

ខេត្តកែវ

184 limits

នានា

$$\lim_{n \rightarrow \infty} \left[1 + \sin\left(\pi\sqrt{2 + 4n^2}\right) \right]^n = e^{\frac{\pi}{2}}$$

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ចូរសរុបលានឱ្យិតិ៍សន្លាបញ្ជីនេះ ៖

$$1) \lim_{x \rightarrow 0} \frac{x \cdot \sqrt[3]{\cos x} - \sin x}{x^5}$$

$$2) \lim_{x \rightarrow 0} \frac{\sin(\sin x) - \sin x}{x^5}$$

$$3) \lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - \sqrt[4]{e^{-x^2}}}{x^4}$$

$$4) \lim_{x \rightarrow 0} \frac{\ln[\cos(x^2)] + \sqrt[6]{1+3x^4} - 1}{x^8}$$

$$5) \lim_{x \rightarrow 0} \frac{\cos 2x - e^{-2x^2 + \frac{4}{3}x^4}}{\tan(x^4)}$$

$$6) \lim_{x \rightarrow 0} \frac{(1+x^2)^{\frac{1}{x}+5} - e^x}{\ln(\cos x)}$$

$$7) \lim_{x \rightarrow 0} \frac{e^{\frac{x}{3}} - \sqrt{\frac{x+3}{3-x}}}{x^3}$$

$$8) \lim_{x \rightarrow 0} \frac{\tan x - x \cdot \sqrt[3]{1+x^2}}{x^5}$$

$$9) \lim_{x \rightarrow 0} \frac{18 \cdot \sqrt[3]{\sin(x^3)} - 18x + x^7}{x^{13}}$$

$$10) \lim_{x \rightarrow 0} \frac{\ln(\cos x) + e^{\frac{x^2}{2}} - 1}{(\sqrt[3]{1+3x} - \sqrt{1+2x}) \cdot \tan^2(\sin x)}$$

$$11) \lim_{x \rightarrow 0} \frac{x\sqrt{1-x^2} - \cos x \cdot \ln(1+x) - \frac{x^2}{2}}{\tan x - \sin x}$$

$$12) \lim_{x \rightarrow 0} \frac{\sin^2 x - x^2 e^{-x} - x^3}{1 - \sqrt{1+x^2} \cdot \cos x}$$

$$13) \lim_{x \rightarrow 0} \frac{\sin(\sin x) - x + \frac{x^3}{3}}{sh(shx) - \tan x}$$

$$14) \lim_{x \rightarrow 0} \frac{\ln(1-x+x^2+x^3+x^4) - \ln(1-x+x^2) + x^3 \cos x - \frac{x^5}{2} \times \sqrt[3]{1+3x}}{x^7}$$

$$15) \lim_{x \rightarrow \infty} \left[2x^4 \ln\left(1 - \frac{1}{x}\right) + \sqrt[3]{8x^9 + 12x^8 + 14x^7 + 15x^6 + 16x^5} \right]$$

$$16) \lim_{x \rightarrow 0} \frac{\cos 2x - e^{-2x^2 - \frac{4}{3}x^4}}{\tan(3x^2) - 3th(x^2)}$$

$$17) \lim_{x \rightarrow 0} \frac{\sqrt[4]{\cos 4x} - \cos(2xe^{x^2})}{(\sin 2x - 2 \tan x)^2}$$

$$18) \lim_{x \rightarrow 0} \frac{(\cos x)^{\sin x} - \sqrt{1-x^3}}{x^6}$$

$$19) \lim_{x \rightarrow 0} \frac{\ln(\cos^2 x) + x^2 \cdot \sqrt[6]{1+x^2 - x^4}}{x^6}$$

$$20) \lim_{x \rightarrow 0} \frac{\cos(\sin x) - \sqrt{1-x^2 + x^4}}{x^4}$$

$$21) \lim_{x \rightarrow 0} \frac{\tan(\sin x) - shx}{\left(\sqrt[5]{\cos x} - 1\right)^2 \cdot \arctan(x^2)}$$

$$22) \lim_{x \rightarrow 0} \frac{e^{x^2} \cos x - chx - e^{-x^2} chx + \cos x}{x^6}$$

$$23) \lim_{x \rightarrow 0} \frac{x^3 \sqrt{1+x} - \sin^3 x - \frac{x^3}{2} \tan x}{\ln(1+x^2) \cdot \left(\sqrt[3]{1+2x^3} - 1\right)}$$

$$24) \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+3 \sin x} - e^{-x^2} - shx}{\arcsin(x^3)}$$

$$25) \lim_{x \rightarrow 0} \frac{e^x - \cos x \cdot \sqrt[3]{1+3x+6x^2}}{\arcsin(2x) \cdot \tan x \cdot sh(3x)}$$

