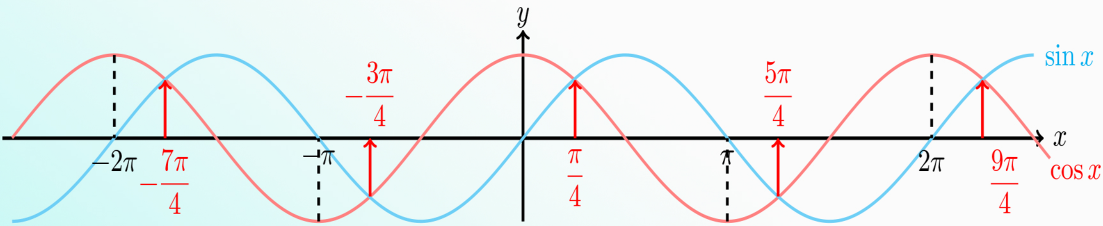
សមីការ និង វិសមីការត្រីកោណមាត្រ



$\cos x > \sin x$ ចម្លើយទូទៅ $-\frac{3\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi \; , \; k \in Z$

ជំនួយក្នុងការសិក្សា

លក្ខណៈរូបមន្ត ទ្រឹស្តីបទ

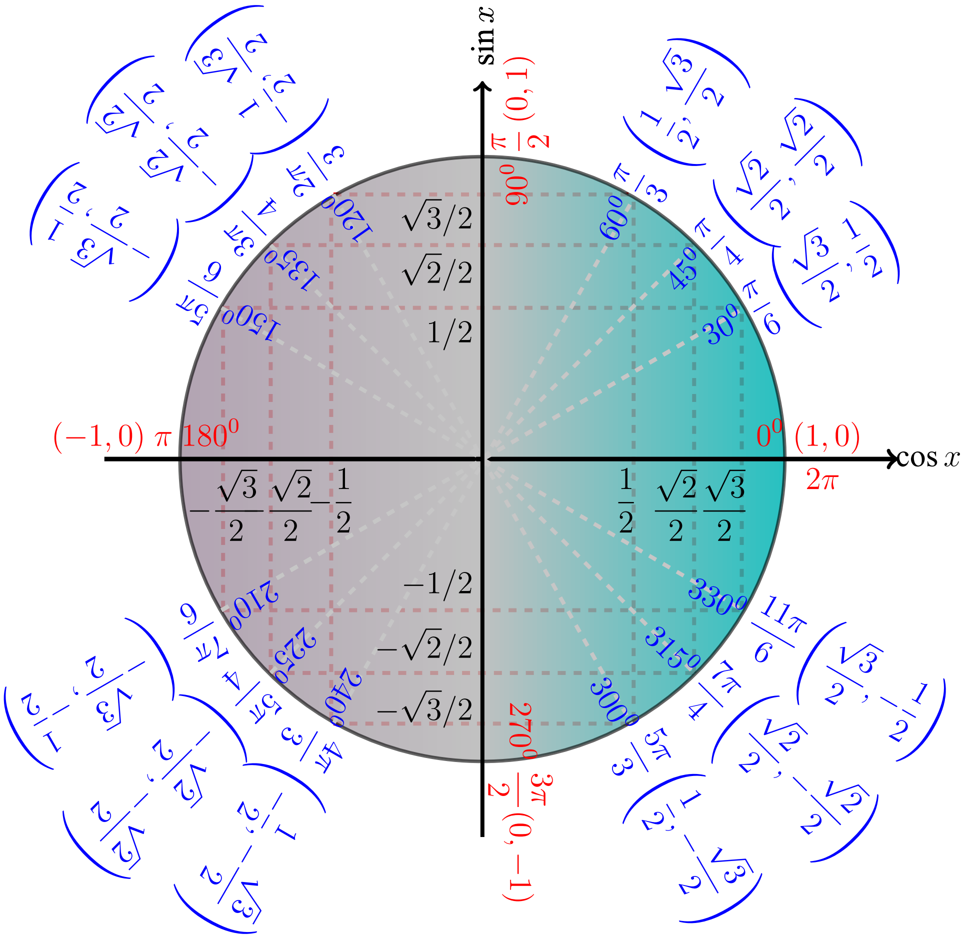
សម្រាយបញ្ជាក់ លំហាត់គំរូ

បោះពុម្ពលើកទី ២

រៀបរៀងដោយ យុយ រាត្រី

fackbook page
Reatrey Math

រង្វង់ត្រីកោណមាត្រ



១. ទំនាក់ទំនងសំខាន់ៗ

១.១ រូបមន្តមុំ បូក និង ដក

1. $\cos(a+b) = \cos a\cos b - \sin a\sin b$ 2. $\cos(a-b) = \cos a\cos b + \sin a\sin b$

3. $\sin(a+b) = \sin a\cos + \sin b\cos a$ 4. $\sin(a-b) = \sin a\cos - \sin b\cos a$

5. $\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a\tan b}$ 6. $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a\tan b}$

7. $\cot(a+b) = \dfrac{\cot a\cot b - 1}{\cot a + \cot b}$ 8. $\cot(a-b) = \dfrac{\cot a\cot b + 1}{\cot b - \cot a}$

១.២ រូបមន្តមុំឌុប និង កន្លះមុំ

ក. $\cos 2x = 2\cos^2 - 1 = 1 - 2\sin^2 x$ ច. $\cos 3x = 4\cos^3 x - 3\cos x$

ខ. $\sin 2x = 2\sin x\cos x$ ឆ. $\sin^2\dfrac{x}{2} = \dfrac{1-\cos x}{2}$

គ. $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ ជ. $\cos^2\dfrac{x}{2} = \dfrac{1+\cos x}{2}$

ឃ. $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$ ឈ. $\tan^2 \dfrac{1-\cos x}{1+\cos x}$

ង. $\sin 3x = 3\sin x - 4\sin^3 x$ ញ. $\tan\dfrac{x}{2} = \dfrac{\sin x}{1+\cos x}$

១.៣ បំលែងផលបូកជាផលគុណ

1. $\cos p + \cos q = 2\cos\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$ 6. $\tan p + \tan q = \dfrac{\sin(p+q)}{\cos p\cos q}$

3. $\cos p - \cos q = -2\sin\dfrac{p+q}{2}\sin\dfrac{p-q}{2}$ 7. $\tan p - \tan q = \dfrac{\sin(p-q)}{\cos p\cos q}$

4. $\sin p + \sin q = 2\sin\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$ 8. $\cot p + \cot q = \dfrac{\sin(p+q)}{\sin p\sin q}$

5. $\sin p - \sin q = 2\sin\dfrac{p-q}{2}\cos\dfrac{p+q}{2}$ 9. $\cot p - \cot q = \dfrac{\sin(q-p)}{\sin p\sin q}$

១.៤ រូបមន្តកន្សោមជាអនុគមន៍ t តាម $t = \tan\dfrac{x}{2}$

ក. $\cos x = \dfrac{1-t^2}{1+t^2}$ ខ. $\sin x = \dfrac{2t}{1+t^2}$ គ. $\tan x = \dfrac{2t}{1-t^2}$

១.៥ បំលែងផលគុណជាផលបូក

១. $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$

២. $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

៣. $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$

៤. $\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$

កំណត់សម្គាល់

រូបមន្តមុំឌុបបានពីរូបមន្តផលបូកជំនួសតម្លៃមុំ $a = b$ និង កន្លះមុំបានមកពីប្តូរតម្លៃមុំ $x \to \frac{x}{2}$

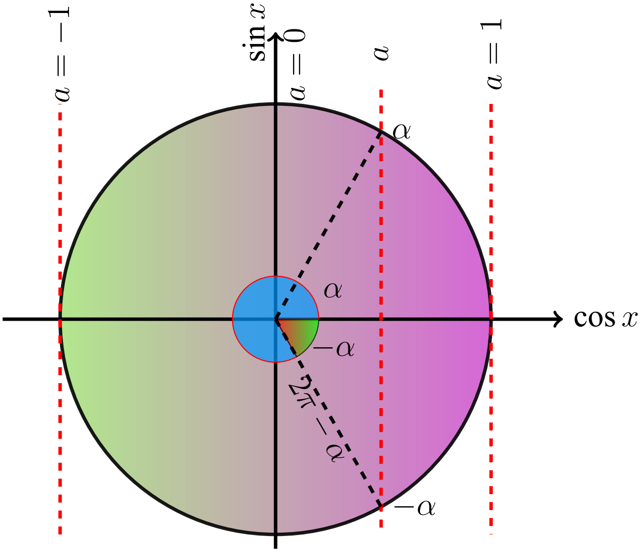
២.សមីការត្រីកោណមាត្រ

២.១ សមីការ $\cos x = a$, a ជាចំនួនពិត

. ○ $|a| > 1$ សមីការ $\cos x = a$ គ្មានចម្លើយ

○ $|a| \leqslant 1$ នាំឲ្យ $x = \pm arc\cos a = \pm\alpha$ ដោយ $\alpha = arc\cos a$

និង $2\pi - \alpha$ ជាមុំផ្ទុយរបស់ α (មួយរង្វង់) ដូចរូប 01



រូប 01

☻ ដូចនេះ $\cos x = a \iff \cos x = \cos\alpha \implies x = \pm\alpha + 2k\pi$, $k\epsilon Z$ ។ (ទូទៅ)

♦ ករណីពិសេស

○ $a = 1$ នោះ $\cos x = 1 \implies x = 2k\pi$, $k\epsilon Z$

○ $a = -1$ នោះ $\cos x = -1 \implies x = (2k+1)\pi$, $k\epsilon Z$

○ $a = 0$ នោះ $\cos x = 0 \implies x = (2k+1)\dfrac{\pi}{2}$, $k\epsilon Z$ ។

▲ ទ្រឹស្តីបទទី ១ $\cos x = 0 \iff x = (2k+1)\dfrac{\pi}{2}$, $k \in Z$

ស្រាយបញ្ជាក់. យើងដឹងថា $\cos x = 0$ នៅពេល x ជាពហុគុណចំនួនសេសនៃ $\dfrac{\pi}{2}$

$\because \cos x = 0 \iff x = \pm\dfrac{\pi}{2}$, $\pm\dfrac{3\pi}{2}$, $\pm\dfrac{5\pi}{2}$, ...

$$\Longleftrightarrow x=(2k+1)\frac{\pi}{2} \ , \ k \in Z$$

$\therefore \cos x = 0 \Longleftrightarrow x=(2k+1)\frac{\pi}{2} \ , \ k \in Z$ ។

▲ ទ្រឹស្តីបទទី ២ បើ $-1 \le a \le 1$ ដែលមាន α មួយដែលធ្វើឲ្យ $\cos x = a$ យើងបានដំណោះស្រាយទូទៅត្រូវបានកំណត់ដោយ $\cos x = \cos\alpha \Longleftrightarrow x = \pm\alpha + 2k\pi \ , k \in Z \ , \ \alpha \in [0,\pi]$ ។

ស្រាយបញ្ជាក់. យើងមាន $\cos x = \cos\alpha \ , \ \alpha \in [0,\pi]$

$$\Longleftrightarrow \cos x - \cos\alpha = 0$$

$$\Longleftrightarrow -2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right) = 0$$

$$\Longleftrightarrow -2\sin\left(\frac{x+\alpha}{2}\right) = 0 \text{ ឬ } \sin\left(\frac{x-\alpha}{2}\right) = 0$$

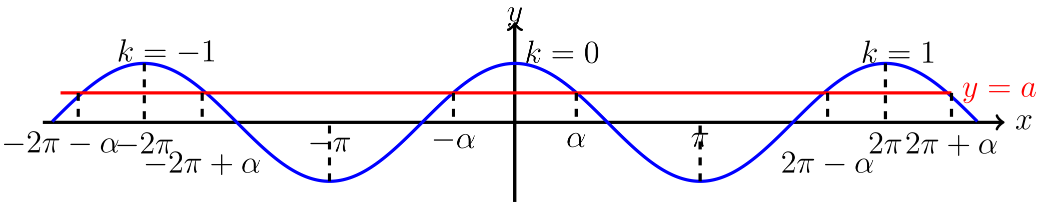
$$\Longleftrightarrow \left(\frac{x+\alpha}{2}\right) = k\pi \quad \text{ ឬ } \left(\frac{x-\pi}{2}\right) = k\pi \ , \ k \in Z$$

$$\Longleftrightarrow x+\alpha = 2k\pi \text{ ឬ } x-\alpha = 2k\pi$$

$$\Longleftrightarrow x = -\alpha + 2k\pi \text{ ឬ } x = \alpha + 2k\pi$$

ដូចនេះ . $\cos x = \cos\alpha \Longleftrightarrow x = \pm\alpha + 2k\pi \ , \ k \in Z \ , \ \alpha \ \in[0,\pi]$ ។

○ **ក្រាបអនុគមន៍** $y = \cos x \ , \ y = a$



លំហាត់គំរូទី១

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\cos x = \frac{1}{2}$

គ . $\cos 2x = \frac{\sqrt{3}}{2}$

ង . $\cos x = \sin x$

ខ . $\cos x = -\frac{\sqrt{2}}{2}$

ឃ . $\cos x = \frac{1}{3}$

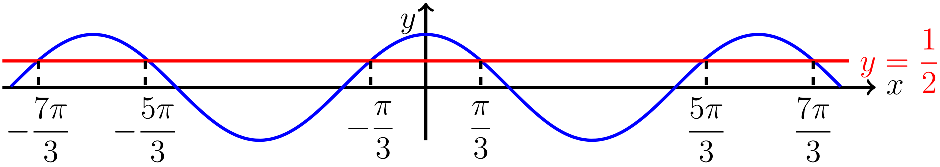
ច . $\cos\left(3x - \frac{\pi}{6}\right) = -1$

ដំណោះស្រាយ.

ក . $\cos x = \frac{1}{2}$

នាំឲ្យ $x = \pm arc\cos\frac{1}{2}$ ដោយ $arc\cos\frac{1}{2} = \frac{\pi}{3}$

យើងបាន $x = \pm\frac{\pi}{3} + 2k\pi\ ,\ k\ \epsilon\ Z$



☻ ដូចនេះ $x = \pm\frac{\pi}{3} + 2k\pi,\quad k\epsilon\ Z$

ខ . $\cos x = -\frac{\sqrt{2}}{2}$

សមីការអាចសរសេរ $\cos x = \cos\frac{3\pi}{4} \Longrightarrow x = \pm\frac{3\pi}{4} + 2k\pi$

☻ ដូចនេះ $x = \pm\frac{3\pi}{4} + 2k\pi,\quad k\epsilon\ Z$ ។

គ . $\cos 2x = \frac{\sqrt{3}}{2}$

សមីការអាចសរសេរ $\cos 2x = \cos\frac{\pi}{3}$

$\Longleftrightarrow 2x = \pm\frac{\pi}{3} + 2k\pi \Longrightarrow x = \pm\frac{\pi}{6} + k\pi,\quad k\epsilon\ Z$

☻ ដូចនេះ $x = \pm\frac{\pi}{6} + k\pi,\quad k\epsilon\ Z$ ។

ឃ . $\cos x = \frac{1}{3}$

នាំឲ្យ $x = \pm arc\cos\frac{1}{3} + 2k\pi$

☻ ដូចនេះ $x = \pm arc\cos\frac{1}{3} + 2k\pi,\quad k\epsilon\ Z$ ។

ង . $\cos x = \sin x$

សមីការអាចសរសេរ $\cos x = \cos\left(\frac{\pi}{2} - x\right) \Longleftrightarrow x = \pm\left(\frac{\pi}{2} - x\right) + 2k\pi$

$\Longleftrightarrow x = \frac{\pi}{2} - x + 2k\pi$ ឬ $x = -\frac{\pi}{2} + x + 2k\pi$ មិនពិត

$\Longleftrightarrow 2x = \frac{\pi}{2} + 2k\pi \Longrightarrow x = \frac{\pi}{4} + k\pi$

☻ ដូចនេះ $x = \frac{\pi}{4} + k\pi,\quad k\epsilon\ Z$ ។

ច . $\cos\left(3x - \frac{\pi}{6}\right) = -1$

នាំឲ្យ $3x - \frac{\pi}{6} = (2k+1)\pi$

$\Longleftrightarrow 3x = \frac{\pi}{6} + 2k\pi + \pi$

$\Longrightarrow x = \frac{7\pi}{18} + \frac{2}{3}k\pi$

☻ ដូចនេះ $x = \frac{7\pi}{18} + \frac{2}{3}k\pi, \quad k\epsilon\, Z$ ។

លំហាត់គំរូទី២

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\cos(3x + \frac{\pi}{3}) = \cos(x - \frac{\pi}{6})$

គ . $\sin^2 x + \sin^2 2x = 1$

ខ . $\cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = \frac{\sqrt{2}}{2}$

ឃ . $\cos 2x - \sin 2x = (\cos x - \sin x)^2$

ដំណោះស្រាយ.

