



លំហាត់គណិតវិទ្យា

សិស្សពូកែថ្នាក់ទី

៩

$$P + P' = 1$$

ដោយ៖ វ៉ាន់ ជានីន
រៀបរៀង៖ បុគ្គ ធម្មរូប

១៨/១២/២០២១

សំណួរទី១ គេឱ្យ a, b, c ជាបីចំនួនពិតដែល $abc = 2020$

ចូរគណនាកន្សោម៖

$$P = \frac{2020a}{ab + 2020a + 2020} + \frac{b}{bc + b + 2020} + \frac{c}{ac + c + 1} - 1 \quad ?$$

និយោគៈស្រាយ

គណនា

$$P = \frac{2020a}{ab + 2020a + 2020} + \frac{b}{bc + b + 2020} + \frac{c}{ac + c + 1} - 1$$

ដោយ $abc = 2020$

គេបាន

$$\begin{aligned} P &= \frac{2020a}{ab + 2020a + 2020} + \frac{b}{bc + b + 2020} + \frac{c}{ac + c + 1} - 1 \\ &= \frac{2020a}{ab + 2020a + 2020} + \frac{ab}{a(bc + b + 2020)} + \frac{abc}{ab(ac + c + 1)} - 1 \\ &= \frac{2020a}{ab + 2020a + 2020} + \frac{ab}{abc + ab + 2020a} + \frac{abc}{abc \cdot a + abc + ab} - 1 \\ &= \frac{2020a}{ab + 2020a + 2020} + \frac{ab}{2020 + ab + 2020a} + \frac{2020}{2020a + 2020 + ab} - 1 \\ &= \frac{2020a + ab + 2020}{2020a + ab + 2020} - 1 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

ដូចនេះ $P = 0$ ។

លំហាត់ទី២ ចូរគណនាផលបូក

$$B = 2020(2021^9 + 2021^8 + \dots + 2021^2 + 2022 + 1) + 1 \quad \text{។}$$

និរន្តរៈស្រាវ

គណនា

$$\begin{aligned} B &= 2020(2021^9 + 2021^8 + \dots + 2021^2 + 2022 + 1) + 1 \\ &= 2020[(2021^9 + 2021^8 + \dots + 2021^2 + 2021 + 1) + 1] + 1 \\ &= 2020(2021^9 + 2021^8 + \dots + 2021^2 + 2021 + 1) + 2020 + 1 \\ &= (2021 - 1)(2021^9 + 2021^8 + \dots + 2021^2 + 2021 + 1) + 2021 \\ &= 2021^{10} - 1 + 2021 \\ &= 2021^{10} + 2020 \end{aligned}$$

លំហាត់ទី៣ គណនា $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$ ។

និរន្តរៈស្រាវ

$$\text{គណនា } \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$$

$$\text{តាង } x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}} \quad , x > 0$$

បើកអង្គទាំងពីរជាការេ

$$\text{គេបាន } x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$$

$$x^2 = 6 + x$$

$$x^2 - x - 6 = 0$$

$$\text{តាម } \Delta = b^2 - 4ac$$

$$= (-1)^2 + 4 \times 1 \times (-6)$$

$$= 1 + 24$$

$$= 25$$

$$\Rightarrow \sqrt{\Delta} = \sqrt{25} = 5$$

$$\Rightarrow x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-(-1) - 5}{2} = -2 < 0$$

$$\Rightarrow x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-(-1) + 5}{2} = 3 > 0$$

យើងបាន $x = 3$ ។

ដូចនេះ $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}} = 3$

លំហាត់ទី៤ ដោយមិនប្រើម៉ាស៊ីនគិតលេខចូរប្រៀប

ធៀបតម្លៃខាងក្រោម៖

១. $2, \sqrt[3]{4}, \sqrt[5]{64}$

២. $2^{30}, 3^{20}, 10^{10}$

៣. $\sqrt[3]{5}, \sqrt{3}, \sqrt[4]{8}$

និរន្តរៈស្រាវ

១. ប្រៀបធៀប $2, \sqrt[3]{4}, \sqrt[5]{64}$

យើងមាន $2 = 2^1$

$$\sqrt[3]{4} = \sqrt[3]{2^2} = 2^{\frac{2}{3}}$$

$$\sqrt[5]{64} = \sqrt[5]{2^6} = 2^{\frac{6}{5}}$$

ដោយ $\frac{2}{3} < 1 < \frac{6}{5}$ នោះ $2^{\frac{2}{3}} < 2^1 < 2^{\frac{6}{5}}$

ដូចនេះ $\sqrt[3]{4} < 2 < \sqrt[5]{64}$

២. ប្រៀបធៀប 2^{30} , 3^{20} , 10^{10}

យើងមាន $2^{30} = 2^{3 \times 10} = (2^3)^{10} = 8^{10}$

$$3^{20} = 3^{2 \times 10} = (3^2)^{10} = 9^{10}$$

$$8 < 9 < 10 \text{ នោះ } 8^{10} < 9^{10} < 10^{10}$$

ដូចនេះ $2^{30} < 3^{20} < 10^{10}$

៣. ប្រៀបធៀប $\sqrt[3]{5}$, $\sqrt{3}$, $\sqrt[4]{8}$

យើងមាន $\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{1 \times 4}{3 \times 4}} = 5^{\frac{4}{12}} = \sqrt[12]{5^4} = \sqrt[12]{625}$

$$\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{6}{12}} = \sqrt[12]{3^6} = \sqrt[12]{729}$$

$$\sqrt[4]{8} = 8^{\frac{1}{4}} = 8^{\frac{3}{12}} = \sqrt[12]{8^3} = \sqrt[12]{512}$$

