\sqrt[3]{xyz}}$ ។

ដូចនេះ $\dfrac{x}{\sqrt[3]{yz}}+\dfrac{y}{\sqrt[3]{xz}}+\dfrac{z}{\sqrt[3]{xy}}\ge\sqrt[3]{x+y+z+24\sqrt[3]{xyz}}$ ។

លំហាត់ទី៣២៤ ចំពោះ គ្រប់ $n\in\mathbb{N}$ ចូរស្រាយបញ្ជាក់ថា $A_n=6789^n-4737^n-59^n+26^n$ ចែកដាច់នឹង 2019 ជានិច្ច ។

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ដំណោះស្រាយ

$2019=3\times 673$ ដែល 3 និង 673 ជាចំនួនបឋមរវាងគ្នា ។

ដោយប្រើឯកលក្ខណៈភាព $a^n-b^n=(a-b)\left(a^{n-1}+a^{n-2}b+\ldots+ab^{n-2}+b^{n-1}\right)$ គេបាន:

$A_A=6789^{\circ}-4737^{\circ}-59^n+26^n=\left(6789^n-4787^{\circ}\right)-\left(59^n-26^n\right)$

$=(6789-4737)q_2-(59-26)q_2=2052q_1-33q_2=3\left(684q_2-11q_2\right)$ ដែល $q_1,q_2\in\mathbb{N}$

គេទាញបាន A_n ចែកដាច់នឹង 3 ។

$A_n=\left(6789^n-59^n\right)-\left(4737^n-26^n\right)$

$=(6789-59)q_3-(4737-26)q_4=6730q_3-4711q_4=673(10q_3-7)$ ដែល $q_3,q_4\in\mathbb{N}$

គេទាញបាន A_n ចែកដាច់នឹង 673 ។

ដូចនេះ យើងសន្និដ្ឋានថា $A_n=6789^n-4737^n-59^n+26^n$ ចែកដាច់នឹង 2019 ជានិច្ចចំពោះគ្រប់ $n\in\mathbb{N}$ ។

លំហាត់ គេអោយ f កំណត់លើ $\mathbb{R}$ ដោយ $f(x)=x^4-x^2+1-\dfrac{1}{x^2+1}$ ។

លំហាត់ទី៣២៥ ចូរគណនាតម្លៃ $P(x)=f(x_1)f(x_2)f(x_3)$ ដោយដឹងថា x_1,x_2,x_3 ជាឫស នៃសមីការ $x^3-3x^2-x+2=0$ ។

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ដំណោះស្រាយ

គណនាតម្លៃ $P(x)=f(x_1)f(x_2)f(x_3)$

តាង $g(x)=x^3-3x^2-x+2=0$ ។ដោយដឹងថា x_1,x_2,x_3 ជាឬសនៃសមីការ $x^3-3x^2-x+2=0$ ។

នោះតាមទ្រឹស្តីបទកត្តា

គេអាចសរសេរ $g(x)=(x-x_1)(x-x_2)(x-x_3)=-(x_1-x)(x_2-x)(x_2-x)=-\prod_{2-2}^{3}(x_2-x)$

គេមាន $f(x)=x^4-x^2+1-\dfrac{1}{x^2+1}=\dfrac{(x^2+1)(x^4-x^2+1)-1}{x^2+1}=\dfrac{x^6+1-1}{x^2+1}=\dfrac{x^6}{x^2+1}=\dfrac{x^6}{(x-1)(x+1)}$

$$P=f(x_1)f(x_2)f(x_3)$$

គេបាន $=\prod_{k=1}^{3}\left[\left(f(x_k)\right]\right]=P=f(x_1)f(x_2)f(x_2)=\prod_{k=1}^{3}\left[\left(f(x_k)\right]=\right]=\prod_{k=1}^{3}\left[\dfrac{x_k^6}{(x_k-i)(x_k+i)}\right]$

$$=\frac{\left[\prod_{k=1}^{3}(x_k)\right]^6}{\prod_{k=1}^{3}(x_k-i)\times\prod_{k=1}^{3}(x_k-i)}$$

$P=\dfrac{[g(0)]^6}{g(i)g(-i)}$ ដោយ g(0)=2, g(i)=-i+3-i+2=5-2 i និង g(-i)=i+3+i+2=5+2 i

ដូចនេះ $P=\dfrac{64}{(5-2i)(5+2i)}=\dfrac{64}{25+4}=\dfrac{64}{29}$ ។

លំហាត់ទី៣២៦

គេអោយកន្សោម $S_n=\sum_{r=1}^{n}e^{rx}\left(r-re^{-x}+e^{-x}\right)$ ដែល x ជាចំនួនពិត និង $x\neq 0$ ។

ចូរស្រាយបញ្ជាក់តាមវិចារកំណើនថាផលបូក $S_n=ne^{nx}$ ចំពោះគ្រប់ចំនួនគត់វិជ្ជមាន n ។

.