ក . $\cos\left(3x + \frac{\pi}{3}\right) = \cos\left(x - \frac{\pi}{6}\right)$

$\Longleftrightarrow 3x + \frac{\pi}{3} = \pm\left(x - \frac{\pi}{6}\right) + 2k\pi$

$\Longleftrightarrow 3x + \frac{\pi}{3} = x - \frac{\pi}{6} + 2k\pi$ ឬ $3x + \frac{\pi}{3} = -x + \frac{\pi}{6} + 2k\pi$

$\Longleftrightarrow 2x = -\frac{\pi}{2} + 2k\pi$ ឬ $4x = -\frac{\pi}{6} + 2k\pi$

$\Longrightarrow x = -\frac{\pi}{4} + k\pi$ ឬ $x = -\frac{\pi}{24} + \frac{1}{2}k\pi$

☻ ដូចនេះ $x = -\frac{\pi}{4} + k\pi$ ឬ $x = -\frac{\pi}{24} + \frac{1}{2}k\pi, \quad k\epsilon\, Z$ ។

ខ . $\cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = \frac{\sqrt{2}}{2}$

$\Longleftrightarrow \cos\left(\frac{\pi}{6} + x\right) = \frac{\sqrt{2}}{2}$ សមីការអាចសរសេរ $\cos\left(\frac{\pi}{6} + x\right) = \cos\frac{\pi}{4}$

$\Longleftrightarrow \frac{\pi}{6} + x = \pm\frac{\pi}{4} + 2k\pi$

$\Longleftrightarrow \frac{\pi}{6} + x = \frac{\pi}{4} + 2k\pi$ ឬ $\frac{\pi}{6} + x = -\frac{\pi}{4} + 2k\pi$

$\Longleftrightarrow x = \frac{\pi}{4} - \frac{\pi}{6} + 2k\pi$ ឬ $x = -\frac{\pi}{4} - \frac{\pi}{6} + 2k\pi$

$\Longrightarrow x=\frac{\pi}{12}+2k\pi$ ឬ $x=-\frac{5\pi}{12}+2k\pi$

☻ ដូចនេះ $x=\frac{\pi}{12}+2k\pi$ ឬ $x=-\frac{5\pi}{12}+2k\pi,\quad k\epsilon\ Z$ ។

គ . $\sin^2 x+\sin^2 2x=1$

$\Longleftrightarrow \frac{1-\cos 2x}{2}+\frac{1-\cos 4x}{2}=1$

$\Longleftrightarrow 1-\cos 2x+1-\cos 4x=2$

$\Longleftrightarrow \cos 2x+\cos 4x=0 \Longleftrightarrow 2\cos 3x\cos x=0$

$\Longleftrightarrow \cos 3x=0$ ឬ $\cos x=0$

$\Longleftrightarrow 3x=(2k+1)\frac{\pi}{2}$ ឬ $x=(2k+1)\frac{\pi}{2}$

$\Longrightarrow x=(2k+1)\frac{\pi}{6}$ ឬ $x=(2k+1)\frac{\pi}{2}$

☻ ដូចនេះ $x=(2k+1)\frac{\pi}{6}$ ឬ $x=(2k+1)\frac{\pi}{2},\quad k\epsilon\ Z$ ។

ឃ . $\cos 2x-\sin 2x=(\cos x-\sin x)^2$

$\Longleftrightarrow \cos 2x-\sin 2x=\cos^2 x-2\sin x\cos x+\sin^2 x$

$\Longleftrightarrow \cos 2x=1 \Longleftrightarrow 2x=2k\pi \Longrightarrow x=k\pi$

☻ ដូចនេះ $x=k\pi\ ,\ k\ \epsilon\ Z$ ។

២.២ សមីការ $\sin x=a\ ,\ a$ ជាចំនួនពិត

. ○ $|a|>1$ សមីការ $\sin x=a$ គ្មានចម្លើយ

○ $|a|\leqslant 1$ នាំឲ្យ $x=arc\sin a$ ឬ $x=\pi-arc\sin a$

ដោយ $\alpha=arc\sin a$ និង $\pi-\alpha$ ជាមុំបន្ថែមរបស់ α (មួយរង្វង់) ដូចរូប 02

☻ ដូចនេះ $\sin x=a \Longleftrightarrow \sin x=\sin\alpha$

$\Longrightarrow x=\alpha+2k\pi$ ឬ $x=\pi-\alpha+2k\pi\ ,\ k\epsilon Z$ ។ (ទូទៅ)

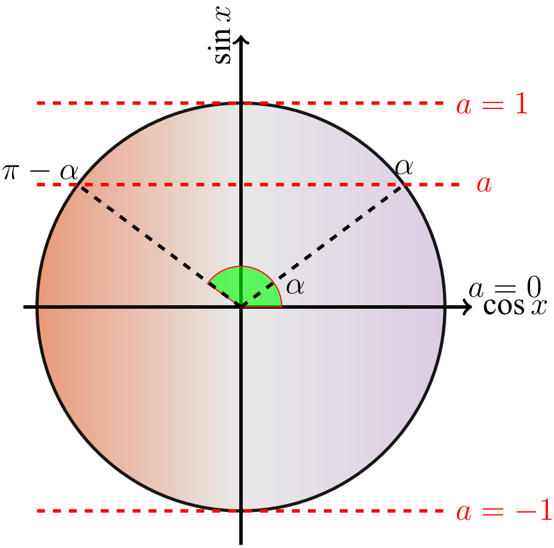
○ អាចសរសេរ $\sin x=\sin\alpha \Longleftrightarrow x=k\pi+(-1)^k\alpha\quad \alpha\ \epsilon\ [-\frac{\pi}{2},\frac{\pi}{2}],\ k\epsilon\ Z$ ។

♦ ករណីពិសេស

○ $a=1$ នោះ $\sin x=1 \Longrightarrow x=\frac{\pi}{2}+2k\pi\ ,\ k\epsilon Z$

○ $a=-1$ នោះ $\sin x=-1 \Longrightarrow x=-\frac{\pi}{2}+2k\pi\ ,\ k\epsilon Z$

$\circ\ a = 0$ នោះ $\sin x = 0 \Longrightarrow x = k\pi \ , \ k\epsilon Z$ ។



រូប 02

▲ ទ្រឹស្តីបទទី ៣ យើងមាន $\sin x = 0 \Longleftrightarrow x = k\pi \ , \ k\epsilon\ Z$

ស្រាយបញ្ជាក់. យើងដឹងថា $\sin x = 0$ នៅពេល x ជាពហុគុណចំនួននៃ π

$\because \sin x = 0 \Longleftrightarrow x = \pm\pi \ , \pm 2\pi \ , \pm 3\pi \ , ...$

$\Longleftrightarrow x = k\pi \ , \ k \ \epsilon \ Z$

$\therefore \sin x = 0 \Longleftrightarrow x = k\pi \ , \ k \ \epsilon \ Z$ ។

▲ ទ្រឹស្តីបទទី ៤ បើ $-1 \le a \le 1$ ដែលមាន α មួយដែលធ្វើឲ្យ $\sin x = a$ យើងបានដំណោះស្រាយទូទៅត្រូវបានកំណត់ដោយ $\sin x = \sin\alpha \Longleftrightarrow x = k\pi + (-1)^k\alpha \, , k \ \epsilon \ Z \, , \ \alpha \ \epsilon \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

ស្រាយបញ្ជាក់. យើងមាន $\sin x = \sin\alpha \ , \ \alpha \ \epsilon \ [-\dfrac{\pi}{2}, \dfrac{\pi}{2}]$

$$\Longleftrightarrow \sin x - \sin\alpha = 0 \Longleftrightarrow 2\cos\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right) = 0$$

$$\Longleftrightarrow \cos\left(\frac{x+\alpha}{2}\right) = 0 \quad \text{ឬ} \quad \sin\left(\frac{x-\alpha}{2}\right) = 0$$

$$\Longleftrightarrow \left(\frac{x+\alpha}{2}\right) = (2k+1)\frac{\pi}{2} \quad \text{ឬ} \quad \left(\frac{x-\alpha}{2}\right) = k\pi \ , \ k \ \epsilon \ Z$$

$$\Longleftrightarrow x + \alpha = (2k+1)\pi \qquad \text{ឬ} \quad x - \alpha = 2k\pi \ , \ k \ \epsilon \ Z$$

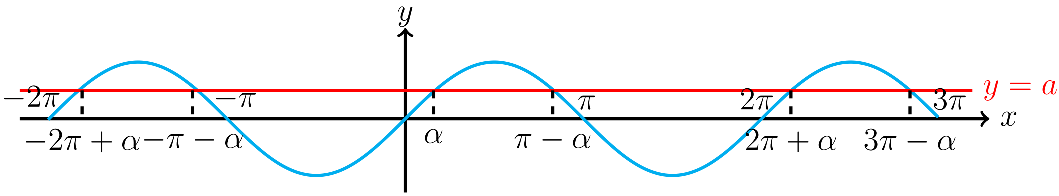
$$\Longleftrightarrow x = (2k+1)\pi - \alpha \qquad \text{ឬ} \quad x = 2k\pi + \alpha \ , \ k \ \epsilon \ Z$$

$\Longleftrightarrow x =$ (ពហុគុណសេសនៃ π)$-\alpha$ ឬ $x =$ (ពហុគុណគូរនៃ π)$+\alpha$

$\Longleftrightarrow x = k\pi + (-1)^k\alpha \ , \ k \epsilon Z$

ដូចនេះ $\sin x = \sin\alpha \Longleftrightarrow x = k\pi + (-1)^k\alpha, \ k \epsilon Z \ , \ \alpha \ \epsilon \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ។

○ **ក្រាបអនុគមន៍** $y = \sin x \ , \ y = a$



. តាមក្រាបសមីការមានសំណុំចម្លើយគឺ៖

$x = \alpha \ , \ 2\pi + \alpha \ , \ 4\pi + \alpha \ , ... = 2k\pi + \alpha$

និង $x = \pi - \alpha \ , \ 3\pi - \alpha \ , \ 5\pi - \alpha \ , ... = (2k+1)\pi - \alpha$

ដូចនេះ (x ជាពហុគុណគូរនៃ π)$+\alpha$ ឬ (ជាពហុគុណសេសនៃ π)$-\alpha$

យើងអាចបង្រួមចម្លើយទាំងពីរដោយឲ្យចម្លើយទូទៅកំណត់ដោយ $x = k\pi + (-1)^k\alpha, \ k \epsilon Z$ ។

លំហាត់គំរូទី៣

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\sin x = \frac{1}{2}$

ខ . $\frac{1}{2}\sin\frac{x}{2} - \frac{\sqrt{3}}{2}\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

គ . $\sin\left(2x + \frac{\pi}{4}\right) = 1$

ឃ . $\sin 2x = \sin\left(\frac{\pi}{3} - x\right)$

ង . $\sin 2x = \sqrt{2}\cos x$

ច . $\sin x \cos^2 x = 1$

ដំណោះស្រាយ.

ក . $\sin x = \frac{1}{2}$

សមីការអាសរសេរ $\sin x = \sin\frac{\pi}{6}$

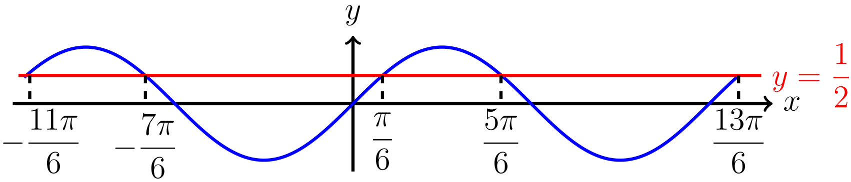
$\Longleftrightarrow x = \frac{\pi}{6} + 2k\pi$ ឬ $x = \pi - \frac{\pi}{6} + 2k\pi$

$\Longleftrightarrow x = \frac{\pi}{6} + 2k\pi$ ឬ $x = \frac{5\pi}{6} + 2k\pi \ , \ k \epsilon Z$

ម្យ៉ាងទៀតសមីការ $\sin x = \sin\frac{\pi}{6}$

យើងអាច យកចម្លើយបង្រួមទូទៅគឺ $x = k\pi + (-1)^k \frac{\pi}{6}\ ,\ k \in Z$

☻ ដូចនេះ $x = k\pi + (-1)^k \frac{\pi}{6}\ ,\ k \in Z$ ។



ខ . $\frac{1}{2}\sin\frac{x}{2} - \frac{\sqrt{3}}{2}\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

$\Longleftrightarrow \sin\frac{x}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{x}{2} = \sin\frac{\pi}{4}$

$\Longleftrightarrow \sin\left(\frac{x}{2} - \frac{\pi}{3}\right) = \sin\frac{\pi}{4}$

$\Longleftrightarrow \frac{x}{2} - \frac{\pi}{3} = k\pi + (-1)^k\frac{\pi}{4} \Longleftrightarrow \frac{x}{2} = k\pi + \frac{\pi}{3} + (-1)^k\frac{\pi}{4}$

$\Longrightarrow x = 2k\pi + \frac{2\pi}{3} + (-1)^k\frac{\pi}{2} = (3k+1)\frac{2\pi}{3} + (-1)^k\frac{\pi}{2}\ ,\ k \in Z$ ។

☻ ដូចនេះ $x = (3k+1)\frac{2\pi}{3} + (-1)^k\frac{\pi}{2}\ ,\ k \in Z$ ។

គ . $\sin\left(2x + \frac{\pi}{4}\right) = 1$

$\Longleftrightarrow 2x + \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi \Longleftrightarrow 2x = \frac{\pi}{2} - \frac{\pi}{4} + 2k\pi$

$\Longleftrightarrow 2x = \frac{\pi}{4} + 2k\pi \Longleftrightarrow x = \frac{\pi}{8} + k\pi\ ,\ k \in Z$

☻ ដូចនេះ $x = \frac{\pi}{8} + k\pi\ ,\ k \in Z$ ។

ឃ . $\sin 2x = \sin\left(\frac{\pi}{3} - x\right)$

$\Longleftrightarrow \frac{\pi}{3} - x = k\pi + (-1)^k 2x \Longleftrightarrow (-1)^k 2x + x = \frac{\pi}{3} - k\pi$

$\Longleftrightarrow x\left[2(-1)^k + 1\right] = \frac{\pi}{3} - k\pi \Longleftrightarrow x = \dfrac{\frac{\pi}{3} - k\pi}{2(-1)^k + 1}\ ,\ k \in Z$

☻ ដូចនេះ $x = \dfrac{\frac{\pi}{3} - k\pi}{2(-1)^k + 1}\ ,\ k \in Z$ ។

ង . $\sin 2x = \sqrt{2}\cos x$

$\Longleftrightarrow 2\sin x\cos x - \sqrt{2}\cos x = 0 \Longleftrightarrow \sqrt{2}\cos x(\sqrt{2}\sin x - 1) = 0$

$$\Longleftrightarrow \cos x = 0 \text{ ឬ } (\sqrt{2}\sin x - 1) = 0 \Longleftrightarrow \sin x = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}$$

$$\Longleftrightarrow x = (2k+1)\frac{\pi}{2} \quad \text{ឬ } x = k\pi + (-1)^k\frac{\pi}{4}\ ,\ k \in Z$$

☻ ដូចនេះ $x = (2k+1)\frac{\pi}{2}$ ឬ $x = k\pi + (-1)^k\frac{\pi}{4}\ ,\ k \in Z$ ។

ច . $\sin x\cos^2 x = 1$

$$\Longleftrightarrow \sin x(1-\sin^2 x) = 1 \Longleftrightarrow 2\sin^3 x - \sin x + 1 = 0$$

$$\Longleftrightarrow (\sin x + 1)(2\sin^2 x - 2\sin x + 1) = 0$$

$$\Longleftrightarrow \sin x + 1 = 0 \text{ ឬ } 2\sin^2 x - 2\sin x + 1 = 0$$

$$\Longleftrightarrow \sin x = -1 \quad \text{ឬ } 2\left[\left(\sin x - \frac{1}{2}\right)^2 + \frac{1}{4}\right] \neq 0$$

$$\Longleftrightarrow x = -\frac{\pi}{2} + 2k\pi,\ k \in Z$$

☻ ដូចនេះ $x = -\frac{\pi}{2} + 2k\pi,\ k \in Z$ ។

លំហាត់គំរូទី៤

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\sin 2x = (\cos x - \sin x)^2$

ខ . $\cos^2 x - \sin x = -1$

គ . $\sin 3x + 5\sin x = 0$

ឃ . $\sin 2x + \sin 4x + \sin 6x = 0$

ដំណោះស្រាយ.