យើងបាន $\sqrt[12]{512} < \sqrt[12]{625} < \sqrt[12]{729}$

ដូចនេះ $\sqrt[4]{8} < \sqrt[3]{5} < \sqrt{3}$

លំហាត់ទី៩ ១. គណនាកន្សោម

$$N = (243)^{0.2} + (81)^{0.25} + (4)^{0.50} - (16)^{0.75} \text{ ។}$$

២. ស្រាយបញ្ជាក់ថា $3^{\sqrt{2}} > 2^{\sqrt{3}}$ ។

និរដេសនា

១. គណនាកន្សោម $N = (243)^{0.2} + (81)^{0.25} + (4)^{0.50} - (16)^{0.75}$

គេបាន $N = (243)^{0.2} + (81)^{0.25} + (4)^{0.50} - (16)^{0.75}$

$$= (3^5)^{0.2} + (3^4)^{0.25} + (2^2)^{0.50} - (2^4)^{0.75}$$

$$= 3 + 3 + 2 - 2^3 = 8 - 8 = 0$$

ដូចនេះ $N = 0$ ។

២. ស្រាយបញ្ជាក់ថា $3^{\sqrt{2}} > 2^{\sqrt{3}}$

យើងដឹងថា $9 > 8$

$$3^2 > 2^3$$

$$3^{\sqrt{2} \times \sqrt{2}} > 2^{\sqrt{3} \times \sqrt{3}} > 2^{\sqrt{3} \times \sqrt{2}}$$

គេបាន $3^{\sqrt{2} \times \sqrt{2}} > 2^{\sqrt{3} \times \sqrt{2}}$

$$(3^{\sqrt{2}})^{\sqrt{2}} > (2^{\sqrt{3}})^{\sqrt{2}}$$

$$3^{\sqrt{2}} > 2^{\sqrt{3}}$$

ដូចនេះ $3^{\sqrt{2}} > 2^{\sqrt{3}}$ ពិត ។

សំណួរទី៦ គេឱ្យ $x^2 + y^2 = 1$ និង $bx^2 = ay^2$ ដែល

$a \neq 0, b \neq 0$ ។

$$\text{ស្រាយបំភ្លឺថា } \frac{x^{2020}}{a^{1010}} + \frac{y^{2020}}{b^{1010}} = \frac{2}{(a+b)^{1010}}$$

និយោគស្រាយ

$$\text{ស្រាយបំភ្លឺថា } \frac{x^{2020}}{a^{1010}} + \frac{y^{2020}}{b^{1010}} = \frac{2}{(a+b)^{1010}}$$

មាន $x^2 + y^2 = 1$

$$\text{និង } bx^2 = ay^2 \Rightarrow \frac{x^2}{a} = \frac{y^2}{b} = \frac{x^2 + y^2}{a+b} = \frac{1}{a+b}$$

$$* \quad \frac{x^2}{a} = \frac{1}{a+b} \Leftrightarrow \left(\frac{x^2}{a} \right)^{1010} = \left(\frac{1}{a+b} \right)^{1010}$$

$$\frac{x^{2020}}{a^{1010}} = \frac{1}{(a+b)^{1010}} \quad (1)$$

$$* \quad \frac{x^2}{a} = \frac{1}{a+b} \Leftrightarrow \left(\frac{y^2}{b} \right)^{1010} = \left(\frac{1}{a+b} \right)^{1010}$$

$$\frac{y^{2020}}{b^{1010}} = \frac{1}{(a+b)^{1010}} \quad (2)$$

យក (1) + (2)

គេបាន

$$\frac{x^{2020}}{a^{1010}} + \frac{y^{2020}}{b^{1010}} = \frac{1}{(a+b)^{1010}} + \frac{1}{(a+b)^{1010}} = \frac{2}{(a+b)^{1010}}$$

ដូចនេះ: $\frac{x^{2020}}{a^{1010}} + \frac{y^{2020}}{b^{1010}} = \frac{2}{(a+b)^{1010}}$

លំហាត់ទី៧ គណនាតម្លៃនៃកន្សោម

$$A = \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \dots + \frac{1}{\sqrt{2020} + \sqrt{2021}} \quad ?$$

នំណោះស្រាយ

គណនា

$$A = \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \dots + \frac{1}{\sqrt{2020} + \sqrt{2021}}$$

គេបាន

$$\begin{aligned}
 A &= \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{2020}+\sqrt{2021}} \\
 &= \frac{1(\sqrt{2}-\sqrt{1})}{(\sqrt{2}+\sqrt{1})(\sqrt{2}-\sqrt{1})} + \frac{1(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} + \dots \\
 &\quad + \frac{1(\sqrt{2021}-\sqrt{2020})}{(\sqrt{2021}+\sqrt{2020})(\sqrt{2021}-\sqrt{2020})} \\
 &= \frac{\sqrt{2}-\sqrt{1}}{(\sqrt{2})^2-(\sqrt{1})^2} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} + \dots \\
 &\quad + \frac{\sqrt{2021}-\sqrt{2020}}{(\sqrt{2021})^2-(\sqrt{2020})^2} \\
 &= \sqrt{2}-\sqrt{1} + \sqrt{3}-\sqrt{2} + \dots + \sqrt{2021}-\sqrt{2020} \\
 &= \sqrt{2021}-1 \\
 \text{ដូចនេះ } A &= \sqrt{2021}-1 \text{ ។}
 \end{aligned}$$

