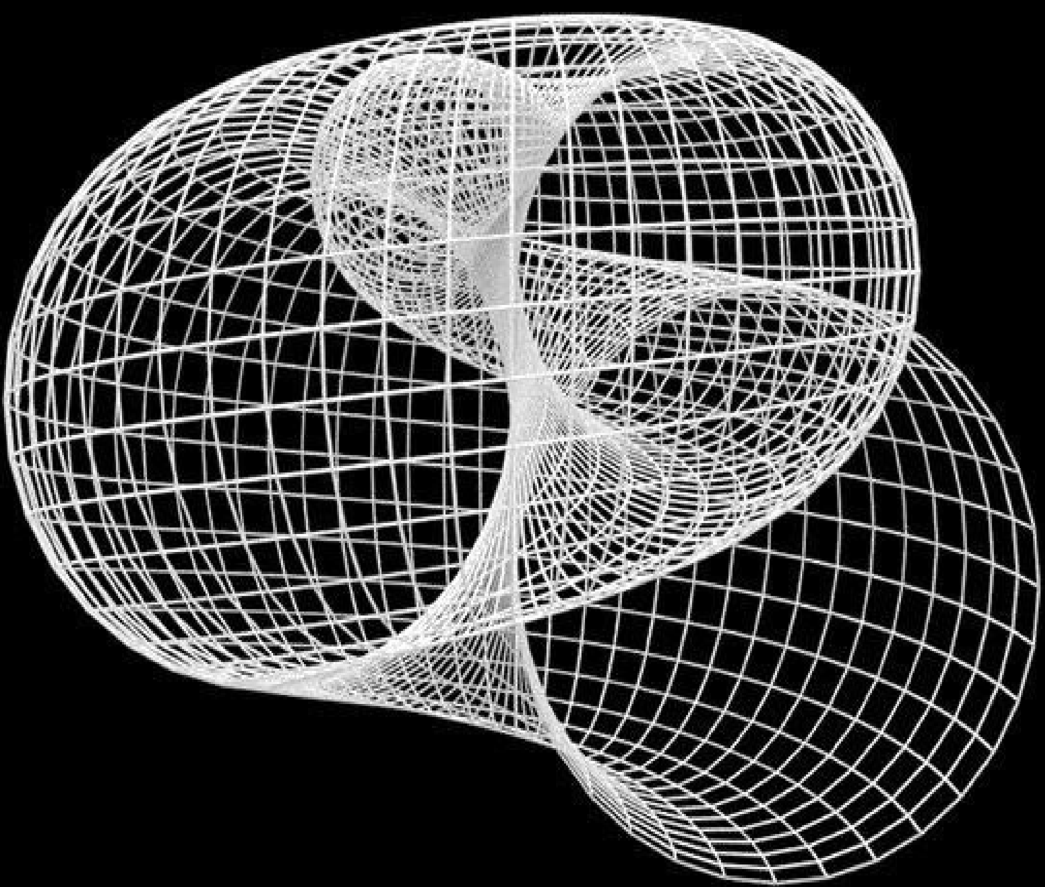
ឆ្នាំទី១

គណិតវិភាគ

ត្រៀមប្រឡងឆមាសទី២



វ៉ន ចាន់រ៉ាក់

លំហាត់

$I.$រកបន្លាយភាពជាប់នៃអនុគមន៍ $f(x)=\dfrac{\ln\left(1+3x^2\right)}{6\sin^2 x}$ ត្រង់ $x_0=0$ ។

$II.$គណនាលីមីតដូចខាងក្រោម៖

1. $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$
2. $\lim\limits_{x\to 1}\left[1+\sin(\pi x)\right]^{\cot \pi x}$
3. $\lim\limits_{x\to 0}\dfrac{\ln(\cos ax)}{\ln(\cos bx)}$

$III.1.$គណនា

$a.\cos\left[\arccos\left(\dfrac{4}{5}\right)+\arcsin\left(\dfrac{12}{13}\right)\right]$

$b.\cos(2\arctan 2)$

2.បង្ហាញថា $3\arctan\left(\dfrac{1}{4}\right)+\arctan\left(\dfrac{5}{99}\right)=\dfrac{\pi}{4}$

$IV.1.$បង្ហាញថា $\left(\dfrac{2x+3}{4x+5}\right)'=\dfrac{\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}}{(4x+5)^2}$

2. គេមានអនុគមន៍ $y=2\cos x+3\sin x$ ។ស្រាយបញ្ជាក់ថា $y''+y=0$ ។

ដំណោះស្រាយ

$I.$រកបន្លាយភាពជាប់នៃអនុគមន៍ $f(x)=\dfrac{\ln\left(1+3x^2\right)}{6\sin^2 x}$ ត្រង់ $x_0=0$