ក . $\sin 2x = (\cos x - \sin x)^2$

$$\Longleftrightarrow \sin 2x = \cos^2 x - 2\sin x\cos x + \sin^2 x$$

$$\Longleftrightarrow 2\sin 2x = 1 \quad \therefore\ \sin^2 x + \cos^2 x = 1\ ,\ 2\sin x\cos x = \sin 2x$$

$$\Longleftrightarrow \sin 2x = \frac{1}{2} = \sin\frac{\pi}{6} \Longleftrightarrow 2x = k\pi + (-1)^k\frac{\pi}{6}$$

$$\Longrightarrow x = \frac{1}{2}\left[k\pi + (-1)^k\frac{\pi}{6}\right]$$

☻ ដូចនេះ $x = \frac{1}{2}\left[k\pi + (-1)^k\frac{\pi}{6}\right]\ ,\ k \in Z$ ។

ខ . $\cos^2 x - \sin x = -1$

$\Longleftrightarrow 1-\sin^2 x-\sin x+1=0 \Longleftrightarrow \sin^2 x+\sin x-2=0$

$\Longleftrightarrow (\sin x-1)(\sin x+2)=0$ ដោយ $\sin x+2\neq 0$

$\Longleftrightarrow \sin x-1=0 \Longleftrightarrow \sin x=1 \Longrightarrow x=\frac{\pi}{2}+2k\pi \;, k\in Z$

☻ ដូចនេះ $x=\frac{\pi}{2}+2k\pi \;, k\in Z$ ។

គ . $\sin 3x+5\sin x=0$

$\Longleftrightarrow 3\sin x-4\sin^3 x+5\sin x=0 \Longleftrightarrow 8\sin x-4\sin^3 x=0$

$\Longleftrightarrow 4\sin x(2-\sin^2 x)=0$ ដោយ $2-\sin^2 x\neq 0$

$\Longleftrightarrow \sin x=0 \Longleftrightarrow x=k\pi \;, k\in Z$

☻ ដូចនេះ $x=k\pi \;, k\in Z$ ។

ឃ . $\sin 2x+\sin 4x+\sin 6x=0$

$\Longleftrightarrow \sin 6x+\sin 2x+\sin 4x=0 \quad \Longleftrightarrow 2\sin 4x\cos 2x+\sin 4x=0$

$\Longleftrightarrow \sin 4x(2\cos 2x+1)=0 \quad \Longleftrightarrow \sin 4x=0$ ឬ $2\cos 2x+1=0$

$\Longleftrightarrow 4x=k\pi$ ឬ $\cos 2x=-\frac{1}{2} \quad \Longleftrightarrow 2x=\pm\frac{2\pi}{3}+2k\pi$

$\Longrightarrow x=\frac{1}{4}k\pi$ ឬ $x=\pm\frac{\pi}{3}+k\pi \;, k\in Z$

☻ ដូចនេះ $x=\frac{1}{4}k\pi$ ឬ $x=\pm\frac{\pi}{3}+k\pi \;, k\in Z$ ។

២.៣ សមីការ $\tan x=a$, a ជាចំនួនពិត

▲ ទ្រឹស្តីបទទី ៥ $\tan x=0 \Longleftrightarrow x=k\pi \;, k\in Z$

ស្រាយបញ្ជាក់. យើងដឹងថា $\tan x=0$ នៅពេល x ជាពហុគុណចំនួននៃ π

$\because \tan x=0 \Longleftrightarrow x=\pm\pi \;, \pm 2\pi \;, \pm 3\pi \;, ...$

$\Longleftrightarrow x=k\pi \;, k\in Z$

$\therefore \tan x=0 \Longleftrightarrow x=k\pi \;, k\in Z$ ។

▲ ទ្រឹស្តីបទទី ៦ តាង a ជាចំនួនពិតនិងមានមុំ α ដែលជាចម្លើយជាក់លាក់នៃសមីការ $\tan x=a$ យើងបានដំណោះស្រាយទូទៅត្រូវបានកំណត់ដោយ $\tan x=\tan\alpha \Longleftrightarrow x=\alpha+k\pi \;, \alpha\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

ស្រាយបញ្ជាក់. យើងមាន $\tan x = \tan\alpha \ , \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

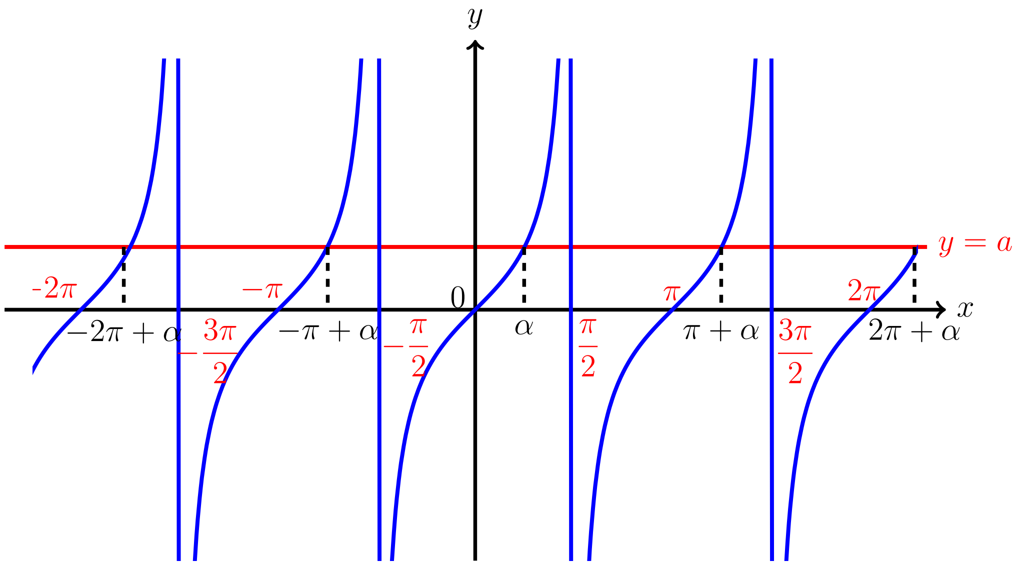
$\iff \frac{\sin x}{\cos x} = \frac{\sin\alpha}{\cos\alpha} \ , \ \cos x \neq 0$

$\iff \sin x\cos\alpha - \sin\alpha\cos x = 0 \iff \sin(x-\alpha) = 0$

$\iff x - \alpha = k\pi \iff x = \alpha + k\pi \ , \ k \in Z$

☻ ដូចនេះ $\tan x = \tan\alpha \iff x = \alpha + k\pi \ , \ k \in Z \ , \ \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ។

○ **ក្រាបអនុគមន៍** $y = \tan x \ , \ y = a$



. វាក៏ច្បាស់ផងដែរពីដំណោះស្រាយតាមក្រាបសមីការមានចម្លើយគឺ៖ $x = \alpha + k\pi \ , \ k \in Z$ ស្រដៀងគ្នាដែរសម្រាប់សមីការ $\cot x = a \iff \cot x = \cot\alpha \implies x = \alpha + k\pi \ , \ k \in Z$ ។ ឬយើងអាចប្រែក្លាយសមីការ $\cot x = a$ ទៅជា $\tan x = \frac{1}{a} \ , a \neq 0$ ដែលអាចដោះស្រាយបាន ។

លំហាត់គំរូទី៨

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\tan x = 1$

ខ . $\tan 3x = \frac{\sqrt{3}}{3}$

គ . $\frac{2\tan x}{1-\tan^2 x} = \sqrt{3}$

ឃ . $\cot\left(\frac{x}{2} - 3\right) = -1$

ង . $\tan^2 x - (1+\sqrt{3})\tan x + \sqrt{3} = 0$

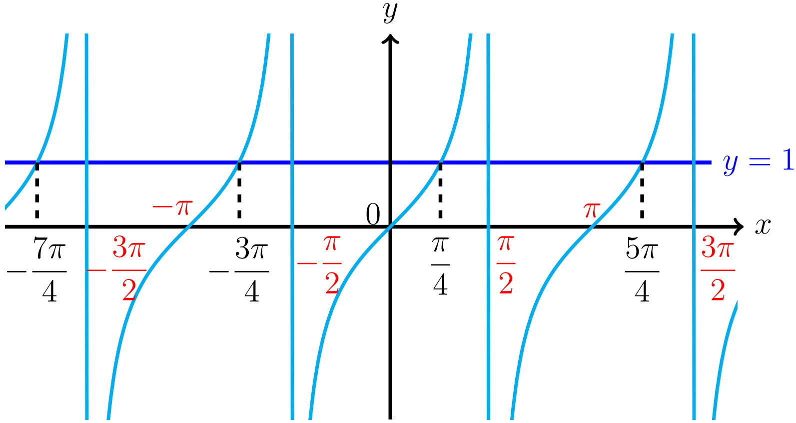
ច . $\tan\left(\frac{\pi}{4} - x\right) + \tan\left(\frac{\pi}{4}\right) = 4$

ដំណោះស្រាយ.

ក . $\tan x = 1$

$\Longleftrightarrow \tan x = \tan\dfrac{\pi}{4} \Longrightarrow x = \dfrac{\pi}{4} + k\pi \ \ , k \epsilon Z$

☻ ដូចនេះ $x = \dfrac{\pi}{4} + k\pi \ \ , k \epsilon Z$ ។



ខ . $\tan 3x = \dfrac{\sqrt{3}}{3}$

$\Longleftrightarrow \tan 3x = \tan\dfrac{\pi}{6} \Longleftrightarrow 3x = \dfrac{\pi}{6} + k\pi$

$\Longrightarrow x = \dfrac{\pi}{18} + \dfrac{1}{3}k\pi \ , k \epsilon Z$

☻ ដូចនេះ $x = \dfrac{\pi}{18} + \dfrac{1}{3}k\pi \ , k \epsilon Z$ ។

គ . $\dfrac{2\tan x}{1-\tan^2 x} = \sqrt{3}$

$\Longleftrightarrow \tan 2x = \tan\dfrac{\pi}{3} \Longleftrightarrow 2x = \dfrac{\pi}{3} + k\pi \Longrightarrow x = \dfrac{\pi}{6} + \dfrac{1}{2}k\pi \ , k \epsilon Z$

☻ ដូចនេះ $x = \dfrac{\pi}{6} + \dfrac{1}{2}k\pi \ , k \epsilon Z$ ។

ឃ . $\cot\left(\dfrac{x}{2} - 3\pi\right) = -1$

$\Longleftrightarrow \cot\left(\dfrac{x}{2} - 3\pi\right) = \cot\left(-\dfrac{\pi}{4}\right) \Longleftrightarrow \dfrac{x}{2} - 3\pi = -\dfrac{\pi}{4} + k\pi$

$\Longleftrightarrow \dfrac{x}{2} = -\dfrac{\pi}{4} + 3\pi + k\pi \Longleftrightarrow \dfrac{x}{2} = \dfrac{11\pi}{4} + k\pi$

$$\Longrightarrow x = \frac{11\pi}{2} + 2k\pi \ , \ k \in Z$$

☻ ដូចនេះ $x = \frac{11\pi}{2} + 2k\pi \ , \ k \in Z$ ។

ង . $\tan^2 x - (1+\sqrt{3})\tan x + \sqrt{3} = 0$

$$\Longleftrightarrow \tan^2 x - \tan x - \sqrt{3}\tan x + \sqrt{3} = 0$$

$$\Longleftrightarrow \tan x(\tan x - 1) - \sqrt{3}(\tan x - 1) = 0$$

$$\Longleftrightarrow (\tan x - 1)(\tan x - \sqrt{3}) = 0$$

$$\Longleftrightarrow \tan x - 1 = 0 \text{ ឬ } \tan x - \sqrt{3} = 0$$

$$\Longleftrightarrow \tan x = 1 \text{ ឬ } \tan x = \sqrt{3}$$

$$\Longleftrightarrow \tan x = \tan\frac{\pi}{4} \text{ ឬ } \tan x = \tan\frac{\pi}{3}$$

$$\Longrightarrow x = \frac{\pi}{4} + k\pi \text{ ឬ } x = \frac{\pi}{3} + k\pi \ , \ k \in Z$$

☻ ដូចនេះ $x = \frac{\pi}{4} + k\pi$ ឬ $x = \frac{\pi}{3} + k\pi \ , \ k \in Z$ ។

ច . $\tan\left(\frac{\pi}{4} - x\right) + \tan\left(\frac{\pi}{4} + x\right) = 4$

$$\Longleftrightarrow \frac{1 - \tan x}{1 + \tan x} + \frac{1 + \tan x}{1 - \tan x} = 4$$

$$\Longleftrightarrow \frac{(1 + \tan x)^2 + (1 - \tan x)^2}{1 - \tan^2 x} = 4$$

$$\Longleftrightarrow 2(1 + \tan^2 x) = 4(1 - \tan^2 x) \Longleftrightarrow 3\tan^2 x = 1$$

$$\Longleftrightarrow \tan^2 x = \frac{1}{3} \Longrightarrow \tan x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$$

យើងបាន $\tan x = \frac{\sqrt{3}}{3} = \tan\frac{\pi}{6}$ ឬ $\tan x = -\frac{\sqrt{3}}{3} = \tan\left(-\frac{\pi}{6}\right)$

នាំឲ្យ $x = \frac{\pi}{6} + k\pi$ ឬ $x = -\frac{\pi}{6} + k\pi \ , \ k \in Z$

☻ ដូចនេះ $x = \pm\frac{\pi}{6} + k\pi \ , \ k \in Z$ ។

លំហាត់គំរូទី៦

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\tan(x - 30^0)\tan 50^0 = 0$

គ . $\tan 5x + \cot 2x = 0$

ខ . $\tan x + \tan 2x + \tan x \tan 2x = 1$

ឃ . $\tan(x - 15^0) = 3\tan(x + 15^0)$

ដំណោះស្រាយ.

ក . $\tan(x-30^0)\tan 50^0 = 0$

$\Longleftrightarrow \tan(x-30^0)=0 \Longleftrightarrow x-30^0 = k\pi$

$\Longrightarrow x = 30^0 + k\pi = \frac{\pi}{6}+k\pi$

ដូចនេះ $x=\frac{\pi}{6}+k\pi\ ,\ k \in Z$

ខ . $\tan x+\tan 2x+\tan x\tan 2x = 1$

សមីការអាចសរសេរ $\tan x+\tan 2x = 1-\tan x\tan 2x$

$\Longleftrightarrow \frac{\tan x+\tan 2x}{1-\tan x\tan 2x}=1 \Longleftrightarrow \tan 3x = 1=\tan\frac{\pi}{4}$

$\Longleftrightarrow 3x=\frac{\pi}{4}+k\pi \Longrightarrow x=\frac{\pi}{12}+\frac{k\pi}{3}\ ,\ k\in Z$

ដូចនេះ $x=\frac{\pi}{12}+\frac{k\pi}{3}\ ,\ k\in Z$ ។

គ . $\tan 5x+\cot 2x=0$

សមីការអាចសរសេរ $\frac{\sin 5x}{\cos 5x}+\frac{\cos 2x}{\sin 2x}=0$

$\Longleftrightarrow \frac{\sin 5x\sin 2x+\cos 5x\cos 2x}{\cos 5x\sin 2x}=0$

$\Longleftrightarrow \frac{\cos(5x-2x)}{\cos 5x\sin 2x}=0$ លក្ខខណ្ឌ $(\cos 5x\neq 0\ ,\ \sin 2x\neq 0)$

យើងបាន $\cos 3x = 0\Longleftrightarrow 3x=\frac{\pi}{2}+k\pi$

$\Longrightarrow x=\frac{\pi}{6}+\frac{k\pi}{3}=(2k+1)\frac{\pi}{6}\ k\in Z$

ដូចនេះ $x=(2k+1)\frac{\pi}{6}\ ,k\in Z$ ។

ឃ . $\tan(x-15^o)=3\tan(x+15^o)$

សមីការអាចសរសេរ $\frac{\tan(x-15^o)}{\tan(x+15^o)}=\frac{3}{1}$

$\Longleftrightarrow \frac{\tan(x-15^o)+\tan(x+15^o)}{\tan(x-15^o)-\tan(x+15^0)}=\frac{3+1}{3-1}$

$\therefore\ \frac{A}{B}=\frac{C}{D}\Longleftrightarrow\frac{A+B}{A-B}=\frac{C+D}{C-D}$

$$\Longleftrightarrow \frac{\dfrac{\sin(x-15^o)}{\cos(x-15^o)}+\dfrac{\sin(x+15^o)}{\cos(x+15^o)}}{\dfrac{\sin(x-15^o)}{\cos(x-15^o)}-\dfrac{\sin(x+15^o)}{\cos(x+15^o)}}=2$$

$$\Longleftrightarrow \frac{\sin(x-15^o)\cos(x+15^o)+\sin(x+15^o)\cos(x-15^o)}{\sin(x-15^o)\cos(x+15^o)-\sin(x+15^o)\cos(x-15^0)}=2$$

$$\Longleftrightarrow \frac{\sin(x-15^o+x+15^o)}{\sin(x-15^o-x-15^o)}=2$$

$$\Longleftrightarrow \frac{\sin 2x}{\sin(-30^o)}=2 \Longleftrightarrow \sin 2x=-1$$

$$2x=-\frac{\pi}{2}+2k\pi \Longrightarrow x=-\frac{\pi}{4}+k\pi \ \ ,k\in Z$$

☻ ដូចនេះ $x=-\dfrac{\pi}{4}+k\pi \ ,k\in Z$ ។

២.៤ សមីការ $\sin^2 x=a$, $\cos^2 x=a$, $\tan^2 x=a$, a ជាចំនួនពិត

. សមីការ $\sin^2 x=\sin^2\alpha$, $\cos^2 x=\cos^2\alpha$, $\tan^2 x=\tan^2\alpha$

$\Longrightarrow x=\pm\alpha+k\pi$, $k\in Z$ ។ (មានចម្លើយទូទៅដូចគ្នា)

កំណត់សម្គាល់

យើងមានរូបមន្តខាងក្រោម៖

ក . $\sin 2x=\dfrac{2\tan x}{1+\tan^2 x}$ ខ . $\cos 2x=\dfrac{1-\tan^2 x}{1+\tan^2 x}$ គ . $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$

ស្រាយបញ្ជាក់.