សំណួរទី៨ ប្រៀបធៀប $\sqrt{2019} + \sqrt{2021}$ និង $2\sqrt{2020}$ ។

និយោគស្រាវ

$$\text{ឧបមា } \sqrt{2019} + \sqrt{2021} < 2\sqrt{2020}$$

លើកអង្គទាំងពីរជាការេ

$$(\sqrt{2019} + \sqrt{2021})^2 < (2\sqrt{2020})^2$$

$$(\sqrt{2019})^2 + 2\sqrt{2019} \times \sqrt{2021} + (\sqrt{2021})^2 < 2^2 (\sqrt{2020})^2$$

$$2019 + 2\sqrt{2019} \times \sqrt{2021} + 2021 < 4 \times 2020$$

$$2020 - 1 + 2\sqrt{2019} \times \sqrt{2021} + 2020 + 1 < 4 \times 2020$$

$$2\sqrt{2019} \times \sqrt{2021} + 2 \times 2020 < 4 \times 2020$$

$$2\sqrt{2019} \times \sqrt{2021} < 2 \times 2020$$

$$\sqrt{2019 \times 2021} < 2020$$

$$\sqrt{(2020-1)(2020+1)} < 2020$$

$$\sqrt{2020^2 - 1^2} < \sqrt{2020^2}$$

$$\sqrt{2020^2 - 1} < \sqrt{2020^2}$$

$$\text{ដូចនេះ } \sqrt{2019} + \sqrt{2021} < 2\sqrt{2020} \quad \text{។}$$

សំណួរទី៩

គេឱ្យកន្សោម $\frac{x^2 + 2y^2}{306} = \frac{x^2 - 2y^2}{294}$ ។ គណនា $\frac{x^2}{y^2}$ ។

ដំណោះស្រាយ

ដោយ

$$\frac{x^2 + 2y^2}{306} = \frac{x^2 - 2y^2}{294} \Leftrightarrow 294(x^2 + 2y^2) = 306(x^2 - 2y^2)$$

$$\text{គេបាន } 294(x^2 + 2y^2) = 306(x^2 - 2y^2)$$

$$294(x^2 + 2y^2) = 306(x^2 - 2y^2)$$

$$49(x^2 + 2y^2) = 51(x^2 - 2y^2)$$

$$49\left(\frac{x^2 + 2y^2}{y^2}\right) = 51\left(\frac{x^2 - 2y^2}{y^2}\right)$$

$$49\left(\frac{x^2}{y^2} + 2\right) = 51\left(\frac{x^2}{y^2} - 2\right)$$

$$49\frac{x^2}{y^2} + 98 = 51\frac{x^2}{y^2} - 102$$

$$51\frac{x^2}{y^2} - 49\frac{x^2}{y^2} = 98 + 102$$

$$2\frac{x^2}{y^2} = 200$$

$$\Rightarrow \frac{x^2}{y^2} = 100$$

$$\text{ដូច្នេះ } \frac{x^2}{y^2} = 100 \text{ ។}$$

សំណួរទី១០

$$\text{ចូរគណនាកន្សោម } B = \sqrt{6 + 4x + \sqrt{81^x + 2(81^x)}} \text{ ។}$$

និយោគស្រាវ

$$\text{គណនាកន្សោម } B = \sqrt{6 + 4x + \sqrt{81^x + 2(81^x)}}$$

$$\begin{aligned} B &= \sqrt{6 + 4x + \sqrt{3 \times 81^x}} \\ &= \sqrt{6 + 4x + \sqrt{3 \times (3^4)^x}} \end{aligned}$$

$$\begin{aligned} \text{គេបាន} &= \sqrt{6 + 4x + \sqrt{3 \times 3^{4x}}} \\ &= \sqrt{6 + 4x + \sqrt{3^{4x+1}}} \\ &= \sqrt{6 + 4} = \sqrt{9} = 3 \end{aligned}$$

ដូចនេះ $B = 3$ ។

សំណួរទី១១

$$\text{រកតម្លៃនៃ } \frac{a}{b-c} \text{ ដោយដឹងថា } \frac{a}{b} = 2020 \text{ និង}$$

$$\frac{b}{c} = 2021 \text{ ។}$$

និយោគស្រាវ

$$\text{រកតម្លៃនៃ } \frac{a}{b-c}$$

$$\text{ដោយ } \frac{b}{c} = 2021 \Rightarrow b = 2021c$$

$$b - c = 2021c - c$$

$$b - c = 2020c$$

និង $\frac{a}{b} = 2020 \Rightarrow a = 2020b$

$$a = 2020 \times 2021c$$

គេបាន $\frac{a}{b-c} = \frac{2020 \times 2021c}{2020c} = 2021$

ដូចនេះ $\frac{a}{b-c} = 2021$ ។

សំណួរទី១២

a និង b ជាចំនួនគត់វិជ្ជមាន។ ដោយដឹងថា

$$a^2 + b^2 = 13 \text{ ។ គណនា } a + b \text{ ។}$$

សំណោះស្រាយ

ដោយ $a^2 + b^2 = 13$

13 ជាផលបូកការេនៃពីរចំនួន

$$13 = 9 + 4$$

$$13 = 4 + 9$$

នោះយើងទាញបាន $\begin{cases} a^2 = 9 \\ b^2 = 4 \end{cases}$ ឬ $\begin{cases} a^2 = 4 \\ b^2 = 9 \end{cases}$