$$26) \lim_{x \rightarrow 0} \frac{\left(1+x\right)^{\frac{1}{x}} - e \cdot \left(\sqrt{1-x+\frac{x^2}{2}} + \sqrt[3]{1+x^2} - 1\right)}{x^3}$$

$$27) \lim_{x \rightarrow 0} \frac{\sin\left(xe^{\frac{x^2}{6}}\right) - x \cos(x^2)}{x^2 \cdot \ln(1+x^3)}$$

$$28) \lim_{x \rightarrow 0} \frac{\ln\left(\frac{\sin x}{x}\right) + e^{\frac{x^2}{6}} - 1}{\ln(\cos x) + \sqrt{1+x^2} - 1}$$

$$29) \lim_{x \rightarrow 0} \frac{e^{-\frac{x^2}{2}} - \sqrt[4]{\cos 2x} \cdot ch(x^3)}{\ln^2(\cos 2x)}$$

$$30) \lim_{x \rightarrow 0} \frac{e^{x^2} \cos x - chx}{x^5 + x^3 \sin^3 x}$$

$$31) \lim_{x \rightarrow 0} \frac{e^{x^2} \cos x - chx + x^5}{x^6 + x^2 \sin^3 x}$$

$$32) \lim_{x \rightarrow 0} \frac{1 - (\cos x)^{\sin x}}{x - \sin(x + x^3)}$$

$$33) \lim_{x \rightarrow 0} \frac{\left[(1+x)^{\frac{1}{x}} - e \right]^2}{\ln(x + \cos x) - x}$$

$$34) \lim_{n \rightarrow \infty} \underbrace{\sin(\sin(\dots \sin x \dots))}_{n \text{ times}} = 0, x \in \mathbb{R}$$

$$35) \lim_{n \rightarrow \infty} \underbrace{\sin(\sin(\dots \sin n \dots))}_{n \text{ times}} = 0$$

$$36) \lim_{n \rightarrow \infty} \frac{1}{n} \left(\underbrace{n! + \left(n! + \dots + (n!)^{\frac{1}{n}} \dots \right)^{\frac{1}{n}}}_{n \text{ times}} \right)^{\frac{1}{n}} = \frac{1}{e}$$

$$37) \lim_{n \rightarrow \infty} e^{-n} \left(1 + n + \frac{n^2}{2} + \dots + \frac{n^n}{n!} \right) = \frac{1}{2}$$

$$38) a_1, a_2, a_3, \dots, a_k > 0$$

$$\text{ñ. } \lim_{n \rightarrow \infty} \left(\frac{a_1^{-n} + a_2^{-n} + \dots + a_k^{-n}}{k} \right)^{\frac{1}{n}} = \min \{ a_1, a_2, a_3, \dots, a_k \}$$

$$\text{2. } \lim_{n \rightarrow \infty} \left(\frac{a_1^n + a_2^n + \dots + a_k^n}{k} \right)^{\frac{1}{n}} = \max \{ a_1, a_2, a_3, \dots, a_k \}$$

$$39) \lim_{x \rightarrow 0} \frac{(1+x)^{15} - (1-x)^{15}}{(1+x)^9 - (1-x)^9} = \frac{5}{3}$$

$$40) \lim_{x \rightarrow 0} \frac{(1 + \tan^2 x)^{\ln \sqrt{\frac{2 \tan x}{\sin 2x}}} - 1}{x^4} = \frac{1}{5}$$

$$41) \lim_{x \rightarrow 0} (1+x)^{\frac{\tan(\sin x) - \sin(\tan x)}{x^4}} = e^{\frac{1}{2}}$$

$$42) \lim_{x \rightarrow 0} (1 + \csc x - \cot x)^{\frac{x \cos\left(\frac{x}{2}\right) - 2}{x^3}} = e^{-\frac{1}{12}}$$

$$43) \lim_{n \rightarrow \infty} \left\{ \frac{n}{p-1} - \left[\left(\frac{n}{n+1} \right)^p + \left(\frac{n}{n+2} \right)^p + \dots \right] \right\}, (p > 1) = \frac{1}{2}$$

$$44) \lim_{n \rightarrow \infty} \frac{1^{\alpha-1} - 2^{\alpha-1} + 3^{\alpha-1} - 4^{\alpha-1} + \dots + (-1)^{n-1} n^{\alpha-1}}{n^\alpha}, (\alpha > 0) = 0$$

$$45) \lim_{x \rightarrow 0} (x!)^{\frac{1}{x}} = e^{-\gamma}$$

$$46) \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - (\sin x)^{m+n+p}}{\sqrt[3]{(1 - \sin^m x)(1 - \sin^n x)(1 - \sin^p x)}}, (m, n, p \neq 0) = \frac{m+n+p}{\sqrt[3]{mnp}}$$