○ យើងមាន $\cos^2 x=\cos^2\alpha$

$$\Longleftrightarrow \frac{1+\cos 2x}{2}=\frac{1+\cos 2\alpha}{2}\Longleftrightarrow \cos 2x=\cos 2\alpha$$

$$\Longleftrightarrow 2x=\pm 2\alpha+2k\pi \Longrightarrow x=\pm\alpha+k\pi \ \ ,\ k\in\ Z$$

☻ ដូចនេះ $x=\pm\alpha+k\pi \ ,\ k\in\ Z$

○ យើងមាន $\sin^2 x=\sin^2\alpha$

$$\Longleftrightarrow \frac{1-\cos 2x}{2}=\frac{1-\cos 2\alpha}{2}\Longleftrightarrow \cos 2x=\cos 2\alpha$$

$$\Longleftrightarrow 2x=\pm 2\alpha+2k\pi \Longrightarrow x=\pm\alpha+k\pi \ \ ,\ k\in\ Z$$

☻ ដូចនេះ $x=\pm\alpha+k\pi \ ,\ k\in\ Z$

○ យើងមាន $\tan^2 x = \tan^2 \alpha$

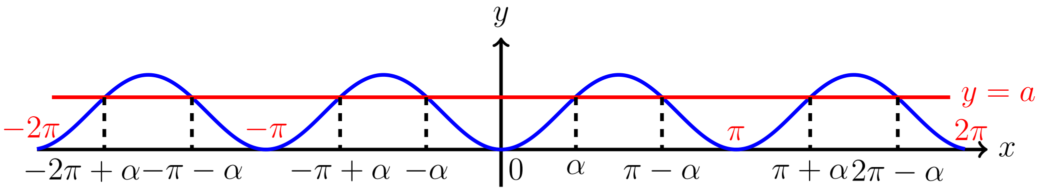
$\Longleftrightarrow \dfrac{1-\tan^2 x}{1+\tan^2 x} = \dfrac{1-\tan^2 \alpha}{1+\tan^2 \alpha}$ តាមរូបមន្ត (ខ) យើងបាន

$\Longleftrightarrow \cos 2x = \cos 2\alpha \Longleftrightarrow 2x = \pm 2\alpha + 2k\pi$

$\Longrightarrow x = \pm\alpha + k\pi \ , \ k \in Z$

☻ ដូចនេះ $x = \pm\alpha + k\pi \ , \ k \in Z$

○ **ក្រាបអនុគមន៍** $y = \sin^2 x \ , \ y = a$



លំហាត់គំរូទី៧

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\sin^2 x = \dfrac{1}{2}$

ខ . $\cos^2 5x = 0$

គ . $4\tan^2 x = 12$

ឃ . $\sin^4 x - \cos^4 x = 0$

ង . $7\cos^2 x + 3\sin^2 x = 4$

ច . $\tan^2 x + \cot^2 x = 2$

ដំណោះស្រាយ.

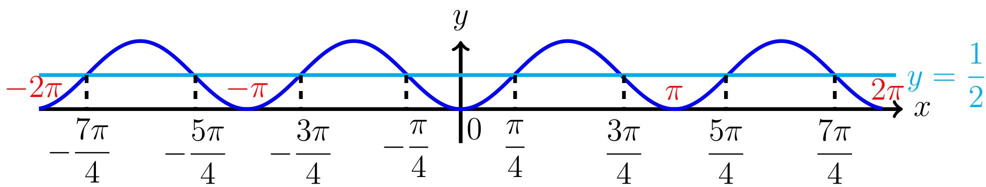
ក . $\sin^2 x = \dfrac{1}{2}$

សមីការអាចសរសេរ $\sin^2 x = \left(\dfrac{\sqrt{2}}{2}\right)^2$

$\Longleftrightarrow \sin^2 x = \left(\sin\dfrac{\pi}{4}\right)^2 = \sin^2\dfrac{\pi}{4}$

$\Longrightarrow x = \pm\dfrac{\pi}{4} + k\pi \ , \ k \in Z$

☻ ដូចនេះ $x = \pm\dfrac{\pi}{4} + k\pi \ , \ k \in Z$ ។



ខ . $\cos^2 5x = 0$

$\iff \cos^2 5x = \cos^2 \frac{\pi}{2} \iff 5x = \pm\frac{\pi}{2} + k\pi$

$\implies x = \pm\frac{\pi}{10} + \frac{k\pi}{5} \quad , \; k \in Z$

☻ ដូចនេះ $x = \pm\frac{\pi}{10} + \frac{k\pi}{5} \quad , \; k \in Z$ ។

គ . $4\tan^2 x = 12$

សមីការអាចសរសេរ $\tan^2 x = 3 = \left(\sqrt{3}\right)^2$

$\tan^2 x = \tan^2 \frac{\pi}{3} \implies x = \pm\frac{\pi}{3} + k\pi \quad , \; k \in Z$

☻ ដូចនេះ $x = \pm\frac{\pi}{3} + k\pi \quad , \; k \in Z$ ។

ឃ . $\sin^4 x - \cos^4 x = 0$

អាចសរសេរ $\left(\sin^2 x\right)^2 - \left(\cos^2 x\right)^2 = 0$

$\iff (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 0$

$\iff \sin^2 x - \cos^2 x = 0 \quad \therefore \; \sin^2 x + \cos^2 x = 1$

$\iff \sin^2 x = \cos^2 x = \sin^2\left(\frac{\pi}{2} - x\right)$

$\iff x = \pm\left(\frac{\pi}{2} - x\right) + k\pi$

$\iff x = \frac{\pi}{2} - x + k\pi$ ឬ $x = -\frac{\pi}{2} + x + k\pi$ មិនពិត

$\iff 2x = \frac{\pi}{2} + k\pi \qquad \implies x = \frac{\pi}{4} + \frac{k\pi}{2} \quad , \; k \in Z$

☻ ដូចនេះ $x = \frac{\pi}{4} + \frac{k\pi}{2} \quad , \; k \in Z$ ។

ង . $7\cos^2 x + 3\sin^2 x = 4$

$\iff 7\cos^2 x + 3(1 - \cos^2 x) = 4 \iff 4\cos^2 x = 1$

$\iff \cos^2 x = \frac{1}{4} = \left(\frac{1}{2}\right)^2 = \cos^2 \frac{\pi}{3}$

$\implies x = \pm\frac{\pi}{3} + k\pi \; , \; k \in Z$

☻ ដូចនេះ $x = \pm\frac{\pi}{3} + k\pi \quad , \; k \in Z$ ។

ច . $\tan^2 x + \cot^2 x = 2$

$$\Longleftrightarrow \tan^2 x + \frac{1}{\tan^2 x} = 2 \Longleftrightarrow \tan^4 x - 2\tan^2 x + 1 = 0$$

$$\Longleftrightarrow \left(\tan^2 x\right)^2 - 2\tan^2 x + 1 = 0$$

$$\Longleftrightarrow (\tan^2 x - 1)^2 = 0 \Longleftrightarrow \tan^2 = 1 = \tan^2 \frac{\pi}{4}$$

$$\Longrightarrow x = \pm\frac{\pi}{4} + k\pi \ \ , \ k \epsilon Z$$

☻ ដូចនេះ $x = \pm\frac{\pi}{4} + k\pi \ , \ k \epsilon Z$ ។

លំហាត់គំរូទី៨

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\sin^2 x = 9$

ខ . $\cos 2x = \cos^2 x$

គ . $2\sin^2 x + \sin^2 2x = 2$

ឃ . $\tan x + \tan 2x + \tan 3x = 0$

ដំណោះស្រាយ.

ក . $\sin^2 x = 9$

$$\Longleftrightarrow \sin^2 x = 3^2 \Longrightarrow x = \pm arc\sin 3 + k\pi$$

☻ ដូចនេះ $x = \pm arc\sin 3 + k\pi \ , \ k \epsilon z$ ។

ខ . $\cos 2x = \cos^2 x$

$$\Longleftrightarrow 2\cos^2 x - 1 = \cos^2 x \Longleftrightarrow \cos^2 x = 1 = \cos^2 0$$

$$\Longleftrightarrow x = k\pi \ \ , \ k \epsilon Z$$

☻ ដូចនេះ $x = k\pi \ \ , \ k \epsilon Z$ ។

គ . $2\sin^2 x + \sin^2 2x = 2$

$$\therefore \ (\sin 2x)^2 = (2\sin x\cos x)^2 = 4\sin^2 x\cos^2 x$$

$$\Longleftrightarrow 2\sin^2 x + 4\sin^2 x\cos^2 x = 2$$

$$\Longleftrightarrow \sin^2 x + 2\sin^2 x\cos^2 x = 1$$

$$\Longleftrightarrow 2\sin^2 x\cos^2 x = 1 - \sin^2 x = \cos^2 x$$

$\iff 2\sin^2 x\cos^2 x - \cos^2 x = 0$

$\iff \cos^2 x(2\sin^2 x - 1) = 0$

$\iff \cos^2 x = 0$ ឬ $2\sin^2 x - 1 = 0 \iff \sin^2 x = \dfrac{1}{2}$

$\implies x = \pm\dfrac{\pi}{2} + k\pi$ ឬ $\sin^2 x = \left(\dfrac{\sqrt{2}}{2}\right)^2 = \sin^2\dfrac{\pi}{4} \implies x = \pm\dfrac{\pi}{4} + k\pi$

☻ ដូចនេះ $x = \pm\dfrac{\pi}{2} + k\pi$, $x = \pm\dfrac{\pi}{4} + k\pi$, $k \in Z$ ។

ឃ . $\tan x + \tan 2x + \tan 3x = 0$

$\iff \tan x + \tan 2x + \tan(2x + x) = 0$

$\iff \tan x + \tan 2x + \dfrac{\tan 2x + \tan x}{1 - \tan x\tan 2x} = 0$

$\iff (\tan x + \tan 2x)\left(1 + \dfrac{1}{1 - \tan 2x\tan x}\right) = 0$

ករណី $\tan x + \tan 2x = 0$

$\iff \tan x + \dfrac{2\tan x}{1 - \tan^2 x} = 0 \iff \tan x\left(1 + \dfrac{2}{1 - \tan^2 x}\right) = 0$

○ $\tan x = 0 \implies x = k\pi$, $k \in Z$

○ $1 + \dfrac{2}{1 - \tan^2 x} = 0 \iff \dfrac{2}{1 - \tan^2 x} = -1$

$\iff 1 - \tan^2 x = -2$

$\iff \tan^2 x = 3 = \left(\sqrt{3}\right)^2 = \tan^2\dfrac{\pi}{3}$

$\implies x = \pm\dfrac{\pi}{3} + k\pi$, $k \in Z$

ករណី $1 + \dfrac{1}{1 - \tan 2x\tan x} = 0$

$\iff 1 - \tan 2x\tan x = -1 \iff \tan 2x\tan x = 2$

$\iff \tan x\left(\dfrac{2\tan x}{1 - \tan^2 x}\right) = 2 \iff \tan^2 x = 1 - \tan^2 x$

$\iff \tan^2 x = \dfrac{1}{2} = \left(\dfrac{\sqrt{2}}{2}\right)^2$

$\implies x = \pm arc\tan\dfrac{\sqrt{2}}{2} + k\pi$, $k \in Z$

☻ ដូចនេះ $x = k\pi$, $x = \pm\dfrac{\pi}{3} + k\pi$, $x = \pm arc\tan\dfrac{\sqrt{2}}{2} + k\pi$, $k \in Z$ ។

តារាងសមីការត្រីកោណមាត្រ

សមីការ	ចម្លើយទូទៅ	សមីការ	ចម្លើយទូទៅ
$\sin x = 0$	$x = k\pi$	$\cos x = 0$	$x = (2k+1)\frac{\pi}{2}$
$\tan x = 0$	$x = k\pi$	$\sin x = 1$	$x = \frac{\pi}{2} + 2k\pi$
$\cos x = 1$	$x = 2k\pi$	$\sin x = -1$	$x = -\frac{\pi}{2} + 2k\pi$
$\cos x = -1$	$x = (2k+1)\pi$	$\tan x = \pm 1$	$x = \pm\frac{\pi}{4} + k\pi$
$\sin x = \sin\alpha$	$x = k\pi + (-1)^k\alpha$	$\cos x = \cos\alpha$	$x = \pm\alpha + 2k\pi$
$\tan x = \tan\alpha$	$x = \alpha + k\pi$	$\sin^2 x = \sin^2\alpha$	$x = \pm\alpha + k\pi$
$\cos^2 x = \cos^2\alpha$	$x = \pm\alpha + k\pi$	$\tan^2 x = \tan^2\alpha$	$x = \pm\alpha + k\pi$

២.៩ សមីការដឺក្រេទី២ ចំពោះអនុគមន៍រង្វង់របស់ x ចំពោះ $a \neq 0$

◇ $a\sin^2 x + b\sin x + c = 0$ 1 ◇ $a\cos^2 x + b\cos x + c = 0$ 2

◇ $a\tan^2 x + b\tan x + c = 0$ 3 ◇ $a\cot^2 x + b\cot x + c = 0$ 4

ស្រាយបញ្ជាក់.

យើងតាង $t = \sin x$ ជំនួសក្នុង 1 និង $t = \cos x$ ជំនួសក្នុង 2 ដែល $-1 \leqslant t \leqslant 1$

យើងបានសមីការ $at^2 + bt + c = 0$ ដោះស្រាយសមីការរួចយកតម្លៃឬសណាដែលផ្ទៀងផ្ទាត់ ។

តាង $t = \tan x$ ជំនួសក្នុង 3 និង $t = \cot x$ ជំនួសក្នុង 4 បានសមីការ $at^2 + bt + c = 0$

លំហាត់គំរូទី៩

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $4 - \cos 2x - 7\sin x = 0$

ខ . $2\cos^2 x - 3\sqrt{2}\cos x + 2 = 0$

គ . $\tan^2\frac{x}{2} - (1-\sqrt{3})\tan\frac{x}{2} - \sqrt{3} = 0$

ឃ . $\frac{1}{\sin^2 x} = \cot x + 3$

ដំណោះស្រាយ.

ក . $4 - \cos 2x - 7\sin x = 0$

ដោយ $\cos 2x = 1 - 2\sin^2 x$

នោះយើងបាន $2\sin^2 x - 7\sin x + 3 = 0$ 1

តាង $t = \sin x$ ដែល $-1 \leqslant t \leqslant 1$

តាម 1 បានសមីការ $2t^2 - 7t + 3 = 0$

$\Delta = (-7)^2 - 4\times 2\times 3 = 49 - 24 = 25 \Longrightarrow \sqrt{\Delta} = 5$

$t_1 = \dfrac{7+5}{2\times 2} = 3$ មិនយក $t_2 = \dfrac{7-5}{2\times 2} = \dfrac{1}{2}$

យើងបាន $\sin x = \dfrac{1}{2} = \sin\dfrac{\pi}{6} \Longrightarrow x = k\pi + (-1)^k\dfrac{\pi}{6}\ ,\ k \epsilon Z$

☻ ដូចនេះ $x = k\pi + (-1)^k\dfrac{\pi}{6}\ ,\ k \epsilon Z$ ។

ខ . $2\cos^2 x - 3\sqrt{2}\cos x + 2 = 0$

តាង $t = \cos x$ ដែល $-1 \leqslant t \leqslant 1$

យើងបានសមីការ $2t^2 - 3\sqrt{2}t + 2 = 0$

$\Delta = (-3\sqrt{2})^2 - 4\times 2\times 2 = 2 \Longrightarrow \sqrt{\Delta} = \sqrt{2}$

$t_1 = \dfrac{3\sqrt{2}+\sqrt{2}}{2\times 2} = \sqrt{2}$ មិនយក $t_2 = \dfrac{3\sqrt{2}-\sqrt{2}}{2\times 2} = \dfrac{\sqrt{2}}{2}$

យើងបាន $\cos x = \dfrac{\sqrt{2}}{2} = \cos\dfrac{\pi}{4} \Longrightarrow x = \pm\dfrac{\pi}{4} + 2k\pi\ ,\ k \epsilon Z$

☻ ដូចនេះ $x = \pm\dfrac{\pi}{4} + 2k\pi\ ,\ k \epsilon Z$

គ . $\tan^2\dfrac{x}{2} - (1-\sqrt{3})\tan\dfrac{x}{2} - \sqrt{3} = 0$

តាង $t = \tan\dfrac{x}{2}$ យើងបានសមីការ $t^2 - (1-\sqrt{3})t - \sqrt{3} = 0$

$\Delta = \left[-(1-\sqrt{3})\right]^2 - 4(-\sqrt{3}) = 1 - 2\sqrt{3} + 3 + 4\sqrt{3} = 4 + 2\sqrt{3}$

$\Longrightarrow \sqrt{\Delta} = \sqrt{4+2\sqrt{3}} = \sqrt{(\sqrt{3})^2 + 2\sqrt{3} + 1^2} = \sqrt{(\sqrt{3}+1)^2} = \sqrt{3}+1$

$t_1 = \dfrac{1-\sqrt{3}+\sqrt{3}+1}{2} = 1\ ;\ t_2 = \dfrac{1-\sqrt{3}-\sqrt{3}-1}{2} = -\sqrt{3}$

∘ $\tan\dfrac{x}{2} = 1 = \tan\dfrac{\pi}{4} \Longleftrightarrow \dfrac{x}{2} = \dfrac{\pi}{4} + k\pi \Longrightarrow x = \dfrac{\pi}{2} + 2k\pi\ ,\ k \epsilon Z$

∘ $\tan\dfrac{x}{2} = -\sqrt{3} = \tan(-\dfrac{\pi}{3}) \Longleftrightarrow \dfrac{x}{2} = -\dfrac{\pi}{3} + k\pi$

$\Longrightarrow x = -\dfrac{2\pi}{3} + 2k\pi\ ,\ k \epsilon Z$

☻ ដូចនេះ $x=\dfrac{\pi}{2}+2k\pi\ ;\ x=-\dfrac{2\pi}{3}+2k\pi\ ,\ k\ \epsilon\ Z$ ។