តាង $g(x)$ជាអនុគមន៍បន្លាយតាមភាពជាប់នៃ $f(x)$ ត្រង់ $x_0=0$

គេមាន៖ $f(x)=\dfrac{\ln\left(1+3x^2\right)}{6\sin^2 x}$

$$\Rightarrow \lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\ln\left(1+3x^2\right)}{6\sin^2 x}=\frac{1}{2}\lim_{x\to 0}\left[\frac{\ln\left(1+3x^2\right)}{3x^2}\times\frac{x^2}{\sin^2 x}\right]=\frac{1}{2}$$

ព្រោះ $\lim_{x\to 0}\dfrac{\ln\left(1+3x^2\right)}{3x^2}=1$ និង $\lim_{x\to 0}\dfrac{x^2}{\sin^2 x}=1$

ដូចនេះ: $g(x)=\begin{cases}\dfrac{\ln\left(1+3x^2\right)}{6\sin^2 x} & if\ \ x\neq 0\\ \dfrac{1}{2} & if\ \ x=0\end{cases}$ ។

$II.$គណនាលីមីតដូចខាងក្រោម៖

1. $\lim_{x\to a}\dfrac{\sin x-\sin a}{x-a}$ រាងមិនកំណត់ $\dfrac{0}{0}$

$$=\lim_{x\to a}\frac{2\sin\left(\frac{x-a}{2}\right)\times\cos\left(\frac{x+a}{2}\right)}{x-a}$$

$$=\lim_{x\to a}\left[\frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}\times\cos\left(\frac{x+a}{2}\right)\right]=\cos a$$

ព្រោះ: $\lim_{x\to a}\dfrac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}=1$

ដូចនេះ $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}=\cos a$ ។

2. $\lim\limits_{x\to 1}\left[1+\sin(\pi x)\right]^{\cot\pi x}$ រាងមិនកំណត់ $(1)^{\infty}$

គេបាន៖ $\lim\limits_{x\to 1}\left[1+\sin(\pi x)\right]^{\cot\pi x}=\lim\limits_{x\to 1}\left[\left(1+\sin(\pi x)\right)^{\frac{1}{\sin(\pi x)}}\right]^{\sin\pi x\tan\pi x}$

$$=e^{\lim\limits_{x\to 1}\cos\pi x}=e^{-1}=\frac{1}{e}$$

ដូចនេះ $\lim\limits_{x\to 1}\left[1+\sin(\pi x)\right]^{\cot\pi x}=\dfrac{1}{e}$ ។

3. $\lim\limits_{x\to 0}\dfrac{\ln(\cos ax)}{\ln(\cos bx)}$ រាងមិនកំណត់ $\dfrac{0}{0}$

គេបាន៖ $\lim\limits_{x\to 0}\dfrac{\ln(\cos ax)}{\ln(\cos bx)}$

$$=\lim_{x\to 0}\frac{\ln\left[1+(\cos ax-1)\right]}{\ln\left[1+(\cos bx-1)\right]}$$

$$=\lim_{x\to 0}\frac{\ln\left[1+(\cos ax-1)\right]}{\cos ax-1}\left\{\frac{\cos bx-1}{\ln\left[1+(\cos bx-1)\right]}\right\}\left(\frac{\cos ax-1}{\cos bx-1}\right)$$

$$=\lim_{x\to 0}\left(\frac{\cos ax-1}{\cos bx-1}\right)=\lim_{x\to 0}\frac{-2\sin^2\left(\frac{ax}{2}\right)}{-2\sin^2\left(\frac{bx}{2}\right)}$$

$$=\lim_{x\to 0}\frac{\sin^2\left(\frac{ax}{2}\right)}{\sin^2\left(\frac{bx}{2}\right)}=\left(\frac{a}{2}\right)^2\times\lim_{x\to 0}\left[\frac{\sin^2\left(\frac{ax}{2}\right)}{\left(\frac{ax}{2}\right)^2}\times\frac{\left(\frac{bx}{2}\right)^2}{\sin^2\left(\frac{bx}{2}\right)}\right]=\left(\frac{a}{2}\right)^2$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\ln(\cos ax)}{\ln(\cos bx)}=\left(\dfrac{a}{2}\right)^2$ ។

III. 1. គណនា៖

$a.\cos\left[\arccos\left(\frac{4}{5}\right)+\arcsin\left(\frac{12}{13}\right)\right]$