$$47) \lim_{x \rightarrow +\infty} \left[\sqrt[n]{(x+a_1)(x+a_2) \times \dots \times (x+a_n)} - \sqrt[m]{(x+b_1)(x+b_2) \times \dots \times (x+b_m)} \right]$$

$$\text{証明 } (a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_m \neq 0) = \frac{1}{n} \sum_{i=1}^n a_i - \frac{1}{m} \sum_{i=1}^m b_i$$

$$48) \lim_{x \rightarrow 0} \frac{\sin \left(2^{-\frac{1}{x^2}} \right)}{2^{-\frac{1}{\sin^2 x}}} = \sqrt[3]{2}$$

$$49) \lim_{x \rightarrow 0} \frac{\ln(1 + \sin x) - \sin[\ln(1 + x)]}{e^{\arctan x} - 1 - \arctan(e^x - 1)} = -\frac{1}{2}$$

$$50) \lim_{n \rightarrow \infty} \left[\frac{1}{n\alpha} + \frac{1}{n\alpha + \beta} + \frac{1}{n\alpha + 2\beta} + \cdots + \frac{1}{n\alpha + (n-1)\beta} \right] = \frac{1}{\beta} \ln\left(\frac{\alpha + \beta}{\alpha}\right)$$

$$51) \lim_{x \rightarrow 2} \frac{(x^2 - 6x + 8)\arctan(x^3 - 8) + 2\ln(x^2 - 4x + 5) + (x-2)^3}{(e^x - e^2)(2 - \sqrt{x+2}) + 2x^2 - 8x + 9 - e^{(x-2)^4}} = \frac{88}{e^2 - 8}$$

$$52) \lim_{x \rightarrow 0} \frac{3x - 3\arctan x - x^3 + x^4}{6\ln(1-x) + 6x + 3x^2 + 2x^3} = -\frac{2}{3}$$

$$53) \lim_{x \rightarrow 0} \frac{\ln|\sin 2x|}{\ln|\sin 3x|}$$

$$54) \lim_{x \rightarrow 0} \frac{\int_0^{x^2} (t^2 + 3t + 2)(\cos t - 1) \sin 2t dt}{x^{12}}$$

$$55) \lim_{x \rightarrow 0} \frac{\int_{x^2}^0 (e^t - 1)^2 \ln(\cos t) dt}{x^{10}}$$

$$56) \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + x\sqrt{x^2 + x}} - \sqrt{x^2 + x\sqrt{x^2 - x}} \right)$$

$$57) \lim_{x \rightarrow \infty} x \left(\sqrt[3]{x^3 + x\sqrt{x^4 + 1} + 2x + 1} + \sqrt[3]{1 - x^2 - x^3} \right)$$

$$58) \lim_{x \rightarrow 0} \frac{\sqrt{1 + 3\sin x} + \sqrt{1 + \tan x} - 2}{\sin 2x}$$

$$59) \lim_{x \rightarrow 0} \frac{1 - \cos x + \ln(1 + \tan^2 2x) + 2(\arcsin x)^3}{1 - \cos 4x + \sin^2 x}$$

$$60) \lim_{x \rightarrow 0} \frac{\arcsin(x^3 + \tan^2 3x) + 2(\arcsin x)^2}{1 - \cos^3 2x + \sin^2 x}$$

$$61) \lim_{x \rightarrow 0} \frac{(x^2 + 2x + 4)(1 - \cos 2x) + (e^{2x} - 1)^2 + x^4}{\ln(\cos 4x) + x^3}$$

$$62) \lim_{x \rightarrow 0} \frac{(x^2 + 3x + 4)\ln(\cos x) + \cos 2x - 1}{(x^2 + 2x + 2)(\sin 2x + x^2)^2}$$

$$63) \lim_{x \rightarrow 0} \frac{(\cos 2x - e^x)(x^2 + 1 - \cos x)}{x(\cos 3x - \cos x)\ln(1 + e^2 - \cos x)}$$

$$64) \lim_{x \rightarrow 1} \frac{(x^3 - 1)\left(x^2 - 2x + \sin \frac{\pi x}{2}\right) + \sin \frac{\pi x}{2} - e^{(x-1)^2}}{(e^{2x} - e^2)(x^4 - 1) + \ln^2 x}$$

$$65) \lim_{x \rightarrow -1} \frac{(x+1)\ln(x+2) + x^3 + 4x^2 + 5x + 1 + e^{(x+1)^2}}{(1 + \cos \pi x)(x^3 + 1) - \sqrt{x^2 + 2x + 2} + 1}$$

$$66) \lim_{x \rightarrow 2} \frac{1 - 2^{4-x^2}}{2\left(\sqrt{2x} - \sqrt{3x^2 - 5x + 2}\right)}$$