ឃ . $\dfrac{1}{\sin^2 x}=\cot x+3$

ដោយ $\dfrac{1}{\sin^2 x}=1+\cot^2 x$

នោះយើងបាន $\cot^2 x-\cot x-2=0$ **1** តាង $t=\cot x$

តាម **1** បានសមីការ $t^2-t-2=0$

$\Delta=(-1)^2-4\times(-2)=1+8=9\Longrightarrow\sqrt{\Delta}=3$

$t_1=\dfrac{1+3}{2}=2\ ,\ t_2=\dfrac{1-3}{2}=-1$

○ $\cot x=2\Longrightarrow x=arc\cot 2+k\pi\ ,\ k\ \epsilon\ Z$

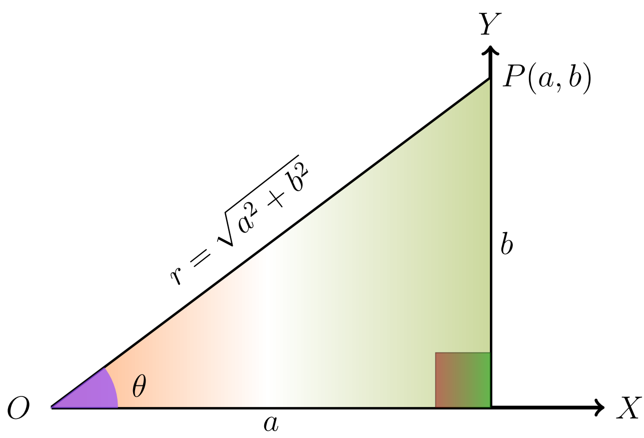
○ $\cot x=-1=\cot(-\dfrac{\pi}{4})$

$\Longrightarrow x=-\dfrac{\pi}{4}+k\pi\ ,\ k\ \epsilon\ Z$

☻ ដូចនេះ $x=arc\cot 2+k\pi\ ,\ x=-\dfrac{\pi}{4}+k\pi\ ,\ k\ \epsilon\ Z$ ។

២.៦ សមីការរាង $a\cos x+b\sin x=c$

. គេដឹងថាមានគូចម្លើយ (a,b) ជាចំនួនពិតដែលមិនសូន្យដែលមាន a ជាធាតុត្រូវគ្នានឹងចំនួនពិត r និង θ ដែល $r>0$។ មើលចំណុច P ដែលមានកូអរដោនេ $P(a,b)$ ក្នុងប្លង់ (xy) ហើយ P ជាចំណុចចល័តលើអ័ក្ស ។ ភ្ជាប់ចំណុច O ទៅចំណុច P ដែលជាប្រវែង r និងមានរង្វាស់ $\angle XOP=\theta$ ដែល θ ជាមុំដែលមានទិសដៅវិជ្ជមាន ។ ដូចរូប **03**



03

. ពីនិយមន័យខាងលើគេមាន $\sin\theta$ និង $\cos\theta$ ដែល $\cos\theta = \frac{a}{r}$ និង $\sin\theta = \frac{b}{r}$

នោះ $a = r\cos\theta$, $b = r\sin\theta$ ។ ☻ ដូចនេះ $r = \sqrt{a^2+b^2}$ និង $\theta = arc\tan\frac{b}{a}$ ។

◇ មុំ θ ត្រូវផ្ទៀងផ្ទាត់ $r\cos\theta = a$, $r\sin\theta = b$ និងស្ថិតលើចំណុច $P(a,b)$ ។

នោះយើងបានសមីការ $r\cos\theta\cos x + r\sin\theta\sin x = c \Longleftrightarrow r\cos(x-\theta) = c$

$\Longrightarrow \cos(x-\theta) = \frac{c}{r}$ សម្រាប់សមីការនេះដោះស្រាយបានលុះត្រាតែ $-1 \leqslant \frac{c}{r} \leqslant 1$

$\Longleftrightarrow -\sqrt{a^2+b^2} \leqslant c \leqslant \sqrt{a^2+b^2}$ បើ $\cos\alpha = \frac{c}{r}$ យើងបានចម្លើយទូទៅផ្តល់ឲ្យ

$x-\theta = 2k\pi \pm \alpha \Longrightarrow x = 2k\pi + \theta \pm \alpha$, ϵZ ។

◇ ហេតុនេះដើម្បីឲ្យសមីការ $a\cos x + b\sin x = c$ មានចម្លើយលុះត្រាតែ $|c| \leqslant \sqrt{a^2+b^2}$

វិធីសាស្ត្រងាយចែកសមីការនេះនឹង $\sqrt{a^2+b^2}$ យើងបាន $\frac{a}{\sqrt{a^2+b^2}}$ និង $\frac{b}{\sqrt{a^2+b^2}}$

ពីមុខ $\cos\theta$ និង $\sin\theta$ ។

♦ ដើម្បីដោះស្រាយសមីការដែលមានរៀង $a\cos x + b\sin x = c$ តាមជំហ៊ានខាងក្រោម៖

1 ទាញរក $a = r\cos\phi$, $b = r\sin\phi$ និង $r = \sqrt{a^2+b^2}$, $\phi = arc\tan\frac{b}{a}$

2 នោះសមីការខាងលើនឹងក្លាយជាសមីការ $r\cos\phi\cos x + r\sin\phi\sin x = c$

$\Longleftrightarrow r\cos(x-\phi) = c \Longrightarrow \cos(x-\phi) = \frac{c}{r} = \frac{c}{\sqrt{a^2+b^2}}$

▷ **ករណីទី១** បើ $|c| > \sqrt{a^2+b^2}$ នោះសមីការ $a\cos x + b\sin x = c$ មិនអាចដោះស្រាយបាន ។

▷ **ករណីទី២** បើ $|c| \leqslant \sqrt{a^2+b^2}$ នោះទាញរក $\frac{c}{\sqrt{a^2+b^2}} = \cos\alpha$ នាំឲ្យបានសមីការ

$\cos(x-\theta) = \cos\alpha \Longleftrightarrow x-\theta = \pm\alpha + 2k\pi \Longrightarrow x = \theta \pm \alpha + 2k\pi$, $k \epsilon Z$ ។

លំហាត់គំរូទី១០

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\sqrt{3}\cos x - \sin x = 1$

ខ . $\sqrt{3}\cos x + \sin x = 0$

គ . $\sqrt{3}\sin x - \cos x = \sqrt{2}$

ឃ . $\sqrt{3}\sin 2x + \cos 2x = \sqrt{2}$

ដំណោះស្រាយ.

ក . $\sqrt{3}\cos x - \sin x = 1$

ដោយ $r = \sqrt{a^2+b^2} = \sqrt{\sqrt{3}^2 + (-1)^2} = \sqrt{4} = 2 > |1| = 1$

យើងបាន $\cos\phi = \frac{a}{r} = \frac{\sqrt{3}}{2}$, និង $\sin\phi = \frac{b}{r} = \frac{1}{2} \Longrightarrow \phi = \frac{\pi}{6}$

សមីការអាចសេសេរ $2\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right) = 1$

$\Longleftrightarrow \cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Longleftrightarrow \cos\left(x + \frac{\pi}{6}\right) = \cos\frac{\pi}{3} \Longrightarrow x + \frac{\pi}{6} = \pm\frac{\pi}{3} + 2k\pi \ , \ k \in Z$

○ $x + \frac{\pi}{6} = \frac{\pi}{3} + 2k\pi \Longrightarrow x = \frac{\pi}{3} - \frac{\pi}{6} + 2k\pi = \frac{\pi}{6} + 2k\pi \ , \ k \in Z$

○ $x + \frac{\pi}{6} = -\frac{\pi}{3} + 2k\pi \Longrightarrow x = -\frac{\pi}{3} - \frac{\pi}{6} = -\frac{\pi}{2} + 2k\pi \ , \ k \in Z$

☻ ដូចនេះ $x = \frac{\pi}{6} + 2k\pi \ , \ x = -\frac{\pi}{2} + 2k\pi \ , \ k \in Z$ ។

ខ . $\sqrt{3}\cos x + \sin x = 0$

ដោយ $r = \sqrt{a^2+b^2} = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2 > 0$

យើងបាន $\cos\phi = \frac{a}{r} = \frac{\sqrt{3}}{2}$, និង $\sin\phi = \frac{b}{r} = \frac{1}{2} \Longrightarrow \phi = \frac{\pi}{6}$

សមីការអាចសេសេរ $2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) = 0$

$\Longleftrightarrow \cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x = 0$

$\Longleftrightarrow \cos\left(x - \frac{\pi}{6}\right) = 0 \Longleftrightarrow x - \frac{\pi}{6} = (2k+1)\frac{\pi}{2}$

$\Longrightarrow x = \frac{\pi}{2} + k\pi + \frac{\pi}{6} = \frac{2\pi}{3} + k\pi \ , \ k \in Z$

☻ ដូចនេះ $x = \frac{2\pi}{3} + k\pi \ , \ k \in Z$ ។

គ . $\sqrt{3}\sin x - \cos x = \sqrt{2}$

អាចសរសេរ $\cos x - \sqrt{3}\sin x = -\sqrt{2}$

ដោយ $r = \sqrt{a^2+b^2} = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2 > |-\sqrt{2}| = \sqrt{2}$

យើងបាន $\cos\phi = \frac{a}{r} = \frac{1}{2}$, និង $\sin\phi = \frac{b}{r} = \frac{\sqrt{3}}{2} \Longrightarrow \phi = \frac{\pi}{3}$

សមីការអាចសេសេរ $2\left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right) = -\sqrt{2}$

$\Longleftrightarrow \cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x = -\frac{\sqrt{2}}{2} = \cos\frac{3\pi}{4}$

$$\Longleftrightarrow \cos\left(x+\frac{\pi}{3}\right)=\cos\frac{3\pi}{4} \Longrightarrow x+\frac{\pi}{3}=\pm\frac{3\pi}{4}+2k\pi \;\;,\; k\in Z$$

○ $x+\frac{\pi}{3}=\frac{3\pi}{4}+2k\pi \Longrightarrow x=\frac{5\pi}{12}+2k\pi \;\;,\; k\in Z$

○ $x+\frac{\pi}{3}=-\frac{3\pi}{4}+2k\pi \Longrightarrow x=-\frac{13\pi}{12}+2k\pi \;\;,\; k\in Z$

☻ ដូចនេះ $x=\frac{5\pi}{12}+2k\pi \;,\; x=-\frac{13\pi}{12}+2k\pi \;\;,\; k\in Z$ ។

ឃ . $\sqrt{3}\sin 2x+\cos 2x=\sqrt{2}$

អាចសរសេរ $\cos 2x+\sqrt{3}\sin 2x=\sqrt{2}$

ដោយ $r=\sqrt{a^2+b^2}=\sqrt{1^2+(\sqrt{3})^2}=\sqrt{4}=2>\sqrt{2}$

យើងបាន $\cos\phi=\frac{a}{r}=\frac{1}{2}$, និង $\sin\phi=\frac{b}{r}=\frac{\sqrt{3}}{2} \Longrightarrow \phi=\frac{\pi}{3}$

សមីការអាចសរសេរ $2\left(\frac{1}{2}\cos 2x+\frac{\sqrt{3}}{2}\sin 2x\right)=\sqrt{2}$

$$\Longleftrightarrow \cos\frac{\pi}{3}\cos 2x+\sin\frac{\pi}{3}\sin 2x=\frac{\sqrt{2}}{2}=\cos\frac{\pi}{4}$$

$$\Longleftrightarrow \cos\left(2x-\frac{\pi}{3}\right)=\cos\frac{\pi}{4} \Longrightarrow 2x-\frac{\pi}{3}=\pm\frac{\pi}{4}+2k\pi \;\;,\; k\in Z$$

○ $2x-\frac{\pi}{3}=\frac{\pi}{4}+2k\pi \Longleftrightarrow 2x=\frac{7\pi}{12}+2k\pi \Longrightarrow x=\frac{7\pi}{24}+k\pi \;\;,\; k\in Z$

○ $2x-\frac{\pi}{3}=-\frac{\pi}{4}+2k\pi \Longleftrightarrow 2x=-\frac{\pi}{12}+2k\pi \Longrightarrow x=-\frac{\pi}{24}+k\pi \;\;,\; k\in Z$

☻ ដូចនេះ $x=\frac{7\pi}{24}+k\pi \;,\; x=-\frac{\pi}{24}+k\pi \;\;,\; k\in Z$ ។

លំហាត់គំរូទី១២

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $2\sin x-3\cos x=-\frac{\sqrt{26}}{2}$

ខ . $3\cos x+4\sin x=5$

ដំណោះស្រាយ.

ក . $2\sin x-3\cos x=-\frac{\sqrt{26}}{2}$

អាចសរសេរ $3\cos x-2\sin x=\frac{\sqrt{26}}{2}$ (1)

ដោយ $r=\sqrt{3^2+(-2)^2}=\sqrt{13}>\frac{\sqrt{26}}{2}$

យើងបាន $\cos\phi = \dfrac{a}{r} = \dfrac{3}{\sqrt{13}}$

និង $\sin\phi = \dfrac{b}{r} = \dfrac{2}{\sqrt{13}} \Longrightarrow \phi = arc\tan\dfrac{b}{a} = arc\tan\dfrac{2}{3}$

តាម $\boxed{1}$ ចែកនឹង $\sqrt{13}$ យើងទទួលបាន

$$\frac{3}{\sqrt{13}}\cos x - \frac{2}{\sqrt{13}}\sin x = \frac{\frac{26}{2}}{\sqrt{13}} = \frac{\sqrt{2}}{2}$$

$$\Longleftrightarrow \cos\phi\cos x - \sin\phi\sin x = \cos\frac{\pi}{4} \Longleftrightarrow \cos(x-\phi) = \cos\frac{\pi}{4}$$

$$\Longleftrightarrow x - \phi = \pm\frac{\pi}{4} + 2k\pi \Longrightarrow x = \pm\frac{\pi}{4} + \phi + 2k\pi \ ,\phi = arc\tan\frac{2}{3} \ , \ k \in Z$$

☻ ដូចនេះ $x = \pm\dfrac{\pi}{4} + \phi + 2k\pi \ ,\phi = arc\tan\dfrac{2}{3}\ , \ k \in Z$ ។

ខ . $3\cos x + 4\sin x = 5$ $\boxed{1}$

ដោយ $r = \sqrt{3^2 + 4^2} = 5 \geqslant 5$

យើងបាន $\cos\phi = \dfrac{a}{r} = \dfrac{3}{5}$ និង $\sin\phi = \dfrac{b}{r} = \dfrac{4}{5} \Longrightarrow \phi = arc\tan\dfrac{b}{a} = arc\tan\dfrac{4}{3}$

តាម $\boxed{1}$ ចែកនឹង 5 យើងទទួលបាន $\dfrac{3}{5}\cos x + \dfrac{4}{5}\sin x = 1$

$$\Longleftrightarrow \cos\phi\cos x + \sin\phi\sin x = 1 \Longleftrightarrow \cos(x-\phi) = 1$$

$$\Longleftrightarrow x - \phi = (2k+1)\frac{\pi}{2} \Longrightarrow x = (2k+1)\frac{\pi}{2} + \phi \ ,\phi = arc\tan\frac{4}{3} \ , \ k \in Z$$

☻ ដូចនេះ $x = (2k+1)\dfrac{\pi}{2} + \phi \ , \ \phi = arc\tan\dfrac{4}{3}\ , \ k \in Z$ ។

២.៧ សមីការរាង $a_0\sin^n x + a_1\sin^{n-1}x\cos x + a_2\sin^{n-2}\cos^2 x + ... + a_n\cos^n x$

◇ $a_0\sin^n x + a_1\sin^{n-1}x\cos x + a_2\sin^{n-2}\cos^2 x + ... + a_n\cos^n x = 0$ $\boxed{1}$

$a_0 \ , \ a_1, ..., \ a_n$ ជាចំនួនពិតនិងមានផលបូកនិទស្សន្ត $\cos x$ និង $\sin x$ តួនីមួយៗស្មើរគ្នាគឺ n ។

. វិធីសាស្ត្រដោះស្រាយ

○ ចែកសមីការ $\boxed{1}$ នឹង $\cos^n x$ ដែល $\cos^n x \neq 0$

○ សមីការនឹងកក្លាយជា $a_0\tan^n x + a_1\tan^{n-1}x + a_2\tan^{n-2}x + ... + a_n = 0$

○ តាង $t = \tan x$ ដោះស្រាយសមីការអថេរ t រួចទាញរក x

លំហាត់គំរូទី១៣

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $3\sin^2 x + 3\sin x\cos x - 6\cos^2 x = 0$

ខ . $3\sin^2 x + 2\sin x\cos x = 2$

គ . $5\sin^2 x - 7\sin x\cos x + 16\cos^2 x = 4$

ឃ . $\sin^3 x - \sin^2 x\cos x - 3\sin x\cos^2 x + 3\cos^3 x = 0$

ដំណោះស្រាយ.