រូបមន្ត $\cos(a+b)=\cos a\cos b-\sin a\sin b$

គេបាន៖ $\cos\left[\arccos\left(\frac{4}{5}\right)+\arcsin\left(\frac{12}{13}\right)\right]$

$$=\cos\left[\arccos\frac{4}{5}\right]\cos\left[\arcsin\frac{12}{13}\right]-\sin\left[\arccos\frac{4}{3}\right]\sin\left[\arcsin\frac{12}{13}\right]$$

$$=\frac{4}{5}\times\sqrt{1-\sin^2\left(\arcsin\frac{12}{13}\right)}-\sqrt{1-\cos^2\left(\arccos\frac{4}{3}\right)}\times\frac{12}{13}$$

$$=\frac{4}{5}\times\sqrt{1-\left(\frac{12}{13}\right)^2}-\sqrt{1-\left(\frac{4}{3}\right)^2}\times\frac{12}{13}$$

$$=-\frac{16}{65}$$

ដូចនេះ៖ $\cos\left[\arccos\left(\frac{4}{5}\right)+\arcsin\left(\frac{12}{13}\right)\right]=-\frac{16}{65}$ ។

$b.\cos(2\arctan 2)$

តាមរូបមន្ត $\cos 2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}$

គេបាន៖ $\cos(2\arctan 2)=\frac{1-\tan^2(\arctan 2)}{1+\tan^2(\arctan 2)}=\frac{1-2^2}{1+2^2}=-\frac{3}{5}$

ដូចនេះ៖ $\cos(2\arctan 2)=-\frac{3}{5}$ ។

2.បង្ហាញថា $3\arctan\left(\frac{1}{4}\right)+\arctan\left(\frac{5}{99}\right)=\frac{\pi}{4}$

គេបាន៖ $3\arctan\left(\frac{1}{4}\right)+\arctan\left(\frac{5}{99}\right)$

$$=\left[\arctan\left(\frac{1}{4}\right)+\arctan\left(\frac{1}{4}\right)\right]+\arctan\left(\frac{1}{4}\right)+\arctan\left(\frac{5}{99}\right)$$

$$=\arctan\left(\frac{\frac{1}{4}+\frac{1}{4}}{1-\frac{1}{4}\times\frac{1}{4}}\right)+\arctan\left(\frac{1}{4}\right)+\arctan\left(\frac{5}{99}\right)$$

$$=\left[\arctan\left(\frac{8}{15}\right)+\arctan\left(\frac{1}{4}\right)\right]\arctan\left(\frac{5}{99}\right)=\arctan\left(\frac{\frac{8}{15}+\frac{1}{4}}{1-\frac{8}{15}\times\frac{1}{4}}\right)=\arctan(1)$$

$$=\arctan\left(\tan\frac{\pi}{4}\right)=\frac{\pi}{4}$$

ដូចនេះ: $3\arctan\left(\frac{1}{4}\right)+\arctan\left(\frac{5}{99}\right)=\frac{\pi}{4}$ ពិត ។

IV.1.បង្ហាញថា $\left(\frac{2x+3}{4x+5}\right)'=\frac{\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}}{(4x+5)^2}$

តាមរូបមន្ត $\left(\frac{u}{v}\right)'=\frac{u'v-v'u}{v^2}$

គេបាន៖ $\left(\frac{2x+3}{4x+5}\right)'=\frac{(2x+3)'(4x+5)-(4x+5)'(2x+3)}{(4x+5)^2}$

$$=\frac{2(4x+5)-4(2x+3)}{(4x+5)^2}=\frac{10-12}{(4x+5)^2}=\frac{5\times2-4\times3}{(4x+4)^2}=\frac{\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}}{(4x+5)'}$$

ដូចនេះ: $\left(\frac{2x+3}{4x+5}\right)'=\frac{\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}}{(4x+5)^2}$ ពិត ។

2.គេមានអនុគមន៍ $y=2\cos x+3\sin x$ ។ស្រាយបញ្ជាក់ថា $y''+y=0$

គេមានអនុគមន៍ $y=2\cos x+3\sin x$

$$\Rightarrow y'=-2\sin x+3\cos x$$
$$\Rightarrow y''=-2\cos x-3\sin x$$
$$\Rightarrow y''+y'=(-2\cos x-3\sin x)+(2\cos x+3\sin x)=0$$

ដូចនេះ: $y''+y=0$ ពិត ។

លំហាត់

I.រកបន្លាយភាពជាប់នៃអនុគមន៍ $f(x)=\frac{\lg_5\left(1+x^2\right)}{1-\cos 2x}$ ត្រង់ $x_0=0$ ។

II.គណនាលីមីតដូចខាងក្រោម៖

1. $\lim\limits_{x\to 0}(1+2\sin x)^{\frac{1}{\sin x}}$
2. $\lim\limits_{x\to -\infty}\frac{\ln\left(1+3^x\right)}{\ln\left(1+2^x\right)}$
3. $\lim\limits_{x\to a}\frac{\cot x-\cot a}{x-a}$