$$67) \lim_{x \rightarrow 0} \frac{(\operatorname{Arctan} x)^2 - \tan x \cdot \sin x}{(1 - \cos x)^2} = -\frac{10}{3}$$

$$68) \lim_{x \rightarrow 0} \frac{(\sin x - x) \cdot \log|x|}{|x|^{\frac{5}{2}}} = 0$$

$$69) \lim_{x \rightarrow 0} \frac{\sqrt{\sin x} - \sqrt{x}}{\sin \sqrt{x} - \sqrt{x}} = 0$$

$$70) \lim_{x \rightarrow 0} \frac{1}{x^2} \left[\frac{\cos(x + x^2)}{1 + x + x^2} - e^{-x} \right] = -1$$

$$71) \lim_{n \rightarrow \infty} \frac{\sqrt{1} \operatorname{Arctan} 1 + \sqrt{2} \operatorname{Arctan} 2 + \dots + \sqrt{n} \operatorname{Arctan} n}{n\sqrt{n}} = \frac{\pi}{3}$$

$$72) \lim_{x \rightarrow 0} \frac{3x - 2 \sin x - \tan x}{3x - 2 \operatorname{sh} x - \operatorname{th} x} = 1$$

$$73) \lim_{x \rightarrow +\infty} \frac{\operatorname{sh}(chx)}{ch(shx)} = 1$$

$$74) \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n \sin\left(\frac{1}{k}\right)}{\sum_{k=1}^n \cos\left(\frac{1}{k}\right)}$$

$$75) \lim_{n \rightarrow \infty} \frac{(n+1)\ln(n!) - 2\ln\left(\prod_{k=2}^n k!\right)}{n^2 + n}$$

$$76) \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n \sqrt{k}}{n\sqrt{n} + (-1)^n n}$$

$$77) \lim_{n \rightarrow \infty} \frac{1}{n^{m+1}} \sum_{k=1}^n \frac{(k+m)!}{k!}, m \in \mathbb{N}$$

$$78) \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n k\sqrt{k} + (-1)^n n}{n^2 \cdot \sqrt{n} + (-1)^{n+1} \cdot (n+1)^2}$$

$$79) \lim_{n \rightarrow \infty} \frac{(\sin x)^{2n+2} + (\cos x)^{2n+2}}{(\sin x)^{2n} + (\cos x)^{2n}}$$

$$80) \lim_{n \rightarrow \infty} \frac{\ln \left[\left(\frac{1}{x} \right)^n + 3^n \right]}{\ln \left[x^n + \left(\frac{1}{3} \right)^n \right]}, \quad x > 0$$

$$81) \lim_{n \rightarrow \infty} \sqrt[n]{\frac{x^{2n} + 3^n + \frac{1}{(2x)^n}}{x^n + \frac{1}{2^n}}}, \quad x > 0$$

$$82) \lim_{n \rightarrow \infty} \left(\lim_{m \rightarrow \infty} \frac{2}{\pi} \arctan \left(m \sin^2 (\pi x \cdot n!) \right) \right), \quad x \in \mathbb{R}$$

$$83) \lim_{n \rightarrow \infty} \left[n^2 \left(1 - \cos \frac{3}{n} \right) + i \left(\sqrt{n^2 + 1} - \sqrt{n} \right) \sin \frac{1}{n} \right]$$

$$84) \lim_{x \rightarrow 0} \frac{\tan^2 x - \tan x}{x^2 - x}$$

$$85) \lim_{x \rightarrow \pi^+} \frac{\sin x + 2\sqrt{x-\pi} \cdot \tan x}{(\pi-x) \sin \frac{x}{2}}$$

$$86) \lim_{x \rightarrow 0} \frac{\sin \left(x + \frac{\pi}{4} \right) - \frac{\sqrt{2}}{2}}{\cos \left(x + \frac{\pi}{4} \right) - \frac{\sqrt{2}}{2}}$$

$$87) \lim_{x \rightarrow 0} \frac{\tan(\sin(\tan x))}{\tan(\tan(\sin x))}$$

$$88) \lim_{x \rightarrow 0} \frac{\cos(\sin x) - \sin(\tan^2 x) - 1}{\tan(\cos^2 x - 1)}$$

$$89) \lim_{x \rightarrow 0} \frac{\cos(x \sin x) - \cos(x \tan x)}{(1 - \cos x \cdot \cos 2x)(1 - \cos 3x)}$$

$$90) \lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} - \cos x + (1+x)^{\frac{5}{4}} - 1}{x}$$

$$91) \lim_{x \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} \cos x\right)}{\sin(\sqrt{\cos x}) - \sin(\sqrt[4]{\cos 2x})}$$