ក . $3\sin^2 x + 3\sin x\cos x - 6\cos^2 x = 0$

ចែកសមីការនឹង $\cos^2 x$ បានសមីការ

$3\tan^2 x + 3\tan x - 6 = 0$ តាង $t = \tan x$

យើងបាន $3t^2 + 3t - 6 = 0 \iff t_1 = 1\ ,\ t_2 = -2$

○ $\tan x = 1 \iff \tan x = \tan\frac{\pi}{4} \Longrightarrow x = \frac{\pi}{4} + k\pi\ ,\ k \in Z$

○ $\tan x = -2 \Longrightarrow x = arc\tan(-2) + k\pi\ ,\ k \in Z$

ដូចនេះ $x = \frac{\pi}{4} + k\pi\ ,\ x = arc\tan(-2) + k\pi\ ,\ k \in Z$

ខ . $3\sin^2 x + 2\sin x\cos x = 2$

សមីការអាចសរសេរ $3\sin^2 x + 2\sin x\cos x = 2(\sin^2 x + \cos^2 x)$

$\iff \sin^2 x + 2\sin x\cos x - \cos^2 x = 0$

ចែកសមីការនឹង $\cos^2 x$ យើងបានសមីការ

$\tan^2 x + 2\tan x - 1 = 0$, តាង $t = \tan x$

យើងបាន $t^2 + 2t - 1 = 0 \iff (t-1)^2 = 0 \Longrightarrow t = 1$

○ $\tan x = 1 \iff \tan x = \tan\frac{\pi}{4} \Longrightarrow x = \frac{\pi}{4} + k\pi\ ,\ k \in Z$

ដូចនេះ $x = \frac{\pi}{4} + k\pi\ ,\ k \in Z$

គ . $5\sin^2 x - 7\sin x\cos x + 16\cos^2 x = 4$

សមីការអាចសរសេរ $5\sin^2 x - 7\sin x\cos x + 16\cos^2 x = 4(\sin^2 x + \cos^2 x)$

$\iff \sin^2 x - 7\sin x\cos x + 12\cos^2 x = 0$

ចែកសមីការនឹង $\cos^2 x$ យើងបានសមីការ

$\tan^2 x - 7\tan x + 12 = 0$, តាង $t = \tan x$

យើងបាន $t^2 - 7t + 12 = 0 \iff t_1 = 3 \ , \ t_2 = 4$

○ $\tan x = 3 \Longrightarrow x = arc\tan 3 + k\pi \ , \ k \in Z$

○ $\tan x = 4 \Longrightarrow x = arc\tan 4 + k\pi \ , \ k \in Z$

☻ ដូចនេះ $x = arc\tan 3 + k\pi \ , \ x = arc\tan 4 + k\pi \ , \ k \in Z$

ឃ . $\sin^3 x - \sin^2 x\cos x - 3\sin x\cos^2 x + 3\cos^3 x = 0$

ចែកសមីការនឹង $\cos^2 x$ បានសមីការ $\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$

$\tan^2 x(\tan x - 1) - 3(\tan x - 1) = 0 \iff (\tan x - 1)(\tan^2 x - 3) = 0$

○ $\tan x - 1 = 0 \Longrightarrow x = \frac{\pi}{4} + k\pi \ , \ k \in Z$

○ $\tan^2 x - 3 = 0 \iff \tan^2 x = 3 = \left(\sqrt{3}\right)^2 = \tan\frac{\pi}{3}$

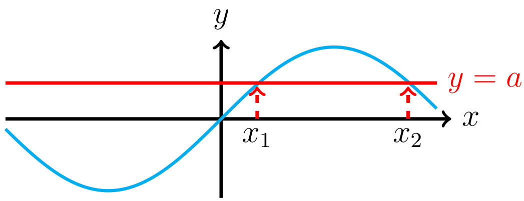
$\Longrightarrow x = \pm\frac{\pi}{3} + k\pi \ , \ k \in Z$

☻ ដូចនេះ $x = \frac{\pi}{4} + k\pi \ , \ x = \pm\frac{\pi}{3} + k\pi \ , \ k \in Z$ ។

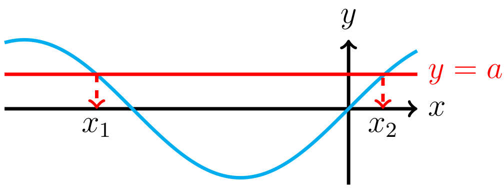
៣.វិសមីការត្រីកោណមាត្រ

◇ ឧបមាយើងត្រូវដោះស្រាយវិសមីការ $f(x) > a$ ឬ $f(x) < a$ យើងត្រូវប្រើក្រាបនៃអនុគមន៍ក្រាប $y = f(x)$ និង $y = a$ ដើម្បីសិក្សាទីតាំងធៀបរវាងក្រាបទាំងពីរទៅតាមតម្លៃ x ។

. បន្ទាប់មកដំណោះស្រាយនៃវិសមីការ $f(x) > a$ វាគឺតម្លៃ x សម្រាប់ចំណុច $(x, f(x))$ នៃក្រាប $y = f(x)$ ស្ថិតនៅខាងលើត្រង់នៃបន្ទាត់ $y = a$ ។



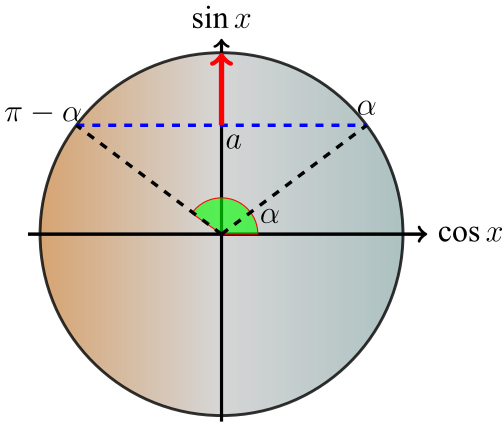
. ស្រដៀងគ្នាដែរដំណោះស្រាយវិសមីការ $f(x) < a$ វាគឺតម្លៃ x សម្រាប់ចំណុច $(x, f(x))$ នៃក្រាប $y = f(x)$ ស្ថិតនៅខាងក្រោមត្រង់នៃបន្ទាត់ $y = a$ ។



៣.១ វិសមីការរាង $\sin x > a$

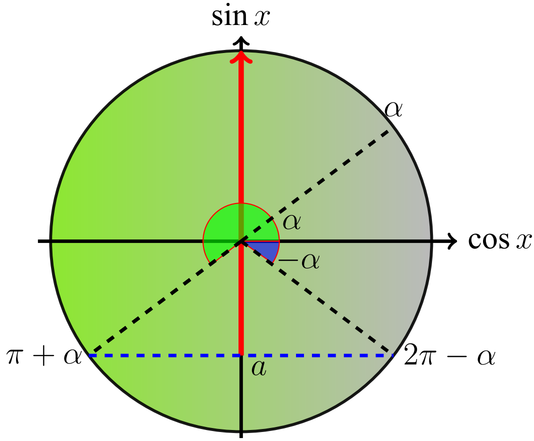
. ឧបមារកតម្លៃ x ដែលបំពេញលក្ខខណ្ឌនូវវិសមីការ $\sin x > a$, $-1 \leqslant a \leqslant 1$ ក្នុងមួយរង្វង់

♦ ករណី $0 < a < 1$



● មើលរូបខាងលើវិសមីការ $\sin x > a$ បានផ្តល់ចម្លើយតម្លៃ $\alpha < x < \pi - \alpha$ យកចម្លើយពីតូចទៅធំនិងប្រាស់ពីទ្រទិនាឡិការ ចម្លើយទូទៅ $\alpha + 2k\pi < x < \pi - \alpha + 2k\pi \ , \ k \epsilon Z$ 1

♦ ករណី $-1 < a < 0$



● មើលរូបខាងលើវិសមីការ $\sin x > a$ បានផ្តល់ចម្លើយតម្លៃ $2\pi - \alpha < x < \pi + \alpha$ តែវាជាតម្លៃពីធំទៅតូចហេតុនេះយើងប្តូរតម្លៃមុំពី $2\pi - \alpha$ ទៅ $-\alpha$ (មុំផ្ទុយគ្នា) នោះបានចម្លើយតម្លៃ $-\alpha < x < \pi + \alpha$ ចម្លើយទូទៅ $-\alpha + 2k\pi < x < \pi + \alpha + 2k\pi$ 2

តែយើងពិនិត្យមើលចម្លើយនៃវិសមីការទាំងពីរជាចម្លើយទូទៅរួមតែមួយ យើងមានចម្លើយទូទៅ 2

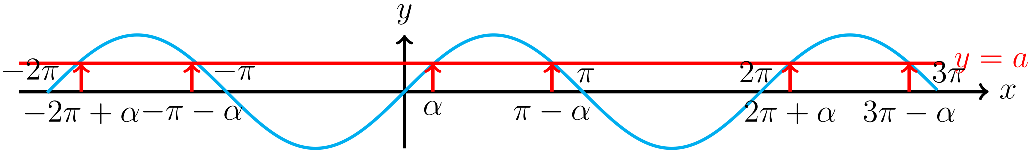
$-\alpha + 2k\pi < x < \pi + \alpha + 2k\pi \ , \ k \ \epsilon Z$ យក $\alpha = -\alpha$

ទទួលបានចម្លើយទូទៅ $-(-\alpha) + 2k\pi < x < \pi - \alpha + 2k\pi$ $\because \sin(-\alpha) = -\sin\alpha$

យើងបាន $\alpha + 2k\pi < x < \pi - \alpha + 2k\pi \ , \ k \epsilon Z$ ។ តាម 1 និង 2

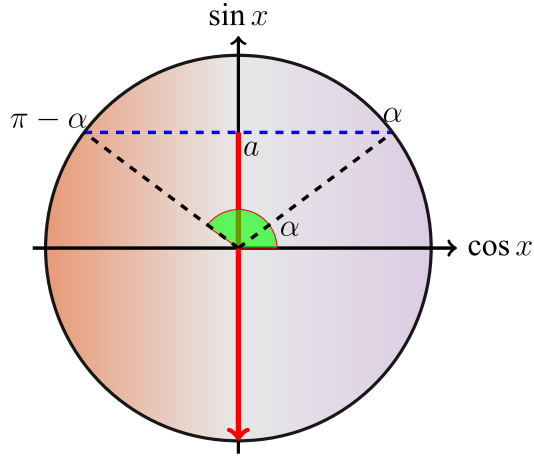
☻ ដូចនេះ $\alpha + 2k\pi < x < \pi - \alpha + 2k\pi \ , \ k \epsilon Z$ ។

○ ក្រាបអនុគមន៍ $\sin x > a$



៣.១ វិសមីការរាង $\sin x < a$

○ ឧបមារកតម្លៃ x ដែលបំពេញលក្ខខណ្ឌនូវវិសមីការ $\sin x < a \ , \ -1 \leqslant a \leqslant 1$ ក្នុងមួយរង្វង់

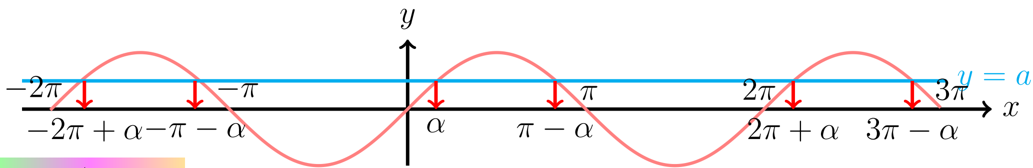


○ តាមរូបខាងលើវិសមីការផ្ដល់ចម្លើយ $\pi-\alpha$ ទៅ α យើងយកទិសដៅច្រាស់ពីទ្រនិចនាឡិការនោះតម្លៃ α ផ្ដល់ឲ្យ $\alpha+2\pi$ ដើម្បីបានតម្លៃមុំពីតូចទៅធំហេតុនេះបានចម្លើយ $\pi-\alpha<x<\alpha+2\pi$ ចម្លើយទូទៅ $\pi-\alpha+2k\pi<x<\alpha+2\pi+2k\pi$ ឬយើងអាចចម្លើយផ្សេងពីនេះយក $k=-1$ រក្សាចម្លើយទូទៅនោះ $\pi-\alpha-2\pi+2k\pi<x<\alpha+2\pi-2\pi+2k\pi$

$\iff -\pi-\alpha+2k\pi<x<\alpha+2k\pi \ , \ k\in Z$

☻ ដូចនេះ $-\pi-\alpha+2k\pi<x<\alpha+2k\pi \ , \ k\in Z$ ។

○ ក្រាបអនុគមន៍ $\sin x<a$



លំហាត់គំរូទី១៤

○ ដោះស្រាយវិសមីការខាងក្រោម៖

ក . $\sin x>\dfrac{1}{2}$ ខ . $\sin x>-\dfrac{\sqrt{3}}{2}$ គ . $\sin x\leqslant\dfrac{1}{2}$ ឃ . $\sin x<-\dfrac{\sqrt{2}}{2}$

ដំណោះស្រាយ.

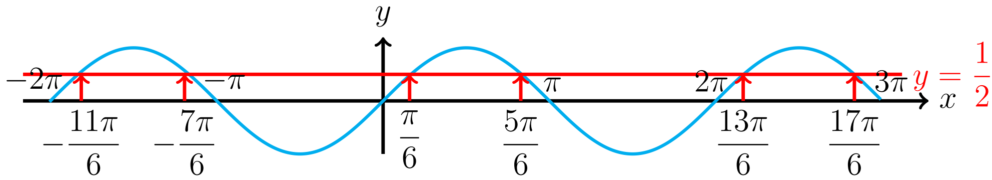
ក . $\sin x>\dfrac{1}{2}$

វិសមីការអាចសរសេរ $\sin x>\sin\dfrac{\pi}{6}$

$$\Longleftrightarrow \frac{\pi}{6} + 2k\pi < x < \pi - \frac{\pi}{6} + 2k\pi \ \ , \ k \in Z$$

$$\Longleftrightarrow \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi \ \ , \ k \in Z$$

☻ ដូចនេះ $\frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi \ \ , \ k \in Z$ ។



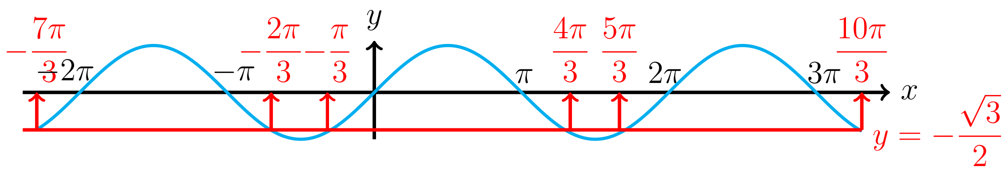
ខ . $\sin x > -\frac{\sqrt{3}}{2}$

វិសមីការអាចសរសេរ $\sin x > \sin\left(-\frac{\pi}{3}\right)$

$$\Longleftrightarrow -\frac{\pi}{3} + 2k\pi < x < \pi - \left(-\frac{\pi}{3}\right) + 2k\pi$$

$$\Longleftrightarrow -\frac{\pi}{3} + 2k\pi < x < \frac{4\pi}{3} + 2k\pi \ \ , \ k \in Z$$

☻ ដូចនេះ $-\frac{\pi}{3} + 2k\pi < x < \frac{4\pi}{3} + 2k\pi \ \ , \ k \in Z$ ។



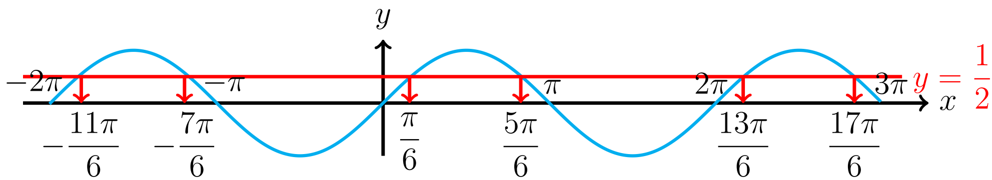
គ . $\sin x \leqslant \frac{1}{2}$

វិសមីការអាចសរសេរ $\sin x \leqslant \sin\frac{\pi}{6}$

$$\Longleftrightarrow -\pi - \frac{\pi}{6} + 2k\pi \leqslant x \leqslant \frac{\pi}{6} + 2k\pi$$

$$\Longleftrightarrow -\frac{7\pi}{6} + 2k\pi \leqslant x \leqslant \frac{\pi}{6} + 2k\pi \ \ , \ k \in Z$$

☻ ដូចនេះ $-\frac{7\pi}{6} + 2k\pi \leqslant x \leqslant \frac{\pi}{6} + 2k\pi \ \ , \ k \in Z$ ។



ឃ . $\sin 2x < -\frac{\sqrt{2}}{2}$

វិសមីការអាចសរសេរ $\sin 2x < \sin\left(-\frac{\pi}{4}\right)$

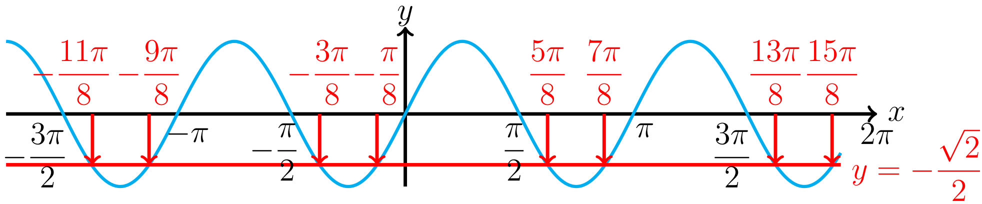
$\Longleftrightarrow -\pi - \left(-\frac{\pi}{4}\right) + 2k\pi < 2x < -\frac{\pi}{4} + 2k\pi$

$\Longleftrightarrow -\pi + \frac{\pi}{4} + 2k\pi < 2x < -\frac{\pi}{4} + 2k\pi \ , \ k \in Z$

$-\frac{3\pi}{8} + 2k\pi < 2x < -\frac{\pi}{8} + 2k\pi$

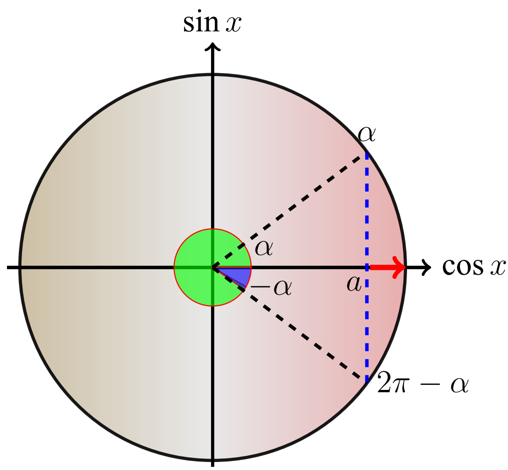
$\Longleftrightarrow -\frac{3\pi}{8} + 2k\pi < x < -\frac{\pi}{8} + k\pi \ , \ k \in Z$