III.1.គណនា៖

ក.$\cos\left[2\arcsin\left(-\frac{2}{3}\right)\right]$

ខ.$\sin\left[\arccos\left(\frac{3}{5}\right)+\arccos\left(\frac{5}{13}\right)\right]$

2.បង្ហាញថា៖

$$4\arctan\left(\frac{1}{5}\right)-\arctan\left(\frac{1}{239}\right)=\frac{\pi}{4}$$

IV.1.គណនា៖ y'_x និង y''_{x^2} នៃអនុគមន៍ $y=f(x)$ កំណត់ដោយ $\begin{cases} x=f'(t) \\ y=tf'(t)-f(t)\end{cases}$ ។

2.គណនា $y^{(n)}$ នៃអនុគមន៍ $y=\frac{1}{x-5}$ ។

ដំណោះស្រាយ

$I.$រកបន្លាយភាពជាប់នៃអនុគមន៍ $f(x)=\dfrac{\lg_5(1+x^2)}{1-\cos 2x}$ ត្រង់ $x_0=0$

– តាង $g(x)$ជាអនុគមន៍បន្លាយតាមភាពជាប់នៃអនុគមន៍ $f(x)$ត្រង់ $x_0=0$

គេមាន $f(x)=\dfrac{\lg_5(1+x^2)}{1-\cos 2x}$

គេបាន៖ $\lim\limits_{x\to0} f(x)=\lim\limits_{x\to0}\dfrac{\lg_5(1+x^2)}{1-\cos 2x}=\lim\limits_{x\to0}\dfrac{\ln(1+x^2)}{\ln 5.(1-\cos 2x)}$

$$=\lim_{x\to0}\frac{\ln(1+x^2)}{\ln 5.2\sin^2 x}=\frac{1}{2\ln 5}\times\lim_{x\to0}\frac{\ln(1+x^2)}{\sin^2 x}$$

$$=\frac{1}{2\ln 5}\lim_{x\to0}\left[\frac{\ln(1+x^2)}{x^2}\times\frac{x^2}{\sin^2 x}\right]$$

$$=\frac{1}{2\ln 5}$$

ព្រោះ $\lim\limits_{x\to0}\dfrac{\ln(1+x^2)}{x^2}=1$ និង $\lim\limits_{x\to0}\dfrac{x^2}{\sin^2 x}=1$

ដូចនេះ៖ $f(x)=\begin{cases}\dfrac{\lg_5(1+x^2)}{1-\cos 2x} & if\ \ x_0\neq0\\ \dfrac{1}{2\ln 5} & if\ \ x_0=0\end{cases}$ ។

$II.$គណនាលីមីតដូចខាងក្រោម៖

1. $\lim\limits_{x\to0}(1+2\sin x)^{\frac{1}{\sin x}}$ រាងមិនកំណត់ $(1)^{\infty}$

គេបាន៖ $\lim\limits_{x\to0}(1+2\sin x)^{\frac{1}{\sin x}}=\lim\limits_{x\to0}\left[(1+2\sin x)^{\frac{1}{2\sin x}}\right]^2=e^2$

ដូចនេះ៖ $\lim\limits_{x\to0}(1+2\sin x)^{\frac{1}{\sin x}}=e^2$

2. $\lim_{x\to-\infty}\frac{\ln\left(1+3^x\right)}{\ln\left(1+2^x\right)}$ រាងមិនកំណត់ $\frac{0}{0}$

គេបាន៖ $\lim_{x\to-\infty}\frac{\ln\left(1+3^x\right)}{\ln\left(1+2^x\right)}$

$$=\lim_{x\to-\infty}\left[\left(\frac{\ln\left(1+3^x\right)}{3^x}\right)\frac{2^x}{\ln\left(1+2^x\right)}\times\left(\frac{3^x}{2^x}\right)\right]$$

$$=\lim_{x\to-\infty}\left[\left(\frac{\ln\left(1+3^x\right)}{3^x}\right)\times\frac{2^x}{\ln\left(1+2^x\right)}\times\frac{1}{\left(\frac{2}{3}\right)^x}\right]=0$$

ព្រោះ៖ $\lim_{x\to-\infty}\frac{\ln\left(1+3^x\right)}{3^x}=1$, $\lim_{x\to-\infty}\frac{2^x}{\ln\left(1+2^x\right)}=1$ និង $\lim_{x\to-\infty}\frac{1}{\left(\frac{2}{3}\right)^x}=0$