$$92) \lim_{x \rightarrow a} \frac{x^{x^a} - a^{x^x}}{x - a}$$

$$93) \lim_{x \rightarrow 0} \frac{\sin x \cdot shx - chx \cdot \cos x + e^{x^2}}{\left(4 + x^2 + x \cdot \sqrt[3]{x} - x^3\right)^{\frac{5}{2}} - 32 \cos x}$$

$$94) \lim_{x \rightarrow 0} \frac{2 \ln(e + \sin x) - e^x - \cos x}{x}$$

$$95) \lim_{x \rightarrow 0} \frac{\sqrt{1 + x \sin x \cdot \cos 2x} - (x^2 - 1)^2}{e^{\tan(x^2)} - \cos x}$$

$$96) \lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - \sqrt[3]{\cos 3x} \cdot \sqrt[4]{\cos 4x}}{\sqrt{chx} - ch3x \cdot ch^2 4x}$$

$$97) \lim_{x \rightarrow 0} \frac{\sqrt[3]{27 + x} - \sqrt[4]{16 - x} - e^x}{\ln(1 + e^x - \cos x)}$$

$$98) \lim_{x \rightarrow 0} \frac{\cos(e^x - \cos x) - \ln(e + x^2)}{\cos x \cdot \cos^2 2x \cdot \cos^3 3x - (1 + x^2)^\pi \cdot (1 - x^2)^e}$$

$$99) \lim_{x \rightarrow 0^+} \frac{2 \sin \left(\sqrt{x^2 + \sqrt{x^3}} \right) + \ln(1+x)}{x + \sqrt{x\sqrt{x}}}$$

$$100) \lim_{x \rightarrow 0} (1 + \cos x)^{(\sin x)^{\tan x}}$$

$$101) \lim_{x \rightarrow 1^+} \left(\frac{chx - \cos x}{shx - \sin x} \right)^{\frac{1}{x-1}}$$

$$102) \lim_{x \rightarrow 0} \left[16 - 7 \sin x + (5x - 3)^3 + \cos x + 2 \cos 2x + 3 \cos 3x + 6^{x+1} \right]^{\frac{x}{\sin x + \cos x - e^x}}$$

$$103) \lim_{x \rightarrow 1} \left[\log_2 (2x^2) + \log_2 x \cdot x^{\log_x (\log_2 x + 1)} + \frac{1}{2} \log_4^2 (x^4) + 2 \right]^{\frac{1}{x}}$$

$$104) \lim_{x \rightarrow 0} \frac{\tan x - thx + \sin \left(sh \left(\sin \left(e^x - \cos x \right) \right) \right)}{\arctan \left(th \left(x^3 - x \right) \right)}$$

$$105) \lim_{x \rightarrow 0} \frac{\sin x - shx + chx - \cos x}{\sin x \cdot \arctan x \cdot (1+x)^{\frac{5}{2}} \cdot [1 + \ln(1+x)]}$$

$$106) \lim_{x \rightarrow 0^+} \frac{\arcsin x + \sin x + \tan x - (1+x)^{\frac{1}{5}} + e^x}{(1+\sqrt{x})^{\sqrt{x}}}$$

$$107) \lim_{x \rightarrow 0} \frac{\arctan \left(\sqrt{\cos x} - \sqrt[3]{chx} \right)}{e^x - \ln(1+x) - \left(1 + \frac{x^2}{4} \right)^{15}}$$

$$108) \lim_{x \rightarrow 0} \frac{\ln(1 + \cos x) - \cos x}{\ln(1 + \sin x) - \sin x}$$

$$109) \lim_{x \rightarrow 0} \frac{\sin(\cos x) - x}{\sin(\sin x) - x}$$

$$110) \lim_{x \rightarrow 1} \tan \left[\arccos x + \sin \left(\frac{x-1}{x+1} \right) \cdot \cos \left(\frac{x+1}{x-1} \right) \right] = 0$$

$$111) \lim_{n \rightarrow \infty} \sqrt[n]{1^\pi + 2^\pi + 3^\pi + \dots + n^\pi} = 1$$

$$112) \lim_{x \rightarrow 1} \frac{x^{1111} + 2x^{2222} - 3x^{3333}}{x^{4444} - 1} = -1$$

$$113) \lim_{n \rightarrow \infty} \frac{(n+1) \sin^{n+1} \left[\frac{\pi}{(n+1)^2} \right]}{n \sin^n \left(\frac{\pi}{n^2} \right)}$$

$$114) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos \left(\frac{x}{2} \right) - \sin \left(\frac{x}{2} \right)}{\cos x}$$