☻ ដូចនេះ $-\frac{3\pi}{8} + 2k\pi < x < -\frac{\pi}{8} + k\pi \ , \ k \in Z$ ។



៣.២ វិសមីការរាង $\cos x > a$

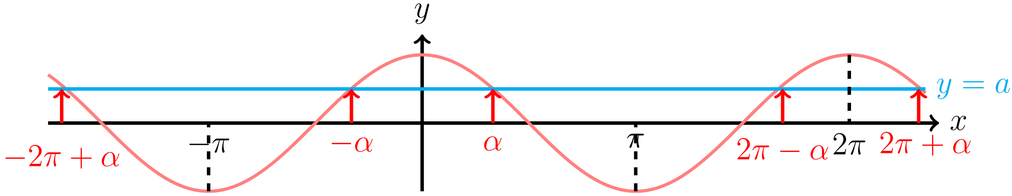
. ឧបមារកតម្លៃ x ដែលបំពេញលក្ខខណ្ឌនូវវិសមីការ $\cos x > a$, $-1 \leqslant a \leqslant 1$ ក្នុងមួយរង្វង់ ។



○ តាមរូបខាងលើវិសមីការផ្តល់ចម្លើយ $2\pi - \alpha$ ទៅ α យើងយកទិសដៅច្រាស់ពីទ្រនិចនាឡិការនោះតម្លៃ $2\pi - \alpha$ ផ្តល់ឲ្យ $-\alpha$ មុំផ្ទុយដើម្បីបានតម្លៃមុំពីតូចទៅធំហេតុនេះបានចម្លើយ $-\alpha < x < \alpha$ ចម្លើយទូទៅ $-\alpha + 2k\pi < x < \alpha + 2k\pi \ , \ k \in Z$

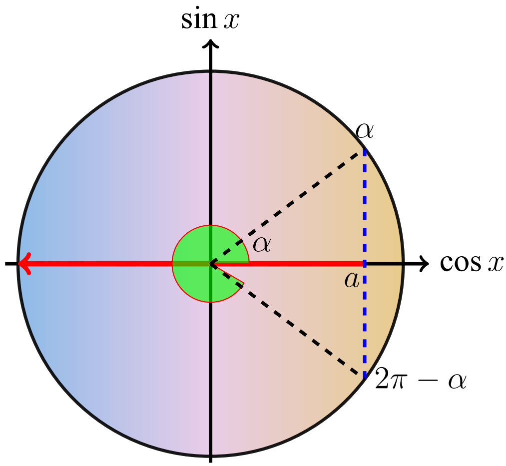
☻ ដូចនេះ $-\alpha + 2k\pi < x < \alpha + 2k\pi \ , \ k \in Z$ ។

○ ក្រាបអនុគមន៍ $\cos x > a$



៣.៣ វិសមីការរាង $\cos x < a$

. ឧបមារកតម្លៃ x ដែលបំពេញលក្ខខណ្ឌនូវវិសមីការ $\cos x < a \ , \ -1 \leqslant a \leqslant 1$ ក្នុងមួយរង្វង់ ។

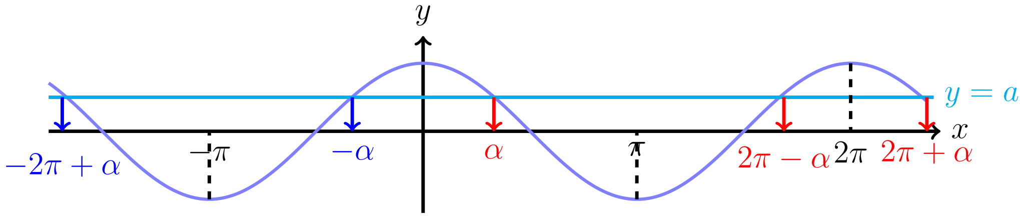


○ តាមរូបខាងលើវិសមីការផ្តល់ចម្លើយ α ទៅ $2\pi - \alpha$ យើងយកទិសដៅច្រាស់ពីទ្រនិចនាឡិការនោះតម្លៃ α ទៅមុំ $2\pi - \alpha$ ជាតម្លៃមុំពីតូចទៅធំហេតុនេះបានចម្លើយ $\alpha < x < 2\pi - \alpha$

ចម្លើយទូទៅ $\alpha + 2k\pi < x < 2\pi - \alpha + 2k\pi \ , \ k \in Z$

☻ ដូចនេះ $\alpha + 2k\pi < x < 2\pi - \alpha + 2k\pi \ , \ k \in Z$ ។

○ ក្រាបអនុគមន៍ $\cos x < a$



លំហាត់គំរូទី១៥

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\cos x > \dfrac{\sqrt{2}}{2}$ គ . $\cos 2x \leqslant \dfrac{1}{2}$ ង . $\cos x < \dfrac{1}{3}$

ខ . $\cos x > -\dfrac{\sqrt{3}}{2}$ ឃ . $\cos x < -\dfrac{\sqrt{3}}{2}$ ច . $\cos x > \sin x$

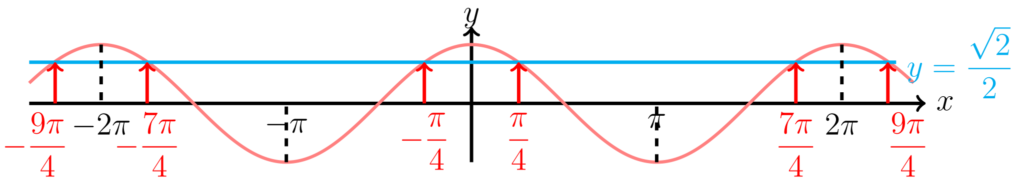
ដំណោះស្រាយ.

ក . $\cos x > \dfrac{\sqrt{2}}{2}$

វិសមីការអាចសរសេរ $\cos x > \cos\dfrac{\pi}{4}$

$\Longleftrightarrow -\dfrac{\pi}{4} + 2k\pi < x < \dfrac{\pi}{4} + 2k\pi \;\; , \; k \in Z$

☻ ដូចនេះ $-\dfrac{\pi}{4} + 2k\pi < x < \dfrac{\pi}{4} + 2k\pi \;\; , \; k \in Z$ ។

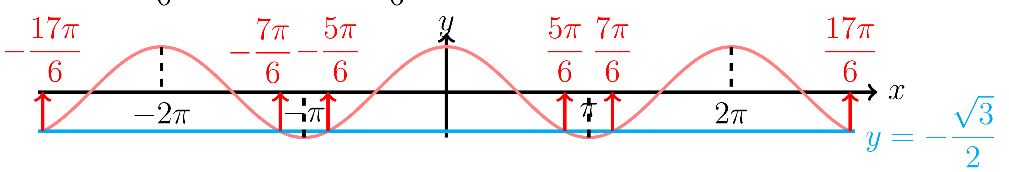


ខ . $\cos x > -\dfrac{\sqrt{3}}{2}$

វិសមីការអាចសរសេរ $\cos x > \cos\dfrac{5\pi}{6}$

$\Longleftrightarrow -\dfrac{5\pi}{6} + 2k\pi < x < \dfrac{5\pi}{6} + 2k\pi \;\; , \; k \in Z$

☻ ដូចនេះ $-\dfrac{5\pi}{6} + 2k\pi < x < \dfrac{5\pi}{6} + 2k\pi \;\; , \; k \in Z$ ។



គ . $\cos 2x \leqslant \dfrac{1}{2}$

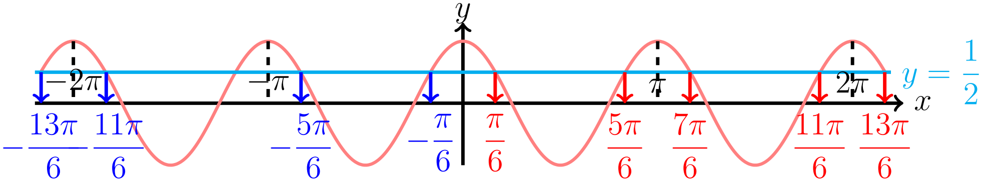
វិសមីការអាចសរសេរ $\cos 2x \leqslant \cos\dfrac{\pi}{3}$

$\Longleftrightarrow \dfrac{\pi}{3} + 2k\pi \leqslant 2x \leqslant 2\pi - \dfrac{\pi}{3} + 2k\pi$

$\Longleftrightarrow \dfrac{\pi}{3}+2k\pi \leqslant 2x \leqslant \dfrac{5\pi}{3}+2k\pi$

$\Longleftrightarrow \dfrac{\pi}{6}+k\pi \leqslant x \leqslant \dfrac{5\pi}{6}+k\pi \ \ , k \in Z$

ដូចនេះ $\dfrac{\pi}{6}+k\pi \leqslant x \leqslant \dfrac{5\pi}{6}+k\pi \ \ , k \in Z$ ។



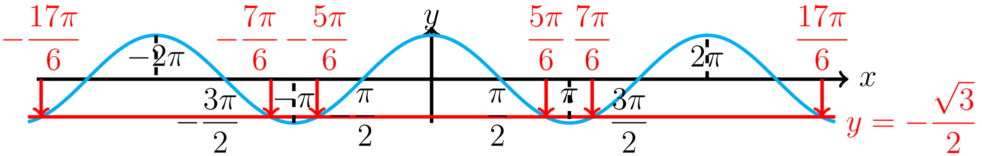
ឃ . $\cos x < -\dfrac{\sqrt{3}}{2}$

វិសមីការអាចសរសេរ $\cos x < \cos\dfrac{5\pi}{6}$

$\Longleftrightarrow \dfrac{5\pi}{6}+2k\pi < x < 2\pi-\dfrac{5\pi}{6}+2k\pi$

$\Longleftrightarrow \dfrac{5\pi}{6}+2k\pi < x < \dfrac{7\pi}{6}+2k\pi$

ដូចនេះ $\dfrac{5\pi}{6}+2k\pi < x < \dfrac{7\pi}{6}+2k\pi \ \ , k \in Z$ ។

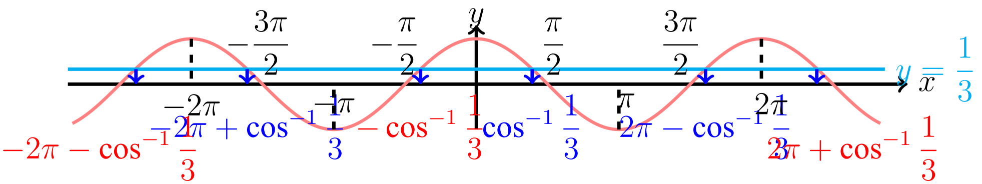


ង . $\cos x < \dfrac{1}{3}$

វិសមីការអាចសរសេរ $\cos x < \cos\left(\cos^{-1}\dfrac{1}{3}\right)$

$\Longleftrightarrow \cos^{-1}\dfrac{1}{3}+2k\pi < x < 2\pi-\cos^{-1}\dfrac{1}{3}+2k\pi$

ដូចនេះ $\cos^{-1}\dfrac{1}{3}+2k\pi < x < 2\pi-\cos^{-1}\dfrac{1}{3}+2k\pi \ \ , k \in Z$ ។



ច . $\cos x > \sin x$

វិសមីការអាចសរសេរ $\cos x - \sin x > 0$

$$\Longleftrightarrow \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right) > 0$$

$$\Longleftrightarrow \cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} > 0$$

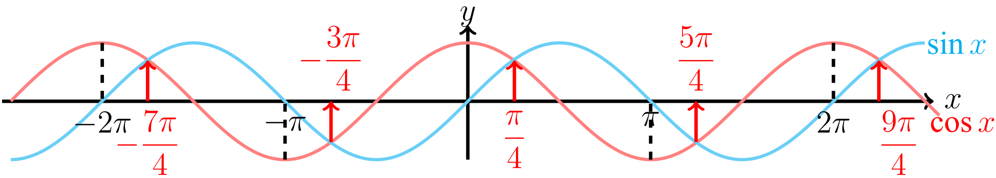
$$\Longleftrightarrow \cos\left(x+\frac{\pi}{4}\right) > 0 \Longleftrightarrow \cos\left(x+\frac{\pi}{4}\right) > \cos\frac{\pi}{2}$$

$$\Longleftrightarrow -\frac{\pi}{2} + 2k\pi < x + \frac{\pi}{4} < \frac{\pi}{2} + 2k\pi$$

$$\Longleftrightarrow -\frac{\pi}{2} - \frac{\pi}{4} + 2k\pi < x < \frac{\pi}{2} - \frac{\pi}{4} + 2k\pi$$

$$\Longleftrightarrow -\frac{3\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi$$

☻ ដូចនេះ $-\frac{3\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi \ , \ k \in Z$ ។



៣.៤ វិសមីការរាង $\tan x > a$

▲ ទ្រឹស្តីបទទី ៧ តាង a ជាចំនួនពិតនិងមានមុំ α ដែលជាចម្លើយជាក់លាក់នៃវិសមីការ $\tan x > a$ យើងបានដំណោះស្រាយទូទៅត្រូវបានកំណត់ដោយ $\tan x > \tan\alpha \Longleftrightarrow k\pi + \alpha < x < k\pi + \frac{\pi}{2}$ $k \in Z \ , \ \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ។

ស្រាយបញ្ជាក់. យើងមាន $\tan x > \tan\alpha \ , \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Longleftrightarrow \frac{\sin x}{\cos x} > \frac{\sin\alpha}{\cos\alpha}$ ដោយ $\cos x \neq 0 \Longrightarrow x \neq (2k+1)\frac{\pi}{2}$

$$\Longleftrightarrow \frac{\sin x\cos\alpha - \sin\alpha\cos x}{\cos\alpha\cos x} > 0 \Longleftrightarrow \frac{\sin(x-\alpha)}{\cos x} > 0 \quad \boxed{1}$$

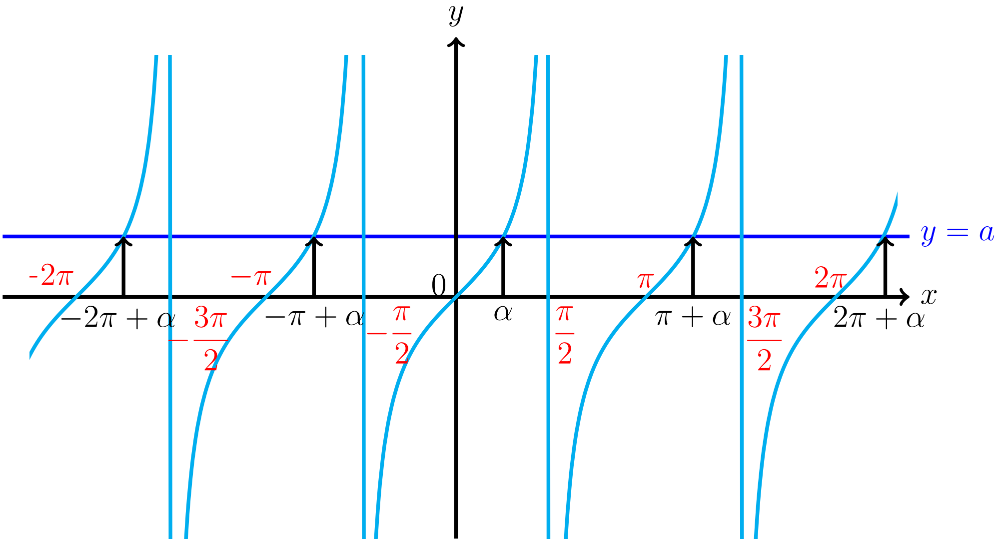
∘ $\sin(x-\alpha) = 0 \Longleftrightarrow x - \alpha = k\pi \Longrightarrow x = k\pi + \alpha$

∘ $\cos x = 0 \Longrightarrow x = k\pi + \frac{\pi}{2}$

តាម $\boxed{1}$ យើងបាន $k\pi + \alpha < x < k\pi + \frac{\pi}{2}$

☻ ដូចនេះ $\tan x > \tan\alpha \Longleftrightarrow k\pi + \alpha < x < k\pi + \frac{\pi}{2} \ , \ k \in Z \ , \ \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ។

○ ក្រាបអនុគមន៍ $\tan x > a$



. វាក៏ច្បាស់ផងដែរពីដំណោះស្រាយតាមក្រាបវិសមីការមានសំណុំចម្លើយគឺ៖ $\tan x > a$

$\Longleftrightarrow k\pi + \alpha < x < k\pi + \dfrac{\pi}{2}\ ,\ k \in Z\ ,\ \alpha \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ ។

៣.៨ វិសមីការរាង $\tan x < a$

▲ ទ្រឹស្តីបទទី ៨ តាង a ជាចំនួនពិតនិងមានមុំ α ដែលជាចម្លើយជាក់លាក់នៃវិសមីការ $\tan x < a$ យើងបានដំណោះស្រាយទូទៅត្រូវបានកំណត់ដោយ $\tan x < \tan\alpha \Longleftrightarrow k\pi - \dfrac{\pi}{2} < x < k\pi + \alpha$ $k \in Z\ ,\ \alpha \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ ។

ស្រាយបញ្ជាក់. យើងមាន $\tan x < \tan\alpha\ ,\alpha \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\Longleftrightarrow \dfrac{\sin x}{\cos x} < \dfrac{\sin\alpha}{\cos\alpha}$ ដោយ $\cos x \neq 0 \Longrightarrow x \neq (2k+1)\dfrac{\pi}{2}$

$\Longleftrightarrow \dfrac{\sin x\cos\alpha - \sin\alpha\cos x}{\cos\alpha\cos x} < 0 \Longleftrightarrow \dfrac{\sin(x-\alpha)}{\cos x} < 0 \quad (1)$