ដូចនេះ៖ $\lim_{x\to-\infty}\frac{\ln\left(1+3^x\right)}{\ln\left(1+2^x\right)}=0$

3. $\lim_{x\to a}\frac{\cot x-\cot a}{x-a}$ រាងមិនកំណត់ $\frac{0}{0}$

គេបាន៖ $\lim_{x\to a}\frac{\cot x-\cot a}{x-a}$

$$=\lim_{x\to a}\frac{\frac{\cos x}{\sin x}-\frac{\cos a}{\sin a}}{x-a}$$

$$=\lim_{x\to a}\frac{\sin a\cos x-\sin x\cos a}{(x-a)\sin x\sin a}=\lim_{x\to a}\frac{\sin(a-x)}{(x-a)\sin x\sin a}$$

$$=-\frac{1}{\sin a}\lim_{x\to a}\frac{\sin(x-a)}{(x-a)}\times\frac{1}{\sin a}$$

$$=-\frac{1}{\sin a}\times\frac{1}{\sin a}=-\frac{1}{\sin^2 a}$$

ដូចនេះ៖ $\lim_{x\to a}\frac{\cot x-\cot a}{x-a}=-\frac{1}{\sin^2 a}$ ។

III.1.គណនា៖

ក.$\cos\left[2\arcsin\left(-\frac{2}{3}\right)\right]$

តាមរូបមន្ត៖ $\cos 2\theta = 1 - 2\sin^2\theta$

គេបាន៖ $\cos\left[2\arcsin\left(-\frac{2}{3}\right)\right]$

$$= 1 - 2\sin^2\left[\arcsin\left(-\frac{2}{3}\right)\right]$$

$$= 1 - 2\left(-\frac{2}{3}\right)^2 = 1 - \frac{8}{9} = \frac{1}{9}$$

ដូចនេះ៖ $\cos\left[2\arcsin\left(-\frac{2}{3}\right)\right] = \frac{1}{9}$ ។

ខ.$\sin\left[\arccos\left(\frac{3}{5}\right) + \arccos\left(\frac{5}{13}\right)\right]$

តាមរូបមន្ត៖ $\sin(a+b) = \sin\cos b + \sin b\cos a$

យើងបាន៖ $\sin\left[\arccos\left(\frac{3}{5}\right) + \arccos\left(\frac{5}{13}\right)\right]$

$$= \sin\left[\arccos\left(\frac{3}{5}\right)\right]\cos\left[\arccos\left(\frac{5}{13}\right)\right] + \sin\left[\arccos\left(\frac{5}{13}\right)\right]\cos\left[\arccos\left(\frac{3}{5}\right)\right]$$

$$= \sqrt{1 - \cos^2\left[\arccos\left(\frac{3}{5}\right)\right]} \times \frac{5}{13} + \sqrt{1 - \cos^2\left[\arccos\left(\frac{5}{13}\right)\right]} \times \frac{3}{5}$$

$$= \sqrt{1 - \left(\frac{3}{5}\right)^2} \times \frac{5}{13} + \sqrt{1 - \left(\frac{5}{13}\right)^2} \times \frac{3}{5}$$

$$= \frac{4}{5} \times \frac{5}{13} + \frac{12}{13} \times \frac{3}{5} = \frac{56}{65}$$

ដូចនេះ៖ $\sin\left[\arccos\left(\frac{3}{5}\right) + \arccos\left(\frac{5}{13}\right)\right] = \frac{56}{65}$ ។

2.បង្ហាញថា៖

$$4\arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right) = \frac{\pi}{4}$$

យើងមាន៖ $4\arctan\left(\frac{1}{5}\right)-\arctan\left(\frac{1}{239}\right)$

$$=2\left[2\arctan\left(\frac{1}{5}\right)\right]-\arctan\left(\frac{1}{239}\right)$$

$$=2\left[\arctan\left(\frac{1}{5}\right)+\arctan\left(\frac{1}{5}\right)\right]-\arctan\left(\frac{1}{239}\right)$$

$$=2\arctan\left(\frac{\frac{1}{5}+\frac{1}{5}}{1-\frac{5}{12}\times\frac{5}{12}}\right)-\arctan\left(\frac{1}{239}\right)$$

$$=\arctan\left(\frac{120}{119}\right)-\arctan\left(\frac{1}{239}\right)=\arctan\left(\frac{\frac{120}{119}+\frac{1}{239}}{1-\frac{120}{119}\times\frac{1}{239}}\right)$$

$$=\arctan(1)=\arctan\left(\tan\frac{\pi}{4}\right)=\frac{\pi}{4}$$

ដូចនេះ៖ $4\arctan\left(\frac{1}{5}\right)-\arctan\left(\frac{1}{239}\right)=\frac{\pi}{4}$ ។