ដោយ $a > 0, b > 0$ គេបាន $\begin{cases} a = 3 \\ b = 2 \end{cases}$ ឬ $\begin{cases} a = 2 \\ b = 3 \end{cases}$

$$\Rightarrow a + b = 5$$

ដូចនេះ $a + b = 5$ ។

លំហាត់ទី១៣ ចូគណនាកន្សោម $A = \sqrt{3\sqrt{5\sqrt{3\sqrt{5\sqrt{3\cdots}}}}}$ ។

និរន្តរៈស្រាវ

គណនាកន្សោម

$$A = \sqrt{3\sqrt{5\sqrt{3\sqrt{5\sqrt{3\cdots}}}}} \quad A > 0$$

លើកអង្គទាំងពីរជាការេ

$$A^2 = 3\sqrt{5\sqrt{3\sqrt{5\sqrt{3\sqrt{5\sqrt{3\cdots}}}}}}$$

$$A^4 = 9 \times 5\sqrt{3\sqrt{5\sqrt{3\sqrt{5\sqrt{3\cdots}}}}}$$

$$A^4 = 45A$$

$$A^4 - 45A = 0$$

$$A(A^3 - 45) = 0$$

$$A = 0 \text{ មិនយក}$$

$$A^3 - 45 = 0$$

$$A^3 = 45$$

$$\Rightarrow A = \sqrt[3]{45} > 0$$

$$\text{ដូចនេះ } A = \sqrt[3]{45}$$

សំណួរទី១៤

ចូរប្រៀបធៀបនៃពីរចំនួនខាងក្រោម៖

១. $A = 2021^4$ និង $B = 2017 \times 2019 \times 2023 \times 2025$

២. 31^{11} និង 17^{14}

សំណោះស្រាយ

១. ប្រៀបធៀប $A = 2021^4$ និង

$B = 2017 \times 2019 \times 2023 \times 2025$

ដោយ $B = 2017 \times 2019 \times 2023 \times 2025$

$$= 2017 \times 2025 \times 2019 \times 2023$$

$$= (2021 - 4)(2021 + 4)(2021 - 2)(2021 + 2)$$

$$= (2021^2 - 4^2)(2021^2 - 2^2)$$

$$= (2021^2 - 16)(2021^2 - 4)$$

$$= 2021^4 - 4 \times 2021^2 - 16 \times 2021^2 + 64$$

$$B = 2021^4 - 20 \times 2021^2 + 64 < 2021^4 = A$$

ដូចនេះ $B < A$ ។

២. ប្រៀបធៀប 31^{11} និង 17^{14}

មាន $31^{11} < 32^{11}$

$$31^{11} < (2^5)^{11} < 2^{55}$$

និង $16^{14} < 17^{14}$

$$(2^4)^{14} < 17^{14}$$

$$2^{56} < 17^{14}$$

$$\text{គេបាន } 31^{11} < 2^{55} < 2^{56} < 17^{14}$$

$$\text{ដូចនេះ } 31^{11} < 17^{14} \quad \text{។}$$

សំណួរទី១៥ គណនាកន្សោម

$$A = \sqrt{7+\sqrt{33}} + \sqrt{7-\sqrt{33}} - \sqrt{22} \quad \text{។}$$

ដំណោះស្រាយ

$$\text{គណនាកន្សោម } A = \sqrt{7+\sqrt{33}} + \sqrt{7-\sqrt{33}} - \sqrt{22}$$

$$\text{តាង } B = \sqrt{7+\sqrt{33}} + \sqrt{7-\sqrt{33}}$$

$$B^2 = \left(\sqrt{7+\sqrt{33}} + \sqrt{7-\sqrt{33}} \right)^2$$

$$B^2 = \left(\sqrt{7+\sqrt{33}} \right)^2 + 2\sqrt{7+\sqrt{33}} \times \sqrt{7-\sqrt{33}} + \left(\sqrt{7-\sqrt{33}} \right)^2$$

$$B^2 = 7 + \sqrt{33} + 2\sqrt{(7+\sqrt{33})(7-\sqrt{33})} + 7 - \sqrt{33}$$

$$B^2 = 14 + 2\sqrt{7^2 - (\sqrt{33})^2}$$

$$B^2 = 14 + 2\sqrt{49-33}$$

$$B^2 = 14 + 2 \times 4 = 22$$

$$\Rightarrow B = \sqrt{22}$$

$$\text{គេបាន } A = \sqrt{22} - \sqrt{22} = 0$$

$$\text{ដូចនេះ } A = 0 \quad \text{។}$$

សំណួរទី១៦

បើ $x - \frac{1}{x} = \sqrt{2019}$ ។ គណនា $x^4 + \frac{1}{x^4}$ ។

ដំណោះស្រាយ

$$\text{គេមាន } x - \frac{1}{x} = \sqrt{2019}$$

លើកអង្គទាំងពីរជាការេ

$$\left(x - \frac{1}{x}\right)^2 = (\sqrt{2019})^2$$

$$x^2 - 2x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 2019$$

$$x^2 - 2 + \frac{1}{x^2} = 2019$$

$$x^2 + \frac{1}{x^2} = 2021$$

លើកអង្គទាំងពីរជាការេ

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 2021^2$$

$$x^4 + 2 \times x^2 \times \frac{1}{x^2} + \frac{1}{x^4} = 2021^2$$

$$x^4 + 2 + \frac{1}{x^4} = 2021^2$$

$$x^4 + \frac{1}{x^4} = 2021^2 - 2$$

$$\text{ដូចនេះ } x^4 + \frac{1}{x^4} = 2021^2 - 2 \text{ ។}$$

សំណួរទី១៧

ស្រាយបំភ្លឺថា

$$(2020)(2022)(2021^2 + 1)(2021^4 + 1)(2021^8 + 1)(2021^{16} + 1) = 2021^{32} - 1$$

និយោគស្រាយ

ស្រាយបំភ្លឺថា

$$(2020)(2022)(2021^2 + 1)(2021^4 + 1)(2021^8 + 1)(2021^{16} + 1) = 2021^{32} - 1$$