ដំណោះស្រាយ

ចំពោះ x ជាចំនួនពិត និង $x\neq 0$ យើងមានកន្សោម:

$$S_n=\sum_{n=1}^{n}e^{rx}\left(r-re^{-x}+e^{-x}\right)=e^{x}\left(1-e^{-x}+e^{-x}\right)+e^{2x}\left(2-2e^{-x}+e^{-x}\right)+\cdots+e^{nx}\left(n-ne^{-x}+e^{-x}\right)$$

បើ $n=1: S_1 = e^x\left(1-e^{-x}+e^{-x}\right)=e^x=1\cdot e^{1.x}$ ពិត

ឧបមាថាវាពិតដល់ $n=k$ គឺ $S_k=\sum_{r=1}^{k} e^{kx}\left(r-re^{-x}+e^{-x}\right)=ke^{kx}$

យើងត្រូវស្រាយថាវាពិតដល់ $n=k+1$ គេបាន៖

$$S_{k+1}=\sum_{r=1}^{k+1} e^{rx}\left(r-re^{-x}+e^{-x}\right)$$

$$=\sum_{r=1}^{k} e^{rx}\left(r-re^{-x}+e^{-x}\right)+e^{(k+1)x}\left[(k+1)-(k+1)e^{-x}+e^{-x}\right]$$

$$=ke^{kx}+e^{kx}\cdot e^x\left(k+1-ke^{-x}-e^{-x}+e^{-x}\right)$$

$$=ke^{kx}+ke^{kx}\cdot e^x+e^{kx}\cdot e^x-ke^{kx}$$

$$=ke^{kx+x}+e^{kx+x}=(k+1)e^{kx+x}=(k+1)e^{(k+1)x}$$ ពិត

ដូចនេះ តាមវិចារកំណើនគេបាន $S_n=ne^{nx}$ គ្រប់ $x\in\mathbb{R}$ ។

លំហាត់ទី៣២៣ ចូរបង្ហាញថា $(\log_a b+\log_b a+2)(\log_a b-\log_{ab} b)\log_b a-1=\log_a b$

.

ដំណោះស្រាយ

របៀបទី១

យើងមាន $(\log_a b+\log_b a+2)(\log_a b-\log_{ab} b)\log_b a-1$

$$=\left(\frac{\ln b}{\ln a}+\frac{\ln a}{\ln b}+2\right)\left(\frac{\ln b}{\ln a}-\frac{\ln b}{\ln a+\ln b}\right)\frac{\ln a}{\ln b}-1$$

$$=\left(\frac{\ln^2 b+2\ln a.\ln b+\ln^2 a}{\ln a\cdot\ln b}\right)\left(\frac{\ln b(\ln a+\ln b)-\ln b\cdot\ln a}{\ln a\cdot\ln a+\ln b}\right)\frac{\ln a}{\ln b}-1$$

$$=\frac{(\ln a+\ln b)^2}{\ln a\cdot\ln b}\left(\frac{\ln b\cdot\ln a+\ln^2-\ln b\cdot\ln a}{\ln a\cdot\ln a+\ln b}\right)\frac{\ln a}{\ln b}-1$$

$$=\frac{(\ln a+\ln b)^2}{\ln a\cdot\ln b}\cdot\frac{\ln^2 b}{\ln a\cdot(\ln a+\ln b)}\frac{\ln a}{\ln b}-1=\frac{\ln a+\ln b}{\ln a}-1=\frac{\ln b}{\ln a}=\log_a b$$

ដូចនេះ $\left(\log_a b+\log_b a+2\right)\left(\log_a b-\log_{ab} b\right)\log_b a-1=\log_a b$ ។

របៀបទី២

គេដឹងថា $\left(\log_a b+\log_b a+2\right)\left(\log_a b-\log_{ab} b\right)\log_b a-1$

គេបាន :

$$\begin{aligned}
&\left(\log_a b+\log_b a+2\right)\left(\log_a b-\log_{ab} b\right)\log_b a-1\\
&=\left(\log_a b+\log_b a+2\right)\left(1-\log_{ab} b\log_b a\right)-1\\
&=\left(\log_a b+\log_b a+2\right)\left(1-\log_{ab} b\log_b a\right)-1\\
&=\left(\log_a b+\log_b a+2\right)\left(1-\log_{ab} a\right)-1\\
&=\left(\log_a b+\frac{1}{\log_a b}+2\right)\left(1-\frac{1}{1+\log_a b}\right)-1\\
&=\left(\frac{\left(\log_a b+1\right)^2}{\log_a b}\right)\left(\frac{\log_a b}{1+\log_a b}\right)-1=1+\log_a b-1=\log_a b
\end{aligned}$$

ដូចនេះ $\left(\log_a b+\log_b a+2\right)\left(\log_a b-\log_{ab} b\right)\log_b a-1=\log_a b$ ។

សូមអភ័យទោសរាល់កំហុសខុសឆ្គងដែលកើតមាន។

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