IV.1.គណនា៖ y''_x និង y''_{x^2} នៃអនុគមន៍ $y=f(x)$ កំណត់ដោយ $\begin{cases} x=f'(t) \\ y=tf'(t)-f(t)\end{cases}$

គេបាន $\begin{cases} x=f'(t) \\ y=tf'(t)-f(t)\end{cases}$

$$\Rightarrow\begin{cases} x'_t=[f'(t)]' \\ y'_t=[tf'(t)-f(t)]'\end{cases}\Rightarrow\begin{cases} [f'(t)]'=f(t) \\ f'(t)+tf''(t)-f'(t)=tf''(t)\end{cases}$$

តាមរូបមន្ត $y'_x=\frac{y'_t}{x'_t}=\frac{tf''(t)}{f''(t)}=t$

តាមរូបមន្ត $y''_{x^2}=\frac{(y'_x)'_t}{x'_t}=\frac{(f')}{f''(t)}=\frac{1}{f''(t)}$

ដូចនេះ: $y'_t = t$ និង $y''_{x^2} = \dfrac{1}{f''(t)}$ ។

2. គណនា $y^{(n)}$ នៃអនុគមន៍ $y = \dfrac{1}{x-5}$

គេមាន $y = \dfrac{1}{x-5}$

$$\Rightarrow y' = -\frac{1}{(x-5)^2} = (-1)^1 \times \frac{1!}{(x-5)^2}$$

$$\Rightarrow y'' = \frac{1\times 2}{(x-5)^3} = (-1)^2 \times \frac{2!}{(x-5)^3}$$

$$\Rightarrow y''' = \frac{1\times 2\times 3}{(x-5)^4} = (-1)^3 \times \frac{3!}{(x-5)^4}$$

$$\Rightarrow y^{(4)} = \frac{1\times 2\times 3\times 4}{(x-5)^5} = (-1)^4 \times \frac{4!}{(x-5)^5}$$

ឧបមាថាពិតដល់ k គឺ $y^{(k)} = (-1)^k \times \dfrac{k!}{(x-5)^{k+1}}$

យើងនឹងស្រាយថាពិតដល់ $k+1$ គឺ $y^{(k+1)} = (-1)^{k+1} \times \dfrac{(k+1)!}{(x-5)^{k+2}}$

គេមាន $y^{(k)} = (-1)^k \times \dfrac{k!}{(x-5)^{k+1}}$

$$\Rightarrow y^{(k+1)} = \left[y(k)\right]' = \left[(-1)^k \times \frac{k!}{(x-5)^{k+1}}\right]'$$

$$(-1)^k \times (-1) \times \frac{k!\times (k+1)}{(x-5)^{k+2}} = (-1)^{k+1} \times \frac{(k+1)!}{(x-5)^{k+2}}$$

ដូចនេះ: $y^{(k)} = (-1)^n \times \dfrac{n!}{(x-5)^{n+1}}$

លំហាត់

$I.$រកបន្លាយតាមភាពជាប់នៃអនុគមន៍ $f(x)=\frac{\lg_4(1+3x^2)}{1-\cos 2x}$ ត្រង់ $x_0=0$

$II.$គណនាលីមីតនៃអនុគមន៍ដូចខាងក្រោម៖

$1.\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{1-\tan x}$

$2.\lim\limits_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

$3.\lim\limits_{x\to 0}\frac{x+1-[x]}{3\sin^2 x}$

$III.$ក. ស្រាយបញ្ជាក់ថា៖ $\arccos x-\arccos y=\arccos\left[xy+\sqrt{(1-x^2)(1-y^2)}\right]$

ខ. ចូរគណនា $\sin(2\arctan 3)$

គ. បង្ហាញថា $4\arctan\left(\frac{1}{5}\right)-\arctan\left(\frac{1}{239}\right)=\frac{\pi}{4}$

$IV.$ ក. គណនា y'_x និង y''_{x^2} នៃអនុគមន៍ $y=f(x)$កំណត់ដោយ៖$\begin{cases}x=2t-t^2\\ y=3t-t^3\end{cases}$

ខ.គេមានអនុគមន៍ $y=\sum\limits_{i=1}^{n}a_i x^i$ ។គណនា $y^{(n)}$ ។

ដំណោះស្រាយ

$I.$ រកបន្លាយតាមភាពជាប់នៃអនុគមន៍ $f(x)=\dfrac{\lg_4\left(1+3x^2\right)}{1-\cos 2x}$ ត្រង់ $x_0=0$