ដោយ

$$\begin{aligned} & (2020)(2022)(2021^2 + 1)(2021^4 + 1)(2021^8 + 1)(2021^{16} + 1) \\ &= (2021 - 1)(2021 + 1)(2021^2 + 1)(2021^4 + 1)(2021^8 + 1)(2021^{16} + 1) \\ &= (2021^2 - 1)(2021^2 + 1)(2021^4 + 1)(2021^8 + 1)(2021^{16} + 1) \\ &= (2021^4 - 1)(2021^4 + 1)(2021^8 + 1)(2021^{16} + 1) \\ &= (2021^8 - 1)(2021^8 + 1)(2021^{16} + 1) \\ &= (2021^{16} - 1)(2021^{16} + 1) \\ &= 2021^{32} - 1 \quad \text{ពិត ។} \end{aligned}$$

សំណួរទី១៨

រកតម្លៃនៃ d ដោយដឹងថា

$$\begin{aligned} a + b + c + d &= 11, \quad 2a + 3c = 19, \quad b + 4d = 22 \\ , \quad 4a + d &= 14 \text{ និង } 5b + 3c = 5 \quad \text{។} \end{aligned}$$

និរន្តរៈស្រាវ

រកតម្លៃ d

យើងមាន $a+b+c+d=11$ (1)

$$2a+3c=19 \quad (2)$$

$$b+4d=22 \quad (3)$$

$$4a+d=14 \quad (4)$$

$$5b+3c=5 \quad (5)$$

$$(1)+(2)+(3)+(4)+(5) : 7a+7b+7c+6d=71$$

$$7a+7b+7c+7d-d=71$$

$$7(a+b+c+d)-d=71$$

$$7 \times 11 - d = 71$$

$$d = 77 - 71$$

$$d = 6$$

ដូចនេះ $d = 6$ ។

សំណួរទី១៩

ដោយដឹងថា

$$(a+b)^2 + (a+3b)^2 - 4(a+b) - 10(a+3b) + 29 = 0$$

គណនាតម្លៃនៃ $2a+4b$ ។

និរន្តរៈស្រាវ

គណនា $2a+4b$

$$\text{មាន } (a+b)^2 + (a+3b)^2 - 4(a+b) - 10(a+3b) + 29 = 0$$

$$(a+b)^2 - 4(a+b) + 4 + (a+3b)^2 - 10(a+3b) + 25 = 0$$

$$(a+b)^2 - 2 \times (a+b) \times 2 + 2^2 + (a+3b)^2 - 2 \times (a+3b) \times 5 + 5^2 = 0$$

$$(a+b-2)^2 + (a+3b-5)^2 = 0$$

$$\text{នោះ } \begin{cases} a+b-2=0 & (1) \\ a+3b-5=0 & (2) \end{cases}$$

$$\text{យក } (1)+(2) : 2a+4b-7=0$$

$$\Rightarrow 2a+4b=7$$

$$\text{ដូចនេះ } 2a+4b=7$$

សំបាត់ទី២០

$$\text{ដោះស្រាយប្រព័ន្ធសមីការ } \begin{cases} -x+y+z=xyz & (1) \\ x-y+z=xyz & (2) \\ x+y-z=xyz & (3) \end{cases}$$

នំណោះស្រាល

$$\text{ដោះស្រាយប្រព័ន្ធសមីការ } \begin{cases} -x+y+z=xyz & (1) \\ x-y+z=xyz & (2) \\ x+y-z=xyz & (3) \end{cases}$$

$$\text{យក } (1)+(2) : 2z=2xyz \Rightarrow z=xyz$$

$$\text{យក } (1)+(3) : 2y=2xyz \Rightarrow y=xyz$$

$$\text{យក } (2)+(3) : 2x=2xyz \Rightarrow x=xyz$$

$$\text{យើងបាន } x=y=z=xyz$$

$$\text{ដោយ } x=xyz$$

$$x = x \times x \times x = x^3$$

$$x - x^3 = 0$$

$$x(1 - x^2) = 0$$

$$x = 0$$

$$1 - x^2 = 0$$

$$\Rightarrow x = \pm 1$$

ដូចនេះ: $x = 0, y = 0, z = 0$

$$x = -1, y = -1, z = -1$$

$$x = 1, y = 1, z = 1 \text{ ។}$$

សំណួរទី២១

ចូរគណនាតម្លៃ

$$S = \frac{2021}{1 \times 2} + \frac{2021}{2 \times 3} + \dots + \frac{2021}{2020 \times 2021}$$

និយាយស្រាយ

$$\begin{aligned} \text{គណនា } S &= \frac{2021}{1 \times 2} + \frac{2021}{2 \times 3} + \dots + \frac{2021}{2020 \times 2021} \\ &= 2021 \left(\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \dots + \frac{1}{2020 \times 2021} \right) \\ &= 2021 \left(\frac{2-1}{1 \times 2} + \frac{3-2}{2 \times 3} + \dots + \frac{2021-2020}{2020 \times 2021} \right) \end{aligned}$$

$$\begin{aligned}
 &= 2021 \left(\frac{2}{1 \times 2} - \frac{1}{1 \times 2} + \frac{3}{2 \times 3} - \frac{2}{2 \times 3} + \dots + \frac{2021}{2020 \times 2021} - \frac{2020}{2020 \times 2021} \right) \\
 &= 2021 \left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{2020} - \frac{1}{2021} \right) \\
 &= 2021 \left(1 - \frac{1}{2021} \right) \\
 &= 2021 - 1 = 2020
 \end{aligned}$$

ដូចនេះ $S = 2020$ ។

សំណួរទី២២

ដោះស្រាយសមីការ៖

$$១. (2^{18} - 2^{17})(2^{15} - 2^{14}) = 2^x$$

$$២. \frac{x}{3} - \frac{x}{6} + \frac{x}{9} - \frac{x}{12} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$$