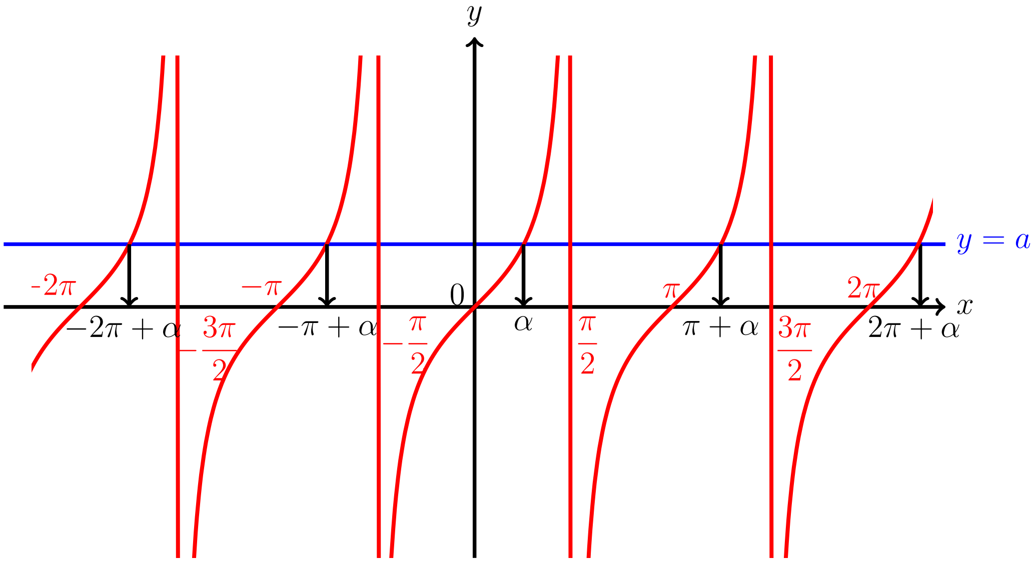
○ $\sin(x-\alpha) = 0 \Longleftrightarrow x - \alpha = k\pi \Longrightarrow x = k\pi + \alpha$

○ $\cos x = 0 \Longrightarrow x = k\pi + \dfrac{\pi}{2}$

តាម (1) យើងបាន $k\pi - \dfrac{\pi}{2} < x < k\pi + \alpha$

☻ ដូចនេះ $\tan x < \tan\alpha \Longleftrightarrow k\pi - \dfrac{\pi}{2} < x < k\pi + \alpha\ ,\ k \in Z\ ,\ \alpha \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ ។

○ ក្រាបអនុគមន៍ $\tan x < a$



. វាក៏ច្បាស់ផងដែរពីដំណោះស្រាយតាមក្រាបវិសមីការមានសំណុំចម្លើយគឺ៖ $\tan x < a$

$\Longleftrightarrow k\pi - \frac{\pi}{2} < x < k\pi + \alpha \ , \ k \in Z \ , \ \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ។

■ ចំពោះវិសមីការ $\cot x > a \Longleftrightarrow \tan x < \frac{1}{a}$

■ ចំពោះវិសមីការ $\cot x < a \Longleftrightarrow \tan x > \frac{1}{a}$

លំហាត់គំរូទី១៦

○ ដោះស្រាយសមីការខាងក្រោម៖

ក . $\tan x > 1$

គ . $\tan 2x > -\sqrt{3}$

ង . $\tan x < 2$

ខ . $\tan\left(x - \frac{\pi}{3}\right) \geqslant \frac{1}{\sqrt{3}}$

ឃ . $\tan x < \sqrt{3}$

ច . $\tan\left(2x - \frac{\pi}{3}\right) < 1$

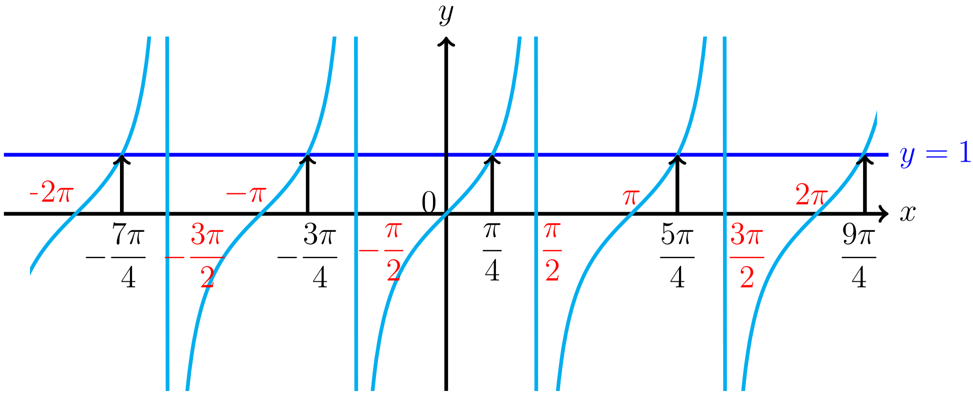
ដំណោះស្រាយ.

ក . $\tan x > 1$

វិសមីការអាចសរសេរ $\tan x > \tan\frac{\pi}{4}$

$\Longleftrightarrow k\pi + \frac{\pi}{4} < x < k\pi + \frac{\pi}{2} \ , \ k \in Z$

☻ ដូចនេះ $k\pi + \frac{\pi}{4} < x < k\pi + \frac{\pi}{2} \ , \ k \in Z$ ។



ខ . $\tan\left(x-\frac{\pi}{3}\right)\geqslant\frac{1}{\sqrt{3}}$

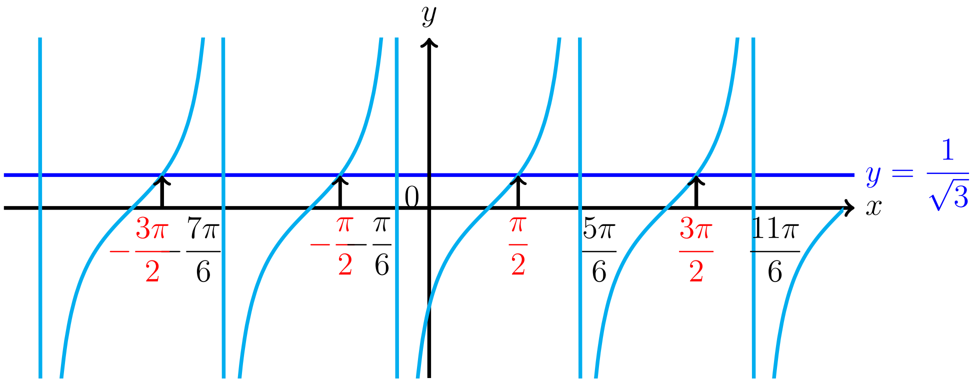
វិសមីការអាចសរសេរ $\tan\left(x-\frac{\pi}{3}\right)\geqslant\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}=\tan\frac{\pi}{6}$

$\Longleftrightarrow k\pi+\frac{\pi}{6}\leqslant x-\frac{\pi}{3}<k\pi+\frac{\pi}{2}$

$\Longleftrightarrow k\pi+\frac{\pi}{6}+\frac{\pi}{3}\leqslant x<k\pi+\frac{\pi}{2}+\frac{\pi}{3}$

$\Longleftrightarrow k\pi+\frac{\pi}{2}\leqslant x<k\pi+\frac{5\pi}{6}$

☻ ដូចនេះ $k\pi+\frac{\pi}{2}\leqslant x<k\pi+\frac{5\pi}{6}$, $k\in Z$ ។



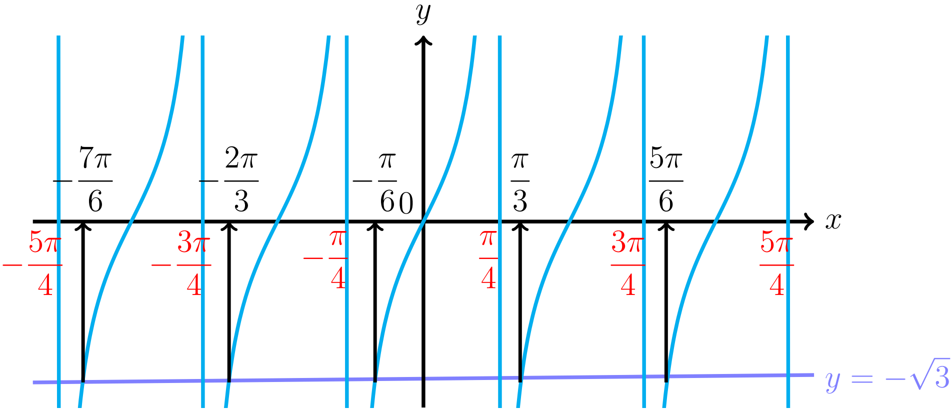
គ . $\tan 2x>-\sqrt{3}$

វិសមីការអាចសរសេរ $\tan 2x>-\sqrt{3}=\tan\left(-\frac{\pi}{3}\right)$

$\Longleftrightarrow k\pi-\frac{\pi}{3}<2x<k\pi+\frac{\pi}{2}$

$$\Longleftrightarrow \frac{k\pi}{2}-\frac{\pi}{6}<x<\frac{k\pi}{2}+\frac{\pi}{4}$$

☻ ដូចនេះ $\frac{k\pi}{2}-\frac{\pi}{6}<x<\frac{k\pi}{2}+\frac{\pi}{4}$, $k\in Z$ ។

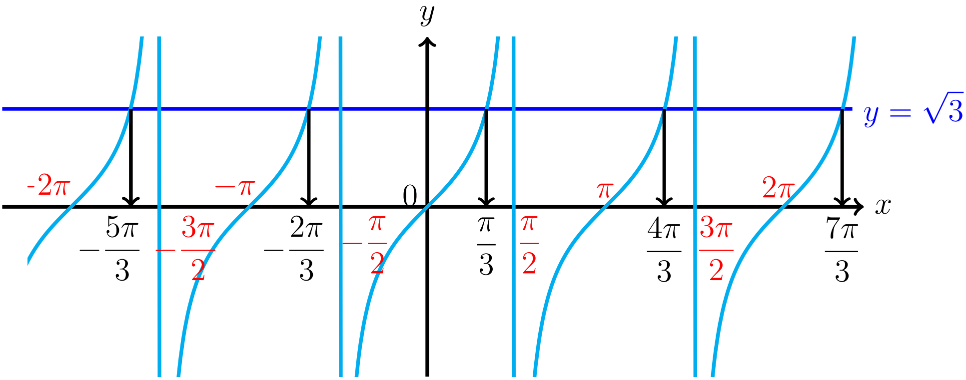


ឃ . $\tan x<\sqrt{3}$

វិសមីការអាចសរសេរ $\tan x<\tan\frac{\pi}{3}$

$$\Longleftrightarrow k\pi-\frac{\pi}{2}<x<k\pi+\frac{\pi}{3} \ , \ k\in Z$$

☻ ដូចនេះ $k\pi-\frac{\pi}{2}<x<k\pi+\frac{\pi}{3}$, $k\in Z$ ។

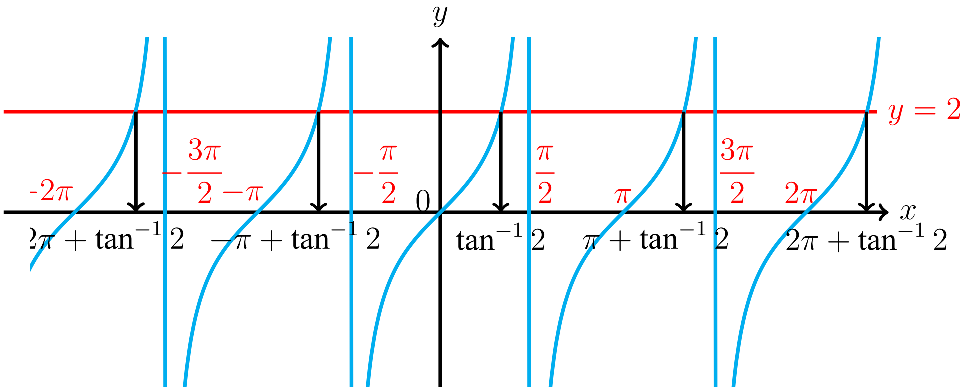


ង . $\tan x<2$

វិសមីការអាចសរសេរ $\tan x<\tan\left(\tan^{-1}2\right)$ $\quad\therefore\ arc\tan 2=\tan^{-1}2$

$$\Longleftrightarrow k\pi-\frac{\pi}{2}<x<k\pi+\tan^{-1}2 \ , \ k\in Z$$

☻ ដូចនេះ $k\pi - \frac{\pi}{2} < x < k\pi + \tan^{-1} 2 \ , \quad k \in Z$ ។



ច . $\tan\left(2x - \frac{\pi}{3}\right) < 1$

វិសមីការអាចសរសេរ $\tan\left(2x - \frac{\pi}{3}\right) < \tan\frac{\pi}{4}$

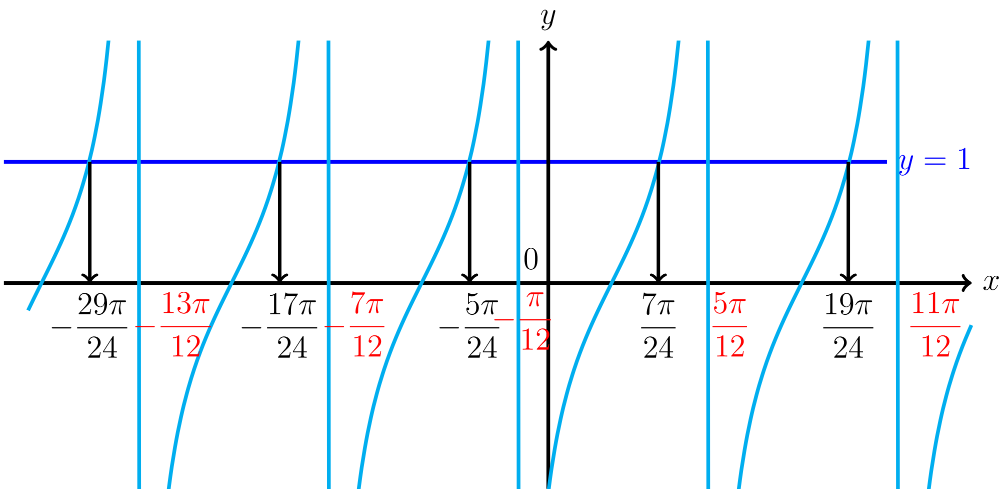
$\Longleftrightarrow k\pi - \frac{\pi}{2} < 2x - \frac{\pi}{3} < k\pi + \frac{\pi}{4}$

$\Longleftrightarrow k\pi - \frac{\pi}{2} + \frac{\pi}{3} < 2x < k\pi + \frac{\pi}{4} + \frac{\pi}{3}$

$\Longleftrightarrow k\pi - \frac{\pi}{6} < 2x < k\pi + \frac{7\pi}{12}$

$\Longleftrightarrow \frac{k\pi}{2} - \frac{\pi}{12} < x < \frac{k\pi}{2} + \frac{7\pi}{24}$

☻ ដូចនេះ $\frac{k\pi}{2} - \frac{\pi}{12} < x < \frac{k\pi}{2} + \frac{7\pi}{24}$ ។



លំហាត់អនុវត្តទី ១

○ ដោះស្រាយវិសមីការខាងក្រោម៖

ក . $\sin x > -\frac{1}{2}$

ខ . $\sin x \geqslant 1$

គ . $\sin x \leqslant \frac{1}{\sqrt{2}}$

ឃ . $\sin x < \frac{\sqrt{3}}{2}$

ង . $\cos x > \frac{\sqrt{3}}{2}$

ច . $\cos x \geqslant -\frac{1}{2}$

ឆ . $\cos x < \frac{1}{\sqrt{2}}$

ជ . $\cos x < \frac{2}{3}$

ឈ . $\tan 2x \leqslant 1$

លំហាត់អនុវត្តទី ២

○ ដោះស្រាយវិសមីការខាងក្រោម៖

ក . $\tan x > \frac{1}{\sqrt{3}}$

ខ . $\tan \frac{x}{2} \geqslant 1$

គ . $\sin 2x < \frac{1}{2}$

ឃ . $\sin 3x < \frac{\sqrt{3}}{2}$

ង . $\cos 5x \leqslant \frac{1}{2}$

ច . $\cos 6x > -\frac{1}{2}$

ឆ . $\sin 5x < -\frac{\sqrt{2}}{2}$

ជ . $\tan 3x > -1$

ឈ . $\tan x > 3$

លំហាត់អនុវត្តទី ៣

○ ដោះស្រាយវិសមីការខាងក្រោម៖

ក . $2\cos x \leqslant -\sqrt{2}$

ខ . $-\sqrt{2}\sin x + 1 \geqslant 0$

គ . $\sqrt{3}\tan x - 1 < 0$

ឃ . $\sin\left(2x + \frac{\pi}{3}\right) > -\frac{\sqrt{3}}{2}$

ង . $\cos\left(\frac{x}{2} - \frac{\pi}{3}\right) \leqslant \frac{1}{\sqrt{2}}$

ច . $\sin x > \cos x$

ឆ . $2\cos\left(\frac{x}{2} + \frac{\pi}{6}\right) \leqslant 1$

ជ . $\frac{5}{4}\sin^2 x + \frac{1}{4}\sin^2 2x > \cos 2x$

លំហាត់អនុវត្តទី ៤

○ ដោះស្រាយវិសមីការខាងក្រោម៖

ក . $\sin x + \cos x > 1$

ខ . $\sin x - \cos x < 1$

គ . $\sqrt{3}\sin x + \cos x > 1$

ឃ . $\sin x - \sqrt{3}\cos x < 1$

ង . $\sqrt{3}\cos x - \sin x > 1$

ច . $\sqrt{3}\cos x + \sin x \leqslant 0$

ឆ . $\sqrt{3}\sin x - \cos x > \sqrt{2}$

ជ . $\sqrt{3}\sin 2x + \cos 2x < \sqrt{2}$

ឈ . $-\sqrt{3}\sin 2x + \cos 2x > -\sqrt{2}$