តាង $g(x)$ ជាអនុគន៍បន្លាយតាមភាពជាប់នៃ $f(x)$ ត្រង់ $x_0=0$

យើងមាន៖ $f(x)=\dfrac{\lg_4\left(1+3x^2\right)}{1-\cos 2x}$

$$\Rightarrow \lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\lg_4\left(1+3x^2\right)}{1-\cos 2x}=\lim_{x\to 0}\frac{\ln\left(1+3x^2\right)}{\ln 4.(1-\cos 2x)}$$

$$=\lim_{x\to 0}\frac{\ln\left(1+3x^2\right)}{\ln 4.\left(2\sin^2 x\right)}=\frac{1}{2\ln 4}\times\lim_{x\to 0}\frac{\ln\left(1+3x^2\right)}{\sin^2 x}$$

$$=\frac{3}{2\ln 4}\times\lim_{x\to 0}\left[\frac{\ln\left(1+3x^2\right)}{3x^2}\times\frac{x^2}{\sin^2 x}\right]=\frac{3}{2\ln 4}$$

ព្រោះ៖ $\lim\limits_{x\to 0}\dfrac{\ln\left(1+3x^2\right)}{3x^2}=1$ និង $\lim\limits_{x\to 0}\dfrac{x^2}{\sin^2 x}=1$

ដូចនេះ៖ $\begin{cases}\dfrac{\log_4\left(1+3x^2\right)}{1-\cos 2x} & if\ \ x_0\neq 0\\ \dfrac{3}{2\ln 4} & if\ \ x_0=0\end{cases}$ ។

$II.$ គណនាលីមីតនៃអនុគមន៍ដូចខាងក្រោម៖

$1.\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\tan x}$ រាងមិនកំណត់ $\left(\dfrac{0}{0}\right)$

គេបាន៖ $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\tan x}$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{1-\tan x} = \lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{\frac{\cos x - \sin x}{\cos x}}$$

$$= \lim_{x\to\frac{\pi}{4}} (\sin x - \cos x) \times \frac{\cos x}{-(\sin x - \cos x)}$$

$$= -\lim_{x\to\frac{\pi}{4}} \cos x = -\frac{\sqrt{2}}{2}$$

ដូចនេះ: $\lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{1-\tan x} = -\frac{\sqrt{2}}{2}$ ។

$2.\lim_{x\to 0} \frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$ រាងមិនកំណត់ $\left(\frac{0}{0}\right)$

គេបាន៖ $\lim_{x\to 0} \frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

$$= \lim_{x\to 0} \frac{\left(\sqrt{2}\right)^2 - \left(\sqrt{1+\cos x}\right)^2}{\sin^2 x} \times \lim_{x\to 0} \frac{1}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0} \frac{2-1-\cos x}{\sin^2 x} \times \frac{1}{2\sqrt{2}} = \lim_{x\to 0} \frac{\sin^2\left(\frac{x}{2}\right)}{2\sin^2\left(\frac{x}{2}\right)}$$

$$= \frac{1}{4\sqrt{2}}$$

ដូចនេះ: $\lim_{x\to 0} \frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x} = \frac{1}{4\sqrt{2}}$ ។

$3.\lim_{x\to 0} \frac{x+1-[x]}{3\sin^2 x}$

ដោយ $0 \le x-[x] < 1$ នោះ៖ $1 \le x+1-[x] < 2$

គេបាន៖ $\lim_{x\to 0} \frac{x+1-[x]}{3\sin^2 x} = +\infty$

ដូចនេះ: $\lim_{x\to 0} \frac{x+1-[x]}{3\sin^2 x} = +\infty$

III.ក. ស្រាយបញ្ជាក់ថា៖ $\arccos x-\arccos y=\arccos\left[xy+\sqrt{\left(1-x^2\right)\left(1-y^2\right)}\right]$

តាមរូបមន្ត៖ $\cos(a-b)=\cos a\cos b+\sin a\sin b$

គេបាន៖

$\cos(\arccos x-\arccos y)=\cos(\arccos x)\cos(\arccos y)+\sin(\arcsin x)\sin(\arcsin y)$

$=xy+\sqrt{1-\cos^2(\arccos x)}.\sqrt{1-\cos^2(\arccos y)}$

$=xy+\sqrt{1-x^2}.\sqrt{1-y^2}$

$=xy+\sqrt{\left(1-x^2\right)\left(1-y^2\right)}$

$\Rightarrow \arccos x-\arccos y=\arccos\left[xy+\sqrt{\left(1-x^2\right)\left(1-y^2\right)}\right]$

ដូចនេះ៖ $\arccos x-\arccos y=\arccos\left[xy+\sqrt{\left(1-x^2\right)\left(1-y^2\right)}\right]$