ដោះស្រាយ

ដោះស្រាយសមីការ

$$១. (2^{18} - 2^{17})(2^{15} - 2^{14}) = 2^x$$

$$(2^{18} - 2^{17})(2^{15} - 2^{14}) = 2^x$$

$$2^{17}(2-1)2^{14}(2-1) = 2^x$$

$$2^{17} \times 2^{14} = 2^x$$

$$2^{31} = 2^x$$

$$\Rightarrow x = 31$$

ដូចនេះ $x = 31$ ។

$$២. \frac{x}{3} - \frac{x}{6} + \frac{x}{9} - \frac{x}{12} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$$

$$\frac{x}{3} - \frac{x}{6} + \frac{x}{9} - \frac{x}{12} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$$

$$\frac{x}{3} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \right) = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$$

$$\frac{x}{3} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \right) = - \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \right)$$

$$\frac{x}{3} = -1$$

$$\Rightarrow x = -3$$

$$\text{ដូចនេះ } x = -3$$

សំណួរទី២៣

គេឱ្យ $a+b+c=1$, $a^2+b^2+c^2=1$ និង

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = m \text{ ចូរគណនា } P = xy + yz + zx \text{ ។}$$

សំណួរទី២៤

$$\text{គណនា } P = xy + yz + zx$$

$$\text{មាន } a+b+c=1 \text{ , } a^2+b^2+c^2=1$$

$$\text{និង } \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = m$$

$$\bullet \quad a+b+c=1 \text{ លើកអង្គទាំងពីរជាការេ}$$

$$(a+b+c)^2 = 1^2$$

$$a^2 + b^2 + c^2 + 2(ab+ac+bc) = 1$$

$$1 + 2(ab+ac+bc) = 1$$

$$2(ab+ac+bc) = 0 \Rightarrow (ab+ac+bc) = 0$$

$$\bullet \quad \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = m$$

$$\frac{x}{a} = m \Rightarrow x = am$$

$$\frac{y}{b} = m \Rightarrow y = bm$$

$$\frac{z}{c} = m \Rightarrow z = cm$$

$$P = xy + yz + zx$$

$$= (am)(bm) + (bm)(cm) + (cm)(am)$$

$$= abm^2 + bcm^2 + cam^2$$

$$= m^2 (ab + bc + ca)$$

$$= m^2 \times 0 = 0$$

ដូចនេះ $P = 0$ ។

លំហាត់ទី២៤

១. ចូរជ្រៀបធៀបចំនួន $A = \sqrt{19}$ និង $6 - \sqrt{3}$ ។

២. ចូរជ្រៀបចំនួនខាងក្រោមតាមលំដាប់ពីតូចទៅធំ

$$a = 2^{88}, b = 3^{55}, c = 5^{44}, d = 7^{33} \text{ ។}$$

និរដេកស្រាវ

១. ជ្រៀបធៀបចំនួន $A = \sqrt{19}$ និង $6 - \sqrt{3}$

ឧបមា $A > B$

$$\text{នោះ } \sqrt{19} > 6 - \sqrt{3}$$

$$\sqrt{19} + \sqrt{3} > 6 \text{ លើកអង្គទាំងពីរជាការេ}$$

$$(\sqrt{19} + \sqrt{3})^2 > 6^2$$

$$19 + 2 \times \sqrt{19} \times \sqrt{3} + 3 > 36$$

$$22 + 2 \times \sqrt{19 \times 3} > 36$$

$$2 \times \sqrt{57} < 36 - 22$$

$$\sqrt{57} > \frac{14}{2}$$

$$(\sqrt{57})^2 > 7^2$$

$$57 > 49 \text{ ពិត}$$

ដូចនេះ $A > B$ ។

២. រៀបចំនួនខាងក្រោមតាមលំដាប់ពីតូចទៅធំ

$$a = 2^{88}, b = 3^{55}, c = 5^{44}, d = 7^{33}$$

$$a = 2^{88} = (2^8)^{11} = 256^{11}$$

$$b = 3^{55} = (3^5)^{11} = 243^{11}$$

$$c = 5^{44} = (5^4)^{11} = 625^{11}$$

$$d = 7^{33} = (7^3)^{11} = 343^{11}$$

$$\text{ដោយ } 243^{11} < 256^{11} < 343^{11} < 625^{11}$$

ដូចនេះ $b < a < d < c$ ។

សំណួរទី២៥

គណនា A ដោយឱ្យលទ្ធផលជាផលគុណនៃ ៣ កត្តាដែល

$$A = \frac{\sqrt{2}-2}{\sqrt{12}+\sqrt{8}+3\sqrt{6}} \text{ ។}$$

ដំណោះស្រាយ

គណនា A ដោយឱ្យលទ្ធផលជាផលគុណ ៣ កត្តា

$$\begin{aligned} A &= \frac{\sqrt{2}-2}{\sqrt{12}+\sqrt{8}+3\sqrt{6}} = \frac{\sqrt{2}-2}{2\sqrt{3}+2\sqrt{2}+(\sqrt{3})^2+\sqrt{6}} \\ &= \frac{\sqrt{2}-2}{2(\sqrt{3}+\sqrt{2})+\sqrt{3}(\sqrt{3}+\sqrt{2})} \\ &= \frac{\sqrt{2}-2}{(\sqrt{3}+\sqrt{2})(2+\sqrt{3})} \\ &= \frac{(\sqrt{2}-2)(\sqrt{3}-\sqrt{2})(2-\sqrt{3})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})(2+\sqrt{3})(2-\sqrt{3})} \\ &= \frac{(\sqrt{2}-2)(\sqrt{3}-\sqrt{2})(2-\sqrt{3})}{(3-2)(4-3)} \\ &= \frac{(\sqrt{2}-2)(\sqrt{3}-\sqrt{2})(2-\sqrt{3})}{1 \times 1} \\ &= (\sqrt{2}-2)(\sqrt{3}-\sqrt{2})(2-\sqrt{3}) \end{aligned}$$

$$\text{ដូចនេះ } A = (\sqrt{2}-2)(\sqrt{3}-\sqrt{2})(2-\sqrt{3}) \text{ ។}$$

លំហាត់ទី២៦

គេឱ្យ $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ និង $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$ គណនាតម្លៃ