$$115) \lim_{n \rightarrow \infty} \frac{(\ln n)^n}{[\ln(n-1)]^{n+1}}$$

$$116) \lim_{n \rightarrow \infty} \left[\sin \left(\frac{\ln 2}{2} \right) + \sin \left(\frac{\ln 3}{3} \right) + \dots + \sin \left(\frac{\ln n}{n} \right) \right]^{\frac{1}{n}} = 1$$

$$117) \lim_{n \rightarrow \infty} n^2 \left[n \sin(2e\pi \cdot n!) - 2\pi \right] = -2\pi \left(1 + \frac{2\pi^2}{3} \right)$$

$$118) \lim_{x \rightarrow 0} \frac{\left[\tan(\tan x) - \sin(\sin x) \right] \cdot \left[e^{\tan(\tan x)} - e^{\sin(\sin x)} \right] \cdot \ln \left(\frac{x}{\sin x} \right)}{\arctan(\arcsin x) \cdot [\arctan(\arcsin x) - \arcsin(\arctan x)]} = 5$$

$$119) \lim_{n \rightarrow \infty} 2^n \cdot \underbrace{\sqrt{2 - \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2}}}}}_{n \text{ times}} = \pi$$

$$120) \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{n}{\frac{1}{2} + \frac{2}{3} + \cdots + \frac{n}{n+1}} \right)^n = e^{\gamma-1}$$

$$121) \lim_{n \rightarrow \infty} e^{-\frac{n}{2}} \cdot \sqrt{n} \cdot \left(\prod_{k=1}^n C_n^k \right)^{\frac{1}{n}} = \frac{e}{\sqrt{2\pi}}$$

$$122) \lim_{n \rightarrow \infty} \left[\sum_{k=1}^n \arctan \left(\frac{1}{k} \right) - \ln n \right] = \frac{1}{2i} \ln \frac{\Gamma(1-i)}{\Gamma(1+i)} \approx 0.30164$$

$$123) \lim_{n \rightarrow \infty} \frac{\int_0^n \frac{|\sin x|}{x} dx}{\ln n} = \frac{2}{\pi}$$

$$124) \lim_{n \rightarrow \infty} \sum_{k=1}^{n^2} \frac{1 + 2^{\frac{1}{2k}} + 3^{\frac{1}{3k}} + \cdots + n^{\frac{1}{nk}}}{n^2 + k^2} = \frac{\pi}{2}$$

$$125) \lim_{n \rightarrow \infty} \frac{\int_0^{\frac{\pi}{2}} \frac{\sin^2(nx)}{\sin x} dx}{\ln n} = \frac{1}{2}$$

$$126) \lim_{n \rightarrow \infty} \frac{\frac{1}{2 \ln 2} + \frac{1}{3 \ln 3} + \cdots + \frac{1}{n \ln n}}{\ln \left[\ln(2 \ln 2 + 3 \ln 3 + \cdots + n \ln n) \right]} = 1$$

$$127) \lim_{n \rightarrow \infty} \int_0^1 \frac{x \sin(nx)}{1 + n^6 x^2} dx = 0$$

$$128) \lim_{n \rightarrow \infty} \frac{1}{2^n} \sum_{k=1}^n C_n^k \sin^2 \left(\pi \sqrt{k^2 + k} \right) = 1$$

$$129) \lim_{x \rightarrow 0} \left(\frac{2 + e^{\frac{1}{x}}}{1 + e^{\frac{4}{x}}} + \frac{\sin x}{|x|} \right) = 1$$

$$130) \lim_{n \rightarrow \infty} \frac{n^3 \cdot \sqrt[n]{2} \left(1 - \cos n^{-2} \right)}{\sqrt{n^2 + 1} - n} = 1$$

$$131) \lim_{n \rightarrow \infty} \frac{1}{n} \left| \sum_{i=1}^n (-1)^{i-1} i \right| = \frac{1}{2}$$

$$132) \lim_{n \rightarrow \infty} n \{ n! \cdot e \} = 1$$

$$133) \lim_{n \rightarrow \infty} \prod_{k=1}^{n-1} \left(\frac{2^k}{2^{k+1} - 1} \right)^{\frac{1}{2^{n-k}}} = \frac{1}{2}$$

$$134) \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2 - 1}} - \sum_{k=2}^n \frac{1}{\sqrt{n^2 - k}} \right) = -1$$

$$135) \lim_{x \rightarrow 0} \frac{\left(\sqrt[10]{1 + \tan x} - 1 \right) \left(\sqrt{1 + x} - 1 \right)}{2x \sin x} = \frac{1}{40}$$

$$136) \lim_{x \rightarrow 0^+} \frac{\sin \left(\sqrt{1 + \sqrt{1 + \sqrt{x}}} - \sqrt{2} \right)}{\sqrt{x}} = \frac{\sqrt{2}}{8}$$