ខ. ចូរគណនា $\sin(2\arctan 3)$

តាមរូបមន្ត៖ $\sin(2\beta)=2\sin\beta\cos\beta$ ឬ $\sin(2\beta)=\dfrac{2\tan\beta}{1+\tan^2\beta}$

គេបាន៖ $\sin(2\arctan 3)=\dfrac{2\tan(\arctan 3)}{1+\tan^2(\arctan 3)}=\dfrac{2\times 3}{1+(3)^2}=\dfrac{3}{5}$

ដូចនេះ៖ $\sin(2\arctan 3)=\dfrac{3}{5}$

គ. បង្ហាញថា $4\arctan\left(\dfrac{1}{5}\right)-\arctan\left(\dfrac{1}{239}\right)=\dfrac{\pi}{4}$

$4\arctan\left(\dfrac{1}{5}\right)-\arctan\left(\dfrac{1}{239}\right)=\dfrac{\pi}{4}$

$$=2\left[2\arctan\left(\frac{1}{5}\right)\right]-\arctan\left(\frac{1}{239}\right)$$

$$=2\left[\arctan\left(\frac{1}{5}\right)+\arctan\left(\frac{1}{5}\right)\right]-\arctan\left(\frac{1}{239}\right)$$

$$=2\arctan\left(\frac{\frac{1}{5}+\frac{1}{5}}{1-\frac{5}{12}\times\frac{5}{12}}\right)-\arctan\left(\frac{1}{239}\right)$$

$$=\arctan\left(\frac{120}{119}\right)-\arctan\left(\frac{1}{239}\right)=\arctan\left(\frac{\frac{120}{119}+\frac{1}{239}}{1-\frac{120}{119}\times\frac{1}{239}}\right)$$

$$4\arctan\left(\frac{1}{5}\right)-\arctan\left(\frac{1}{239}\right)=\arctan(1)=\arctan\left(\tan\frac{\pi}{4}\right)=\frac{\pi}{4}$$

ដូចនេះ៖ $4\arctan\left(\frac{1}{5}\right)-\arctan\left(\frac{1}{239}\right)=\frac{\pi}{4}$ ។

IV. ក. គណនា y'_x និង y''_{x^2} នៃអនុគមន៍ $y=f(x)$ កំណត់ដោយ៖ $\begin{cases}x=2t-t^2\\ y=3t-t^3\end{cases}$

គេមាន៖ $\begin{cases}x=2t-t^2\\ y=3t-t^3\end{cases}\Rightarrow\begin{cases}x'_t=\left(2t-t^2\right)'=2-2t\\ y'_t=\left(3t-t^3\right)'=3-3t^2\end{cases}$

តាមរូបមន្ត៖ $y'_x=\frac{y'_x}{x'_t}\Rightarrow\frac{3-3t^2}{2-2t}=\frac{3\left(1-t^2\right)}{2(1-t)}=\frac{3(1-t)(1+t)}{2(1-t)}=\frac{3(1+t)}{2}$

តាមរូបមន្ត៖ $y''_{x^2}=\frac{\left(y'_x\right)'_t}{x'_t}\Rightarrow\frac{\left(\frac{3(1+x)}{2}\right)'}{2(1-t)}=\frac{\frac{3}{2}}{2(1-t)}=\frac{3}{4(1-t)}$

ដូចនេះ៖ $y'_t=\frac{3(1+t)}{2}$ និង $y''_{x^2}=\frac{3}{4(1-t)}$ ។

ខ.គេមានអនុគមន៍ $y=\sum_{i=1}^{n}a_ix^i$ ។គណនា $y^{(n)}$

គេមាន៖ $y=\sum_{i=1}^{n}a_ix^i=a_1x+a_2x^2+a_3x^3+a_4x^3+a_5x^4+...+a_nx^n$

$\Rightarrow y'=a_1+2a_2x+3a_3x^2+4a_4x^3+5a_5x^4+...+na_nx^{(n-1)}$

$\Rightarrow y''=2a_2+3.2.a_3x+4.3.a_4x^2+5.4.a_5x^3+...+n(n-1).a_nx^{(n-2)}$

$\Rightarrow y'''=3.2.1.a_3+4.3.2a_4x+5.4.3a_5x^2+...+n(n-1)(n-2).a_nx^{(n-3)}$

$\Rightarrow y^{(4)}=4.3.2.1.a_4+5.4.3.2.a_5x+...+n(n-1)(n-2)(n-3).a_nx^{(n-4)}$

....

....

$\Rightarrow y^{(n-1)}=n(n-1)(n-2)(n-3)\times...\times2\times a_nx$

$\Rightarrow y^{(n)}=n(n-1)(n-2)(n-3)\times...\times2\times1\times a_n=n!\times a_n$