នៃកន្សោម $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ ។

ដំណោះស្រាយ

គណនាតម្លៃនៃកន្សោម $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$

យើងមាន $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$

$$\frac{ayz + bxz + cxy}{xyz} = 0$$

$$\Rightarrow ayz + bxz + cxy = 0$$

ហើយ $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$$\left(\frac{x}{a} + \frac{y}{b} + \frac{z}{c} \right)^2 = 1^2$$

$$\left(\frac{x}{a} \right)^2 + \left(\frac{y}{b} \right)^2 + \left(\frac{z}{c} \right)^2 + 2 \left(\frac{xy}{ab} + \frac{xz}{ac} + \frac{yz}{bc} \right) = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} + 2 \left(\frac{cxy + bxz + ayz}{abc} \right) = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} + 2 \left(\frac{0}{abc} \right) = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\text{ដូចនេះ } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ ។}$$

សំខាន់ទី២៧

$$\text{សម្រួលកន្សោម } A = \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} \quad ។$$

និរន្តរៈស្រាវ

សម្រួលកន្សោម

$$A = \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

$$\text{ដោយ } a^3 + b^3 + c^3 - 3abc$$

$$= a^3 + 3a^2b + 3ab^2 + b^3 + c^3 - 3a^2b + 3ab^2 - 3abc$$

$$= (a+b)^3 + c^3 - 3ab(a+b+c)$$

$$= (a+b+c) \left[(a+b)^2 - (a+b)c + c^2 \right] - 3ab(a+b+c)$$

$$= (a+b+c) \left[(a+b)^2 - (a+b)c + c^2 - 3ab \right]$$

$$= (a+b+c) (a^2 + 2ab + b^2 - ac - bc + c^2 - 3ab)$$

$$= (a+b+c) (a^2 + b^2 + c^2 - ab - ac - bc)$$

$$\text{នោះ } A = \frac{(a+b+c)(a^2 + b^2 + c^2 - ab - ac - bc)}{a^2 + b^2 + c^2 - ab - bc - ca} = a+b+c$$

$$\text{ដូចនេះ } A = a+b+c \quad ។$$

សំណួរទី២៨

១.បង្ហាញថា $81^6 - 9 \cdot 27^7 - 9^{11}$ ចែកដាច់នឹង 5 ។

២.បង្ហាញថា $1^{2021} + 2^{2021} + 3^{2021} + 4^{2021} + 5^{2021}$ ចែកដាច់នឹង 5 ។

និដោះស្រាយ

១.បង្ហាញថា $81^6 - 9 \cdot 27^7 - 9^{11}$ ចែកដាច់នឹង 5

$$\begin{aligned} 81^6 - 9 \cdot 27^7 - 9^{11} &= (3^4)^6 - 3^2 \cdot (3^3)^7 - (3^2)^{11} \\ &= 3^{24} - 3^{23} - 3^{22} \\ &= 3^{22}(3^2 - 3 - 1) \\ &= 3^{22} \times 5 \end{aligned}$$

ដោយ $3^{22} \times 5$ ជាពហុគុណនៃ 5

ដូចនេះ $81^6 - 9 \cdot 27^7 - 9^{11}$ ចែកដាច់នឹង 5 ។

២.បង្ហាញថា $1^{2021} + 2^{2021} + 3^{2021} + 4^{2021} + 5^{2021}$ ចែកដាច់នឹង 5

$$1^{2021} + 2^{2021} + 3^{2021} + 4^{2021} + 5^{2021}$$

$$= (1^{2021} + 4^{2021}) + (2^{2021} + 3^{2021}) + 5^{2021}$$

$$\bullet \quad 1^{2021} + 4^{2021} = (1+4)(1^{2020} - 1^{2019} \times 4 + \dots - 1 \times 4^{2019} + 4^{2020})$$

$$\text{តាង } M = 1^{2020} - 1^{2019} \times 4 + \dots - 1 \times 4^{2019} + 4^{2020}$$

$$\text{នោះ } 1^{2021} + 4^{2021} = 5M$$

$$\bullet \quad 2^{2021} + 3^{2021} = (2+3)(2^{2020} - 2^{2019} \times 3 + \dots - 2 \times 3^{2019} + 3^{2020})$$

$$\text{តាង } N = 2^{2020} - 2^{2019} \times 3 + \dots - 2 \times 3^{2019} + 3^{2020}$$

នោះ: $2^{2021} + 3^{2021} = 5N$

គេបាន $1^{2021} + 2^{2021} + 3^{2021} + 4^{2021} + 5^{2021}$

$= 5M + 5N + 5^{2021}$

$= 5(M + N + 5^{2020})$

ដោយ $5(M + N + 5^{2020})$ ជាពហុគុណនៃ 5

ដូចនេះ $1^{2021} + 2^{2021} + 3^{2021} + 4^{2021} + 5^{2021}$ ចែកដាច់នឹង 5 ។

សំណួរទី២៩

គេឱ្យ $x = \sqrt{3+\sqrt{5}}$, $y = \sqrt{3-\sqrt{5}}$ គណនា $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$ ។

សំណោះ: ប្រាកដ

គណនា $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

$= \frac{(\sqrt{x}+\sqrt{y})(\sqrt{x}+\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}$