$$137) \lim_{n \rightarrow \infty} \sin \left(\pi \sqrt{n^2 + 1} \right) = 0$$

$$138) \lim_{x \rightarrow 2^+} \frac{\left[x \right]^2 - 4}{x^2 - 4} = 0$$

$$139) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \sum_{k=1}^n x^k \right)^n, \left(|x| < 1 \right) = e^{\frac{x}{1-x}}$$

$$140) \lim_{x \rightarrow a} \left(\frac{\sin x}{\sin a} \right)^{\frac{1}{x-a}}, (a \neq k\pi, k \in \mathbb{Z}) = e^{\cot a}$$

$$141) \lim_{x \rightarrow +\infty} x^{\frac{7}{4}} \cdot \left(\sqrt[4]{x+1} + \sqrt[4]{x-1} - 2 \cdot \sqrt[4]{x} \right) = -\frac{3}{16}$$

$$142) \lim_{x \rightarrow 0} \frac{\ln(\sin^2 x + e^x) - x}{\ln(x^2 + e^{2x}) - 2x} = 1$$

$$143) \lim_{x \rightarrow 1} \frac{1-x}{5-x-\sqrt{x}-\sqrt[3]{x}-\sqrt[4]{x}-\sqrt[5]{x}}$$

$$144) \lim_{x \rightarrow 1} \frac{\sqrt{\sqrt{x}-1}}{\sqrt[4]{x-1}} = 0$$

$$145) \lim_{x \rightarrow 1} \frac{x^{2n} - 3 + 2x^{-2n}}{x^{2n} - 4 + 3x^{-2n}}, n \neq 0 = \frac{1}{2}$$

$$146) \lim_{x \rightarrow n} \frac{\lceil 3x \rceil - 3\lceil x \rceil}{x - n}, n \in \mathbb{Z} = \text{इस उम्मीद का}$$

$$147) \lim_{x \rightarrow 0} \frac{\lceil x^2 + 2x + 1 \rceil - \lceil x + 1 \rceil^2}{x^2 + 2x} = 0$$

$$148) \lim_{x \rightarrow 0} \frac{2m - \sqrt[2m-1]{x+1} + \sqrt[2m+1]{x-1}}{\sqrt[2m+1]{x+1} + \sqrt[2m-1]{x-1}}, m \in \mathbb{N}$$

$$149) \lim_{\substack{x \rightarrow a \\ y \rightarrow a}} \frac{(x-y)a^m + (a-x)y^m - (a-y)x^m}{(x-y)(a-x)(a-y)}$$

$$150) \lim_{x \rightarrow 2a} \frac{x^2 - ax - 2a^2 + 2x - 4a}{x^2 - 5ax + 6a^2 - x + 2a}$$

$$151) \lim_{x \rightarrow \infty} \frac{\sqrt[5]{x^4} \cdot \sqrt[5]{x^4} \cdot \sqrt[5]{x^4} \dots}{\sqrt[4]{x^5} \cdot \sqrt[4]{x^5} \cdot \sqrt[4]{x^5} \dots}$$

$$152) \lim_{x \rightarrow 0} \frac{\sin\left(\frac{\pi}{4} + x\right) - \sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} - x\right)} \times \frac{2(1 - \cos x)}{x^2} = -1$$

$$153) \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2(9x - 2\pi) - \sin^2(5x - 2\pi)}{\cos 2x}$$

$$154) \lim_{x \rightarrow \sqrt{n}} \left(\sqrt[n]{\frac{\sqrt{n}}{x - \sqrt{n}}} + 1 \right) \cdot \sqrt[n]{x^2 - n} = \sqrt[n]{2n}$$

$$155) \lim_{x \rightarrow +\infty} \frac{(\ln(\ln x))^{(x^{\ln x})}}{(\ln x)^{x^{(\ln(\ln x))}}}$$

$$156) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{sgn}(\sin 2x)}{\operatorname{sgn}(\cos 3x)}$$

$$157) \lim_{x \rightarrow 0^+} \left(x^{\sqrt{x}} - \sqrt{x}^x \right) = 0$$

$$158) \lim_{x \rightarrow +\infty} \left(x^{\sqrt{x}} - \sqrt{x}^x \right) = -\infty$$

$$159) \lim_{x \rightarrow +\infty} \frac{a^{(b^x)}}{b^{(a^x)}}, (1 < a < b) = +\infty$$

$$160) \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\ln(\sin x) - \ln(\cos x)}$$

$$161) \lim_{x \rightarrow \infty} \left[2 - \frac{\ln(x+1)}{x} \right]^{\frac{1}{x}} = 1$$

$$162) \lim_{x \rightarrow 0} \frac{\sin(\sin x) - \sin(\sin(\sin x))}{\sin x \cdot \sin(\sin x) \cdot \sin(\sin(\sin x))} = \frac{1}{6}$$

