គណនាលីមីតនៃស្វ៊ីតខាងក្រោម៖

1. $\lim\limits_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\dots+\frac{1}{3n}\right)$

2. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+(n+1)^2}\right)$

3. $\lim\limits_{n\to\infty}\left(\frac{1^2}{1^3+n^3}+\frac{2^2}{2^3+n^3}+\dots+\frac{n^2}{n^3+n^3}\right)$

4. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)^{\frac{1}{2}}\left(1+\frac{3}{n}\right)^{\frac{1}{3}}\dots\left(1+\frac{n}{n}\right)^{\frac{1}{n}}$

5. $\lim\limits_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\dots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$

6. $\lim\limits_{n\to\infty}\left\{\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\dots n}{n\cdot n\cdot n\cdot n\dots n}\right\}^{\frac{1}{n}}$

ដំណោះស្រាយ

1. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+(n+1)^2}\right)$

$we\ have\ \lim\limits_{n\to\infty}\sum\limits_{r=0}^{2n}\frac{1}{n+r}$

$$=\lim_{n\to\infty}\sum_{r=0}^{2n}\frac{1}{n\left[1+\left(\frac{r}{n}\right)\right]}=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n}\frac{1}{1+\left(\frac{1}{n}\right)}$$

$$=\int_0^2\frac{1}{1+x}dx=\left[\ln(1+x)\right]_0^2=(\ln 3)$$

$$Hence:\boxed{\lim_{n\to\infty}\left(\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+(n+1)^2}\right)=ln3}$$

2. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+(n+1)^2}\right)$

$we\ have\ \lim\limits_{n\to\infty}\sum\limits_{r=0}^{n+1}\frac{n}{n^2+r^2}$

$$=\lim_{n\to\infty}\sum_{r=0}^{n+1}\frac{n}{n^2\left[1+\left(\frac{r}{n}\right)^2\right]}=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n+1}\frac{1}{1+\left(\frac{r}{n}\right)^2}$$

$$=\int_0^1\frac{1}{1+x^2}dx=\left[\arctan(x)\right]_0^1=\frac{\pi}{4}$$

$$Hence:\boxed{\lim_{n\to\infty}\left(\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+(n+1)^2}\right)=\frac{\pi}{4}}$$

3. $\lim_{n\to\infty}\left(\frac{1^2}{1^3+n^3}+\frac{2^2}{2^3+n^3}+\ldots+\frac{n^2}{n^3+n^3}\right)$

we have $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r^2}{r^3+n^3}\right)$

$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^2}{n^3\left(1+\left(\frac{r}{n}\right)^3\right)}=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^3\right)}$

$=\int_0^1\frac{x^2}{1+x^3}dx=\frac{1}{3}\int_0^1\frac{d\left(1+x^3\right)}{\left(1+x^3\right)}=\frac{1}{3}\left[\ln\left(1+x^3\right)\right]_0^1$

$=\frac{1}{3}\ln 2$

Hence : $\boxed{\lim_{n\to\infty}\left(\frac{1^2}{1^3+n^3}+\frac{2^2}{2^3+n^3}+\ldots+\frac{n^2}{n^3+n^3}\right)=\frac{1}{3}\ln 2}$

4. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)^{\frac{1}{2}}\left(1+\frac{3}{n}\right)^{\frac{1}{3}}\ldots\left(1+\frac{n}{n}\right)^{\frac{1}{n}}$

we let $p=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)^{\frac{1}{2}}\left(1+\frac{3}{n}\right)^{\frac{1}{3}}\ldots\left(1+\frac{n}{n}\right)^{\frac{1}{n}}$

$\log p=\lim_{n\to\infty}\left[\log\left(1+\frac{1}{n}\right)+\frac{1}{2}\log\left(1+\frac{2}{n}\right)+\frac{1}{3}\log\left(1+\frac{3}{n}\right)+\ldots+\frac{1}{n}\log\left(1+\frac{n}{n}\right)\right]$

$\log p=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r}\log\left(1+\frac{r}{n}\right)$

$\log p=\int_0^1\frac{1}{x}\log(1+x)\,dx=\int_0^1\frac{1}{x}\left[x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\right]dx$

$$\log p=\int_0^1\left(1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+\dots\right)dx=\left[x-\frac{x^2}{4}+\frac{x^3}{9}-\frac{x^4}{16}+\dots\right]_0^1$$

$$\log p=1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\dots\infty=\frac{\pi^2}{12},\ from\ trigonometry.$$

$$\Rightarrow p=e^{\frac{\pi^2}{12}}$$

$$Hence:\boxed{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)^{\frac{1}{2}}\left(1+\frac{3}{n}\right)^{\frac{1}{3}}\dots\left(1+\frac{n}{n}\right)^{\frac{1}{n}}=e^{\frac{\pi^2}{12}}}$$

5 . $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\dots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$

$$we\ let\ a=\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\dots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$$

$$\log a=\lim_{n\to\infty}\frac{1}{n}\log\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\dots\left(1+\frac{n^2}{n^2}\right)\right]$$

$$\log a=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\log\left(1+\frac{k^2}{n^2}\right)=\int_0^1\log\left(1+x^2\right)dx$$

$$\log a=\left[x\log\left(1+x^2\right)\right]_0^1-\int_0^1\frac{2x^2}{1+x^2}dx=\log 2+\frac{1}{2}(\pi-4),\ Photomath$$

$$\Rightarrow a=2e^{\frac{\pi-4}{2}}$$

$$Hence:\boxed{\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\dots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}=2e^{\frac{\pi-4}{2}}}$$

6. $\lim\limits_{n\to\infty}\left\{\dfrac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\ldots n}{n\cdot n\cdot n\cdot n\ldots n}\right\}^{\frac{1}{n}}$

$$let\ \log m=\lim_{n\to\infty}\frac{1}{n}\left[\log\left(\frac{1}{n}\right)+\log\left(\frac{2}{n}\right)+\log\left(\frac{3}{n}\right)+\ldots+\log\left(\frac{n}{n}\right)\right]$$

$$\log m=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)=\int_{0}^{1}\log x\,dx=-1$$

$$\Rightarrow m=\frac{1}{e}$$

$$Hence:\ \boxed{\lim_{n\to\infty}\left\{\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\ldots n}{n\cdot n\cdot n\cdot n\ldots n}\right\}^{\frac{1}{n}}=\frac{1}{e}}$$