$= \frac{x+y+2\sqrt{xy}}{x-y}$

• $x = \sqrt{3+\sqrt{5}} = \sqrt{2\left(\frac{3+\sqrt{5}}{2}\right)}$

$$\begin{aligned}
 &= \sqrt{\left(\frac{6+2\sqrt{5}}{2}\right)} = \sqrt{\left(\frac{5+2\times\sqrt{5}\times\sqrt{1+1}}{\sqrt{2}}\right)} \\
 &= \frac{\sqrt{(\sqrt{5})^2 + 2\times\sqrt{5}\times\sqrt{1+1}}}{\sqrt{2}} = \frac{\sqrt{(\sqrt{5}+1)^2}}{\sqrt{2}} \\
 &= \frac{\sqrt{5}+1}{\sqrt{2}} = \frac{\sqrt{2}(\sqrt{5}+1)}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{10}+\sqrt{2}}{2}
 \end{aligned}$$

សម្រាយដូចខាងលើ $y = \sqrt{3-\sqrt{5}} = \frac{\sqrt{10}-\sqrt{2}}{2}$

$$x+y = \frac{\sqrt{10}+\sqrt{2}}{2} + \frac{\sqrt{10}-\sqrt{2}}{2} = \sqrt{10}$$

$$x-y = \frac{\sqrt{10}+\sqrt{2}}{2} - \frac{\sqrt{10}-\sqrt{2}}{2} = \sqrt{2}$$

$$\begin{aligned}
 xy &= \sqrt{3+\sqrt{5}} \times \sqrt{3-\sqrt{5}} \\
 &= \sqrt{(3+\sqrt{5})(3-\sqrt{5})} \\
 &= \sqrt{3^2 - (\sqrt{5})^2} = \sqrt{9-5} = 2
 \end{aligned}$$

យើងបាន $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{x+y+2\sqrt{xy}}{x-y} = \frac{\sqrt{10}+2\sqrt{2}}{\sqrt{2}}$

$$\begin{aligned}
 &= \frac{\sqrt{2}(\sqrt{5}+2)}{\sqrt{2}} \\
 &= \sqrt{5}+2
 \end{aligned}$$

ដូចនេះ $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \sqrt{5}+2$ ។

សំខាន់ទី៣០ ដោះស្រាយសមីការ

$$១. \underbrace{3^x + 3^x + 3^x + \dots + 3^x}_{\text{តួ } 27} = 3^{1000} \text{ ។}$$

$$២. \underbrace{2003^y + 2003^y + \dots + 2003^y}_{\text{តួ } 2003} = 1 \text{ ។}$$

និយោគស្រាវ

ដោះស្រាយសមីការ

$$១. \underbrace{3^x + 3^x + 3^x + \dots + 3^x}_{\text{តួ } 27} = 3^{1000}$$

$$27 \times 3^x = 3^{1000}$$

$$3^3 \times 3^x = 3^{1000}$$

$$3^{3+x} = 3^{1000}$$

$$\text{គេបាន } 3+x=1000 \Rightarrow x=1000-3=997$$

$$\text{ដូចនេះ } x=997 \text{ ។}$$

$$២. \underbrace{2003^y + 2003^y + \dots + 2003^y}_{\text{តួ } 2003} = 1$$

$$2003 \times 2003^y = 1$$

$$2003^{1+y} = 2003^0$$

$$\text{គេបាន } 1+y=0 \Rightarrow y=-1$$

$$\text{ដូចនេះ } y=-1 \text{ ។}$$

លំហាត់ទី៣១ ចូរសម្រួលកន្សោម

$$A = \sqrt{4 + \sqrt{10 + 2\sqrt{5}}} + \sqrt{4 - \sqrt{10 + 2\sqrt{5}}} \text{ ។}$$

និយោគស្រាវ

$$\text{សម្រួលកន្សោម } A = \sqrt{4 + \sqrt{10 + 2\sqrt{5}}} + \sqrt{4 - \sqrt{10 + 2\sqrt{5}}}$$

លើកអង្គទាំងពីរជាការេ

$$A^2 = \left(\sqrt{4 + \sqrt{10 + 2\sqrt{5}}} - \sqrt{4 - \sqrt{10 + 2\sqrt{5}}} \right)^2$$

$$A^2 = 4 + \sqrt{10 + 2\sqrt{5}} - 2\sqrt{4 + \sqrt{10 + 2\sqrt{5}}} \times \sqrt{4 - \sqrt{10 + 2\sqrt{5}}} + 4 - \sqrt{10 + 2\sqrt{5}}$$

$$A^2 = 8 + 2\sqrt{(4 + \sqrt{10 + 2\sqrt{5}})(4 - \sqrt{10 + 2\sqrt{5}})}$$

$$A^2 = 8 + 2\sqrt{16 - (10 + 2\sqrt{5})}$$

$$A^2 = 8 + 2\sqrt{6 - 2\sqrt{5}}$$

$$A^2 = 8 + 2\sqrt{(\sqrt{5})^2 - 2 \times \sqrt{5} \times \sqrt{1} + 1}$$

$$A^2 = 8 + 2\sqrt{(\sqrt{5} - 1)^2}$$

$$A^2 = 8 + 2(\sqrt{5} - 1)$$

$$A^2 = 6 + 2\sqrt{5}$$

$$\Rightarrow A = \sqrt{6 + 2\sqrt{5}} = \sqrt{(\sqrt{5})^2 - 2 \times \sqrt{5} \times \sqrt{1} + 1} = \sqrt{(\sqrt{5} - 1)^2} = \sqrt{5} - 1$$

$$\text{ដូចនេះ } A = \sqrt{5} - 1 \text{ ។}$$