$$163) \lim_{x \rightarrow 0} \frac{e^{(1+x)^{\frac{1}{x}}} - (1+x)^{\frac{e}{x}}}{x^2} = \frac{e^{e+1}}{8}$$

$$164) \lim_{x \rightarrow 0} \frac{\sin x \cdot [\sin x - \sin(\sin x)]}{x^4} = \frac{1}{6}$$

$$165) \lim_{n \rightarrow \infty} n^2 \times \frac{\sqrt[n]{1+n} - \sqrt[n+1]{n}}{\ln(1+n)} = 1$$

$$166) \lim_{x \rightarrow 3} \frac{\sin(x^x) - \sin(3^x)}{3^{x^x} - 3^{3^x}} = \frac{\cos 9}{3^9 \ln 3}$$

$$167) \lim_{x \rightarrow \frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x} = \ln a$$

$$168) \lim_{x \rightarrow -\infty} \frac{x^3 \left[\cos\left(\frac{\pi x + 1}{x}\right) - \sin^2\left(\frac{\pi x + 1}{x}\right) + 1 \right]}{1 + 2x} = -\frac{1}{4}$$

$$169) \lim_{x \rightarrow 0^+} \frac{2x \cdot (\arcsin x)^2 + \tan x - \sin x}{x^3} = 2$$

$$170) \lim_{n \rightarrow \infty} \sqrt[n]{2 - \frac{(-1)^{n+1} + (-1)^{n+2} + \dots + (-1)^{2n}}{n}}$$

$$171) \lim_{x \rightarrow 0} \frac{6 \sin(\sin(\sin(\sin x)))) - 6x + 5x^3}{x^5}$$

$$172) \lim_{x \rightarrow 0} \frac{6 \tan(\pi \sin x) - 6\pi x - (2\pi^3 - \pi)x^3}{6(\sin(\pi(\sin(\pi x)))) - 6\pi^2 x + (\pi^6 + \pi^4)x^3}$$

$$173) \lim_{x \rightarrow 0} \frac{-24 \sin^2 \left(\frac{\tan(\sin(e^x - 1))}{2} \right) + 5x^4 + 6(x^3 + x^2)}{x^5}$$

$$174) \lim_{x \rightarrow 0} \frac{48 \sin \left(\sin \left(\sin \left(e^{\frac{\sin x}{2}} - 1 \right) \right) \right) - 24x - 6x^2 + 3x^3}{12 \left(\sqrt[3]{2x^4 + x^3 + x^2 + 1} - \sqrt[3]{x^3 + 1} \right) - 3x^2 + x^3}$$

$$175) \lim_{x \rightarrow 0} \frac{x^2}{\sqrt[n]{x^3 + 1} - \sqrt[n]{x^2 + 1}} = -n$$

$$176) \lim_{x \rightarrow 0} \frac{\cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x - \frac{x}{\sin x}}{x^2} = -\frac{512}{3}$$

$$177) \lim_{x \rightarrow 0} \frac{10^{2x} + 10^{-2x} - 2}{10^{2x} - 10^{-2x}} = 0$$

$$178) \lim_{n \rightarrow \infty} \frac{2^n}{\sqrt{n!}} = 0$$

$$179) \lim_{x \rightarrow \sqrt{10}} \frac{\sqrt{7+2x} - (\sqrt{5} + \sqrt{2})}{x^2 - 10} = \frac{\sqrt{5} - \sqrt{2}}{6\sqrt{10}}$$

$$180) \lim_{x \rightarrow \sqrt{6}} \frac{\sqrt{5+2x} - (\sqrt{3} + \sqrt{2})}{x^2 - 6} = \frac{\sqrt{3} - \sqrt{2}}{2\sqrt{6}}$$

$$181) \lim_{x \rightarrow \sqrt{2}} \frac{\sqrt{3+2x} - (\sqrt{2} + 1)}{x^2 - 2} = \frac{\sqrt{2} - 1}{2\sqrt{2}}$$

$$182) \lim_{x \rightarrow 0} \left\{ \frac{\lambda + \ln[(1+a_1x)(1+a_2x)(1+a_3x) \times \dots \times (1+a_nx)]}{\lambda + \ln[(1+b_1x)(1+b_2x)(1+b_3x) \times \dots \times (1+b_nx)]} \right\}^{\frac{1}{x}}$$

ଯେତୁ $\lambda \neq 0$ & $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n > 0$

$$183) \lim_{n \rightarrow \infty} \left[1 + \sin(\pi \sqrt{2 + 4n^2}) \right]^n = e^{\frac{\pi}{2}}$$

$$184) \lim_{n \rightarrow \infty} \sin(\pi \sqrt{2 + 4n^2}) = 0$$